Econophysics of adaptive power markets: When a market does not dampen fluctuations but amplifies them

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The average economic agent is often used to model the dynamics of simple markets, based on the assumption that the dynamics of a system of many agents can be averaged over in time and space. A popular idea that is based on this seemingly intuitive notion is to dampen electric power fluctuations from fluctuating sources (as, e.g., wind or solar) via a market mechanism, namely by variable power prices that adapt demand to supply. The standard model of an average economic agent predicts that fluctuations are reduced by such an adaptive pricing mechanism. However, the underlying assumption that the actions of all agents average out on the time axis is not always true in a market of many agents. We numerically study an econophysics agent model of an adaptive power market that does not assume averaging *a priori*. We find that when agents are exposed to source noise via correlated price fluctuations (as adaptive pricing schemes suggest), the market may amplify those fluctuations. In particular, small price changes may translate to large load fluctuations through catastrophic consumer synchronization. As a result, an adaptive power market may cause the opposite effect than intended: Power demand fluctuations are not dampened but amplified instead.

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I. INTRODUCTION

Modern power markets face the challenge to satisfy a continuous demand for electricity, despite fluctuating energy sources as, e.g., solar or wind [1–6]. It has been proposed to reduce fluctuations in power markets via time-varying pricing schemes, in order to stimulate the shift of energy consuming activities with flexible execution times as, e.g., washing or heating, to times with excess supply [7–10]. From the perspective of a standard economic theory this is a simple picture: A specific value of the price leads to a predictable total demand. Consequently, there is an equilibrium price, where demand and supply are balanced [11]. As a result, one would expect that part of the demand thereby is shifted to times with lower prices [7]. Thus the market would act as a low pass filter for power fluctuations, an elegant idea at first sight, indeed.

However, real markets often behave differently than the single representative agent of standard economic theory [12,13], most prominently illustrated by crashes of stock markets and similar phenomena resulting from interactions among a large number of agents [14–18]. Even in markets where agents do not interact directly, they may exhibit coordinated behavior. For example, the actions of consumers may self-organize on the time axis, with catastrophic synchronization as a possible result. In that case, averaging over the dynamics of many agents over time is not appropriate because the central limit theorem's assumption of independent agents is not given. The market, instead of acting like a low pass filter that dampens fluctuations, turns into a generator for catastrophic time series.

In fact, problems with the central limit theorem in dynamical systems with many degrees of freedom are well known from different fields and often are related to time series that exhibit large fluctuations. Such phenomena have been discussed, for example, in the contexts of earthquakes, rice piles, stock markets, solar flares, and mass extinctions [19–21]. These systems have in common that fluctuations with broad or power-law size distributions occur that do not need a full mechanism of self-organized criticality (SOC) at work. Instead, coherent stochastic noise acting on a system with many agents may suffice to explain such power-law-distributed fluctuations [19]. Agents can react to the coherent noise in a way that causes their actions to synchronize at rare events. As a result, power-law-distributed event sizes appear even for narrow (and even Gaussian) distributions of the coherent noise [21].

In this paper we study whether this mechanism may be at work in markets, or more specifically, in power markets. Collective behavior of agents in a market can be treated with agent-based models allowing for individual behavior of agents. Agent-based models constructed on simple rules of individual behavior in markets have been shown to exhibit many features of real markets [15–18,22]. We here study one of the simplest possible agent-based models for an adaptive power market.

Our toy model consists of independent agents reacting to a predefined global price time series. Their rare consumption events set in once the actual price is below an individual highest acceptable price. The highest acceptable prices of each agent are updated with a stochastic process to account for saturation after consumption and growing need for electricity in times without consumption. This is to model rare consumption events with flexible execution time, while the base demand connected to time-fixed activities is ignored in this study. We analyze the effect of demand synchronization at low prices. As a result, the total demand can exceed the average demand by several orders of magnitude. To prove the robustness of this behavior, we analyze the demand distribution and the demand curve (demand over price [11]) for different price time series with and without correlation. We find the behavior of our artificial market to be in sharp contrast to standard economic theory. A sensitive demand curve and saturation effects question the application of equilibrium prices.

II. MODEL DESCRIPTION

Let us now define the power market agent model. We analyze an artificial market consisting of one power provider

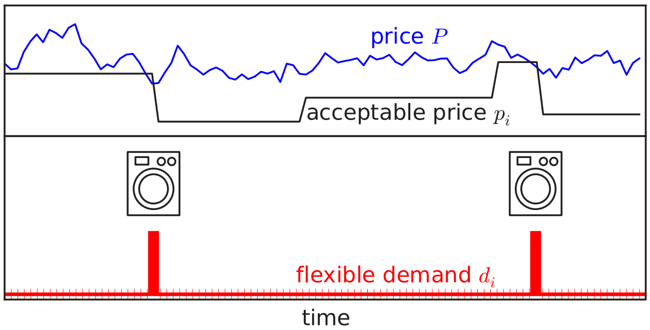

FIG. 1. (Color online) Sketch of the model dynamics for an individual agent. Top: Price time series (blue) together with the price acceptance of the agent (black). Bottom: Demand of the agent. With increasing time we see a consumption event with lowering the highest acceptable price (saturation), followed by two increments (growing need) and another consumption.

and N power consumers. The power provider has to deal with time-dependent supply $S(t)$ (e.g., due to weather dependent electricity production). In addition, the power provider has to guarantee balance of supply and demand, as mismatch may cause blackouts. Assuming a smoothly decreasing demand for increasing prices $D(P)$, the power provider could control demand using time-dependent prices $P(t) = D^{-1}[S(t)]$ with the inverse function D^{-1} of $D(P)$, implicating $D[P(t)] = S(t)$.

Therefore, in every (integer) time step t the power provider sets a price $P(t)$ (with time average $\bar{P} = 1$) visible to all consumers. The individual demand of an agent i is defined as

$$d_i(t) = \begin{cases} 1 & \text{if } P(t) \leqslant p_i(t), \\ 0 & \text{if } P(t) > p_i(t), \end{cases} \tag{1}$$

with its individual highest acceptable price $p_i(t)$, as illustrated in Fig. 1. It is initialized with $p_i(t=0) = \text{rand}(0,1)$ and evolves according to

$$p_i(t+1) = \begin{cases} \text{rand}(0,p_i(t)), & \text{if } P(t) \leqslant p_i(t), \\ \text{rand}(p_i(t),1), & \text{else with prob. } f, \\ p_i(t), & \text{else.} \end{cases} \tag{2}$$

The term $\text{rand}(a,b)$ denotes a random number uniformly drawn from the half-open interval $[a,b)$. The first case $[P(t) \leqslant p_i(t)]$ corresponds to power consumption at time t. As a consequence, the acceptable price will then also be lowered to represent saturation. The second case, rare increases of the highest acceptable price p_i with probability $f \ll 1$, is to model the increasing need for power-consumption with time. This stochastic evolution of $p_i(t)$ is inspired by the coherent noise model by Newman and Sneppen [19], where a resilience threshold toward catastrophic events is evolved in time. In contrast to Ref. [19], where all replacements are chosen out of the whole interval between zero and one, here the values p_i may not increase for the first case in Eq. (2) or decrease for the second case.

The total demand $D(t) = \sum_{i=1}^{N} d_i(t)$ is satisfied by the power provider. We avoid including an additional contribution of time-fixed activities $D_{\text{base}}(t)$ into this model, since this part is not the focus of the present study and would not change the overall dynamics. To analyze the capabilities of the power provider to shape demand time series $D(t)$, we use different types of noisy time series $P(t)$. We take independent identically distributed prices out of a Gaussian distribution with mean $\bar{P} = 1$ and different standard deviations σ_P. Additionally, to consider correlations over time (as they are known for common price time series and for weather phenomena), we use a Langevin equation as a particularly simple realization,

$$P(t+1) - P(t) = -\upsilon_0[P(t) - \bar{P}] + \sigma_0 \xi(t), \tag{3}$$

with an independent normally distributed random variable $\xi(t)$ (the blue solid line in Fig. 3 shows the Gaussian density for such a time series).

III. SYNCHRONIZATION AT LOW PRICES

Figure 2 on top shows a section of a price time series according to Eq. (3) with $\upsilon_0 = 0.2$ and $\sigma_0 = 0.1$. In the bottom panel, we see the according demand divided by the average demand. The average demand $\bar{D} = \frac{1}{T+1} \sum_{t=0}^{T} D(t)$ for the system with $f = 10^{-3}$, $N = 10^6$ agents, and a simulation time of $T = 10^7$ (plus 10^3 initial time steps for reaching a stationary state) was calculated to be $\bar{D} = 979$. Therefore, a single agent demands on average $\bar{d} = \bar{D}/N = 9.79 \times 10^{-4} \approx f$. The parameter f indicates the rareness of consumption. The time series of $D(t)/\bar{D}$ shows demand peaks more than two orders of magnitude above the average demand dominating the whole time series. This is due to synchronization: At low prices, many agents demand at the same time. As a result, the prices, fluctuating in a narrow range, cause a broadly distributed demand time series with extreme events.

In Fig. 3 we see the density of highest acceptable prices $p_i(t)$ averaged over time and agents (black circles),

$$\rho_p(P) = \sum_{i=1}^{N} \sum_{t=0}^{T} \Delta_{p_i(t),P}/N(T+1), \tag{4}$$

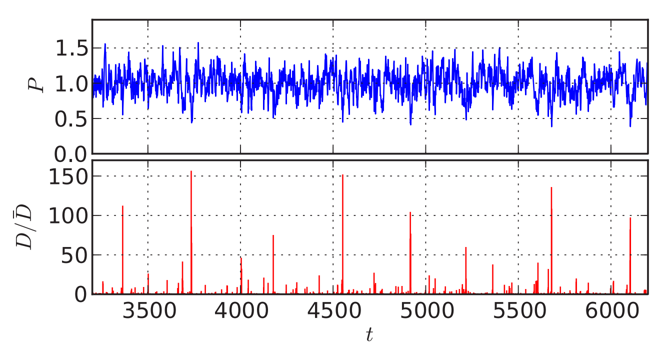

FIG. 2. (Color online) The Gaussian distributed-price time series $P(t)$ shown on the top [generated with Eq. (3) and values $\upsilon_0 = 0.2$, $\sigma_0 = 0.1$] translates into a broadly distributed total-demand time series $D(t)$ (shown on the bottom, divided by average demand $\bar{D}$). Simulations were performed for $N = 10^6$ agents with rare consumption parameter $f = 10^{-3}$. At low prices, consumers execute their rare consumption activities in a synchronized fashion, leading to total demand D far above the average demand $\bar{D}$.

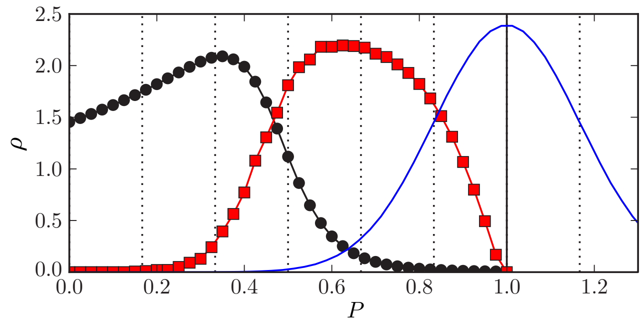

FIG. 3. (Color online) Density of prices for the time series as described in the caption of Fig. 2 (blue solid line). The average price is indicated with a vertical line together with multiples of one standard deviation (dotted lines). The density of highest acceptable prices $p_i(t)$ [Eq. (4), black circles] shows a concentration at prices far below the average price. The density of load bought at certain prices [Eq. (5), red squares] shows a maximum at rare price events more than two standard deviations below the average price. Simulation results are shown for $N = 10^6$ and $f = 10^{-3}$ (as in Fig. 2) and $T = 10^7$ time steps.

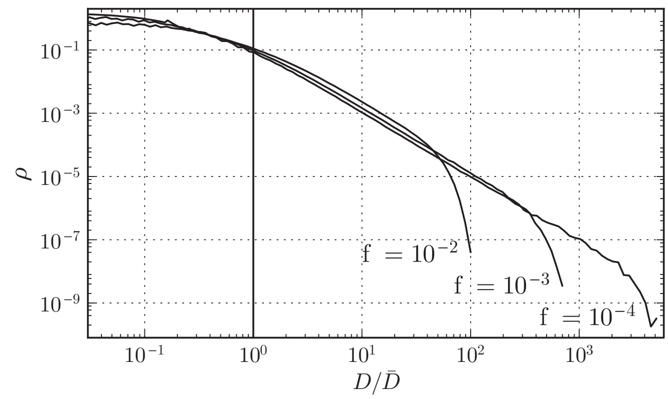

FIG. 4. The demand density $\rho(D/\bar{D})$ is shown for independent Gaussian distributed prices ($\sigma_P = 1/6$), with $N = 10^6$, $T = 10^7$, and different values of f as indicated. Even for a parameter $f = 10^{-2}$, where agents consume rather often, highest total demand is far above the average. With $f = 10^{-3}$ and $f = 10^{-4}$, we see convergence toward a power law with exponent $\alpha = 2$.

the density of total loads consumed at certain prices (red squares),

$$\rho_D(P) = \sum_{t=0}^{T} D(t)\Delta_{P(t),P}/\bar{D}, \quad (5)$$

and the price distribution (blue solid line). The binning of data values p [$p_i(t)$ or $P(t)$, respectively] to intervals between $P = 0, \frac{1}{40}, \frac{2}{40}, \frac{3}{40}, \ldots, 1$ is realized with $\Delta_{p,P}$: $\Delta_{p,P} = 1$ if $P < p \leqslant P + \frac{1}{40}$ and $\Delta_{p,P} = 0$ if otherwise. The average price $\bar{P}$ is indicated with a black vertical line, and multiples of one standard deviation of the price distribution are indicated with dotted vertical lines. We observe that only a small fraction of the demands are executed within one standard deviation of the average price, 35% of the price events only lead to 18% of the demand. This part is due to agents who need to consume power very soon. The average price for consumers $\sum_t P(t)D(t)/\sum_t D(t) = 0.65$ is much lower. Due to synchronization effects, rare events below $\bar{P} - 3\sigma_P = 0.5$ constituting only 0.14% of the time series lead to a part of 16% of the total demand. In conclusion, the agents indeed consume at low costs and their strategy is beneficial. Additionally, the strategy represents individual needs, implemented by random moves of the individual highest acceptable prices.

Let us finally discuss how the rareness of consumption influences extreme synchronization of demand. In Fig. 4, the distribution of demand D is shown for independent Gaussian distributed prices with $\sigma_P = 1/6$ and different rareness of consumption ($f = 10^{-2}$, $f = 10^{-3}$, and $f = 10^{-4}$). All simulations in this study are done with $N = 10^6$ and $T = 10^7$. Even in the case $f = 10^{-2}$, where consumers buy on average in one of one hundred time steps, maximum demands are almost two orders of magnitude larger than the average demand. For rarer consumption (smaller values of f), the distribution of loads clearly gets the shape of a truncated power law with exponent $\alpha = 2$, as expected from Ref. [19]. The results are qualitatively the same, if the parameter f is drawn individually for every consumer [replacing f by individual f_i in Eq. (2)]. For values f_i taken out of a uniform distribution in the half-open interval $(0,10^{-3}]$ we found the same power law with exponent $\alpha = 2$ and slightly increased cutoff compared to constant $f = 10^{-3}$.

IV. ROBUST OCCURRENCE OF HIGH DEMAND

With Fig. 5 we can see how our artificial power market reacts to price time series of different types, as shown on the top. On the bottom, the according load densities are shown (shifted for better visibility) for a system with $f = 10^{-3}$. The results emphasize the robustness of synchronization in our artificial market. On top the result for Gaussian distributed prices with $\sigma_P = 1/6$ is shown (A). Below, the same type of price time series with $\sigma_P = 1/20$ is used with similar results (B). We found that changing the standard deviation of the prices σ_P leads to the same dynamics, only with buying events at different typical prices. From a study on coherent noise models [21] we can conclude that using other distributions for the prices (exponential, power laws) should not change the results considerably. The third case is a correlated time series generated with Eq. (3) ($v_0 = 0.2$ and $\sigma_0 = 0.1$) (C). The same type of broadly distributed demand emerges. We also tested a real-price time series by using the Dow Jones index (detrended daily closure values 1900–2007, accordingly the model was evolved only for about $T = 29\,000$ time steps), with similar results (D). In conclusion, this means that for our artificial market the synchronization of consumers occurs for very different price time series and can hardly be avoided. This contrasts to the picture of a controllable market of standard economy, where $D(t)$ would converge to a suitable expected value $D[P(t)]$ for a large number of consumers N. This would hold with the central limit theorem, if the individual demands $d_i(t)$ would be statistically independent, as assumed in models of power markets inspired by standard economy [23]. However, we studied a market model where in contrary the individual demands $d_i(t)$ are strongly correlated. Generalized central limit theorems for random variables with correlations teach us about large fluctuations being present

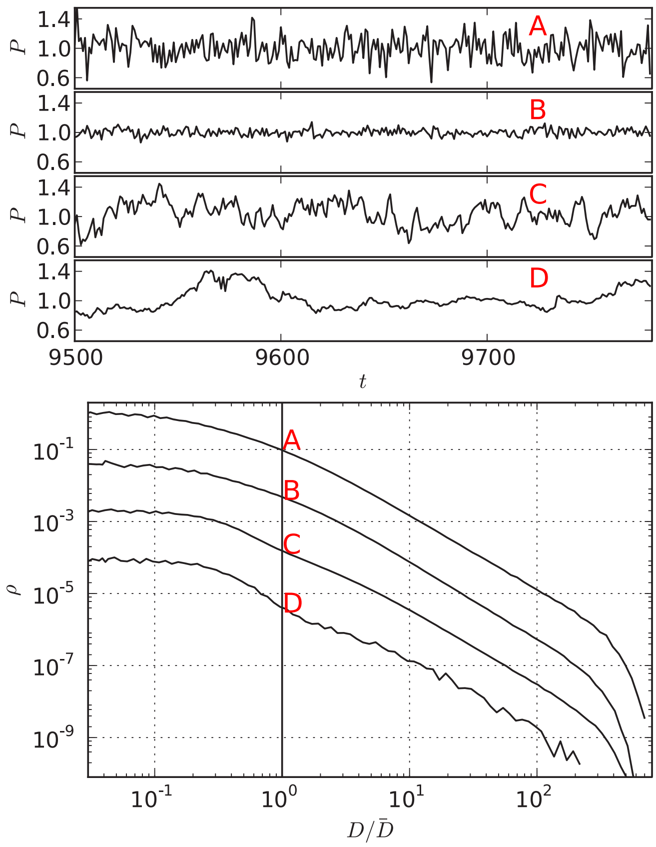

FIG. 5. (Color online) The price time series shown on the top have widely differing features. We used uncorrelated Gaussian distributed prices with $\sigma_P = 1/6$ (A, as used in Fig. 4) and $\sigma_P = 1/20$ (B), as well as a time series with correlations over short times (C, as used in Fig. 2) and the Dow Jones (D, locally detrended). The according demand density for these cases is shown on the bottom (shifted for better visibility. $N = 10^6$, $f = 10^{-3}$, $T = 29\,000$ in case D and $T = 10^7$ else). Synchronization of demand occurs robustly.

in different time scales [24]. In our model, fluctuations appear due to rare consumption at low prices, even without elaborated strategies or interactions among agents as studied in financial market models [14–18,25].

Finally, let us discuss the demand curve that is the basic tool for the power provider to set prices. In Fig. 6 we see a binning of events according to the rescaled demand $D/\bar{D}$ and price P. The counts of events are shown with color values in logarithmic scale. On the left we see results for uncorrelated prices and $\sigma_P = 1/6$ (case A in Fig. 5). Due to the distribution of prices P, events with low prices are generally rare, but if they occur, they lead to high demand. The average demand for a certain price interval according to this binning is indicated with a dashed line. This is the so-called demand curve frequently used in standard economics to calculate equilibrium prices. The demand spans more than three orders of magnitude within about four standard deviations of the price (we checked that the same holds for the smaller value $\sigma_P = 1/20$). Smooth changes of the price lead to drastic changes of the demand. This is in sharp contrast to standard economics and limits the feasibility of equilibrium prices. Additionally, the demand values span more than an order of magnitude for many price

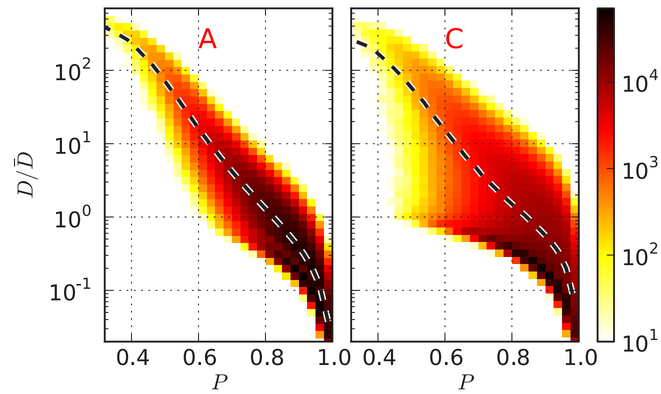

FIG. 6. (Color online) Binning of events by price intervals and demand intervals ($N = 10^6$, $f = 10^{-3}$, $T = 10^7$). Counts of events are shown with color values in logarithmic scale for Gaussian-distributed uncorrelated prices (left, case A in Fig. 5) and correlated prices (right, case C in Fig. 5). The dashed lines show the average demand for price intervals (the so-called demand curve) with an almost exponential dependence in both cases. The demand curve is a crucial tool for guessing equilibrium prices; therefore, an exponential dependence together with large fluctuations indicates difficulties for the power provider.

values. This is due to saturation effects (only few agents buy at a low price, if a lower price recently occurred). In the right panel of Fig. 6, results for the correlated price time series are shown. The demand curve is not changed considerably, while the distribution of demands for certain prices is broadened. Due to saturation effects, consecutive low prices lead to shrinking loads.

V. SUMMARY AND OUTLOOK

We studied a simple agent-based model of an electricity market with variable prices and studied collective effects when consumers aim for lowest prices. In particular, we consider consumption with time-flexible execution as, e.g., washing or heating. Time-variable consumption is modeled with a stochastic process for individual highest acceptable prices. As a central quantity, the total demand emerging in our artificial market has been analyzed.

Our main observation is that the rare consumption events of the consumers in the market tend to strongly synchronize at low prices. This leads to peak demands exceeding the average demand by several orders of magnitude. These frequent extreme events account for a considerable part of the average demand over time. We find that high demands occur robustly for different types of price time series, as long as the pricing noise hits the consumers coherently. We find power-law-distributed demands with large extreme events, both, for uncorrelated price time series as well as for correlated time series. The catastrophic behavior of the system appears to be hardly to prevent. In earlier power market models [7,10,23,26], correlations among consumers have usually not been considered. Indeed, a classical model with a single utility-maximizing agent, only, demonstrates how to set prices such that demand is shifted to desired times [7]. In some agent-based models with detailed description of power distribution [23], the time-dependent demand of individual users is modeled as independent; therefore, the total demand converges to a predefined shape. Our results complement a different line of research emphasizing increased fluctuations in complex power markets. In Ref. [26], the role of feedback loops between consumers, suppliers, and distribution was emphasized, and in Ref. [10], it was shown that the feedback between suppliers and consumers can increase market fluctuations as well.

Finally, we question the concept of equilibrium prices in the context of our artificial market. As the system shows an exponential growth of demand when prices drop, equilibrium prices can hardly establish. Demands take on a wide range of values, even at the same price.

While these are results from a statistical physics-inspired toy model for an electricity power market with fluctuating energy sources and an adaptive pricing scheme, they may provide a lesson for real markets as well. In particular, they seem to indicate that the, at first sight, brilliant idea to use market mechanisms as a low-pass filter for fluctuating electricity sources (e.g., by communicating price information to consumers through the so-called smart meters) may not only break down under certain conditions. More importantly, they also can lead to catastrophic consequences when a basic prerequisite fails: Breakdown of the central limit theorem when consumers do not act statistically independently.

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