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Universality in spectral statistics of open quantum graphs

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The quantum evolution maps of closed chaotic quantum graphs are unitary and known to have universal spectral correlations matching predictions of random matrix theory. In chaotic graphs with absorption the quantum maps become nonunitary. We show that their spectral statistics exhibit universality at the soft edges of the spectrum. The same spectral behavior is observed in many classical nonunitary ensembles of random matrices with rotationally invariant measures.

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Since the 1970s the physics community has paid significant attention to the particularities of the energy spectrum of quantum systems with chaotic behavior in the classical limit. In 1984 Bohigas *et al.* conjectured [1] that the spectral fluctuations of closed chaotic Hamiltonian systems are universal and coincide with those of one of three canonical random matrix ensembles (RMEs) [2]. Based on semiclassical considerations, the validity of the Bohigas-Giannoni-Schmit (BGS) conjecture has been established by now on the physical level of rigor [3–5].

Various physical phenomena might lead to the opening of quantum systems: attaching external leads to quantum dots, dissipation through Ohmic losses, partial reflection of microwaves at the boundaries of dielectric microcavities, etc. As a result, many theoretical and experimental studies have focused on open chaotic systems whose wave dynamics are described by nonunitary evolution operators [6]. It is of great interest to know whether (and under what conditions) open chaotic systems exhibit universal properties. So far, the vast majority of studies in this regard have been restricted to regimes of weak opening, where the mean dwell time of the particle in the system grows in the semiclassical limit. In such a case, the quantum characteristics are insensitive to system-specific features and can be successfully studied within the RME theory approach. For instance, transport properties of quantum dots with a finite number of open channels were shown to be universal [7] and in agreement with the RME theory predictions [8]. In a strongly open system, particle can leave the system phase space before it is effectively explored and hence the standard universality assumptions break down. In the present article we establish a form of spectral universality that holds in strongly open chaotic systems even if the mean dwell time stays finite in the semiclassical limit.

In what follows we focus on the model of quantum graphs with broken time-reversal symmetry. Quantum graphs were proposed as a paradigm for the study of compact [9] and scattering [10] quantum chaotic systems. They were also studied experimentally in the presence of absorption [11]. Let us briefly describe a standard construction of quantum graphs with V vertices connected by B bonds (see, e.g., [12] for details). At bonds $b = 1,\dots,B$ the waves ψ_b satisfy the free Schrödinger equation $(-id_{x_b} + \mathcal{A}_b)^2\psi_b(x_b) = k^2\psi_b(x_b)$, where $x_b \in [0,L_b]$ measures the distance along the bond b and $\mathcal{A}_b$ is a constant vector potential introduced to break the time-reversal symmetry. The corresponding general solution is a superposition of two plane waves propagating in opposite directions $\psi_b(x_b) = e^{-i\mathcal{A}_b x_b}(e^{ikx_b}a_{b+} + e^{-ikx_b}a_{b-})$. The constants $a_{b\pm}$ for different bonds are then connected at the vertices by means of the scattering matrices σ^v, $v = 1,\dots,V$. To proceed further one introduces the associated directed graph Γ with the double number $N = 2B$ of bonds $(b,\bar{b})$ carrying waves with the positive and negative momenta separately such that $L_b = L_{\bar{b}}$ and $\mathcal{A}_b = -\mathcal{A}_{\bar{b}}$. The complete spectral information is carried by the $N \times N$ quantum map $U(k) = S\Lambda(k)$, where the internal scattering matrix S depends on the graph's structure, σ^v, and $\mathcal{A}_b$, while the dependence on the energy k^2 is entirely stored in the diagonal part $\Lambda(k) = \mathrm{diag}\{e^{ikL_j}\},\ j \in \{b,\bar{b}\}$. Note that S also fixes the classical map F on Γ, whose matrix elements $F_{ij} = |S_{ij}|^2$ specify the classical transition probabilities between the bonds of the graph. The spectrum $\{k_n\}$ of the system is provided by solutions of the secular equation $\det[I - U(k_n)] = 0$. If all σ^v are unitary and $\mathcal{A}_b$ are real then the resulting quantum map U is unitary and the system's spectrum is real. It is possible to open the system by either attaching external leads to the graph or introducing absorption at bonds (or vertices) violating the aforementioned conditions. In any such case the resulting internal scattering matrix S and U are no longer unitary and we colloquially refer to Γ as an open quantum graph.

An appealing feature of quantum graphs is the exact trace formula connecting the density of states $d(k) = \sum_n \delta(k - k_n)$ with the traces of the quantum map $U(k)$. As a result, the two-point spectral correlation function can be expressed as the discrete Fourier transform of the spectral form factor $\langle|\mathrm{Tr}U(k)|^2\rangle_k$, where $\langle\cdot\rangle_k \equiv \lim_{K\to\infty}\frac{1}{K}\int_0^K dk(\cdot)$ is the average over the wave number. Furthermore, it was shown in [13] that for graphs with rationally independent bond lengths (which we assume throughout the paper) the average over k can be traded for the averages over independent parameters kL_b, $b = 1,\dots B$. Therefore, the spectral correlations in individual quantum graphs can be found by solving the same problem for the ensemble of matrices $U_{\boldsymbol{\phi}} \equiv S\Lambda_{\boldsymbol{\phi}}$, $\Lambda_{\boldsymbol{\phi}} = \mathrm{diag}\{e^{i\phi_b}\}_{b=1}^N$, and $\phi_b = \phi_{\bar{b}}$, where the averages are taken over the flat probability measure $\prod_{b=1}^B \frac{d\phi_b}{2\pi}$. It is worth noting that the constraint $\phi_b = \phi_{\bar{b}}$ can actually be removed by the additional averaging over the vector potential. After adding a common real phase α to each $\mathcal{A}_b$ the ensemble of $U(k)$ with the average over both k

and α becomes equivalent to the one of matrices $S\Lambda_{\boldsymbol{\phi}}$, where all components of the vector $\boldsymbol{\phi} = (\phi_1, \ldots \phi_N)$ are independent and the average $\langle\cdot\rangle_{\boldsymbol{\phi}}$ is over the measure $d\nu \equiv \prod_{k=1}^{N} \frac{d\phi_k}{2\pi}$ in the domain $\phi_k \in [0,2\pi]$.

The case of graphs with unitary S has been analyzed by both semiclassical [14,15] and supersymmetry methods [16]. It was demonstrated that under certain condition on the spectral gap of the classical map, the BGS conjecture holds, i.e., in the presence (absence) of time-reversal symmetry, the spectrum of $U_{\boldsymbol{\phi}}$ has the same spectral statistics as Gaussian orthogonal (unitary) random matrix ensembles. For strongly open quantum graphs the eigenvalues $\{z_k\}_{k=1}^{N}$ of $U_{\boldsymbol{\phi}}$ are not confined to the unit circle, but rather are distributed isotropically over the complex plane. The isotropy follows immediately from the invariance of $d\nu$ under the rotation $\phi_k \to \phi_k + \phi$, $k = 1, \ldots, N$. Typically, with the increase of the graph's dimension z_k become more and more concentrated in an annulus whose boundaries are referred to as the inner (outer) spectral edge. As we show below, a spectral universality holds at the $1/\sqrt{N}$ neighborhood of these edges. For the sake of simplicity of the exposition, we formulate the result for the outer edge and then discuss its extension to the inner edge.

Main result. Let $\Gamma^{(N)}$ be an infinite sequence of open quantum graphs with $S^{(N)}\Lambda_{\boldsymbol{\phi}}$ and $F^{(N)}$ being their $N \times N$ quantum and classical evolution, respectively. For the matrix $F^{(N)}$ we denote by λ, $\bar{\chi}$, and χ the largest eigenvalue and the corresponding left and right eigenvectors normalized by $(\bar{\chi},\chi) = N$. We will consider the spectrum $\{z_k\}_{k=1}^{N}$ of the rescaled quantum propagator $S\Lambda_{\boldsymbol{\phi}}$, $S \equiv \frac{1}{\sqrt{\lambda}} S^{(N)}$ in the limit $N \to \infty$, under the following conditions: (i) the large spectral gap of $F \equiv F^{(N)}/\lambda$, with the next to the largest eigenvalue λ_2 satisfying $1 - |\lambda_2| = O(N^{-\kappa})$ for $\kappa < \frac{1}{2}$, and (ii) the strong nonunitarity of S, with the parameter $\mu^{(N)} = \frac{1}{N}\mathrm{Tr}[(S\mathcal{X}S^{\dagger}\bar{\mathcal{X}})^2 - (\mathcal{X}\bar{\mathcal{X}})^2]$ having a strictly positive limit $\mu = \lim_{N\to\infty} \mu^{(N)} > 0$, where $\mathcal{X}_{i,j} = \chi_i \delta_{i,j}$ and $\bar{\mathcal{X}}_{i,j} = \bar{\chi}_i \delta_{i,j}$ are the diagonal matrices constructed from χ and $\bar{\chi}$. Assuming conditions (i) and (ii) hold, the spectral density $\tilde{\rho}(z) = \frac{1}{N}\langle \sum_{k=1}^{N} \delta(z - z_k)\rangle_{\boldsymbol{\phi}}$ is a function of $r = |z|$ only and $\rho(r) \equiv 2\pi r \tilde{\rho}(r)$ has a universal form at $1/\sqrt{N}$ in the vicinity of the edge $|z| = 1$:

$$\rho\left(1 + \frac{s}{\sqrt{N}}\right) = \frac{1}{\mu}\mathrm{erfc}\left(\frac{s}{\sqrt{2\mu}}\right) + O(N^{-\epsilon}) \qquad (1)$$

for $\epsilon > 0$. In particular, $\rho(1) = \mu^{-1} + O(N^{-\epsilon})$. The form factor $\mathcal{K}(n) = \frac{1}{N}\langle |\mathrm{Tr}(S\Lambda)^n|^2\rangle_{\boldsymbol{\phi}}$ demonstrates the universal asymptotics

$$\sqrt{N}\mathcal{K}(n) = \frac{2}{\mu t}\sinh\left(\frac{\mu t^2}{2}\right) + O(N^{-\epsilon}) \qquad (2)$$

in the limit where $t = n/\sqrt{N}$ is fixed and $N \to \infty$.

A few remarks are in order. First, note that the spectral density of the Ginibre unitary ensemble [17] and of other strongly nonunitary ensembles [18,19] with rotationally invariant measures demonstrate the same soft edge universal form (1). Only the scaling parameter μ depends on the specifics of these ensembles. Second, for finite values of μ the mean distance between eigenvalues of S is of order $1/\sqrt{N}$ (recall that in the unitary case it is $1/N$). Because of the differences in the mean level distance, the semiclassical limit $n \sim \sqrt{N}$ and $N \to \infty$ considered here differs from the limit $n \sim N$ and $N \to \infty$ usually considered in the unitary case. Third, the asymptotics for the inner edge can be established by considering inverse matrices $(S^{(N)})^{-1}\Lambda^{*}_{\boldsymbol{\phi}}$ whose spectrum $\{z_k^{-1}\}_{k=1}^{N}$ has the density $\rho'(r) = r^{-2}\rho(1/r)$. This inversion maps the inner edge to the outer and (1) and (2) become applicable to ρ' with the parameter μ being defined by the matrices S^{-1}. Fourth, by Eq. (1) the outer and the inner edges of the nonrescaled quantum maps $S^{(N)}\Lambda_{\boldsymbol{\phi}}$ are given by $\sqrt{\lambda}$ and $1/\sqrt{\lambda'}$, respectively, where λ and λ' are the highest eigenvalues of the classical maps $|S^{(N)}|^2_{i,j}$ and $|(S^{(N)})^{-1}|^2_{i,j}$. If $\lambda' = \infty$ (e.g., S is not invertible) then the inner edge does not exist. Fifth, condition (i) on the spectral gap of the classical map is analogous to Tanner's condition [20] in the unitary case. The difference between $\kappa < 1/2$ (nonunitary) and $\kappa < 1$ (unitary) is due to the different time scales involved. Condition (i) is satisfied for many important classes of graphs (see, e.g., [12] and examples below). Condition (ii) implies strong nonunitarity of S. If, for instance, the number of open channels in a scattering graph is fixed, then $\lim_{N\to\infty} \mu^{(N)} = 0$ and condition (ii) is violated. Note that $\mu^{(N)} \geqslant 0$ always holds and $\mu^{(N)} \equiv 0$ if S is unitary.

Comparison with numerics. Before turning to the derivation of Eqs. (1) and (2), let us compare these results with the numerically calculated spectral density and form factor of $S\Lambda_{\boldsymbol{\phi}}$ averaged over $\boldsymbol{\phi}$. We consider several families of matrices S (or, equivalently, quantum graphs). (a) Connectivity graphs. Given a graph Γ let S be its edge-connectivity matrix, i.e., the element $S_{i,j}$ is either 1 if the ith edge of Γ is connected to the jth edge through some vertex or 0 otherwise. This choice is of special interest due to the connection with the problem of length degeneracies in metric graphs [21]. When Γ is a d-regular connected graph it is straightforward to see that $\lambda = d$ and $\mathcal{X} = \bar{\mathcal{X}} = \mathbb{1}$, implying $\mu = d - 1$. The comparison of (1) and (2) with numerics for such a graph is shown in Figs. 1(a) and 1(b). (b) Doubly stochastic graphs [Fig. 2(a)]. Let S be such that F is a doubly stochastic matrix i.e., $\sum_i F_{i,j} = \sum_j F_{i,j} = 1$. This can be achieved, for instance, by taking $\sigma^{v}_{i,j} = |u^{v}_{i,j}|$, where u^{v} are arbitrary unitary matrices. It is known that these matrices almost surely satisfy the required spectral gap condition [22]. As in the previous example, the highest eigenvector χ of F is uniform, while $\lambda = 1$. This gives $\mu^{(N)} = \frac{1}{N}\mathrm{Tr}(SS^{\dagger})^2 - 1$ (compare with the result of [19] for RMEs with unitary invariant measures). (c) Damped quantum maps were suggested in [23] as toy models for open quantum systems. They are represented as products $U_{\mathcal{M}} \cdot D$, where the $N \times N$ unitary matrix $U_{\mathcal{M}}$ is a quantization of a classical map $\mathcal{M}$ and D is a smooth diagonal matrix modeling absorption. Here we checked a particular case of Walsh's quantized baker's map whose quantization for $N = 2^p$ can be written as $U^{(p)}\Lambda_{\boldsymbol{\phi}}$, where $U^{(p)}_{i,j} = \frac{1}{\sqrt{2}}(\delta_{i,2j-1 \bmod N} - \delta_{i,2j} + \delta_{i,2j-N})$ and $\boldsymbol{\phi}$ is arbitrary (see [24]). The matrix $S^{(p)} = U^{(p)}D$ can in turn be interpreted as the scattering matrix for the de Brujin graph with absorption. We compared the spectral density of matrices $S^{(p)}\Lambda_{\boldsymbol{\phi}}$ with (1) and found good agreement for both inner and outer edges [see Fig. 2(b)]. Note that in this case, as follows from numerics, the parameter $\mu^{(N)}$ grows with N and the spectral density at the edges converges to zero at $N \to \infty$.

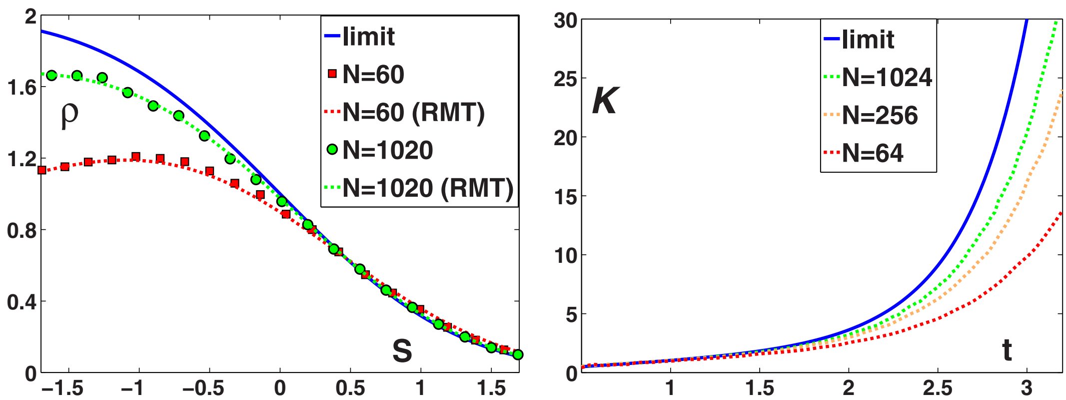

FIG. 1. (Color online) Shown on the left is the spectral density of 2-regular connectivity graphs ($\mu = 1$) vs the universal function (1) [solid (blue) line]; the two dashed lines are analytic results from [18] for RMEs of truncated unitary matrices of the same dimensions. On the right are the spectral form factors for the same family of graphs plotted as dashed lines vs (2) [solid (blue) line].

This observation agrees with the phenomenon of eigenvalue clustering near a typical value shown in [23].

Derivation of Eqs. (1) and (2). By definition the form factor $\mathcal{K}(n)$ can be represented as the double sum over n-periodic trajectories γ and γ' of the graph having the same length:

$$\frac{N}{n}\mathcal{K}(n) = \left\langle \left| \sum_{\gamma} A_\gamma e^{i(\boldsymbol{n}_\gamma,\boldsymbol{\phi})} \right|^2 \right\rangle_{\boldsymbol{\phi}} = \sum_{\gamma,\gamma'} A_\gamma A^*_{\gamma'} \delta_{\boldsymbol{n}_\gamma,\boldsymbol{n}_{\gamma'}}. \quad (3)$$

Here $\boldsymbol{n}_\gamma$ is an integer-valued N-dimensional vector whose elements n_b indicate the number of times γ visits the bond b ($\sum_b n_b = n$). The amplitudes A_γ are products of the matrix elements S_{ij} taken along the path γ and include the multiplicity factors that are 1 for prime periodic orbits. Following the standard semiclassical prescription [25], we analyze first the diagonal $\gamma = \gamma'$ contribution in (3). Leaving out only prime periodic orbits and assuming a long trajectory limit ($n \sim \sqrt{N} \gg 1$) yields

$$\sum_{i_1 \ldots i_n} |S_{i_1 i_2}|^2 \cdots |S_{i_n i_1}|^2 = \mathrm{Tr} F^n = 1 + O(N^{-1/2+\kappa}),$$

where we used condition (i) on the spectral gap of F.

To calculate the next contribution one takes into account pairs of self-crossing trajectories. In each pair the partners γ and γ' possess the same vectors $\boldsymbol{n}_\gamma = \boldsymbol{n}_{\gamma'}$, but traverse the bonds in a different order (see Fig. 3). Note that, because of broken time-reversal symmetry, only trajectories with an even number of encounters contribute in (3). Furthermore, since n is set to be of the same order as $\sqrt{N}$, only encounters with two entering and two exiting loops should be considered. In sharp contrast to the unitary case (where the relevant scale is $n \sim N$), here the diagrams with l-encounters for $l > 2$ contribute to the subleading order $N^{-\epsilon}$ only.

The contribution from trajectories with $2m$ encounters can be split into a product of three factors coming from encounters

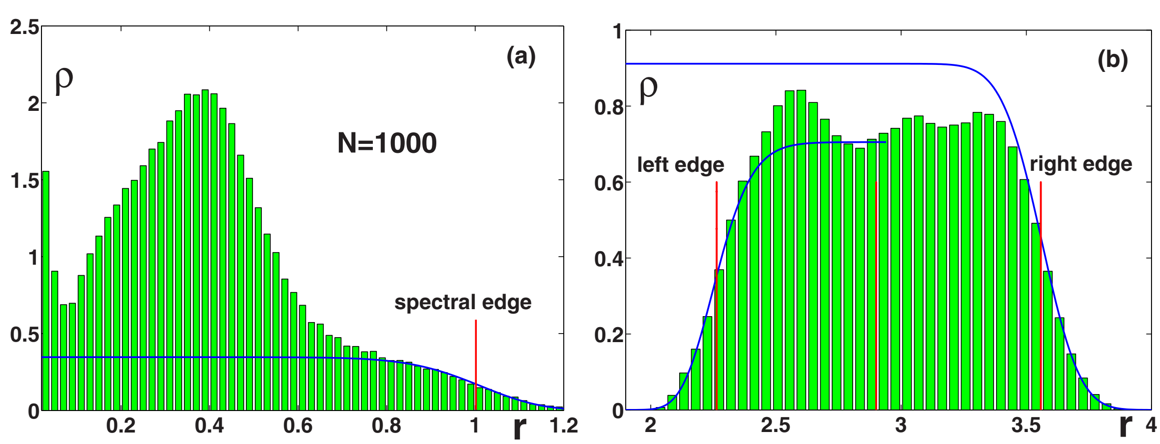

FIG. 2. (Color online) The (nonrescaled) spectral densities of quantum maps $S^{(N)}\Lambda_{\boldsymbol{\phi}}$ vs asymptotics (1) (solid blue lines) for (a) doubly stochastic 10-regular graph with $N = 1000$, where each vertex matrix σ^v is fixed by a random choice of 10×10 unitary matrix u^v, and (b) a damped de Brujin graph with $N = 2^7$, $D_{i,j} = \delta_{i,j} f(2j\pi/N)$, and $f(x) = 3.2 + \sin(x) + \sin(2x) + \sin(3x)$. The parameters are $\mu = 0.1542$ and $\lambda_1 = 12.6578$ for the outer edge and $\mu' = 0.3133$ and $\lambda'_1 = 0.1952$ for the inner edge. The inner $\sqrt{1/\lambda'_1}$ and outer $\sqrt{\lambda_1}$ radii are depicted by vertical solid (red) left and right lines. The (red) line in the middle shows the mean value of $\log f(x)$, the point, where ρ clusters at $N \to \infty$ (see [23]).

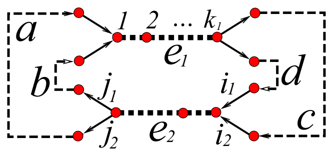

FIG. 3. (Color online) Diagram of periodic orbits with two 2-encounters e_1,e_2 and four loops a,b,c,d ($m=1$) contributing to (3). The two orbits of the same length are represented by the sequences $[ae_1ce_2be_1de_2]$ and $[ae_1de_2be_1ce_2]$.

$\mathcal{N}_{\rm enc}$, loops $\mathcal{N}_{\rm loop}$ connecting them, and the combinatorics $\mathcal{N}_{\rm comb}$. The latter takes into account all possible reconnections of loops and encounters, i.e., the number of different diagrams. For the diagrams with 2-encounters it is known [26] to be $\mathcal{N}_{\rm comb}=\frac{(4m)!}{2^{2m}(2m+1)!}$. Given $2m$ encounters there are $\frac{n^{4m}}{(4m)!}$ (to the leading order of n) choices to fix the lengths $\ell_1,\dots,\ell_{4m}$ of the loops connecting them such that the total length is fixed $\sum_{i=1}^{4m}\ell_i=n-2\sum_{i=1}^{2m}(k_i+1)$, where k_i is the length (i.e., the number of vertices) of the ith encounter. The contribution from all possible paths of the length $\ell\gg 1$ connecting jth and ith bonds is given by $\sum_{i_1\cdots i_{\ell-1}}F_{ii_1}\cdots F_{i_{\ell-1}j}=\bar{\chi}_i\chi_j+O(N^{-1/2+\kappa})$. This yields, for the total contribution from $4m$ loops with fixed entering and exiting bonds,

$$\mathcal{N}_{\rm loop}=\frac{n^{4m}}{(4m)!}\left(\prod_{r=1}^{2m}\bar{\chi}_{i_r}\chi_{j_r}\right)+O(N^{-1/2+\kappa}).$$

Given that incoming (i_1,i_2) and outgoing (j_1,j_2) bonds of an encounter are fixed, the total contribution from all possible paths connecting them is

$$\mathcal{N}_{\rm enc}^{(1)}=\left(1-\delta_{i_1,i_2}\right)\left(1-\delta_{j_1,j_2}\right)S_{i_1j_1}S^*_{i_1j_2}S_{i_2j_2}S^*_{i_2j_1},$$
$$\mathcal{N}_{\rm enc}^{(k)}=\sum_{i,j}\left(1-\delta_{i_1,i_2}\right)\left(1-\delta_{j_1,j_2}\right)F_{i_1i}F_{i_2i}[Q^k]_{ij}F_{jj_1}F_{jj_2}$$

for encounters of the lengths $k=1$ (containing a single vertex) and $k>1$, respectively. Here Q is an axillary matrix with the elements $Q_{ij}=F_{ij}^2$. Combining these expressions with the factors $\bar{\chi}_{i_1}\bar{\chi}_{i_2}\chi_{j_1}\chi_{j_2}$ from $\mathcal{N}_{\rm loop}$ and taking the sum over the indices gives for each encounter of the length k the following result:

$$\sum_{j_1,j_2,i_1,i_2}\bar{\chi}_{i_1}\bar{\chi}_{i_2}\chi_{j_1}\chi_{j_2}\mathcal{N}_{\rm enc}^{(1)}={\rm Tr}[(S\mathcal{X}S^\dagger\bar{\mathcal{X}})^2-2(\bar{\mathcal{X}}\mathcal{X})^2+\bar{\mathcal{X}}^2Q\mathcal{X}^2]\quad\text{for}\quad k=1,$$

$$\sum_{j,j_1,j_2,i,i_1,i_2}\bar{\chi}_{i_1}\bar{\chi}_{i_2}\chi_{j_1}\chi_{j_2}\mathcal{N}_{\rm enc}^{(k)}={\rm Tr}[\bar{\mathcal{X}}^2Q^k\mathcal{X}^2-2\bar{\mathcal{X}}^2Q^{k+1}\mathcal{X}^2+\bar{\mathcal{X}}^2Q^{k+2}\mathcal{X}^2]\quad\text{for } k>1.$$

After summing up over all k, taking into account $\mathcal{N}_{\rm comb}$ and the remaining combinatorial factor from $\mathcal{N}_{\rm loop}$ we arrive at

$$\mathcal{K}(n)=\frac{n}{N}\sum_{m=0}^{\infty}\frac{n^{4m}(\mu^{(N)})^{2m}}{(2N)^{2m}(2m+1)!}+O(N^{-1/2+\kappa}),\qquad(4)$$

which is the Taylor expansion of Eq. (2). Finally, the spectral density can be restored through the relationship

$$\rho(r)=\frac{2}{\pi r}\lim_{\varepsilon\to 0}{\rm Im}R_\varepsilon(r^{-2}),\qquad R_\varepsilon(r)=\sum_{n=1}^{\infty}(re^{i\varepsilon})^n\mathcal{K}(n)$$

by substituting (2) and transforming the sum into an integral. The saddle-point approximation to this integral in the regime $n\sim\sqrt{N}$ results in Eq. (1).

Formally, Eqs. (1) and (2) can be derived also using the supersymmetry approach of [16]. To this end, the function $R_\varepsilon(r)$ is represented as the integral over supersymmetric fields. The result then follows by leaving out only the zero-mass mode in the saddle-point approximation. Contrary to the setup of unitary S, however, even in the best case scenario of graphs with finite spectral gaps in their classical evolution maps, the contribution of massive modes cannot be discounted on the basis of the rough estimation suggested in [16]. Due to a lack of space, we omit the detailed discussion of the supersymmetry approach.

We have shown that the spectral density and the form factor of the quantum evolution map for strongly open quantum graphs demonstrate universal behavior at the spectrum edges at the scale of the mean distance between eigenvalues. We conjecture that higher-order spectral correlations exhibit similar universality as well. In a sense, our result can be seen as an extension of the well established universality for closed quantum graphs. From the semiclassical point of view, the strongly nonunitary case differs from the unitary one in the time scales involved: $\sqrt{N}$ rather than N. This results in the exclusion of all diagrams with l-encounters for $l>2$. In a transitional case with weak unitarity breaking, where $\mu^{(N)}\sim N^{-1}$, these diagrams must be actually taken into account since they contribute to the same order (in n/N) as (4).

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