Dynamical stability of dipolar Bose-Einstein condensates with temporal modulation of the *s*-wave scattering length

S. Sabari,[1,2] Chandroth P. Jisha,[3,*] K. Porsezian,[2,†] and Valeriy A. Brazhnyi[3]
[1]*Department of Physics, University of Pune, Pune 411007, Maharashtra, India*
[2]*Department of Physics, Pondicherry University, Puducherry 605014, Puducherry, India*
[3]*Centro de Física do Porto, Faculdade de Ciências, Universidade do Porto, R. Campo Alegre 687, Porto 4169-007, Portugal*
(Received 20 May 2015; revised manuscript received 25 July 2015; published 14 September 2015)

We study the stabilization properties of dipolar Bose-Einstein condensate by temporal modulation of short-range two-body interaction. Through both analytical and numerical methods, we analyze the mean-field Gross-Pitaevskii equation with short-range two-body and long-range, nonlocal, dipolar interaction terms. We derive the equation of motion and effective potential of the dipolar condensate by variational method. We show that there is an enhancement of the condensate stability due to the inclusion of dipolar interaction in addition to the two-body contact interaction. We also show that the stability of the dipolar condensate increases in the presence of time varying two-body contact interaction; the temporal modification of the contact interaction prevents the collapse of dipolar Bose-Einstein condensate. Finally we confirm the semi-analytical prediction through the direct numerical simulations of the governing equation.

DOI: 10.1103/PhysRevE.92.032905 PACS number(s): 05.45.Yv, 03.75.Lm

I. INTRODUCTION

The experimental realization of Bose-Einstein condensates (BECs) of ^{52}Cr [1–4], ^{164}Dy [5,6], and ^{168}Er [7] with long-range dipole-dipole (DD) interaction superposed on the short-range atomic interaction marks a major development in ultracold quantum gases. Because of the long-range nature and anisotropic character of the DD interaction, the dipolar BEC possesses many distinct features and new exciting phenomena such as the dependence of stability on the trap geometry [1,2], new dispersion relations of elementary excitations [8–10], unusual equilibrium shapes, roton-maxon character of the excitation spectrum [10–17], and novel quantum phases, including supersolid and checkerboard phases [18–20]. These features arise due to the interplay between the *s*-wave contact interaction and dipolar interaction.

The ability to tune the contact interactions using Feshbach resonance has attracted a considerable interest in the study of dipolar BECs [10–12,14,21–32]. One of particular interest is macroscopically excited BEC, such as solitons. Solitons are localized waves that propagate over long distances without change in shape or attenuation. The existence of solitonic solutions is a common feature of nonlinear wave equations, so that solitons appear in many diverse physical systems. The theoretical description of a dilute weakly interacting dipolar BEC can be formulated by including a nonlocal DD interaction term in the Gross-Pitaevskii (GP) equation [1–4,33–35]. The nonlinear terms in the GP equation characterized by both DD interaction and the two-body *s*-wave scattering length can support both dark and bright matter-wave solitons. In the conventional BEC, bright (dark) matter-wave solitons form when the negative (attractive) *s*-wave scattering length exactly balance with the dispersion and the attractive contact interaction [36–38]. In the dipolar BECs, a nonlocal DD interaction term is involved in the nonlinear part with the *s*-wave contact interaction. DD interaction has been a subject of active investigation in disparate physical systems during the past decades. DD interaction plays a crucial role in the physics of solitons and modulational instability [39,40]. The new prospect for the formation of matter-wave bright solitons in BECs are suggested by the presence of DD interaction. Hence, in the presence of DD interaction, one can get a bright soliton even for positive (repulsive) *s*-wave scattering length ($a > 0$), which can be controlled by means of the Feshbach resonance with a tunable time-dependent magnetic field [34]. Further, in recent years, study of temporal and spatial modulated nonlinearities have attracted considerable attention in several areas, for example, nonlinear physics [41], optics [42–47], and conventional BECs [48–52].

In conventional BECs, the periodic temporal modification of the atomic scattering length achieved by Feshbach resonance has been used to stabilize the bright solitons in higher dimensions. Through the GP equation with constant and oscillatory part of the two-body interaction, Saito and Ueda stabilized the trapless matter-wave bright solitons in 2D by temporal modulation of contact interaction [51]. Adhikari examined the problem and stabilized the untrapped soliton in 3D and the vortex soliton in 2D by temporal modulation of contact interaction [49]. Abdullaev *et al.* studied the stability of trapped BECs in 2D and 3D with periodic modulation of contact interaction [50]. We studied the stability of the 3D BEC with constant and oscillatory part for both the two- and three-body interactions in our previous work [48]. Besides, Wang *et al.* [53] and Wu *et al.* [52] discussed 2D stable solitons and vortices for BECs with spatially modulated cubic nonlinearity and a harmonic trapping potential, respectively.

The objective of the present work is to study the significance of both constant and oscillatory part of short-range contact interaction on the stability of dipolar BECs. The effective strength of the DD interactions can be controlled by adjusting the orientation of the dipoles with respect to the axis of the linear trap, while the strength of the contact interactions may be effectively tuned by means of the Feshbach resonance, as shown in the condensate of ^{52}Cr and ^{168}Dy atoms.

*cpjisha@gmail.com
†ponzsol@gmail.com

Studies of dipolar BECs with periodic temporal modification of the s-wave scattering length has not been reported so far and could lead to interesting results. In the present work, we investigate the stability of the dipolar matter wave with both constant and oscillatory part of the two-body s-wave scattering length. From our theoretical analysis, we suggest that one can increase the stability of the dipolar BEC by considering the oscillatory part of the contact interaction. This is the main result of this paper.

A numerical study of the time-dependent GP equation with nonlocal DD interaction term is of interest, as this can provide solutions to many stationary and time-evolution problems. The time-independent GP equation yields only the solution of stationary problems. As our principal interest is in time-evolution problems, we shall consider the time-dependent equation in this paper. In the present study we analyze the stability of the dipolar BECs and point out that a temporal modification of the contact interaction can lead to a stabilization of the dipolar system. In this paper, in addition to analytical studies, we also perform numerical verification for the stability of a dipolar BEC. In particular, by analyzing the GP equation using the variational method and direct numerical integration, we analyze the stability properties of the dipolar BEC with constant and oscillatory part of the contact interactions. Our analysis strongly suggests that the inclusion of oscillatory part of the contact interaction can help stabilize the dipolar BEC.

The organization of the paper is as follows. In Sec. II, we present a brief overview of the mean-field model. Then, we discuss the variational study of the problem and point out the possible stabilization of a dipolar BEC in 2D with and without the oscillatory part of the two-body contact interaction. In Sec. III, we report the numerical results of the time-dependent GP equation through split-step Crank-Nicholson (SSCN) method. Finally, we give the concluding remarks in Sec. IV.

II. MODEL EQUATION AND VARIATIONAL EQUATIONS

Consider a dipolar BEC of N particles with mass m and magnetic dipole moment μ. At sufficiently low temperatures the ground and excited states of the condensate is described by the time-dependent, dimensionless GP equation with nonlocal DD interaction term [1–4,33–35],

$$i\frac{\partial\psi(\mathbf{r},\tau)}{\partial\tau} = \left[-\frac{1}{2}\nabla^2 + V(\mathbf{r}) + 4\pi a(t)N|\psi(\mathbf{r},\tau)|^2 + N\int U_{\rm dd}(\mathbf{r}-\mathbf{r}')|\psi(\mathbf{r}',\tau)|^2 d^3r'\right]\psi(\mathbf{r},\tau), \quad (1)$$

with DD interaction $U_{\rm dd}(\mathbf{X}) = 3a_{\rm dd}(1-3\cos^2\theta)/\mathbf{X}^3$, $\mathbf{X} = \mathbf{r}-\mathbf{r}'$. Here θ is the angle between $\mathbf{X}$ and the polarization direction z and the normalization is $\int|\psi(\mathbf{r},\tau)|^2 d^3\mathbf{r} = 1$. The constant $a_{\rm dd} = \mu_0\bar{\mu}^2 m/(12\pi\hbar^2)$ is a length characterizing the strength of the DD interaction and its experimental value for ^{52}Cr is $16a_0$, for ^{164}Dy is $66a_0$, and for ^{168}Er is $130a_0$ [1–4], with a_0 the Bohr radius, $\bar{\mu}$ the (magnetic) dipole moment of a single atom, and μ_0 the permeability of free space. The dimensionless 3D harmonic trap is $V(\mathbf{r}) = (\Omega_\rho^2\rho^2 + \Omega_z^2 z^2)/2$, where $\mathbf{r} = (\rho,z)$, with ρ the radial coordinate and z the axial coordinate. In Eq. (1) length is measured in units of characteristic harmonic oscillator length $l = \sqrt{\hbar/m\omega}$, angular frequency of trap in units of ω, time t in units of ω^{-1}, and energy in units of $\hbar\omega$.

In the following, we consider Eq. (1) with strong axial trap ($\Omega_z \gg \Omega_\rho$), the dipolar BEC is assumed to be in the disk-shape. Then, the governing equation is the 2D GP equation with a nonlocal DD interaction term. Equation (1) can be written in the dimensionless form as follows [34]:

$$i\frac{\partial\phi(\rho,t)}{\partial t} = \left[-\frac{1}{2}\nabla_\rho^2 + \frac{\Omega_\rho^2\rho^2}{2} + g(t)|\phi(\rho,t)|^2 + g_{\rm dd}\int\frac{d^2k_\rho}{(2\pi)^2}e^{i\mathbf{k}_\rho\cdot\rho}n(\mathbf{k}_\rho)h_{2D}\left(\frac{k_\rho d_z}{\sqrt{2}}\right)\right]\phi(\rho,t), \quad (2)$$

where, $g(t) = 4\pi Na(t)/\sqrt{2\pi}d_z$, $g_{\rm dd} = 4\pi Na_{\rm dd}/\sqrt{2\pi}d_z$, $n(\mathbf{k}_\rho) = \int e^{i\mathbf{k}_\rho\cdot\rho}|\phi(\rho)|^2 d\rho$, $d_z = \sqrt{1/\Omega_z}$, $h_{2D} = 2 - 3\sqrt{\pi}\xi e^{\xi^2}erfc(\xi)$, and $\mathbf{k}_\rho = (k_x,k_y)$. The dipolar term has been written in the Fourier space after taking the convolution of the corresponding variables [34].

In order to obtain the governing equation of motions of the condensate parameters, we use the variational approach with the Gaussian trial wave function for the solution of Eq. (2):

$$\phi(\rho) = \frac{1}{R\sqrt{\pi}}\exp\left(-\frac{\rho^2}{2R^2} + i\beta\rho^2\right). \quad (3)$$

The Lagrangian density for generating Eq. (2) is

$$\mathcal{L} = \frac{i}{2}(\phi_t\phi^* - \phi_t^*\phi) - \frac{|\nabla_\rho\phi|^2}{2} - \frac{\Omega_\rho^2\rho^2|\phi|^2}{2} - a(t)\mathcal{N}|\phi|^4 - a_{\rm dd}\mathcal{N}|\phi|^2\int\frac{d^2k_\rho}{(2\pi)^2}e^{ik_\rho\cdot\rho}n(k_\rho)h_{2D}\left(\frac{k_\rho d_z}{\sqrt{2}}\right), \quad (4)$$

where, $\mathcal{N} = (2\sqrt{2\pi}/d_z)N$. The trial wave function Eq. (3) is substituted in the Lagrangian density and the effective Lagrangian is calculated by integrating the Lagrangian density as $L_{\rm eff} = \int\mathcal{L}\,dz$. The Euler-Lagrangian equation for the variational parameter R and β yield the following equation for width R,

$$\frac{\partial^2 R}{\partial t^2} = \frac{1}{R^3} - \Omega_\rho^2 R + \frac{N}{\sqrt{2\pi}}\frac{[2a(t) - a_{\rm dd}g(\kappa)]}{R^3 d_z}, \quad (5)$$

with $g(\kappa) = 2 - 7\kappa^2 - 4\kappa^4 + 9\kappa^4 d(\kappa)/(1-\kappa^2)^2$, $d(\kappa) = {\rm atanh}\sqrt{1-\kappa^2}/\sqrt{1-\kappa^2}$ and $\kappa = R/d_z$.

Since, we consider a periodic modulation of the s-wave interaction of the form $a(t) = \epsilon_0 + \epsilon_1\sin(\omega t)$ on the stability of dipolar BEC, where ϵ_0 and ϵ_1 are the amplitudes of constant and oscillating part of s-wave contact interaction, respectively, a Kapitza averaging scheme can be used to treat these oscillatory terms [54]. Such a modulation of the contact interaction is possible by manipulating an external magnetic or optical field near a Feshbach resonance [41–44,46,48–53].

After including the oscillating nonlinearity in the contact interaction part, we get the following second-order differential equation for the evolution of the width for radial coordinates [34],

$$\frac{\partial^2 R}{\partial t^2} = \frac{1}{R^3} - \Omega_\rho^2 R + \frac{N}{\sqrt{2\pi}} \frac{[2(\epsilon_0 + \epsilon_1 \sin(\omega t)) - a_{dd} g(\kappa)]}{R^3 d_z}, \tag{6}$$

Now R can be separated into a slowly varying part $A(t)$ and a rapidly varying part $B(t)$ by $R(t) = A(t) + B(t)$. When $\omega \gg 1$, $B(t)$ becomes of the order of ω^{-2}. Keeping the terms of the order of up to ω^{-2} in $B(t)$, one may obtain the following equations of motion for $A(t)$ and $B(t)$ [54],

$$\ddot{B}(t) = \frac{2N\epsilon_1 \sin(\omega t)}{\sqrt{2\pi} d_z A(t)^3}, \tag{7}$$

$$\ddot{A}(t) = \frac{1}{A(t)^3} - \Omega_\rho^2 A(t) + \frac{N}{\sqrt{2\pi} d_z} \frac{[2\epsilon_0 - a_{dd}\, g(\kappa(t))]}{A(t)^3} - \frac{6N}{\sqrt{2\pi}} \frac{\epsilon_1 \langle B(t) \sin(\omega t) \rangle}{d_z A(t)^4}, \tag{8}$$

where $\langle \cdots \rangle$ denotes the time average over the rapid oscillation. From Eq. (7) we can get $B(t) = -2N\epsilon_1 \sin(\omega t)/[\sqrt{2\pi} d_z\, \omega^2 A(t)^3]$ and substituting it into Eq. (8), we obtain the following equation of motion for the slowly varying part:

$$\ddot{A} = \frac{1}{A^3} - \Omega_\rho^2 A + \frac{N}{\sqrt{2\pi} d_z} \frac{[2\epsilon_0 - a_{dd}\, g(\kappa)]}{A^3} + \frac{3N^2\epsilon_1^2}{\pi d_z \omega^2 A^7}. \tag{9}$$

The variational approximation suggests that the effect of the DD interaction is to reduce the constant part of the contact interaction for $a_{dd} > 0$. Immediately, one can conclude that the system effectively becomes attractive for $a_{dd} > \epsilon_0$. So one can have the formation of bright soliton even for positive (repulsive) scattering length a, provided that $a_{dd} > \epsilon_0$. The effective potential $U(A)$ corresponding to the above equation of motion can be written as

$$U(A) = \frac{1 + \Omega_\rho^2 A^4}{2A^2} + \frac{N}{2\sqrt{2\pi} d_z} \frac{[2\epsilon_0 - a_{dd}\, f(\kappa)]}{A^2} + \frac{N^2 \epsilon_1^2}{2\pi d_z \omega^2 A^6}, \tag{10}$$

where $f(\kappa) = [1 + 2\kappa^2 - 3\kappa^2 d(\kappa)]/(1 - \kappa^2)$. Now, we analyze the nature of the effective potential $U(A)$ versus A in the presence and absence of a_{dd} and ϵ_1. At this point, it is worth mentioning that Eq. (6), and hence Eq. (10), could be considered formally equivalent to the equation governing a charged particle in an inhomogeneous electric-field (particularly if only the two-body interaction terms are considered) resulting in the so-called ponderomotive force [55]; the concept was used to design the Paul trap [56] and can also be used to trap laser light in plasma [57].

We concentrate in the study of a weak trap such that $\Omega_\rho \ll 1$. In this case, the dipolar condensate is generally unstable for low values of the constant two-body interaction term. In Figs. 1–3, we show the stability properties of the dipolar condensate by considering the effects of both constant and oscillatory part of the s-wave contact interaction enlightening the effect of each terms on the stability of the condensate. Figure 1(a) shows that the effect of the dipolar term as well as the constant two-body interaction term (because of the catastrophic collapse of the 2D condensate in the presence of constant two-body interaction alone) is to expand the BEC as there is no potential minimum (see the dashed and dotted lines). A condensate with only two-body interaction (no dipolar term) can be stabilized in the presence of oscillatory two-body interaction (see dash-dotted line). But a dipolar condensate is considerably stabilized in the presence of constant and oscillating two-body terms as can be seen from the solid lines. Figure 1(b) further stresses this point where the effect of the amplitude of the constant part of the two-body interaction in the stabilization of the condensate is illustrated.

In Fig. 2, we show the importance of the oscillating nonlinearity on the stability of the condensates. For low values of constant contact interaction, in the absence of oscillating nonlinearity, no minimum in the potential depth appears in the effective potential (dotted, dashed, dash-dotted) curves. But if we switch on the oscillating nonlinearity, due to the confining nature of the oscillating term, the system can now have a minimum in the effective potential even for smaller values of

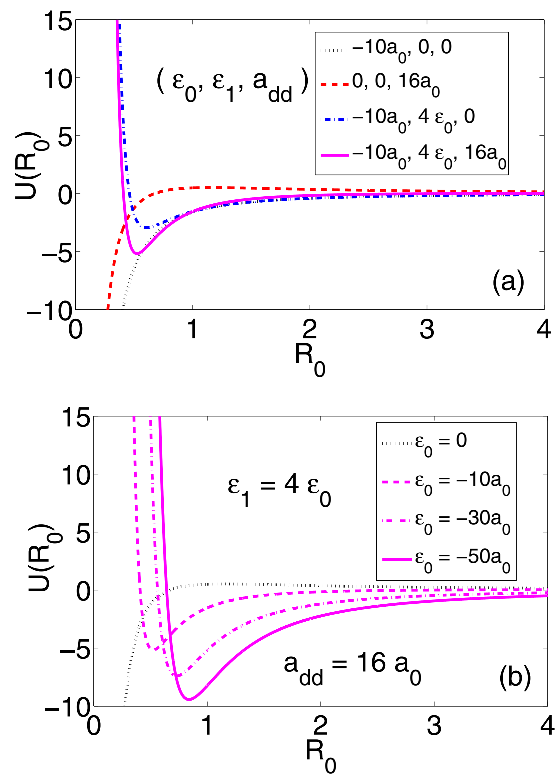

FIG. 1. (Color online) The effective potential using Eq. (10) showing the stability properties of the condensate. (a) Dotted curve represents the potential in the presence of constant part of the contact interaction alone, dashed curve for dipolar interaction alone, dash-dotted curve for both constant and oscillatory part of the contact interaction alone, and continuous curve for dipolar with both constant and oscillatory part of the contact interactions. (b) Effective potential curves for different values of constant part of contact interaction in the presence of dipolar interaction and oscillatory part of contact interaction as marked. Other parameters are taken as $a_{dd} = 16a_0$, $\epsilon_0 = -10a_0$, $\epsilon_1 = 4\epsilon_0$, $\omega = 10\pi$, $N = 10\,000$, $d_z = 1$, and $\Omega_\rho = 0.04$ (weak trapping).

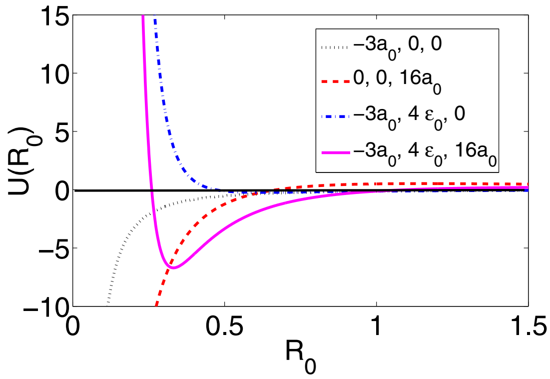

FIG. 2. (Color online) The effective potential using Eq. (10) showing the importance of the oscillating nonlinearity. Dotted curve represents the potential in the presence of constant part of the contact interaction alone, dashed curve for dipolar interaction alone, dash-dotted curve for both constant and oscillatory part of the contact interaction alone, and continuous curve for dipolar with both constant and oscillatory part of the contact interactions. Other parameters are taken as $\omega = 10\pi$, $N = 10\,000$, $d_z = 1$, and $\Omega_\rho = 0.04$ (weak trapping). The values of ϵ_0, ϵ_1, and $a_{\rm dd}$ are as marked in the figure.

constant contact interaction, say $\epsilon_0 = -3a_0$. Furthermore, in Fig. 3, the effect of the amplitude of the oscillatory part of the two-body interaction as well as the frequency of oscillation is depicted. The condensate is more stable for lower amplitude of ϵ_1 and for higher frequency of oscillation ω. It is evident from Figs. 1–3, that there is a stability region for the dipolar BEC for relatively smaller values of two-body constant contact interaction ($\epsilon_0 = -3a_0$) as well as an enhancement in the stability region of dipolar BEC for relatively large values of the two-body constant contact interaction ($\epsilon_0 = -10a_0$) due to the inclusion of oscillatory part of the contact interaction. In the following, we confirm these predictions using direct numerical integration of the governing equation.

III. NUMERICAL RESULTS

We solve the GP Eq. (2) by employing real-time propagation with split-step Crank-Nicolson method applied to the diffraction operator [58]. The DD interaction is evaluated by fast Fourier transform [12,13]. The typical discretized space and time steps for the numerical grid is 0.05 and 0.005, respectively. In the following, we present results for ^{52}Cr atoms which has a moderate dipole moment with $a_{\rm dd} = 16a_0$ [1–4]. Here, we use Eq. (3) as an initial wave function for dynamical evaluation of the system with $R(=R_0) = 0.25$ and $\beta = 0$. The condensate consists of 10 000 atoms.

We consider different dynamical regimes wherein we alternatively study the effects of inclusion of the time-dependent periodic two-body interaction as well as the DD interaction so as to understand their effects on the BEC dynamics. In Fig. 4, we illustrate the dynamical evolution properties of the dipolar BEC in the absence of both constant and oscillatory part of contact interaction, i.e., $\epsilon_0 = 0$, and $\epsilon_1 = 0$ but with a nonzero DD interaction. As can be seen from the input and output profiles after $t = 50$, over time, the condensate collapses. Next, we study what is the effect of the two-body interaction term

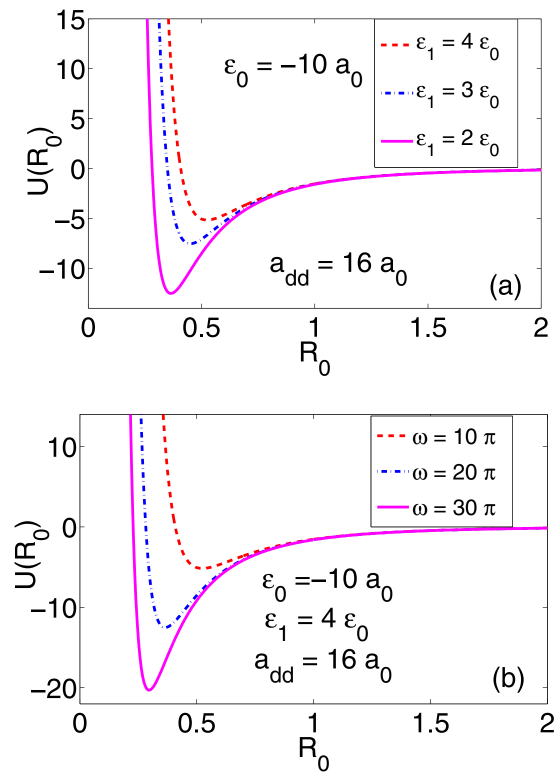

FIG. 3. (Color online) The effective potential using Eq. (10) showing the stability properties of the condensate. (a) Potential curves for varying values of the amplitude of oscillating nonlinearity ($\epsilon_1 = 4\epsilon_0$ for dashed curve, $\epsilon_1 = 3\epsilon_0$ for dash-dotted curve, and $\epsilon_1 = 2\epsilon_0$ for solid curve). (b) Potential curves for varying values of frequency of oscillation of the two-body interaction term ($\omega = 10\pi$ for dashed curve, $\omega = 20\pi$ for dash-dotted curve, and $\omega = 30\pi$ for solid curve). Other parameters are taken as $a_{\rm dd} = 16a_0$, $\epsilon_0 = -10a_0$, $N = 10\,000$, $d_z = 1$, and $\Omega_\rho = 0.04$ (weak trapping). It can be clearly seen that time-periodic two-body interactions help in stabilization of the DD BEC.

by including the contact terms. The time dynamics of the condensate for various $\epsilon_{0,1}$ values is depicted in Fig. 5. As can be seen, the instability of the condensate is progressively

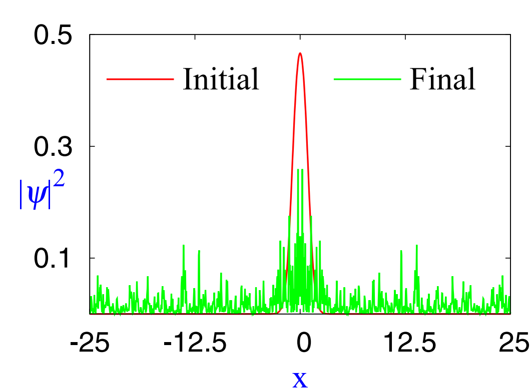

FIG. 4. (Color online) Transverse profiles of the dipolar BEC without constant and oscillatory part of contact interaction at the input (solid line) and after $t = 50$ (distorted curve) showing the instability of the BEC in time in the presence of DD interaction alone. Parameters are taken as $N = 10\,000$, $a_{\rm dd} = 16a_0$, $\epsilon_0 = 0$, and $\epsilon_1 = 0$.

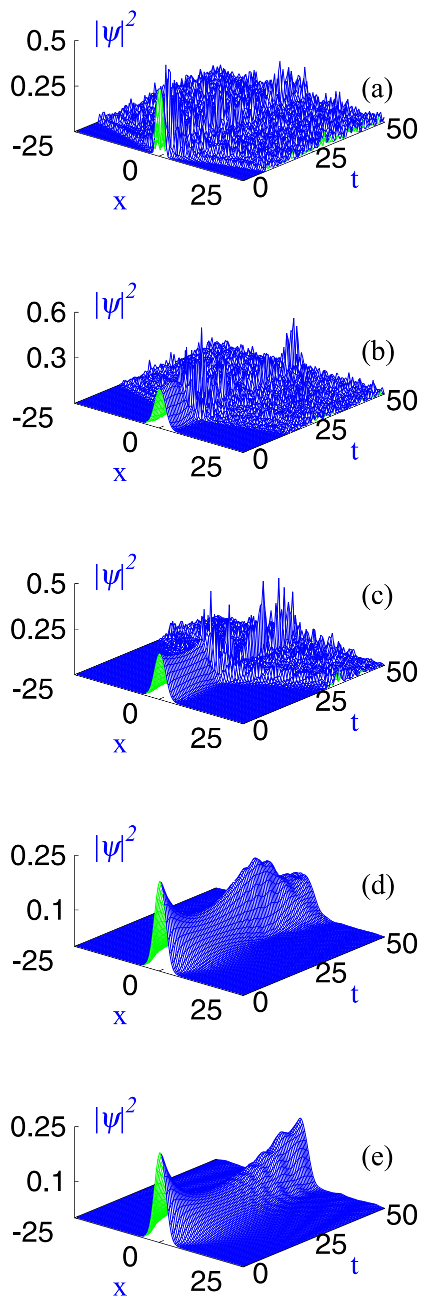

FIG. 5. (Color online) Stabilization of the dipolar BEC with $N = 10\,000$ atoms by tuning the constant part of the contact interaction alone. (a) $\epsilon_0 = 0a_0$, (b) $\epsilon_0 = -6a_0$, (c) $\epsilon_0 = -7a_0$, (d) $\epsilon_0 = -8a_0$, and (e) $\epsilon_0 = -10a_0$. Other parameters are $a_{\rm dd} = 16a_0$, $\epsilon_1 = 4\epsilon_0$, and $\omega = 10\pi$.

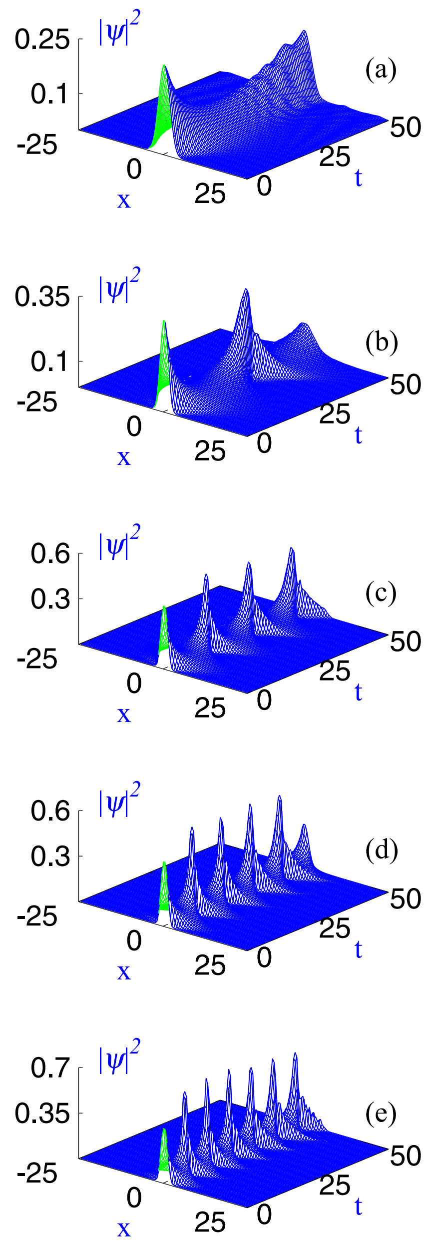

FIG. 6. (Color online) Stabilization of the dipolar BEC with $N = 10\,000$ atoms by tuning the frequency of oscillation of the time-periodic contact interaction. (a) $\omega = 10\pi$, (b) $\omega = 14\pi$, (c) $\omega = 16\pi$, (d) $\omega = 18\pi$, and (e) $\omega = 20\pi$. Other parameters are $a_{\rm dd} = 16a_0$, $\epsilon_0 = -10a_0$, and $\epsilon_1 = 4\epsilon_0$.

decreasing with increasing two-body interaction; both the constant and oscillatory part. Here the frequency of oscillation of the time-periodic term is kept constant at $\omega = 10\pi$. It is clear that oscillating contact interaction can help in stabilizing the condensate, because the effect of the oscillatory term is to induce an additional potential due to Kapitza averaging [47,54] the profile and magnitude of which depends on the frequency of oscillation.

Now we illustrate the effect of varying frequency of oscillation of the time-periodic oscillatory part of the contact interaction keeping all other parameters, i.e., amplitude of the constant and oscillatory part of the contact interaction as well as the dipolar interaction as constant. The results are depicted in Fig. 6 for various frequency of oscillation ω. As can be seen, increase in oscillation frequency further helps in the stabilization.

IV. CONCLUSION

In conclusion, we have theoretically investigated the stability of dipolar BECs by considering constant and oscillatory part of the contact interaction, that is the nonlinearity due to the two-body interaction is considered to be time-periodic. Such periodic behavior can be obtained by modulating the scattering length using Feshbach resonance. The effect of the oscillatory term in the contact interaction is to provide an additional confining potential which helps in the stabilization of the dipolar condensate. To illustrate this, we first performed a variational analysis on the governing equation and obtained the equations of motion. For this a Kapitza averaging scheme was employed to separate the slow and fast varying components in the contact interaction term. Using this we derived the effective potential which is experienced by the condensate. A minimum in the potential signifies a possible stable state. We obtained the potential using experimental values for the dipolar BEC ^{52}Cr and found that the fast varying contact interaction term can help in stabilizing the BEC. To further prove this point, we performed direct dynamical evolution of the condensate using the harmonic oscillator solution to understand the stability properties. As from variational analysis, numerically also we conclude that the dipolar BEC is stabilized and also the stability of the dipolar condensate is highly enhanced by the time-periodic contact interaction in addition to the constant part of the contact interaction.

ACKNOWLEDGMENTS

S.S. thanks University Grants Commission (UGC) for offering support through a Post-Doc under the Dr. D.S. Kothari Post Doctoral Fellowship Scheme. J.C.P. gratefully acknowledges Fundação para a Ciência e a Tecnologia, POPH-QREN, and FSE (FCT, Portugal) for support through the fellowship SFRH/BPD/77524/2011. J.C.P. and K.P. thank DST-FCT for support through an India-Portugal bilateral project. K.P. thanks the National Board for Higher Mathematics (NBHM), Indo-French Centre for the Promotion of Advanced Research (IFCPAR), Department of Science and Technology (DST) and Council of Scientific & Industrial Research (CSIR), Government of India, for the financial support through major projects.

[1] Koch, L. Tobias, M. Thierry, F. Jonas, G. Bernd, and P. A. Axel, Nat. Phys. **4**, 218 (2008).
[2] T. Lahaye, C. Menotti, L. Santos, M. Lewenstein, and T. Pfau, Rep. Prog. Phys. **72**, 126401 (2009).
[3] Lahaye, K. Thierry, F. Tobias, F. Bernd, M. Marco, G. Jonas, G. Axel, and P. A. Stefano, Nature **448**, 672 (2007).
[4] A. Griesmaier, J. Stuhler, T. Koch, M. Fattori, T. Pfau, and S. Giovanazzi, Phys. Rev. Lett. **97**, 250402 (2006).
[5] M. Lu, N. Q. Burdick, S. H. Youn, and B. L. Lev, Phys. Rev. Lett. **107**, 190401 (2011).
[6] S. H. Youn, M. Lu, U. Ray, and B. L. Lev, Phys. Rev. A **82**, 043425 (2010).
[7] K. Aikawa, A. Frisch, M. Mark, S. Baier, A. Rietzler, R. Grimm, and F. Ferlaino, Phys. Rev. Lett. **108**, 210401 (2012).
[8] R. M. Wilson, S. Ronen, and J. L. Bohn, Phys. Rev. Lett. **104**, 094501 (2010).
[9] C. Ticknor, R. M. Wilson, and J. L. Bohn, Phys. Rev. Lett. **106**, 065301 (2011).
[10] L. Santos, G. V. Shlyapnikov, and M. Lewenstein, Phys. Rev. Lett. **90**, 250403 (2003).
[11] L. Santos, G. V. Shlyapnikov, P. Zoller, and M. Lewenstein, Phys. Rev. Lett. **85**, 1791 (2000).
[12] K. Góral and L. Santos, Phys. Rev. A **66**, 023613 (2002).
[13] S. Yi and L. You, Phys. Rev. A **63**, 053607 (2001).
[14] S. Yi and L. You, Phys. Rev. A **67**, 045601 (2003).
[15] O. Dutta and P. Meystre, Phys. Rev. A **75**, 053604 (2007).
[16] S. Ronen, D. C. E. Bortolotti, and J. L. Bohn, Phys. Rev. Lett. **98**, 030406 (2007).
[17] N. G. Parker and D. H. J. O'Dell, Phys. Rev. A **78**, 041601 (2008).
[18] M. Baranov, Phys. Rep. **464**, 71 (2008).
[19] O. Tieleman, A. Lazarides, and C. Morais Smith, Phys. Rev. A **83**, 013627 (2011).
[20] K. Zhou, Z. Liang, and Z. Zhang, Phys. Rev. A **82**, 013634 (2010).
[21] K. Góral, K. Rzążewski, and T. Pfau, Phys. Rev. A **61**, 051601 (2000).
[22] S. Yi and L. You, Phys. Rev. A **61**, 041604 (2000).
[23] X. Guo-Yong, Z. Yong-Sheng, L. Jian, and G. Guang-Can, J. Opt. B **5**, 208 (2003).
[24] S. Giovanazzi, P. Pedri, L. Santos, A. Griesmaier, M. Fattori, T. Koch, J. Stuhler, and T. Pfau, Phys. Rev. A **74**, 013621 (2006).
[25] Y. Y. Lin, R.-K. Lee, Y.-M. Kao, and T.-F. Jiang, Phys. Rev. A **78**, 023629 (2008).
[26] D. H. J. O'Dell, S. Giovanazzi, and C. Eberlein, Phys. Rev. Lett. **92**, 250401 (2004).
[27] P. Pedri and L. Santos, Phys. Rev. Lett. **95**, 200404 (2005).
[28] I. Tikhonenkov, B. A. Malomed, and A. Vardi, Phys. Rev. Lett. **100**, 090406 (2008).
[29] L. Young-S, P. Muruganandam, and S. Adhikari, J. Phys. B: At. Mol. Opt. Phys. **44**, 101001 (2011).
[30] N. R. Cooper, E. H. Rezayi, and S. H. Simon, Phys. Rev. Lett. **95**, 200402 (2005).
[31] J. Zhang and H. Zhai, Phys. Rev. Lett. **95**, 200403 (2005).
[32] E. H. Rezayi, N. Read, and N. R. Cooper, Phys. Rev. Lett. **95**, 160404 (2005).
[33] P. Muruganandam, R. Kishor Kumar, and S. Adhikari, J. Phys. B: At. Mol. Opt. Phys. **43**, 205305 (2010).
[34] P. Muruganandam and S. Adhikari, Laser Phys. **22**, 813 (2012).
[35] T. Lahaye, J. Metz, B. Fröhlich, T. Koch, M. Meister, A. Griesmaier, T. Pfau, H. Saito, Y. Kawaguchi, and M. Ueda, Phys. Rev. Lett. **101**, 080401 (2008).
[36] K. E. Strecker, G. B. Partridge, A. G. Truscott, and R. G. Hulet, Nature **417**, 150 (2002).
[37] L. Khaykovich, F. Schreck, G. Ferrari, T. Bourdel, J. Cubizolles, L. Carr, Y. Castin, and C. Salomon, Science **296**, 1290 (2002).
[38] U. Al Khawaja, H. T. C. Stoof, R. G. Hulet, K. E. Strecker, and G. B. Partridge, Phys. Rev. Lett. **89**, 200404 (2002).
[39] W. Krolikowski, O. Bang, J. J. Rasmussen, and J. Wyller, Phys. Rev. E **64**, 016612 (2001).
[40] O. Bang, W. Krolikowski, J. Wyller, and J. J. Rasmussen, Phys. Rev. E **66**, 046619 (2002).
[41] Y. Kivshar and G. Agrawal, *Quantum Theory of Many-Particle Systems* (Academic Press, San Diego, 1971).

[42] I. Towers and B. A. Malomed, J. Opt. Soc. Am. B **19**, 537 (2002).
[43] J. Zeng and B. A. Malomed, Phys. Rev. A **85**, 023824 (2012).
[44] S. K. Adhikari, Phys. Rev. E **71**, 016611 (2005).
[45] P. A. Subha, C. P. Jisha, and V. C. Kuriakose, Pramana **69**, 229 (2007).
[46] C. Dai, X. Wang, and J. Zhang, Ann. Phys. **326**, 645 (2011).
[47] A. Alberucci, L. Marrucci, and G. Assanto, New J. Phys. **15**, 083013 (2013).
[48] S. Sabari, R. Raja, K. Porsezian, and P. Muruganandam, J. Phys. B: At. Mol. Opt. Phys. **43**, 125302 (2010).
[49] S. K. Adhikari, Phys. Rev. A **69**, 063613 (2004).
[50] F. K. Abdullaev, J. G. Caputo, R. A. Kraenkel, and B. A. Malomed, Phys. Rev. A **67**, 013605 (2003).
[51] H. Saito and M. Ueda, Phys. Rev. Lett. **90**, 040403 (2003).
[52] L. Wu, L. Li, J.-F. Zhang, D. Mihalache, B. A. Malomed, and W. M. Liu, Phys. Rev. A **81**, 061805 (2010).
[53] D.-S. Wang, X.-H. Hu, J. Hu, and W. M. Liu, Phys. Rev. A **81**, 025604 (2010).
[54] L. Landau and E. M. Lifshitz, *Mechanics* (Pergamon-Oxford, Oxford, 1960).
[55] H. Boot and R. Harvie, Nature **180**, 1187 (1957).
[56] W. Paul, Rev. Mod. Phys. **62**, 531 (1990).
[57] M. Tzoufras, F. S. Tsung, W. B. Mori, and A. A. Sahai, Phys. Rev. Lett. **113**, 245001 (2014).
[58] P. Muruganandam and S. Adhikari, Comput. Phys. Commun. **180**, 1888 (2009).