Electromagnetic waves in a model with Chern-Simons potential

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We investigated the appearance of Chern-Simons terms in electrodynamics at the surface or interface of materials. The requirement of locality, gauge invariance, and renormalizability in this model is imposed. Scattering and reflection of electromagnetic waves in three different homogeneous layers of media is determined. Snell's law is preserved. However, the transmission and reflection coefficient depend on the strength of the Chern-Simons interaction (connected with Hall conductance), and parallel and perpendicular components are mixed.

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I. INTRODUCTION

Space-time homogeneity and isotropy are typical for usual quantum field theory models of elementary particles. It is a natural assumption in the study of various processes with simplest excitations of quantum vacuum. However, it is not suitable for modeling the interaction of quantum fields with macroscopic objects, changing essentially the vacuum properties. In this case, quantum macro-effects may appear in dynamics of material bodies which cannot be explained in the framework of classical physics. Theoretically, this problem was first considered in 1948 by Casimir, who showed that quantum vacuum fluctuations cause the attraction between two perfectly conducting parallel plates of an uncharged capacitor [1]. This phenomenon, called the Casimir effect (CE), is observed experimentally [2–5], and the results obtained empirically for materials with high conductivity are, with a high degree of accuracy, in agreement with theoretical ones [6,7]. At typical distances of 10–1000 nm for the CE both quantum and classical features of the system become essential. Their combination forms a special nanophysics. Investigations of it are not only of general theoretical interest, they are also important for the development of new technical devices, in view of the increasing trend toward their miniaturization.

Although there are numerous papers devoted to the theoretical problems of the CE [6,8], they are often based on simplified models of a free scalar field theory or free electromagnetic field with fixed boundary conditions, applying only to investigations of some particular aspect of the CE, and ignoring usually specificity of quantum electrodynamics. Such models are not suitable for a complete description of a wide range of nanophysical phenomena occurring in the system as a result of the interaction of quantum degrees of freedom with the material body of a given shape (classic defect). The results presented in our paper were obtained within the Symanzik approach [9] for construction of quantum field theory models when there are spatial inhomogeneities with sharp boundaries. They are described by an additional action functional (action of the defect) that is concentrated in the region of space where the macroscopic object is located. In quantum electrodynamics the interaction of photons with the defect modeling background field is completely determined by the requirements of the locality, gauge invariance, renormalizability, and is described by the Chern-Simons action functional with a dimensionless constant characterizing the material properties of the surface [10]. It affects the Casimir force, which is nonuniversal and can be not only attractive but also repulsive for a flat capacitor [10]. It is shown also that in this model the static electric charge interacting with the surface defect generates a magnetic field, and the stable straight-line current creates an electric field [10]. The calculated Casimir-Polder potential for a neutral atom near a flat surface allowed us to find the parity-violating corrections to the previously known results [11]. Based on the earlier proposed model [10] we studied in this paper the electromagnetic waves in three layers of matter with magnetic susceptibilities μ_1, μ_2, μ_3 and permittivities ε_1, ε_2, ε_3 separated by two parallel material planes $x_3 = \pm l/2$ whose Chern-Simons interaction with the electromagnetic field is characterized by coupling constants a_1, a_2. We can show that the interfaces of such a kind have finite Hall conductance, simply connected with Chern-Simons permittivity.

II. STATEMENT OF PROBLEM

For the formulation and investigation of the model it is convenient to use the notations $\check{\alpha}$ and $\mathbf{a}$ for three- and two-component arrays, respectively. We define also the scalar product and the $*$ composition of them: $\check{\alpha}\check{\beta} = \alpha_1\beta_1 + \alpha_2\beta_2 + \alpha_3\beta_3$, $\mathbf{ab} = a_1b_1 + a_2b_2$, $\check{\alpha} * \check{\beta} = (\alpha_1\beta_1,\alpha_2\beta_2,\alpha_3\beta_3)$, $\mathbf{a} * \mathbf{b} = (a_1b_1,a_2b_2)$.

Let us introduce the arrays:

$$\check{\theta}_l \equiv (\theta(-l/2 - x_3),\theta(l/2 - |x_3|),\theta(x_3 - l/2)),$$
$$\mathbf{d}_l \equiv (\delta(x_3 + l/2),\delta(x_3 - l/2)).$$

Here $\theta(\alpha)$ and $\delta(\alpha)$ are Heaviside step-function and Dirac δ function. The scalar products of $\check{\theta}_l$ with $\check{\beta} = (\beta_1,\beta_2,\beta_3)$ and $\mathbf{d}_l$ with $\mathbf{c} = (c_1,c_2)$ are defined as

$$\mathcal{F}(\beta_1,\beta_2,\beta_3) = \mathcal{F}(\check{\beta}) \equiv \check{\beta}\check{\theta}_l, \quad \mathcal{D}(c_1,c_2) = \mathcal{D}(\mathbf{c}) \equiv \mathbf{c}\mathbf{d}_l.$$

Then one obtains

$$\frac{\partial}{\partial x_3}\mathcal{F}(\check{\beta}) = \mathcal{F}\left(\frac{\partial}{\partial x_3}\check{\beta}\right) + \mathcal{D}[\mathbf{s}(\check{\beta})],$$
$$\mathcal{F}(\check{\beta})\mathcal{F}(\check{\gamma}) = \mathcal{F}(\check{\beta} * \check{\gamma}),\ \mathcal{F}(1,1,1) = 1.$$

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where $\mathbf{s}(\breve{\beta}) \equiv (\beta_2 - \beta_1, \beta_3 - \beta_2)$. The model [10] of the photon field A_μ interacting with the two-dimensional material surface described by equation $\Phi(x) = 0$ can be generalized for the considered defining the action functional as

$$S(A) = -\frac{1}{4} G_{\mu\nu} F^{\mu\nu} + S_\phi(A). \tag{1}$$

Here, $F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu$, $G_{\mu\nu} \equiv \mathcal{E}(x_3) F_{\mu\nu}$, if $\mu = 0$ or $\nu = 0$, and $G_{\mu\nu} \equiv \mathcal{M}^{-1}(x_3) F_{\mu\nu}$ if $\mu \neq 0$, $\nu \neq 0$ with $\mathcal{E}(x_3) \equiv \mathcal{F}(\breve{\varepsilon})$, $\mathcal{M}(x_3) \equiv \mathcal{F}(\breve{\mu})$.

The functional $S_\phi(A)$ describes the interaction of the two-dimensional material objects (defects) with the photon field. The defects lie, in our case, at two parallel planes $x_3 = l_i$ with $\mathbf{l} = (-l/2, +l/2)$. Using the notation $\Phi_j(x) = x_3 - l_j$ we can write the action of the defects as $S_\phi(A) = S_1(A) + S_2(A)$, where

$$S_j(A) = \frac{a_j}{2} \int \partial_\mu \Phi_j(x) A_\nu(x) \tilde{F}^{\mu\nu}(x) \delta(\Phi_j(x)) dx$$
$$= \frac{a_j}{2} \int A_\nu(x) \tilde{F}^{3\nu}(x) \delta(\Phi_j(x)) dx, \quad j = 1,2.$$

Here, $\tilde{F}^{\mu\nu}$ is the dual field tensor $\tilde{F}^{\mu\nu} = \epsilon^{\mu\nu\lambda\rho} F_{\lambda\rho}$, and $\epsilon^{\lambda\mu\nu\rho}$ is the totally antisymmetric tensor, $\epsilon^{0123} = 1$.

The Chern-Simons action $S_{\rm CS}(V)$ of the abelian gauge field V_α ($\alpha = 0,1,2$) in the three-dimensional space-time is the integral of the invariant-3 form [12],

$$S^{(3)}_{\rm CS}(V,\chi) = \chi \int V_\alpha \partial_\beta V_\gamma \epsilon^{\alpha\beta\gamma} d^3x,$$

where $\epsilon^{\alpha\beta\gamma}$ is the Levi-Civita tensor ($\epsilon^{012} = 1$), and χ is a constant. Using the notations $V^{(j)}_\alpha = A_\alpha|_{x_3 = l_j}$, we can present the action $S_j(A)$ of the defect layers in our four-dimensional model in the form $S_j(A) = -S^{(3)}_{\rm CS}(V^{(j)}, a_j)$.

Both the Abelian and non-Abelian Chern-Simons terms are used in many different models [13]. They can generate the interaction potential of nonrelativistic particles moving in the two-dimensional space [14] and enable one to construct the gauge invariant theory of the three-dimensional massive gauge fields [15,16]. The gauge-field models of such a kind are relevant for the fractional quantum Hall effect [17] and probably also for high-T_C superconductivity [18].

In the four-dimensional gauge-field theory one studies the models with the Chern-Simons-like action of the form $S^{(4)}_{\rm CS}(A,k) = k_\mu A_\nu \tilde{F}^{\mu\nu}$, including a constant vector k_μ [19]. The functional $S^{(4)}_{\rm CS}(A,k)$ is gauge invariant, but violates the Lorentz symmetry. It describes the effects of spontaneous Lorentz invariance breaking in the so-called standard-model extension [20] and is used for a modification of the Maxwell theory [19].

Comparing the actions $S^{(3)}_j(V,\chi)$, $S^{(4)}_{\rm CS}(A,k)$ in the above-mentioned models with the Chern-Simons terms $S_\Phi(A)$ in Eq. (1), one notes that the coupling constants a_j in $S_j(A)$ are dimensionless, but the parameter χ and the vector k_μ have the dimension of mass.

Therefore, in the (2+1)-dimensional Maxwell-Chern-Simons theory, the photon has a "topological" mass $m = \chi$ [16]. The Casimir force f between parallel lines in this model is the same as in the theory of the free scalar field with mass m. It is attractive, and

$$f = -\frac{1}{16\pi l^3} \int_{2ml}^{\infty} \frac{y^2 dy}{e^y - 1},$$

where l is the distance between the lines [16]. For small ml, $f \sim -(8\pi l^3)^{-1}[\zeta(3) + ml - (ml)^2]$ (ζ denotes here the Riemann zeta function), and for the large distances ($ml \gg 1$), one obtains $f \sim -(8\pi l^3)^{-1}[2(ml)^2 + 2(ml) + 1]e^{-2ml}$.

Analogous results exist in the Maxwell-Proca-Chern-Simons theory [21], and a similar nontrivial distance dependence of the Casimir force between two parallel conducting plates is obtained also in the standard-model extension [22]. On the other hand, in the four-dimensional theory of the quantum electromagnetic field with the Chern-Simons defect actions $S_j(A)$, the power function describes exactly the dependence of the Casimir force $F_{\rm Cas}$ from the distance l between two parallel planes in vacuum [10]:

$$F_{\rm Cas} = -\frac{\pi^2}{240 l^4} C(a_1, a_2). \tag{2}$$

Here, the function $C(a_1,a_2)$ is expressed in terms of the polylogarithm $\mathrm{Li}_4(z)$ [10].

For identical defect planes ($a_1 = a_2 = a$), the factor $f(a) = C(a,a)$ is an even function of a, $f(0) = f(a_0) = 0$, with $a_0 \approx 0.5892$, and $\lim_{|a|\to\infty} f(a) = 1$ in accordance with the Casimir force for perfectly conducting plates [1]. If $0 < |a| < a_0$, then $f(a) < 0$, and the Casimir force is repulsive. If $|a| > a_0$, then $0 < f(a) < 1$, and the Casimir force is attractive. It vanishes for $a = 0, |a| = a_0$.

The presented results demonstrate the difference between the proposed model Eq. (1) and the other ones constructed with the inclusion of Chern-Simons terms in the Lagrange density.

The Euler-Lagrange equations for the action functional $S(A)$ Eq. (1) are written as modified Maxwell's equations:

$$\frac{\delta S(A)}{\delta A_\nu} = \partial_\xi G^{\xi\nu} + \mathcal{D}(\mathbf{a}) J^\nu = 0. \tag{3}$$

We use the notations $J^\nu \equiv \epsilon^{3\nu\sigma\rho} F_{\sigma\rho}$, $\mathbf{a} \equiv (a_1, a_2)$. We construct the general solution of Eq. (3), analyze its properties, and consider processes of plain-wave scattering.

Action Eq. (1) and the Euler-Lagrange Eq. (3) are invariant under gauge transformation $A_\mu(x) \to A_\mu(x) + \partial_\mu \varphi(x)$. Thus, the solution of Eq. (3) is defined up to a gauge transformation. We fix it by choosing the temporal gauge $A_0 = 0$. Then the vector-potential $A^\mu = (0, \vec{A})$ yields the electric field $\vec{E} = -\partial_0 \vec{A}$ and the magnetic induction $\vec{B} = \vec{\partial} \times \vec{A}$.

We solve Eq. (3) using the Fourier transform over coordinates $x_0 = ct$, x_1, x_2 for the vector-potential A_μ:

$$A_\mu(x) = \frac{1}{(2\pi)^{\frac{3}{2}}} \int e^{i\overline{px}} A_\mu(x_3, \overline{p}) d\overline{p}$$
$$= \frac{2\,\mathrm{Re}}{(2\pi)^{\frac{3}{2}}} \int \theta(p_0)[e^{i\overline{px}} A_\mu(x_3, \overline{p})] d\overline{p}.$$

Here and later we use the notation $\overline{p}$ for vector $\overline{p} = (p_0, p_1, p_2)$, $\overline{px} = p_0 x_0 - p_1 x_1 - p_2 x_2$. Re denotes the real part and $\omega = c p_0$ the frequency.

III. SOLUTION OF EULER-LAGRANGE EQUATIONS

With the gauge condition $A_0 = 0$, Eq. (3) for $\vec{A}(x_3,\overline{p})$ are equivalent to the following ones

$$(\partial_3 \mathcal{E}\mathcal{P}^{-2}\partial_3 + \mathcal{E})\rho = \frac{2i}{p^0}\mathcal{D}(\mathbf{a})\tau, \tag{4}$$

$$(\partial_3 \mathcal{M}^{-1}\partial_3 + \mathcal{M}^{-1}\mathcal{P}^2)\tau = -2ip^0\mathcal{D}(\mathbf{a})\rho, \tag{5}$$

$$A_3 = \mathcal{P}^{-2}\partial_3\rho, \tag{6}$$

where

$$\rho \equiv ip_1A_1 + ip_2A_2, \quad \tau \equiv ip_2A_1 - ip_1A_2,$$
$$\mathcal{P} \equiv \mathcal{F}(\kappa_1,\kappa_2,\kappa_3), \quad \kappa_i \equiv \sqrt{p_0^2\varepsilon_i\mu_i - p_1^2 - p_2^2}.$$

By definition the real part of κ_j is chosen to be nonnegative, and if it vanishes, then $\kappa_j = -i|\kappa_j|$.

If in the Symanzik approach one takes into account polarization effects arising from the interaction of quantum fluctuations of the fermion fields with the defect and electromagnetic field, then with the accuracy sufficient for our problem, the constants ϵ_i, μ_i, a_i will be functions of $\bar{p}$ in Eqs. (4)–(6). We assume that these effective characteristics of real materials are used in Eqs. (4)–(6) in the further calculations.

The fields ρ,τ are found from Eqs. (4) and (5). The components A_1, A_2 of the vector-potential $\vec{A}$ are expressed by ρ and τ,

$$A_1 = -i(\rho\, p_1 + \tau\, p_2)\mathfrak{p}^{-2}, \quad A_2 = i(\tau\, p_1 - \rho\, p_2)\mathfrak{p}^{-2}, \tag{7}$$

where $\mathfrak{p}^2 = p_1^2 + p_2^2$. The electromagnetic field $\vec{A}(x_3,\bar{p})$ in the considered medium is characterized by the mutually orthogonal vectors $\vec{p}_\parallel = (p_1,p_2,0)$, $\vec{p}_\perp = (p_2,-p_1,0)$, $\vec{t} = (0,0,1)$. The vectors $\vec{p}_\parallel$, $\vec{t}$ define the plane of incidence. In virtue of Eqs. (6) and (7), the vector potential $\vec{A} = (A_1, A_2, A_3)$ can be presented in the form $\vec{A} = \vec{A}_\parallel + \vec{A}_\perp$, where $\vec{A}_\parallel$ is parallel to the plane of incidence, and $\vec{A}_\perp$ is perpendicular to it:

$$\vec{A}_\parallel(x_3,\bar{p}) = (-i\vec{p}_\parallel\mathfrak{p}^{-2} - \vec{t}\,\mathcal{P}^{-2}\partial_3)\rho(x_3,\bar{p}), \tag{8}$$

$$\vec{A}_\perp(x_3,\bar{p}) = -i\vec{p}_\perp\mathfrak{p}^{-2}\tau(x_3,\bar{p}). \tag{9}$$

Since in our gauge $\vec{E}(\bar{p},x_3) = -ip_0\vec{A}(\bar{p},x_3)$, the field $\rho(x_3,\bar{p})$, ($\tau(x_3,\bar{p})$) describe plane waves whose electric field vectors are parallel (perpendicular) to the plane of incidence. Equations (4) and (5) show that the Chern-Simons defects mix parallel and transverse components of the phonon field.

Let us introduce the notations $\mathbf{f}(x_3) = (\rho(x_3),\tau(x_3))$ and define

$$\mathbf{K} = \begin{pmatrix} \mathcal{E}\mathcal{P}^{-2} & 0 \\ 0 & \mathcal{M}^{-1} \end{pmatrix}, \quad \mathbf{C} = \begin{pmatrix} 0 & p_0^{-1} \\ -p_0 & 0 \end{pmatrix},$$

$$\mathbf{L}_i = \begin{pmatrix} e_i & 0 \\ 0 & m_i \end{pmatrix}, \quad e_i = \frac{\varepsilon_i}{\kappa_i}, \quad m_i = \frac{\kappa_i}{\mu_i}, \quad i = 1,2,3.$$

Then we can present Eqs. (4) and (5) in a compact form:

$$(\partial_3\mathbf{K}\partial_3 + \mathbf{K}\mathcal{P}^2)\mathbf{f} = 2i\mathcal{D}(\mathbf{a})\mathbf{C}\mathbf{f}. \tag{10}$$

We conclude that $\mathbf{f}$ is continuous at $x_3 = l_j$,

$$\mathbf{f}_j(l_j) = \mathbf{f}_{j+1}(l_j), \tag{11}$$

since a discontinuity would yield a δ' function on the left-hand side of Eq. (10), which is absent on the right-hand side. Due to Eq. (7), $A_{1,2}$ is continuous at the defects. Thus, the derivatives $\partial_{0,1,2}A_{1,2}$ are continuous, which implies the continuity of the components $E_{1,2}$ and B_3.

Introducing $\mathbf{f}(x_3) = \mathcal{F}(\check{\mathbf{f}}(x_3))$ with $\check{\mathbf{f}}(x_3) = (\mathbf{f}_1(x_3),\mathbf{f}_2(x_3),\mathbf{f}_3(x_3))$, we integrate Eq. (10) from $x_3 = l_j - \eta$ to $x_3 = l_j + \eta$ with infinitesimal η:

$$\frac{\mathbf{L}_{j+1}}{\kappa_{j+1}}\partial_3\mathbf{f}_{j+1}(l_j) - \frac{\mathbf{L}_j}{\kappa_j}\partial_3\mathbf{f}_j(l_j) = 2ia_j\mathbf{C}\mathbf{f}(l_j). \tag{12}$$

Within the layers $x_3 \neq \pm l_j$, Eq. (10) is written as $(\partial_3^2 + \kappa_i^2)\mathbf{f}_i(x) = 0$ and yields

$$\mathbf{f}_i = \mathbf{f}_i^+ + \mathbf{f}_i^-, \quad \mathbf{f}_i^\pm = (\rho_i^\pm,\tau_i^\pm) = \mathbf{c}_i^\pm e^{\mp i\kappa_i x_3}. \tag{13}$$

For real κ_i the solution with the upper (lower) sign describes a plane wave moving in positive (negative) x_3-direction.

It follows from Eqs. (13), (8), and (9) that $\vec{A} = \vec{A}^+ + \vec{A}^-$ and

$$\vec{A}_\parallel^\pm(x_3) = -\frac{i\vec{p}_\parallel^{\,\pm}\rho^\pm(x_3)}{\mathfrak{p}^2}, \quad \vec{A}_\perp^\pm(x_3) = -\frac{i\vec{p}_\perp\tau^\pm(x_3)}{\mathfrak{p}^2}, \tag{14}$$

with

$$\vec{p}_\parallel^{\,\pm} \equiv \vec{p}_\parallel \mp \mathfrak{p}^2\mathcal{P}^{-1}\vec{t}, \quad (\vec{p}_\parallel^{\,\pm})^2 = \mathcal{P}_0^2\mathcal{P}^{-2}\mathfrak{p}^2, \tag{15}$$

$$\mathcal{P}_0 \equiv \mathcal{P}|_{p_1=p_2=0} = p_0\mathcal{F}(n_1,n_2,n_3), \quad n_i = \sqrt{\varepsilon_i\mu_i}. \tag{16}$$

Since $\partial_3\mathbf{f}_j = i\kappa_j\tilde{\mathbf{f}}_j$, where $\tilde{\mathbf{f}}_j \equiv \mathbf{f}_j^- - \mathbf{f}_j^+$, the condition Eq. (12) can be written as

$$\mathbf{L}_{j+1}\tilde{\mathbf{f}}_{j+1}(l_j) - \mathbf{L}_j\tilde{\mathbf{f}}_j(l_j) = 2a_j\mathbf{C}\mathbf{f}_j(l_j), \quad j = 1,2. \tag{17}$$

These equations describe the discontinuity of the components $H_{1,2}$ of the magnetic field and D_3 of the dielectric displacement due to the currents a_jJ^ν in Eq. (3):

$$D_{3,j+1} - D_{3,j} = -a_jJ_j^0 = -2a_jB_{3,j}, \tag{18}$$

$$H_{1,j+1} - H_{1,j} = -a_jJ_j^2 = 2a_jE_{1,j}, \tag{19}$$

$$H_{2,j+1} - H_{2,j} = a_jJ_j^1 = 2a_jE_{2,j}. \tag{20}$$

In order to solve Eqs. (11) and (17) it is convenient to introduce the following 2×2 matrices:

$$\mathbf{T}_j^{\alpha\beta} = \mathbf{1} + \alpha\mathbf{L}_{j+1}^{-1}(\beta\mathbf{L}_j - 2a_j\mathbf{C}), \quad j = 1,2, \quad \alpha,\beta = \pm 1,$$

and four-component vectors $\mathbf{U}_j = (\mathbf{u}_j^+,\mathbf{u}_j^-)$, $\mathbf{V}_j = (\mathbf{v}_j^+,\mathbf{v}_j^-)$ with $\mathbf{u}_j^\pm = \mathbf{f}_j^\pm(l_j)$, $\mathbf{v}_j^\pm = \mathbf{f}_{j+1}^\pm(l_j)$. Then we obtain from Eqs. (11), (13), and (17) the relations between the $\mathbf{V}$ and $\mathbf{U}$ by means of the transfer matrices T:

$$\mathbf{V}_j = T_j\mathbf{U}_j, \quad \mathbf{U}_2 = T_l\mathbf{V}_1, \quad \mathbf{V}_2 = T\mathbf{U}_1, \quad T = T_2T_lT_1,$$
$$T_l = \begin{pmatrix} e^{-il\kappa_2}\mathbf{1} & 0 \\ 0 & e^{il\kappa_2}\mathbf{1} \end{pmatrix}, \quad T_j = \frac{1}{2}\begin{pmatrix} \mathbf{T}_j^{++} & \mathbf{T}_j^{+-} \\ \mathbf{T}_j^{-+} & \mathbf{T}_j^{--} \end{pmatrix}.$$

One has for nonactive media (real ε, μ, and a)

$$G_j = T_j^\dagger G_{j+1}T_j, \; T_l^\dagger G_jT_l = G_j, \quad T^\dagger G_3T = G_1,$$
$$\mathbf{U}_1^*G_1\mathbf{U}_1 = \mathbf{V}_1^*G_2\mathbf{V}_1 = \mathbf{U}_2^*G_2\mathbf{U}_2 = \mathbf{V}_2^*G_3\mathbf{V}_2. \tag{21}$$

Here †, ∗ denote the hermitian conjugation of the matrix and the complex conjugation of the vector components,

$$G_j \equiv \begin{pmatrix} \frac{\mathrm{Re}\kappa_j}{|\kappa_j|}\mathbf{g}_j & \frac{\mathrm{Im}\kappa_j}{|\kappa_j|}\mathbf{g}_j \\ -\frac{\mathrm{Im}\kappa_j}{|\kappa_j|}\mathbf{g}_j & -\frac{\mathrm{Re}\kappa_j}{|\kappa_j|}\mathbf{g}_j \end{pmatrix}, \quad \mathbf{g}_j \equiv \begin{pmatrix} p_0 e_j & 0 \\ 0 & m_j/p_0 \end{pmatrix},$$

$\mathrm{Re}\kappa_j$ ($\mathrm{Im}\kappa_j$) is the real (imaginary) part of κ_j.

For a complete analysis of the propagation of waves in the considered medium it is enough to assume that in the region $x_3 > l/2$ there are no waves moving in the negative direction of x_3 axis. This restriction obeys $\mathbf{f}_3^-(l/2) = \mathbf{v}_2^- = 0$, since for real $\kappa_3, \mathbf{f}_3^-(l/2)$ is the amplitude of the wave moving from $x_3 = +\infty$ to the plane $x_3 = l/2$, and for imaginary κ_3 the field must decay exponentially for $x_3 \to +\infty$. Then $\mathbf{V}_2 = T\mathbf{U}_1$ yields

$$\mathbf{T}^{-+}\mathbf{u}_1^+ + \mathbf{T}^{--}\mathbf{u}_1^- = 0, \quad \mathbf{v}_2^+ = \mathbf{T}^{++}\mathbf{u}_1^+ + \mathbf{T}^{+-}\mathbf{u}_1^-, \qquad (22)$$

where $\mathbf{T}^{\pm\pm}$ denote the corresponding 2×2—submatrices of the 4×4—matrix T.

For real κ_1 the amplitude of the incident wave propagating in the region $x_3 < -l/2$ in the positive x_3 direction is $\mathbf{c}_{\mathrm{in}} = \mathbf{c}_1^+ = \mathbf{u}_1^+ e^{-i\kappa_1 l/2}$. The amplitude of the reflected wave is $\mathbf{c}_r = \mathbf{c}_1^- = \mathbf{u}_1^- e^{i\kappa_1 l/2}$ and that of the transmitted wave is given by $\mathbf{c}_t = \mathbf{c}_3^+ = \mathbf{v}_2^+ e^{i\kappa_3 l/2}$ for real κ_3. The amplitudes $\mathbf{c}_r$, $\mathbf{c}_t$ are obtained from Eq. (22):

$$\mathbf{c}_r = -e^{i\kappa_1 l}(\mathbf{T}^{--})^{-1}\mathbf{T}^{-+}\mathbf{c}_{\mathrm{in}}, \qquad (23)$$

$$\mathbf{c}_t = e^{i(\kappa_3+\kappa_1)l/2}[\mathbf{T}^{++} - \mathbf{T}^{+-}(\mathbf{T}^{--})^{-1}\mathbf{T}^{-+}]\mathbf{c}_{\mathrm{in}}. \qquad (24)$$

If κ_3 is imaginary, then $\mathbf{c}_r$ yields again the amplitude of the reflected wave (total reflection), whereas $\mathbf{c}_t$ describes the amplitude of the decaying wave.

If both κ_1 and κ_3 are imaginary, then the waves are totally reflected at both $x_3 = \pm l/2$. The waves obey $\mathbf{v}_2^- = \mathbf{u}_1^+ = 0$. Then Eq. (22) can have a nonzero solution only if κ_3 is imaginary (since by Eq. (21), $\mathbf{V}_2^* G_3 \mathbf{V}_2 = \mathbf{U}_1^* G_1 \mathbf{U}_1 = 0$), and $\det \mathbf{T}^{--} = 0$ with

$$\begin{aligned} \mathbf{T}^{--} &= \tfrac{1}{4}(\mathbf{T}_2^{-+}e^{-i\kappa_2 l}\mathbf{T}_1^{+-} + \mathbf{T}_2^{--}e^{+i\kappa_2 l}\mathbf{T}_1^{--}) \\ &= \tfrac{1}{4}[\mathbf{T}_2^{--}(e^{2i\kappa_2 l}\mathbf{1} - \mathbf{R}_2\mathbf{R}_1)e^{-i\kappa_2 l}\mathbf{T}_1^{--}], \qquad (25) \\ \mathbf{R}_2 &= -(\mathbf{T}_2^{--})^{-1}\mathbf{T}_2^{-+},\ \mathbf{R}_1 = \mathbf{T}_1^{+-}(\mathbf{T}_1^{--})^{-1}. \end{aligned}$$

The matrices $\mathbf{R}_j$ describe the total reflection of the waves coming from the center to l_j, $\mathbf{v}_1^+ = \mathbf{R}_1\mathbf{v}_1^-$, $\mathbf{u}_2^- = \mathbf{R}_2\mathbf{u}_2^+$. These matrices differ by a similarity transformation from unitary matrices $\mathbf{O}_j = \mathbf{g}_2^{1/2}\mathbf{R}_j\mathbf{g}_2^{-1/2}$. Thus, one obtains electromagnetic waves propagating in layer 2 as soon as one of the two eigenvalues $e^{i\phi}$ of the unitary matrix $\mathbf{O}_2\mathbf{O}_1$ agrees with $e^{2i\kappa_2 l}$.

If κ_j is real, then the functions $\mathbf{f}_j^{\pm}(x_3)e^{i\bar{p}\bar{x}}$ describe plane waves propagating in the medium with constants ε_j, μ_j in directions of vectors $\vec{p}_j^{\pm} = (p_1, p_2, \pm\kappa_j)$ with velocity $v_j = cp_0/|\vec{p}_j^{\pm}| = c/n_j$. For the angle ϑ_j between $\vec{p}_j$ and the x_3 axis, $\sin\vartheta_j = \mathfrak{p}/|\vec{p}_j| = \mathfrak{p}/(p_0 n_j)$, and this equality yields Snell's law $\sin\vartheta_j/\sin\vartheta_k = n_k/n_j$. The component $v_j^{3\pm}$ of the wave front velocity v_j is equal to $v_j^{3\pm} = \pm v_j\kappa_j/|\vec{p}_j^{\pm}| = \pm c\kappa_j/(p_0 n_j^2)$.

The electric field vector of the wave propagating in the jth layer in the positive (negative) direction of the x_3 axis is $\vec{E}_j^+ = -ip_0\vec{A}_j^+$ ($\vec{E}_j^- = -ip_0\vec{A}_j^-$), and the corresponding energy density is $\varepsilon_j|\vec{E}_j^+|^2$ ($\varepsilon_j|\vec{E}_j^-|^2$). The energy current density propagating in the positive x_3 direction is $I_j = I_j^+ - I_j^-$, $I_j^{\pm} = v_j^{3+}\varepsilon_j|\vec{E}_j^{\pm}|^2$. In virtue of Eqs. (14)–(16),

$$I_j^{\pm} = I_{\rho j}^{\pm} + I_{\tau j}^{\pm}, \quad I_{\rho j}^{\pm} = \frac{p_0^3 e_j|\rho_j^{\pm}|^2}{\mathfrak{p}^2}, \quad I_{\tau j}^{\pm} = \frac{p_0 m_j|\tau_j^{\pm}|^2}{\mathfrak{p}^2}.$$

If we denote $U_3 \equiv V_2$, then $I_j = \mathfrak{p}^2 U_j^* G_j U_j/p_0^2$. The energy is conserved in the nonactive medium, therefore the quantity I_j is independent of x_3 and $I_j = I_k$ (in agreement with Eq. (21)). It follows from Eq. (21) that the energy current I_j vanishes in case of total reflection, since $V_2^* G_3 V_2 = 0$ by imaginary κ_3 and $\mathbf{v}_2^- = 0$.

If κ_j is imaginary, then, similarly as in a wave-guide, the waves propagate in the jth layer parallel to the plane $x_3 = 0$ in direction of vector $\vec{p}_{\|}$. Due to the boundary conditions given by the matrices $\mathbf{O}_i$, the relation between ω and $\vec{p}_{\|}$ will be changed.

IV. CONCLUSION

The Chern-Simons interaction at $x_3 = l_i$ does not change Snell's law. However, the reflection and transmission coefficients depend on the strengths a_i of these interactions. They lead to a mixing between the parallel and perpendicular components of the electromagnetic waves and they change the relation between frequency and wave vector for waves between two totally reflecting media. Consequently, such interactions will also modify the strength of the Casimir effect. A search for surfaces and layers showing such a behavior is of particular interest.

The electromagnetic polarization effects governed by the defect action $S_\Phi(A)$ are concentrated on the planes $x_3 = \pm l/2$. Outside the planes $x_3 = \pm l/2$, Eq. (3) describe electromagnetic waves with the usual dispersion relations, and the polarization mixing is defined by the boundary condition Eqs. (18)–(20).

In contrast, the Chern-Simons modification of the (3+1)-dimensional Maxwell theory [19] generated by the translation invariant action $S^{(4)}(A,p)$ yields circularly polarized plane waves. There, the Chern-Simons term coupling is given by the four-vector p^μ. The plane waves with four-vector k^μ obey the dispersion relation $(p^\mu p_\mu)^2 + (p^\mu p_\mu)(k^\mu k_\mu) - (k^\mu p_\mu)^2 = 0$. The velocity of the wave propagation depends on its polarization.

The presented results may be verified experimentally. In this way, it is possible to determine the Chern-Simons permittivity $a(\bar{p})$. It has a simple physical meaning. Comparing Eq. (3) with the usual inhomogeneous Maxwell equations in media, we see that $j_{i,k} = -a_i J_i^k$ with $i,k = 1,2$ in Eqs. (19) and (20) can be interpreted as components of currents $\vec{j}_i = (j_{i,1}, j_{i,2}, 0)$ generated by the electric field $\vec{E}_i = (E_{i,1}, E_{i,2}, 0)$ in the planes $x_3 = l_i$. It follows from Eqs. (19) and (20), $j_{i,k} = \sigma_{i,k1}E_{i,1} + \sigma_{i,k2}E_{i,2}$ with $\sigma_{i,kl} = -\sigma_{i,lk}$, $k,l = 1,2$, and $\sigma_{i,12} = -2a_i$.

Thus, $\vec{j}_i$ is the Hall current in the plane $x_3 = l_i$ and the strength of the Chern-Simons interaction $a_i(\bar{p}) = \sigma_{i,21}/2$ defines the Hall conductivity $\sigma_{i,kl}$. In SI units $\sigma_{i,12} = -a_i e^2/(h\alpha)$ with the fine structure constant α and

$h/e^2 = R_K = 25812{,}8\ldots\Omega$. Note also that Eqs. (19) and (20) can be considered as standard boundary conditions connecting the magnetic field with the Hall current on the surfaces between layers. It follows from Eq. (18) that the magnetic induction B_3 in the considered processes generates a jump in the dielectric displacement D_3 on the boundaries of the layers. It can be considered as a manifestation of the magnetoelectric effect.

In the framework of this approach, we have shown that our two-dimensional material body interacting with an electromagnetic field has nonzero Hall conductivity. Using the results of our calculations, it can be found by optical experiments. In this way one can predict also the Casimir force, if as in the case $\epsilon_i = \mu_i = 1$, with $i = 1,2,3$, its theoretical dependence from the parameters of the model is known. It follows from the above presented results that the Casimir force [10] between two planes in vacuum with equal Hall conductivity σ_{12} is attractive, if $|\sigma_{12}| > 2.065$ and repulsive otherwise.

If the Chern-Simons permittivity of the plane depends on the wave vector $\bar{p}$, the result Eq. (2) obtained for constant parameters a_i needs to be corrected. The formalism [10] used for calculation of F_{Cas} presented in Eq. (2) enables one to do it. The problem is reduced to replacing $a_i \to a_i(\bar{p})$ in the photon propagator, which is included in the integral over $\bar{p}$ yielding the Casimir energy. Corrections to the Casimir force obtained in this way could be considered as contributions of van der Waals forces between the planes analogous to those arising between material slabs [7,23].

When a is finite, the Chern-Simons potential breaks the time and space parity. It is the case also for the interaction of photons with (2+1)-dimensional Dirac fields modeling two-dimensional materials [25,26].

In this paper we have considered only the case of inactive media ($\text{Im}a_j = \text{Im}\epsilon_j = \text{Im}\mu_j = 0$). Using complex values of the model parameters and also taking into account the defect contribution of the (3+1)-dimensional Dirac field [24], one can construct within the Symanzik approach in quantum electrodynamics a model for a wide class of quantum macroscopic phenomena in systems with two-dimensional space inhomogeneities. In such models one can investigate the Hall effect, plasmonics, nanophotonics, topological insulators, properties of two-dimensional materials, doping, thin films, and sharp interfaces.

One places high emphasis on these problems, and many important results are obtained in the study of them [26,27]. The comprehensive model, built within the proposed approach and based on fundamental physical principles, seems to be suitable for this research field. We expect that it provides an opportunity to obtain more accurate quantitative results than those that have been achieved to date by use of other theoretical assumptions. An investigation of such models will enable us to understand more deeply the relationship between different nanophysical effects.

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APPENDIX: DETAILED RESULTS AND COMMENTS

We give an obvious form of matrices used in our calculations. They are functions of $\check{e} = (e_1,e_2,e_3)$, $\check{m} = (m_1,m_2,m_3)$ and can be written as

$$\mathbf{M}(\check{e},\check{m}) = \begin{pmatrix} f(\check{e},\check{m}) & g(\check{e},\check{m}) \\ -p_0^2 g(\check{m},\check{e}) & f(\check{m},\check{e}) \end{pmatrix}. \tag{A1}$$

Thus, $\mathbf{M}$ is completely defined by its elements $\{\mathbf{M}\}_{11} = f(\check{e},\check{m})$ and $\{\mathbf{M}\}_{12} = g(\check{e},\check{m})$.

The matrices $\mathbf{T}_j^{\pm\pm}$ and their inverses are given by

$$\{\mathbf{T}_j^{\alpha\beta}\}_{11} = 1 + \alpha\beta\frac{e_j}{e_{j+1}}, \quad \{\mathbf{T}_j^{\alpha\beta}\}_{12} = -\alpha\frac{2a_j}{e_{j+1}p_0},$$

$$\{(\mathbf{T}_j^{\alpha\beta})^{-1}\}_{11} = \frac{1+\alpha\beta\frac{m_j}{m_{j+1}}}{\det(\mathbf{T}_j^{\alpha\beta})}, \quad \{(\mathbf{T}_j^{\alpha\beta})^{-1}\}_{12} = \frac{\alpha\frac{2a_j}{e_{j+1}p_0}}{\det(\mathbf{T}_j^{\alpha\beta})},$$

$$\det(\mathbf{T}_j^{\alpha\beta}) = \frac{4a_j^2 + (e_{j+1}+\alpha\beta e_j)(m_{j+1}+\alpha\beta m_j)}{e_{j+1}m_{j+1}}.$$

The matrices $\mathbf{T}^{\pm\pm}$ satisfy

$$\mathbf{T}^{\pm\pm} = \cos(\kappa_2 l)\mathbf{Z}_1^{\pm\pm} + i\sin(\kappa_2 l)\mathbf{Z}_2^{\pm\pm},$$

$$\{\mathbf{Z}_1^{\alpha\beta}\}_{11} = \frac{\alpha\beta e_1 + e_3}{2e_3}, \quad \{\mathbf{Z}_1^{\alpha\beta}\}_{12} = -\frac{\alpha(a_1+a_2)}{e_3 p_0},$$

$$\{\mathbf{Z}_2^{\alpha\beta}\}_{11} = \frac{4\alpha a_1 a_2 e_2 - (\alpha e_2^2 + \beta e_1 e_3)m_2}{2e_2 m_2 e_3},$$

$$\{\mathbf{Z}_2^{\alpha\beta}\}_{12} = \frac{\alpha\beta a_2 e_2 m_1 + a_1 e_3 m_2}{e_2 m_2 e_3 p_0}, \quad \alpha,\beta = \pm 1.$$

The relation Eqs. (23) and (24) for the amplitudes $\mathbf{c}_t$, $\mathbf{c}_r$ can be written as $\mathbf{c}_r = -e^{i\kappa_1 l}\mathbf{T}_r\mathbf{c}_{\text{in}}$, $\mathbf{c}_t = e^{i(\kappa_3+\kappa_1)l/2}\mathbf{T}_t\mathbf{c}_{\text{in}}$ with

$$\mathbf{T}_r = (\mathbf{T}^{--})^{-1}\mathbf{T}^{-+}, \quad \mathbf{T}_t = \mathbf{T}^{++} - \mathbf{T}^{+-}(\mathbf{T}^{--})^{-1}\mathbf{T}^{-+}.$$

Using the notations

$$\varphi(a,b) = a\cos(\kappa_2 l) + i\,b\sin(\kappa_2 l),$$

$$\psi(a,b,c) = b(a+c)\cos(\kappa_2 l) + i\,(ac+b^2)\sin(\kappa_2 l),$$

$$e_i^\alpha = \varphi(\alpha e_2, e_i), \quad m_i^\beta = \varphi(\beta m_2, m_i), \quad \varphi_i^{\alpha\beta} = e_i^\alpha m_i^\beta,$$

$$e^\alpha = \psi(e_1,\alpha e_2,\alpha e_3), \quad m^\beta = \psi(m_1,\beta m_2,\beta m_3),$$

$$\psi^{\alpha\beta} = e^\alpha m^\beta, \quad \alpha,\beta = \pm 1,$$

one can write the matrices $\mathbf{T}_t$, $\mathbf{T}_r$ in the following form:

$$\{\mathbf{T}_t\}_{11} = \frac{2e_1[e_2 m^+ - 4i\,a_1 a_2 m_2\sin(\kappa_2 l)]}{z},$$

$$\{\mathbf{T}_t\}_{12} = -\frac{4m_1(a_2 m_2 e_1^+ + a_1 e_2 m_3^+)}{p_0 z},$$

$$\{\mathbf{T}_r\}_{11} = \frac{1}{z}\{8a_1a_2e_2m_2 + \psi^{-+} + 4[a_1^2\varphi_3^{++} - a_2^2\varphi_1^{-+} - 4a_1^2a_2^2\sin^2(\kappa_2 l)]\},$$

$$\{\mathbf{T}_r\}_{12} = \frac{4m_1\{a_2e_2m_2 + a_1[\varphi_3^{++} - 4a_2^2\sin^2(\kappa_2 l)]\}}{p_0 z},$$

where

$$z = 4e_2m_2e_3m_3\det\mathbf{T}^{--} = \psi^{++} + 8a_1a_2e_2m_2 + 4[a_2^2\varphi_1^{++} + a_1^2\varphi_3^{++} - 4a_1^2a_2^2\sin^2(\kappa_2 l)].$$

The reflection matrices $\mathbf{R}_i$ defined by Eq. (25) are

$$\mathbf{R}_1 = \mathbf{T}_1^{+-}(\mathbf{T}_1^{--})^{-1}, \quad \mathbf{R}_2 = -(\mathbf{T}_2^{--})^{-1}\mathbf{T}_2^{-+},$$
$$\{\mathbf{R}_i\}_{11} = -\frac{r_i^{-+}}{r_i^{++}}, \quad \{\mathbf{R}_i\}_{12} = -\frac{4a_i m_2}{r_i^{++}},$$
$$r_1^{\alpha\beta} = 4a_1^2 + (e_1 + \alpha e_2)(m_1 + \beta m_2),$$
$$r_2^{\alpha\beta} = 4a_2^2 + (e_3 + \alpha e_2)(m_3 + \beta m_2), \quad \alpha,\beta = \pm 1.$$

Multiplication and the inverse of matrices of the form of Eq. (A1) yield matrices of the same type. Because $\mathbf{g}_2$ does not belong to this class of matrices, this is also the case for the matrices $\mathbf{O}_j = \mathbf{g}_2^{1/2}\mathbf{R}_j\mathbf{g}_2^{-1/2}$:

$$\mathbf{O}_j = -\frac{1}{r_j^{++}}\begin{pmatrix} r_j^{-+} & 4a_j\sqrt{e_2 m_2} \\ -4a_j\sqrt{e_2 m_2} & r_j^{+-} \end{pmatrix}. \tag{A2}$$

The $r^{\pm\pm}$ obey

$$r_j^{+-}r_j^{-+} + 16a_j^2 e_2 m_2 = r_j^{++}r_j^{--}. \tag{A3}$$

If a_1, a_2, e_2, m_2 are real, and e_1, e_3, m_1, m_3 are imaginary, then $(r^{-+})^* = r^{+-}$, $(r^{++})^* = r^{--}$, and it follows from Eqs. (A2) and (A3) that the matrices $\mathbf{O}_1, \mathbf{O}_2, \mathbf{O} = \mathbf{O}_2\mathbf{O}_1$ are unitary, and

$$\mathbf{O} = \frac{1}{R}\begin{pmatrix} P & Q \\ -Q^* & P^* \end{pmatrix},$$

where

$$R = r_1^{++}r_2^{++}, \quad P = r_1^{-+}r_2^{-+} - 16a_1a_2e_2m_2,$$
$$Q = 4\sqrt{e_2m_2}(a_1r_2^{-+} + a_2r_1^{+-}), \quad PP^* + QQ^* = RR^*.$$

The eigenvalues $\lambda_{1,2}$ of the matrix $\mathbf{O}$ read

$$\lambda_{1,2} = \frac{-P - P^* \pm \sqrt{(P-P^*)^2 - 4QQ^*}}{2R} = e^{i(\zeta+\eta_{1,2})},$$
$$\tan(\zeta) = -\frac{\mathrm{Im}R}{\mathrm{Re}R}, \quad \tan(\eta_{1,2}) = \mp\frac{\sqrt{(\mathrm{Im}P)^2 + |Q|^2}}{\mathrm{Re}P}.$$

They coincide for $\mathrm{Im}P = 0$, $Q = 0$. In this case, $\eta_{1,2} = 0$,

$$r_2^{-+} = -\frac{a_2}{a_1}r_1^{+-}, \quad P = -\frac{a_2}{a_1}r_1^{++}r_1^{--} = P^*.$$

The boundary condition Eqs. (18)–(20) can be proved directly from Eqs. (6) and (7). Using the relations $\vec{D} = \varepsilon\vec{E}$, $\vec{B} = \mu\vec{H}$, $\vec{E} = -\partial_0\vec{A}$, $\vec{B} = \vec{\partial}\times\vec{A}$, $\mathfrak{p}^2 + \kappa^2 = p_0^2\varepsilon\mu$ and notations $\varepsilon/\kappa = e$, $\kappa/\mu = m$, we obtain $D_3 = -p_0 e\tilde{\rho}$,

$$H_1 = \frac{p_1 m\tilde{\tau} - p_2 e\tilde{\rho}p_0^2}{\mathfrak{p}^2}, \quad H_2 = \frac{p_1 e\tilde{\rho}p_0^2 + p_2 m\tilde{\tau}}{\mathfrak{p}^2}.$$

It follows from $J^\nu = \epsilon^{3\nu\sigma\rho}F_{\sigma\rho}$ that $J^0 = 2\tau$,

$$J^1 = 2\frac{p_0(p_1\tau - p_2\rho)}{\mathfrak{p}^2}, \quad J^2 = 2\frac{p_0(p_1\rho + p_2\tau)}{\mathfrak{p}^2}.$$

Thus, in virtue of Eq. (17), the equalities Eqs. (18)–(20) are fulfilled.

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