Reply to "Comment on 'Temperature-dependent orientational ordering on a spherical surface modeled with a lattice spin model' "

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In Romano's Comment [Phys. Rev. E **91**, 046501 (2015)] on Sec. II of our paper [Phys. Rev. E **90**, 022502 (2014)], the author suggests that our findings concerning the nature of the ordering transition of our modified Lebwohl-Lasher model with two-dimensional planar rotators on a planar lattice are inconsistent with known mathematical results. We argue in this Reply that our findings are in fact in agreement with previous mathematical and simulation results and that the criticisms raised by Romano have no impact on the results presented in our paper.

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In his Comment [1], the author provides some remarks and criticisms of Sec. II of our paper [2] concerning the nature of the two-dimensional ordering transition. He claims that some of our results do not agree with mathematical results. Here, we argue that our simulation results are not in conflict with mathematical results and that the criticisms raised in Ref. [1] are due to a misunderstanding or an issue with wording.

Our study is concerned—as the title and abstract state clearly—with ordering on the surface of a sphere. We were motivated by recent experiments where droplets with nematic shells were prepared with a radius of about 50 μm [3]. Due to topological reasons, no perfect order exists on a spherical surface even for zero temperature. Therefore, we follow earlier work [4] and focus instead on the local orientational ordering (see Eq. (15) in Ref. [2]).

From computer simulations, we find that the local orientational order increases smoothly with decreasing temperature, see Figs. 3 and 4 in Ref. [2]. The planar case that we treat in Sec. II of Ref. [2] merely serves as a reference for the spherical surface. We use comparable system sizes as in the spherical case, and we are explicitly *not* interested in the limit of infinite system size. From standard Monte Carlo simulations we find that not only the local, but also the usual Maier-Saupe orientational order parameter (Eq. (3) in Ref. [2]) increases around the same temperature for this finite-size model. Therefore, we believe, the orientational ordering found in the planar model is helpful to understand the local ordering in the spherical case. Our numerical findings concerning orientational ordering in the planar model for both square and triangular lattices are consistent with earlier Monte Carlo simulations [5,6].

It is worth noting that the length-scale-dependent spatial fluctuations obtained in our simulations allow us to extract a consistent value of the corresponding Frank elastic constant, see Fig. 2 and theoretical estimate Eq. (18) in Ref. [2]. There, we take the long-wavelength limit by considering fluctuations on the scale of the system, again without taking the infinite system size limit.

We fully agree with the Comment made in Ref. [1] that no true long-range order exists in the planar Lebwohl-Lasher model in the thermodynamic limit and the transition we observe is perhaps more appropriately referred to as a pseudotransition. We apologize if our wording or headings were misleading, however, we made *no* claims concerning the thermodynamic limit. Since the available mathematical results (see Ref. [1] for a summary of those results) only deal with the thermodynamic limit, we are not in conflict with those results about this transition. Instead, we merely stated numerical results for finite systems comparable to the system sizes we investigated in the spherical case. Therefore the comments raised in Ref. [1] do not affect any of the results presented in our paper [2].

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