Formation, spin-up, and stability of field-reversed configurations

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Formation, spontaneous spin-up, and stability of θ-pinch formed field-reversed configurations are studied self-consistently in three dimensions with a multiscale hybrid model that treats all plasma ions as full-orbit collisional macroparticles and the electrons as a massless quasineutral fluid. The end-to-end hybrid simulations reveal poloidal profiles of implosion-driven fast toroidal plasma rotation and demonstrate three well-known discharge regimes as a function of experimental parameters: the decaying stable configuration, the tilt unstable configuration, and the nonlinear evolution of a fast growing tearing mode.

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I. INTRODUCTION

The field-reversed configuration (FRC) [1,2] is a magnetic confinement scheme in a cylindrical geometry where a compact toroidal plasmoid is trapped by the predominantly poloidal magnetic field reversed on axis by a self-induced toroidal current. The FRC is characterized by an extreme range of magnetic field with a relatively small magnetic field in the core plasma and large fields at the wall. The most notable technique for FRC formation [1,2] is the field-reversed θ-pinch discharge (FRTD). In a FRTD, a preionized plasma with an embedded reverse (bias) axial magnetic field is created prior to formation. The direction of the axial field at the wall is then quickly reversed by applying a large voltage pulse in the active θ coil over a time scale shorter than the inductive response time of the plasma. The imploding (forward) field rapidly pushes the plasma off the wall, creates radially propagating shocks, and reconnects with the trapped bias field, resulting in a toroidal closed-line magnetic topology inside a simply connected surface (separatrix). With a proper choice of discharge parameters this leads to the formation of stable, elongated, and wall-detached plasma objects that retain their axisymmetric structure over a long time [1,2]. However, the internal structure of high-temperature FRCs, namely, their self-induced rotation and instabilities, still remain poorly understood due to the lack of (i) comprehensive diagnostics and (ii) robust (physically accurate and numerically stable) simulation tools.

Magnetohydrodynamics (MHD) models cannot capture large-orbit ion effects near the *X* points and, importantly, near the *O* point (where the plasma density reaches its maximum) and the separatrix (where the thickness of the scrape-off layer becomes comparable to the ion Larmor radius). As a result, contrary to experimental evidence, three-dimensional (3D) MHD and Hall MHD simulations predict destruction of unsustained FRCs by the tilt mode regardless of their shapes [1,2]. Therefore, while MHD is capable of providing a qualitative description of the formation, acceleration, and translation of axisymmetric FRCs [2] and Hall MHD is sufficient (albeit with relatively high viscosity and resistivity) to yield the sustainment of 3D FRCs with energetic ion beams [3] and rotating magnetic fields [4], the 3D formation and stability of θ-pinch formed FRCs can only be studied in full detail with kinetic ion models [2]. Full-particle codes are still incapable of simulating large-scale magnetized objects such as FRCs under realistic experimental conditions. Therefore, the most promising candidates for kinetic FRC modeling are hybrid codes that combine the particle-in-cell description for plasma ions with the electron fluid approach and radiation-free Maxwell's equations [5–7]. Until now, however, 3D FRTD simulations have been out of reach of hybrid codes because of severe numerical difficulties that arise in such models due to large variations in the plasma density and magnetic field. As a result, hybrid simulations were only applied to axisymmetric [5] and infinitely long [8] discharge models or focused on exploring the 3D stability of FRC equilibria obtained analytically under *ad hoc* assumptions [6].

By conducting end-to-end 3D simulations of θ-pinch discharges with a hybrid code, this study aims at addressing two important physics questions: (i) May a toroidal plasma spin-up be caused by magnetic-field implosion itself rather than be attributed to external (e.g., end shorting of open field lines) and dissipative (particle and flux loss) effects? (ii) Can hybrid simulations reproduce discharge conditions that result in stable and unstable plasma configurations as observed in FRTD experiments? The rest of the paper briefly describes a computational setup and discusses simulation results that provide positive answers to both questions.

II. PHYSICAL MODEL

As noted above, hybrid simulations of FRTD plasmas encounter stiff numerical time scales due to short-wavelength whistler oscillations generated in low-density and/or high-magnetic-field-gradient (coil) regions. In a computer simulation a time-accurate time integration is required in such regions in order to properly describe fast plasma transit dynamics during magnetic implosion. This puts stringent stability conditions on a global multiscale hybrid system that must be followed computationally for hundreds of ion cyclotron periods. To avoid long and potentially unstable calculations, previous 3D hybrid studies of FRC equilibria have used analytical states constructed under a number of simplifying initial assumptions [6]: (i) no poloidal currents (and therefore no toroidal field), (ii) no plasma rotation, (iii) the plasma current is carried by electrons only. While those hybrid studies did provide valuable insight into the growth and stabilization of various toroidal modes in analytical FRCs, they came short of reproducing some well-known properties

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of experimental FRCs, notably, their spontaneous toroidal spin-up, self-similar stable evolution (decay), and emergence of toroidal magnetic fields and tearing instabilities. On the other hand, previous hybrid simulations of θ-pinch-driven FRC formation [5,8] were performed in two dimensions and did not include ion-ion collisions, which prove to be an important factor in establishing the FRC plasma equilibrium during the formation phase.

The simulations presented in this paper have been performed with an asynchronous hybrid simulation code HYPERS (hybrid parallel event-resolved simulator), specifically developed to handle multiscale plasma systems [7]. HYPERS enables self-adaptive local time increments for all computational variables (fields and particles) and preserves zero divergence of the magnetic field to within roundoff errors. This code has been enhanced with an ion-collision grid-based Coulomb model [9]. In this model ion velocities at each cell are scattered with self-consistent (temperature- and density-dependent) collision rates so that the process leads to their Maxwellization while preserving the net momentum and energy. In θ-pinch discharges ion-ion collisions play an important role as they lead to a fast isotropization of ion heating, which is predominantly driven in the poloidal plane. The isotropization process is especially critical in the early stage of FRC formation when the plasma is sufficiently cold and the ion-ion collision and ion cyclotron frequencies are of the same order. The effect of ion-ion collisions always remains strong near the O point, where ion scattering contributes to establishing a smooth peaked density profile. Since the electrons in our hybrid simulations are represented as a massless quasineutral fluid, we apply an *ad hoc* (Chodura) resistivity model typically used in MHD simulations [10]:

$$\eta_{\mathrm{Ch}} = \max\left[\frac{4\pi\,\nu_{\mathrm{Ch}}}{\omega_{pe}^2}, \eta_{\min}\right],$$
$$\nu_{\mathrm{Ch}} = c_{\mathrm{Ch}}\omega_{pi}\left[1 - \exp\left(f_{\mathrm{Ch}}\frac{v_{ed}}{v_s}\right)\right], \quad v_{ed} = |\mathbf{J}|/en_e, \qquad (1)$$

where $f_{\mathrm{Ch}} = 0.3$, $c_{\mathrm{Ch}} = 0.1$–0.2, v_s is the ion sound speed, $\mathbf{J}$ is the total current density, ω_{pi} and ω_{pe} are the ion and electron plasma frequencies, respectively, n_e is the electron number density, and $\eta_{\min}$ is a limiting classical (Spitzer) value.

III. THE FRTD SETUP

HYPERS uses a uniform Cartesian mesh for discretizing field and particle equations. This mesh approximates the cylindrical wall of a FRTD device as a staircase. At the beginning of a simulation a bias magnetic field $B_z^{\mathrm{bias}}(z) < 0$ is assumed to be frozen in the plasma. It remains nearly constant in the solenoidal part of the discharge chamber and is shaped up near the axial ends by two adjoining coils that produce axial fields $B_z^{\mathrm{end}} = 0.4$ kG. To force a prescribed temporal evolution of the axial magnetic field $B_z^c(t)$ at the cylindrical wall with the radius R_w, an axisymmetric toroidal electric field E_θ^c is applied during a quarter period $t_{1/4} = 4\ \mu$s inside the solenoidal coil:

$$E_\theta^c = \begin{cases} \frac{r}{R_s}E_\theta^s & \text{for } R_w \leqslant r \leqslant R_s \\ \frac{R_s}{r}E_\theta^s & \text{for } r \geqslant R_s, \end{cases} \qquad (2)$$

$$E_\theta^s = -\frac{R_s}{2}\frac{\partial}{\partial t}B_z^c, \quad B_z^c(z,t) = \Delta B_c(z)\sin\frac{\pi t}{2t_{1/4}}, \qquad (3)$$

where r is the radial distance and $R_s \geqslant R_w$ is the radius of a numerical shell. Note that Eq. (2) substituted into Faraday's law results in $\partial B_z/\partial t = 0$ for $r > R_s$.

For simplicity, the fully ionized deuterium plasma is assumed to have initially homogeneous density and temperature profiles. The simulations were conducted with typical FRTD parameters [2,11]: the initial plasma (electron) density $n_{e0} = 5 \times 10^{13}$ cm^{-3}, the electron and ion temperatures $T_e = T_i = 5$ eV, the axial bias field -0.6 kG $\leqslant B_z^{\mathrm{bias}} \leqslant -0.2$ kG, and the magnetic-field increment [see Eq. (3)] at the axial plane of symmetry $\Delta B_c = 1.3$ kG $- B_z^{\mathrm{bias}}$. The computational domain of $21(x) \times 21(y) \times 80(z)$ initial proton inertial lengths $\lambda_p \approx 3.2$ cm was discretized into a Cartesian grid of $80(x) \times 80(y) \times 240(z)$ cells, with 50 ion macroparticles seeded randomly per cell. These mesh and particle resolutions were verified to produce accurate and converging results. The field boundary conditions were assumed to be open ended (resistive) in the axial direction and perfectly conducting at the cylindrical wall of the device. Particles were absorbed at all boundaries. In what follows simulation times and velocities are normalized using the ion cyclotron frequency Ω_{ci} and Alfvén velocity v_A computed for deuterium ions with respect to the reference value of the magnetic field $B_0 = 1$ kG and the initial plasma density n_{e0}. All magnetic fields are normalized by B_0.

Stability properties of FRC plasmas are commonly characterized with two dimensionless parameters: the elongation $E = L_s/R_s$, where L_s and R_s are the separatrix half length and radius, respectively, and the parameter $S^* = R_s/\lambda_i$, where λ_i is the characteristic ion skin depth calculated with respect to the maximum plasma density [2]. The theoretical threshold for the kinetic stability of FRCs to the tilt mode can be roughly estimated as $S^*/E \leqslant 3.5$ [2].

IV. SIMULATION RESULTS

To analyze stability properties of FRCs, three parameter regimes were identified in this study based on initial values of the bias field B_z^{bias} and Chodura resistivity c_{Ch}. The first simulation was initialized with a weaker bias field $B_z^{\mathrm{bias}} = -0.2$ kG and a lower resistivity value $c_{\mathrm{Ch}} = 0.1$. As shown in Fig. 1, this simulation produces an axisymmetric column of plasma, with two plasma plumes being pushed towards each other by the tension of magnetic-field lines reconnecting near the end coils. These plumes are accompanied in transit with toroidal fields generated inside the separatrix through the stretching of the poloidal magnetic field by a sheared toroidal electron flow [5]. The largest antisymmetric fields are eventually found to reconnect at the center plane while dissipating to small values near the FRC axial ends (Fig. 1). The plumes in this case form an elongated FRC with $S^* \approx 9$, $E \approx 4.2$, and $S^*/E \approx 2$, which remains stable to the tilt mode in agreement with the parameter range mentioned above. Radial profiles of some quantities, obtained by cutting through one of the toroidal field maxima, are shown in Fig. 2.

Note that the density and magnetic-field profiles in Fig. 2 look similar to experimentally observed profiles [2]. The FRC

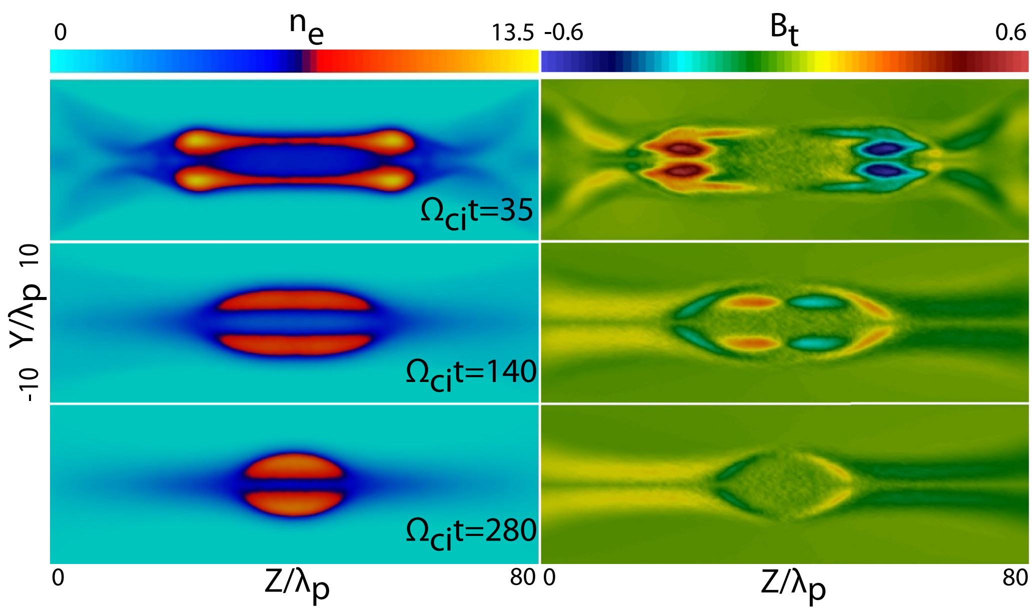

FIG. 1. (Color online) Temporal evolution of the plasma density n_e and self-generated toroidal field B_t in a stable, axisymmetric field reversed θ-pinch discharge.

in this case is found to preserve axisymmetry for a long time ($>400\Omega_{ci}^{-1}$). It slowly decays due to particle and flux losses, with approximately 50% of its particle inventory being lost through the axial ends after the FRC is formed ($t \sim 140\Omega_{ci}^{-1}$). The peak ion temperature $T_i \approx 120$ eV is observed during the formation. The electron temperature evolves adiabatically ($T_e \sim n_e^{\gamma-1}$ and $\gamma = 5/3$) and reaches its maximum at $T_e \approx$ 25 eV.

It should be emphasized that the earlier 3D hybrid studies of analytically initialized FRCs [6] suggested that the kinetic stabilization of the tilt mode should be accompanied by *a significant expansion* of the plasma in the axial direction, a physical scenario not supported by experimental evidence and results presented in Fig. 1. This difference in FRC behavior may be due to the enhanced stability of FRC equilibria with self-generated rotation and toroidal magnetic fields.

The Hall physics, being responsible for generating toroidal fields, also causes the FRC ions to spontaneously rotate [see Figs. 2(a) and 3] due to a radial Hall electric field (driven by $\partial B_z/\partial r$) that arises during a shocklike magnetic-field penetration into the plasma in the radial direction [8]. In three dimensions (Fig. 3) magnetic-field lines reconnect at the plasma axial ends, so resulting magnetic-field gradients $\partial B_r/\partial z$ contribute to the radial electric field and affect the plasma rotation. Note that the plasma spin-up shown in Fig. 3 is generated on a much faster time scale compared to the spin-ups due to resistive field decay [6], field-line end shorting [12], and particle losses [13].

Importantly, the ion rotation profiles shown in Figs. 2(a) and 3 reveal that the rotation remains diamagnetic on the outside of the FRC but changes direction near the separatrix and becomes paramagnetic near the geometric axis. This spin-up mechanism appears to be a universal intrinsic feature of the FRTD technique and may explain some recent experimental evidence [14] that points to an *internal* source of plasma rotation in FRCs, as opposed to the conventional theories,

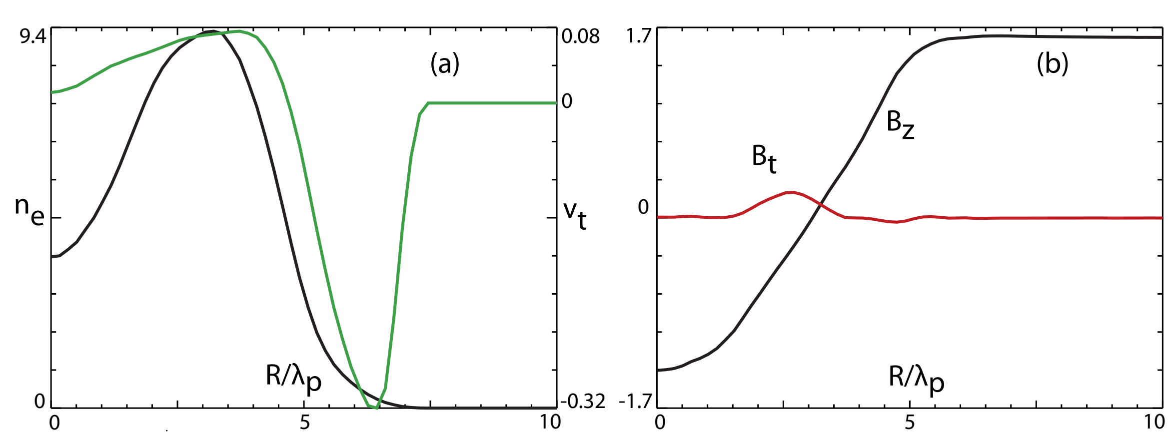

FIG. 2. (Color online) Stable FRC profiles at $\Omega_{ci}t = 140$ cut radially through one of the toroidal field maxima: (a) plasma density (black line) and toroidal velocity [light gray (green) line] and (b) axial (black line) and toroidal [light gray (red) line] magnetic fields.

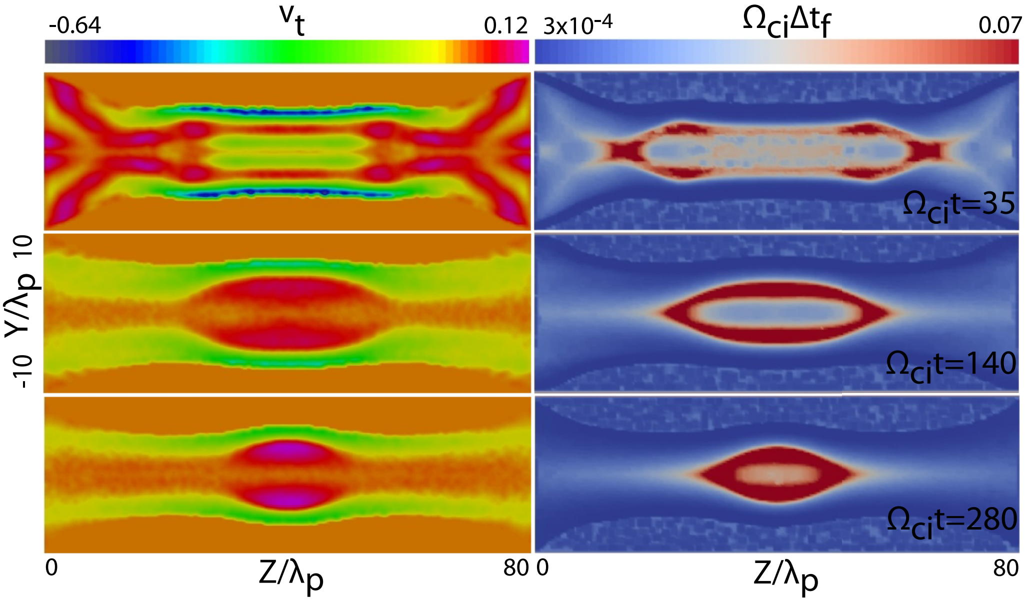

FIG. 3. (Color online) Temporal evolution of the plasma toroidal velocity v_t (left) and local field time increment Δt_f (right) in a stable, axisymmetric field-reversed θ-pinch discharge.

which assume that the rotation is driven by external forces and radially transferred to the plasma core through viscous interactions [2]. In particular, this experimental investigation found that the FRC plasma near its axial plane of symmetry rotates in the paramagnetic direction just after the formation phase. In time, the rotation accelerates in the diamagnetic direction and eventually reverses its sign, which suggests that it is caused by fast dynamic effects. Interestingly, the paramagnetic rotation was not observed in these experiments near the axial ends of the FRC. This seems to be consistent with the pattern of rotation velocity shown in Fig. 3, where the radial profile of ion rotation near the axial ends always remains predominantly diamagnetic due to the strong action of reconnecting (closing) magnetic-field lines.

Figure 3 also demonstrates a unique simulation feature: an instantaneous distribution of event-driven numerical time increments adaptively selected by the HYPERS code. Throughout a simulation these increments may vary by two orders of magnitude, with the smallest ones occurring in the FRC edge region, where the Hall-driven physics would become difficult to follow explicitly in conventional (time-stepped) hybrid and Hall MHD simulations.

The second physical regime of FRC formation is modeled by applying a stronger axial bias field $B_z^{\text{bias}} = -0.6$ kG and imposing a slightly larger resistivity $c_{\text{Ch}} = 0.2$. This results in a larger magnetic-field tension force near the end coils and produces a fatter and shorter FRC equilibrium with $E \approx 2.7$, $S^* \approx 11.2$, and $S^*/E \approx 4$. These parameters fall

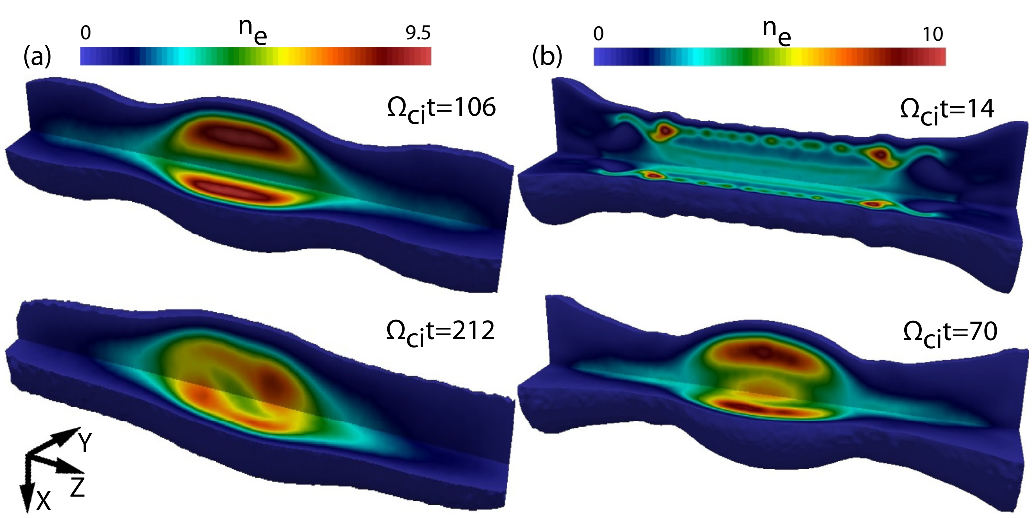

FIG. 4. (Color online) Temporal evolution of the plasma density n_e (isovolumes) in two unstable discharges where axisymmetric configurations are disrupted by (a) a late tilt instability of the FRC with $S^*/E \approx 4$ and (b) an early magneto-Taylor instability of the plasma column.

out of the stability range mentioned above. As expected from theory, this FRC gets quickly destroyed by the disruptive tilt mode [Fig. 4(a)]. This disruption, however, looks more like a classical kink rather than the sliding mode observed in MHD and hybrid simulations of FRCs with no initial poloidal flows and toroidal rotation.

Finally, the third simulation regime is obtained by keeping the same (larger) bias field $B_z^{\mathrm{bias}} = -0.6$ kG and reducing the Chodura resistivity to the same value as in the first case $c_{\mathrm{Ch}} = 0.1$. In this case [Fig. 4(b)] the plasma column is quickly disrupted by short-wavelength magnetic-field perturbations. This tearing process results in forming a strongly twisted configuration that is far from equilibrium and consequently loses axisymmetric containment due to fast growing toroidal modes. Since the initial perturbations are very short ranged, they can be efficiently dissipated or mitigated by larger plasma resistivity during the early stages of the discharge, as shown in the previous case with $c_{\mathrm{Ch}} = 0.2$.

Tearing modes were the first ones observed in θ-pinch discharges together with rotational and tilt modes. The occurrences of tearing instabilities *very early* during the discharge, however, cannot be attributed to resistive modes as argued in [2] since the resistive modes grow on times scales an order of magnitude too slow to explain the available experimental observations. A more plausible theoretical explanation for the observed rapid tearing and magnetic island formation was found in early axisymmetric studies of FRTD plasmas [15,16]. Namely, the rapid tearing is caused by a Kruskal-Schwarzschild interchange instability driven by a radial ion reflection off the reverse bias field towards the magnetic-field null in the field-reversal layer (current sheath). The present study demonstrates the nonlinear evolution of this magneto-Taylor instability in three dimensions and shows that it may be effectively prevented or greatly mitigated by high plasma resistivity that naturally arises in the early stages of a θ-pinch discharge due to implosion-driven plasma microinstabilities.

V. CONCLUSION

Ion cyclotron, kinetic, and collisional effects play an important role in the physics of FRCs. Kinetic ion-fluid electron simulations have been applied to model the FRC formation and stability in three-dimensional field-reversed θ-pinch discharges. These hybrid simulations treat all plasma ions as full-orbit particles and incorporate realistic experimental conditions such a θ-pinch electric-field drive, axial coils, and collisional physics in an open-ended geometry with a cylindrical wall. The computational studies have reproduced different aspects of the θ-pinch driven formation of FRCs and successfully demonstrated a strong parametric dependence of their stability to the frequently observed tilt and magneto-Taylor modes. Importantly, the simulations have also revealed temporal dynamics and poloidal profiles of a spontaneous plasma spin-up driven by the radial electric field that arises due to the plasma Hall response to the fast imploding magnetic field.

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