Performance of quantum Otto refrigerators with squeezing

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The performance of a quantum Otto refrigerator coupled to a squeezed cold reservoir has been evaluated using the χ figure of merit. We have shown that squeezing can enhance the coefficient of performance (COP) dramatically, surpassing the Carnot COP defined by the initial temperatures of the heat baths. Furthermore, when the squeezing parameter approaches its maximum value, the work input vanishes while the cooling rate remains finite, in apparent contravention of the second law of thermodynamics. To explain this phenomenon, we have shown that squeezing renders the thermal bath into a nonequilibrium state and the temperature of the bath becomes frequency dependent. Thereby, a correlation to the Carnot COP has been deduced. The results reveal that the COP under the maximum χ figure of merit is of the Curzon-Ahlborn style that cannot surpass the actual Carnot COP, and is thus consistent with the second law of thermodynamics.

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I. INTRODUCTION

The optimization of real thermodynamic cycles has attracted increasing attention because of fuel depletion and the need to save energy. Refrigerators are used widely in our everyday lives, and they operate between two heat reservoirs at temperatures T_1 and T_2 ($T_1 > T_2$). The coefficient of performance (COP) for a traditional refrigerator is constrained by the Carnot limit, $\varepsilon_C = T_2/(T_1 - T_2)$, according to the second law of thermodynamics [1]. However, in attaining the Carnot COP, the cooling rate decreases because of the infinite cycle duration, which is unrealistic for actual applications. Pioneered by Curzon and Ahlborn [2], who considered the cycle duration, many methods and models have been developed to study practical heat devices, such as the endoreversible model, low-dissipation model, and the irreversible model based on the Onsager relation [3–8], and they have provided useful results.

The maximum power output is often adopted as the main criterion for optimizing real heat engines. For refrigerators, however, the minimum power input is not an appropriate optimization criterion [9], and much effort has been devoted to optimizing refrigerators under different figures of merit. Jiménez de Cisneros *et al.* [10] studied the COP at the maximum COP figure of merit through the linear irreversible model. By maximizing the per-unit-time COP of endorevesible refrigerators, Velasco *et al.* [11] obtained the upper bound of COP, $\varepsilon_{CA} = \sqrt{\varepsilon_C + 1} - 1$, i.e., the Curzon-Ahlborn (CA) coefficient of performance. Yan and Chen [12] conducted optimization with a target function $\varepsilon \dot{Q}_c$, the χ figure of merit for refrigerators proposed by de Tomás *et al.* [13], with $\dot{Q}_c$ being the cooling load rate of the refrigerators and ε the COP. The χ figure of merit, defined as $\chi = zQ_{in}/t_{cycle}$, becomes the maximum power and $\varepsilon \dot{Q}_c$ figures of merit for heat engines and refrigerators, respectively. Here z is the converter efficiency (efficiency for heat engines and COP for refrigerators), Q_{in} is the heat absorbed by the system, and t_{cycle} denotes the time duration for a cycle. Taking χ as the target function in a low-dissipation model, Wang *et al.* [9] proposed that the COP at maximum χ was bounded between 0 and $(\sqrt{9 + 8\varepsilon_C} - 3)/2$. Under the conditions of symmetric dissipations, the CA coefficient of performance may be retrieved. It has also been obtained in refrigerators with nonisothermal processes [14].

Quantum thermodynamics offers a new way to study microscopic heat devices. Based on the quantum analogs of the classical thermodynamic cycles, many quantum thermodynamic cycles, such as the quantum Carnot cycle, Otto cycle, and the Brayton cycles, have been constructed [15,16] and are often studied within spin or coupled systems, harmonic oscillator systems, and ideal quantum gases [17–19]. The literature on quantum Otto cycles, especially for heat engines and refrigerators, is extensive [20–24]. In the field of thermodynamics in small systems, an important question is the validity of the thermodynamic laws in a system involving only a few particles. Many investigations have been performed to reexamine the validity of the laws and principles of thermodynamics in small systems. Recently, Roßnagel *et al.* [25] studied the quantum Otto engine coupled to a high-temperature squeezed thermal reservoir and claimed an efficiency at the maximum power output exceeding the Carnot efficiency. Furthermore, they also proposed a concept to realize it experimentally. Quan [26] studied the maximum efficiency of ideal heat engines in a small system and proposed a working-substance-dependent correction to the Carnot efficiency at the nanoscale. Whether the efficiency of a heat engine can be larger than the standard Carnot efficiency in a quantum system is still under discussion [27–29].

Originating in quantum optics, squeezing has also been studied in quantum thermodynamics [30–32]. However, the use of squeezed thermal baths in quantum thermodynamics has been largely unexplored [25]. In this work, the performance of a quantum Otto refrigeration cycle coupled to a low-temperature squeezed thermal reservoir based on a time-dependent harmonic oscillator has been investigated under the χ figure of merit [25], while still treating the high-temperature reservoir as purely thermal. The model of quantum Otto refrigerators with squeezing is described in Sec. II. In Sec. III, we show that with the absence of squeezing, the COP at the maximum χ figure of merit is the CA coefficient of performance and squeezing enhances the COP

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dramatically, surpassing the Carnot COP defined by the initial temperatures of the heat baths for large squeezing parameters. Furthermore, we show that squeezing renders the thermal bath into a nonequilibrium state, and in the presence of squeezing the temperature of the bath becomes frequency dependent. Therefore, a Carnot COP can no longer be defined by the initial temperatures of the heat baths. The actual Carnot COP, however, may be defined and has been deduced. The results show that the COP under the maximum χ figure of merit is also of the CA style, and has been shown not to exceed the actual Carnot COP.

II. QUANTUM OTTO REFRIGERATOR WITH SQUEEZED RESERVOIR

The quantum Otto refrigerator is the reversed form of the heat engine cycle, shown in Fig. 1; it consists of four consecutive processes like the traditional Otto cycle [15,18,25,33,34]. In this paper, the working medium is a single harmonic oscillator whose frequency is time dependent and varies between ω_2 and $\omega_1(\omega_2 < \omega_1)$. The isochoric processes can be treated as constant frequency processes. The oscillator is coupled to two heat baths at temperatures T_2 and $T_1(T_1 > T_2)$. Here we introduce the two inverse temperatures $\beta_1 = 1/K_B T_1$ and $\beta_2 = 1/K_B T_2$, with K_B the Boltzmann constant. Therefore the Carnot coefficient of performance may be written as $\varepsilon_C = 1/(\beta_2/\beta_1 - 1)$. As depicted in Fig. 1, the cycle starts in a thermal state A, at frequency ω_1, and temperature T_1. The average energy, $\langle H \rangle_A = \hbar\omega_1 \langle n(\beta_1) \rangle$, which is calculated based on the quantum number distribution, reads

$$\langle H \rangle_A = \frac{\hbar\omega_1}{2} \coth\left(\frac{\hbar\omega_1\beta_1}{2}\right), \tag{1}$$

where $\hbar$ represents Planck's constant. In the process A to B, the frequency decreases from ω_1 to ω_2. The duration of this process is τ_{AB}. It is an isentropic expansion process, whose transformation is unitary for an isolated system and whose von Neumann entropy is constant. The mean energy at point B can be calculated by solving the Schrodinger equation for the driven quantum oscillator, which is given by [35–37]

$$\langle H \rangle_B = \frac{\hbar\omega_2}{2} \coth\left(\frac{\hbar\omega_1\beta_1}{2}\right) Q_2^*, \tag{2}$$

where Q_2^* is a dimensionless adiabaticity parameter in the process, characterizing the speed of the transformation [37]. It equals unity for an adiabatic process that is much slower than the typical time scales of the system and increases with the degree of nonadiabaticity. $Q_2^* = 1$ for adiabatic and $Q_2^* > 1$ for nonadiabatic compression or expansion.

In the process B to C, the system is coupled to a squeezed thermal reservoir at temperature T_2, where the squeezing parameter is denoted as r. The system is then relaxed to a nondisplaced squeezed thermal state with mean phonon number $\langle n(\beta_2,r) \rangle = \langle n \rangle + (2\langle n \rangle + 1)\sinh^2 r$ [25,30], where $\langle n \rangle = [\exp(\hbar\omega_2\beta_2) - 1]^{-1}$ is the thermal occupation number. We assume the duration of this interaction to be much shorter than that of the isentropic process, and thus the frequency remains constant. The time duration of this process is denoted as τ_{BC}. The mean energy at point C is

$$\langle H \rangle_C = \frac{\hbar\omega_2}{2} \coth\left(\frac{\hbar\omega_2\beta_2}{2}\right) \alpha, \tag{3}$$

where $\alpha = \langle n(\beta_2,r) \rangle / \langle n \rangle$, reflecting the change of the thermal occupation number due to the squeezing. Similar to the analysis in [25], the temperature of the cold bath is assumed to be unaffected by the squeezing. After the isentropic compression process C to D, the frequency is brought back to its initial value ω_1 and the mean energy at point D reads

$$\langle H \rangle_D = \frac{\hbar\omega_1}{2} \coth\left(\frac{\hbar\omega_2\beta_2}{2}\right) Q_1^* \alpha \tag{4}$$

where Q_1^* is also a dimensionless adiabaticity parameter in the process, characterizing the speed of the transformation in that process of duration τ_{CD}. $Q_1^* = 1$ for adiabatic and $Q_1^* > 1$ for nonadiabatic compression or expansion.

In the final isochoric process D to A, the system is coupled to the hot thermal bath with no squeezing applied. The duration of this process is denoted by τ_{DA}. Because of the stochastic nature of this process, the squeezed state is thermalized in this process. Therefore the heat absorbed from the cold-squeezed reservoir and that released to the hot reservoir are, respectively, given by

$$Q_h = \langle H \rangle_D - \langle H \rangle_A = \frac{\hbar\omega_1}{2} \coth\left(\frac{\hbar\omega_2\beta_2}{2}\right) Q_1^* \alpha - \frac{\hbar\omega_1}{2} \coth\left(\frac{\hbar\omega_1\beta_1}{2}\right), \tag{5}$$

$$Q_c = \langle H \rangle_C - \langle H \rangle_B = \frac{\hbar\omega_2}{2} \coth\left(\frac{\hbar\omega_2\beta_2}{2}\right) \alpha - \frac{\hbar\omega_2}{2} \coth\left(\frac{\hbar\omega_1\beta_1}{2}\right) Q_2^*. \tag{6}$$

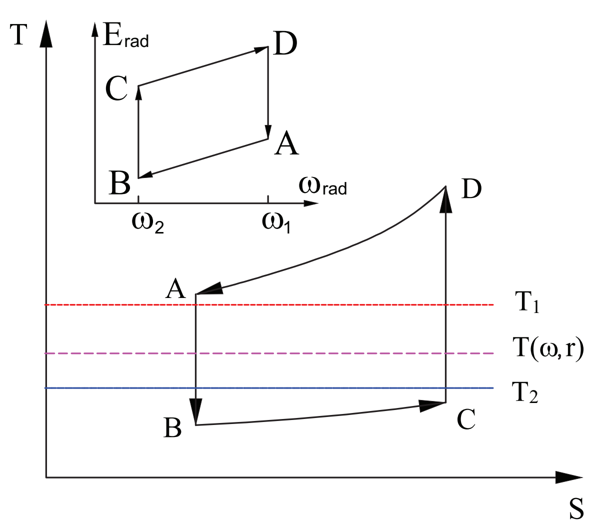

FIG. 1. (Color online) T-S diagram of the quantum Otto cycle. The insert shows the corresponding energy-frequency diagram. Squeezing is applied to the cold reservoir in the process B to C.

In Ref. [25], a realistic proposal for a quantum Otto heat engine was conceived, which consists of a single ion confined in a linear Paul trap and coupled to laser reservoirs. The radio frequency electrodes that create the confining potential are title towards the trap axis. Due to the geometry the axial frequency is fixed, while the radio frequency is a function of the axial position. The bath acts on the radial modes only. A change in the variance of the radial state of the ion leads to a displacement in axial direction, which corresponds to the piston in a heat engine. However, as the radial trap frequency is much higher than the axial one, work provided for the radial mode cannot be transferred to the axial mode. This means that the squeezing process in the radial direction does not perform any work resulting in axial motion. The displacement in the axial direction is only because of a large change in the variance of the radial motional state. The phase of the squeezed state is scrambled each time, and without having any information about this phase, it is impossible to extract work in either direction. Following Ref. [25], the energy required to maintain the squeezed state has not been considered. The work input may accordingly be calculated as $W = Q_h - Q_c$, along with the COP $\varepsilon = Q_c/W$. The refrigeration rate is $R = W/\tau_{\text{cycle}}$, where $\tau_{\text{cycle}} = \tau_{AB} + \tau_{BC} + \tau_{CD} + \tau_{DA}$ denotes the time corresponding to a complete cycle through the states. According to the aforementioned literature, $Q^*_{1,2} \geqslant 1$ [35], where the equality holds when the isotropic processes are adiabatic. In addition, the maximum cooling and COP occur only in situations where $Q^*_1 = Q^*_2 = 1$. For simplicity, we assume that the cycle duration is fixed and the isentropic processes are adiabatic. Now, consider the high-temperature limit $\hbar\omega_i\beta_i \ll 1$. Equations (5) and (6) are then reduced to

$$Q_h = \langle H\rangle_D - \langle H\rangle_A = \frac{\omega_1}{\omega_2\beta_2}(1+2\sinh^2 r) - \frac{1}{\beta_1}, \tag{7}$$

$$Q_c = \langle H\rangle_C - \langle H\rangle_B = \frac{1}{\beta_2}(1+2\sinh^2 r) - \frac{\omega_2}{\omega_1\beta_1}. \tag{8}$$

Therefore the work input may be rewritten as

$$W = \frac{\omega_1}{\omega_2\beta_2}(1+2\sinh^2 r) - \frac{1}{\beta_1} - \frac{1}{\beta_2}(1+2\sinh^2 r) + \frac{\omega_2}{\omega_1\beta_1}. \tag{9}$$

For a refrigerator, the heat absorbed and the work input must be positive, i.e., $Q_c > 0$, $W > 0$. Based on Eqs. (8) and (9), we thus have

$$\frac{\omega_2}{\omega_1} < \frac{\beta_1}{\beta_2}(1+2\sinh^2 r), \quad \text{where} \quad r \leqslant \operatorname{arcsin}h\left(\sqrt{\frac{1}{2\varepsilon_C}}\right). \tag{10}$$

III. PERFORMANCE UNDER THE χ CRITERION

A. Thermal bath with squeezing: Assumption of equilibrium state

The χ criterion can be regarded as a relatively appropriate criterion for optimizing refrigerators [13]. This figure of merit takes both the COP and the cooling rate into consideration. Thus we have

$$\chi = \frac{\frac{1}{\beta_2}(1+2\sinh^2 r) - \frac{\omega_2}{\omega_1\beta_1}}{\left(\frac{\omega_1}{\omega_2} - 1\right)t_{\text{cycle}}}. \tag{11}$$

According to Eq. (11), for a prescribed cycle duration χ attains a maximum value when the frequencies satisfy the relation

$$\frac{\omega_2}{\omega_1} = 1 - \sqrt{1 - \frac{\beta_1}{\beta_2}(1+2\sinh^2 r)}. \tag{12}$$

As a result, the corresponding COP is given by

$$\varepsilon_\chi = \frac{1}{\sqrt{1 - \frac{\beta_1}{\beta_2}(1+2\sinh^2 r)}} - 1. \tag{13}$$

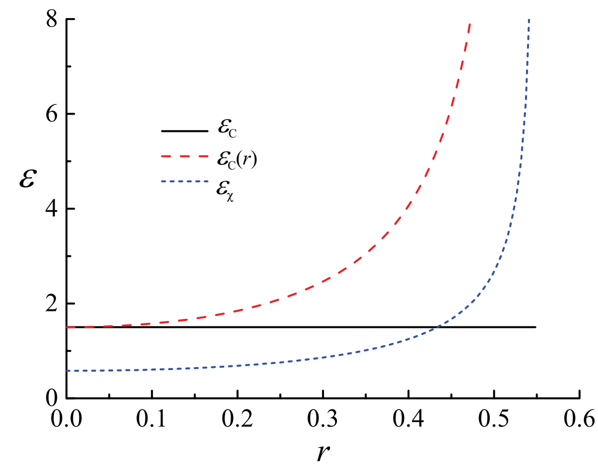

FIG. 2. (Color online) Variation of COP at maximum χ figure of merit with the squeezing parameter r where $\beta_1/\beta_2 = 0.6$.

For the cold reservoir without squeezing ($r = 0$), we recovered the CA coefficient of performance $\varepsilon_{\text{CA}} = \sqrt{1+\varepsilon_C} - 1$ [13,14]. According to Eq. (13), however, ε_χ approaches the Carnot COP ε_C when $r = \operatorname{arcsin}h[\sqrt{1/2(\varepsilon_C+1)}]$. Provided r is very large, the COP surpasses the Carnot COP as shown in Fig. 2. Furthermore, when $r \to \operatorname{arcsin}h(\sqrt{1/2\varepsilon_C})$, $Q_c \to 1/\beta_1$ and $W \to 0$. This implies that heat is transferred from the low-temperature reservoir to the high-temperature reservoir without any work input. The same phenomenon has been demonstrated earlier as well [25]. The quantum Otto heat engine is coupled to two thermal baths. The cold bath is an equilibrium system while the hot bath was assumed to be a squeezed thermal bath characterized by its temperature and the squeezing parameter r. In that study, at a large enough squeezing parameter the efficiency of the heat engine at maximum power output approaches unity. It predicts that work may be extracted from a single heat source without causing any effects on the other. These two results seem to violate the second law of dynamics.

B. Thermal bath with squeezing: Assumption of nonequilibrium state

The models adopted in Ref. [25] and this paper are based on the assumption that squeezing does not affect the bath's temperature. Alicki [38] argued that in the presence of squeezing, a thermal bath is in a nonequilibrium state and therefore its temperature will be frequency dependent. The Carnot COP should not then be defined by the initial temperatures of the heat baths. Consider a quantum harmonic oscillator thermometer with its frequency weakly coupled to a stationary bath by means of the interaction Hamiltonian $H_{\text{int}} = (a + a^{+}) \otimes B$, where a, a^{+} are the bosonic or fermionic annihilation and creation operators, respectively, and B is the bath observable. The bath drives the thermometer to a thermal equilibrium state at the frequency-dependent temperature $T(\omega,r)$ induced by the squeezing, which is determined by the following relation:

$$\exp\left(-\frac{\hbar\omega}{2K_B T(\omega,r)}\right) = \frac{G(-\omega)}{G(\omega)}, \quad \text{where} \quad G(\omega) = \int_{-\infty}^{+\infty} e^{i\omega t}\langle B(r,t)B(r)\rangle_{\text{bath}} dt. \quad (14)$$

As to the equilibrium bath, Eq. (14) is satisfied with a fixed $T(\omega,r)$ for an arbitrary $B(r)$ (Kubo-Martin-Schwinger condition). By ergodic averaging, the temperature $T(\omega,r)$ measured by the harmonic thermometer linearly coupled to the oscillator bath is given by the formula [38]

$$\exp\left(-\frac{\hbar\omega}{K_B T(\omega,r)}\right) = \frac{\langle n\rangle + (2\langle n\rangle + 1)\sinh^2 r}{1 + \langle n\rangle + (2\langle n\rangle + 1)\sinh^2 r}, \quad \text{where} \quad \langle n\rangle = \frac{1}{\exp(\hbar\omega\beta_2) - 1}. \quad (15)$$

According to above equation, the equilibrium temperature is $T(\omega,r) = T_2$ when $r = 0$. Furthermore, according to Eq. (15), $T(\omega,r)$ increases monotonously with increasing r. Under the high-temperature limit $\hbar\omega_i\beta_i \ll 1$, Eq. (15) reduces to

$$T(\omega,r) = (1 + 2\sinh^2 r)T_2. \quad (16)$$

When $r \to \arcsin h(\sqrt{1/2\varepsilon_C})$, $T(\omega,r) \to T_1$, which is in accordance with the fact that the temperature of the cold reservoir cannot exceed that of the hot reservoir for a refrigerator. The upper bound of the squeezing parameter is the same as in Eq. (10); therefore Eq. (16) is justified. In the presence of squeezing, the refrigerator must be considered as operating between temperatures $T(\omega,r)$ and T_1. The actual Carnot COP should then be written as $\varepsilon_C(r) = 1/[T_1/T(\omega,r) - 1]$. As $T(\omega,r) \geqslant T_1, \varepsilon_C(r) \geqslant \varepsilon_C$. Meanwhile, Eq. (13) becomes $\varepsilon_\chi = \sqrt{1 + \varepsilon_C(r)} - 1 < \varepsilon_C(r)$, as depicted in Fig. 2. Therefore we arrive at the conclusion that squeezing does not allow the COP to surpass the actual Carnot COP. In addition, the COP under the maximum χ figure of merit is still of the CA style.

IV. CONCLUSIONS

The performance of a quantum Otto refrigerator coupled to a squeezed cold reservoir has been studied under the χ figure of merit. We have shown that with the absence of squeezing, the COP at maximum χ figure of merit is the CA coefficient of performance and squeezing can enhance the COP dramatically, surpassing the Carnot COP defined by the initial temperatures of the heat baths for a large squeezing parameter. Furthermore, when the squeezing parameter approaches its maximum value, heat is directly transferred from the low-temperature reservoir to the high-temperature reservoir without any work input, in apparent contravention of the second law of thermodynamics. To explain this phenomenon, we have shown that squeezing renders the thermal bath into a nonequilibrium state. In the presence of squeezing, the temperature of the bath is frequency dependent. Under such circumstances, the Carnot COP should therefore not be defined by the initial temperatures of the heat baths. The actual Carnot COP has been deduced. The results show that the COP under the maximum χ figure of merit is also of the CA style and does not exceed the actual Carnot COP, thereby demonstrating consistency with the second law of thermodynamics.

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