Pseudorelativistic effects on solitons in quantum semiconductor plasma

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A theory for nonlinear excitations in quantum plasmas is presented for narrow-gap semiconductors by considering the combined effects of quantum and pseudorelativity. The system is governed by a coupled Klein-Gordon equation for the collective wave functions of the conduction electrons and Poisson's equation for the electrostatic potential. This gives a closed system, including the effects of charge separation, quantum tunneling, and pseudorelativity. By choosing the typical parameters of semiconductor InSb, the quasistationary soliton solution, which is a multipeaked dark soliton, is obtained numerically and shows depleted electron densities correlated with a localized potential. The dynamical simulation result shows that the dark soliton is stable and has a multipeaked profile, which is consistent with the quasistationary solution. The present model and results may be useful in understanding the nonlinear properties of semiconductor plasma on an ultrafast time scale.

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I. INTRODUCTION

The collective dynamics of a quantum electron gas in a semiconductor are of great importance in possible applications to the field of quantum computing [1]. Accordingly, quantum semiconductor plasma physics has attracted much attention due to its potential applications in semiconductor nanostructures, such as spintronics, nanotubes, quantum dots, and quantum wells [2], where quantum effects are very important due to the fact that the de Broglie wavelength of charge carriers can be comparable to the characteristic spatial scales of the system in modern miniature semiconductor structures. These quantum effects can produce new interesting physical phenomena of electrostatic and electromagnetic waves in semiconductor quantum plasmas [3,4], which can be investigated by quantum hydrodynamic (QHD) equations, of which the Bohm potential stands for the quantum tunneling effect [5]. More recently, the two-stream instability in a quantum semiconductor plasma was studied using the QHD model [6]. The instability in an electron beam pumped GaAs semiconductor can arise due to the excitation of electron-hole pairs [7]. The longitudinal waves and electromagnetic surface waves were all modified by the quantum corrections [8,9]. The quantum effects would also reduce the threshold electric field for the onset of parametric amplification [10].

On the other hand, in Kane's model the dispersion relation of narrow-gap semiconductors shows that the conduction electron is a relativistic one with effective velocity-dependent mass, where the effective speed plays the part of the speed of light, which is several orders smaller than the speed of light in vacuum [11]. The large nonlinear optic effects in n-doped narrow-gap semiconductors come from the nonparabolicity of conduction electrons [12]. Accordingly, the latter can be used to simulate electron acceleration in the wake field [13] or to simulate the beat wave generation of plasmons [14].

The collective effect plays a crucial role in the investigation of electron dynamics on an ultrafast time scale [15]. As the short electron pulse propagates in the metallic nanowires, the stable wake field may be excited for a reasonable set of experimentally accessible parameters, which can be used to produce radiation in the extreme-ultraviolet range [16]. Recently, experiments and simulations have shown the existence of nonlinear collective excitation, such as solitons, in the semiconductor microcavity [17,18]. The properties of bright polariton solitons in semiconductor microcavity operation are affected by the exciton-photon coupling [19]. Experimental observations of solitons have also been made in a GaAs slab [20]. The dark solitons can withstand perturbations and turbulence during a considerable time in the quantum semiconductor plasma, where three typical semiconductors, GaAs, GaSb, and GaN, are studied and the solitons in GaN seem to be more stable than in GaAs and GaSb [21]. The quantum effects strongly affect the modulational instability of envelope solitons in semiconductor plasma [22]. The features of solitary waves in different semiconductors (GaAs, GaSb, GaN, and InP) are different depending upon the quantum effects [23].

However, very little work has been done regarding the soliton in semiconductor plasmas consisting of conduction electrons with nonparabolicity. Electromagnetic solitons can appear in dense n-doped InSb semiconductor plasma due to pseudorelativistic nonlinear effects [24–26], but the above research does not consider the quantum effects on electromagnetic solitons. Accordingly, the influence of the combination of pseudorelativity and quantum effects on solitons is investigated in narrow-gap semiconductors in this paper. In Sec. II, by introducing the quantum energy operator and momentum operator into Kane's dispersion relation of narrow-gap semiconductors, the Klein-Gordon equation (KGE) is derived for pseudorelativistic semiconductor quantum plasma. In Sec. III, the KGE is solved by using a numerical method and a steady soliton solution with a multipeaked profile is obtained. To evaluate the stability of the dark soliton, the dynamics is also investigated by a numerical method. In the final section, the conclusion and importance of the present results are given.

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II. BASIC EQUATIONS

Narrow-gap semiconductors have a large degree of nonparabolicity. To give a basic model of the conduction electrons in a narrow-gap semiconductor, we consider Kane's dispersion relation to stand for the nonparabolicity of conduction electrons, which is a relativistic relation with effective velocity-dependent mass. The Kane's dispersion relation of narrow-gap semiconductors is given by [11]

$$\mathcal{E} = (\mathbf{p}^2 c_*^2 + m_*^2 c_*^4)^{1/2}, \tag{1}$$

where $\mathcal{E}$ is the energy of a conduction-band electron, $\mathbf{p}$ is quasimomentum, $c_* = (E_g/2m_*)^{1/2}$ plays the part of the speed of light, with m_* being the effective mass of the electron at the bottom of the conduction band, and E_g is the width of the gap separating the valence and the conduction band. For semiconductor InSb the speed of light $c_* \approx 3\times 10^{-3}c$, with c being the speed of light in vacuum [11]. Nonparabolicity of the conduction band leads to electron velocity-momentum dependence, as one can obtain by $\mathbf{v} = \partial\mathcal{E}/\partial\mathbf{p}$. Then the expression of velocity was $\mathbf{v} = \mathbf{p}/m_*(1+p^2/m_*c_*^2)^{1/2}$ [13], which was similar to the velocity of the relativistic electron gas plasma. In this paper, the quantum effects will also be considered for relativistic electrons in a narrow-gap semiconductor. By the substitution $\mathcal{E} \to i\hbar\partial/\partial t + e\phi$ and $\mathbf{p} \to -i\hbar\nabla$ in (1), where $\hbar$ is the Planck constant divided by 2π, we obtain the KGE for an ensemble of conduction electrons in narrow-gap semiconductor as

$$\left(i\hbar\frac{\partial}{\partial t} + e\phi\right)^2 \psi - \hbar^2 c_*^2 \nabla^2\psi + m_*^2 c_*^4 \psi = 0, \tag{2}$$

where ψ represents an ensemble of conduction electrons, ϕ represents the scalar potentials, and e is the magnitude of the electron charge. Equation (2) can be reduced to a collective Schrödinger for an ensemble of electrons in the nonrelativistic limit, which was already used to investigate the collective nonlinear excitation in electron-hole semiconductor plasma [21].

Here Eq. (2) neglects degeneracy of electrons, which appears due to the Pauli exclusion principle and is important in dense matters. It has already been shown that the nonlinearity of degeneracy can support the formation of stable dark solitons and vortices in nonrelativistic electron gas plasmas [27]. But in the present work, the nonlinear excitation has to be investigated for pseudorelativistic quantum semiconductor plasmas. The present KGE model (2) also neglects the effects of exchange potential due to electron spin, which describes the interactions between the electrons. The presence of exchange potential strongly affects the collective breather modes in semiconductor quantum wells [28], but the main limitation of the KGE model is that the spin effect is neglected. The advantage of the present KGE model is that it can take quantum and pseudorelativistic effects on an equal footing and the electric charge density can be calculated self-consistently from the KGE. The corresponding electric charge densities are now obtained as

$$\rho_e = \frac{-e}{2m_* c_*^2}\left[\psi^*\left(i\hbar\frac{\partial}{\partial t} + e\phi\right)\psi + \psi\left(i\hbar\frac{\partial}{\partial t} + e\phi\right)^*\psi^*\right]. \tag{3}$$

The right-hand side of Eq. (3) is multiplied by the electron charge $-e$. Accordingly, ρ_e can be interpreted as the electric charge density rather than a probability density, since ρ_e is neither positive nor negative now. The systems are closed by Poisson's equation

$$\nabla^2\phi = -\frac{1}{\varepsilon}(\rho_e + en_0), \tag{4}$$

where ε is the effective dielectric permittivity of the lattice. For the InSb semiconductor, the effective dielectric is $\varepsilon = 14\varepsilon_0$, with ε_0 being the vacuum dielectric constant [24], and n_0 is the unperturbed density of the electrons and also of the positive neutralizing ionic background. Equations (2) and (4) are our desired system that describes pseudorelativistic semiconductor plasma collective interactions in the quantum regime. The KGE for an ensemble of electrons in the gas plasma is already obtained to investigate the intense laser pulse interaction with quantum gas plasma [29]. For electrostatic waves, due to the charge separation between the conduction electrons and the positive ionic background, the quantum effects are important at short wavelengths. If the wavelength is comparable to the characteristic length $\hbar/m_*c_* \approx 7.2\times 10^{-9}$ m, the pseudorelativistic effects will also be important for the conduction electron. Accordingly, the combined quantum effects and relativity will be important for nonlinear collective excitation in nanoscale in the narrow-gap semiconductor plasmas.

We first discuss the linear properties of the system (2) and (4). By linearizing the system (2) and (4) and using the Fourier modes, the dispersion relation was already obtained in the gas plasmas [29,30] and in a relativistic bosonic gas [31,32], which shows that in the classical limit $\hbar \to 0$ the dispersion relation reduces to the Langmuir oscillations, and in the nonrelativistic limit the dispersion relation reduces to the ordinary dispersion relation of electrostatic modes in quantum plasmas without the degeneracy effects. The dispersion relation can be obtained alternatively by a kinetic equation, which is exactly equivalent to KGE and describes the space-time evolution of a spinless charged particle in plasmas, where a new dispersion relation was obtained with first-order quantum corrections and can also reduce to the ordinary dispersion relation of electrostatic modes in the nonrelativistic limit [33]. However, it should be noted that the new dispersion relation has a degeneracy term [33]. In Refs. [29] and [30], the dispersion relation also gives a pair branch mode. The latter will be important in the ultradense gas plasmas, such as in the astrophysics environment. However, in our system, the relativistic effect is a pseudo effect. Then we just consider the plasma mode in this paper and the pair production is safely neglected.

III. THE STEADY SOLITON AND ITS DYNAMICS

In the following, the possibility of nonlinear excitations is investigated by system (2) and (4) in one dimension. For convenience, a new wave function $\Psi(z,t)$ is introduced by the transformations $\psi(\mathbf{r},t) = \Psi(z,t)\exp(-im_*c_*^2 t/\hbar)$. To study quasi-steady-state structures propagating with a constant speed v_0, the physical quantity is assumed that $\phi = \phi(\xi)$, where $\xi = z - v_0 t$. It is convenient to introduce the eikonal

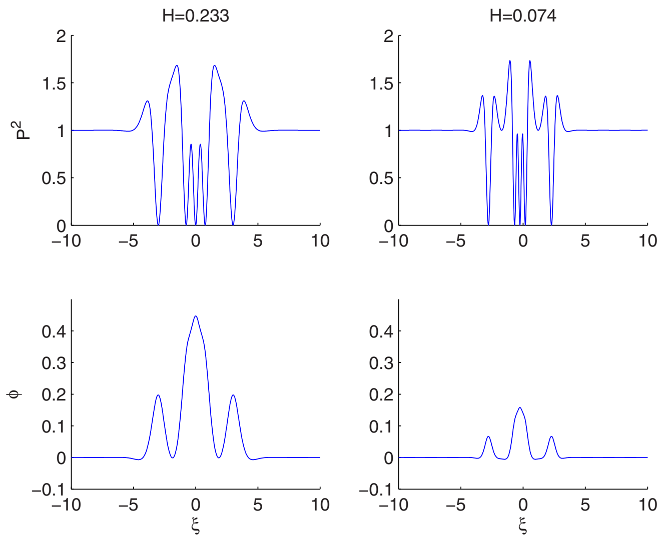

FIG. 1. (Color online) The spatial profiles of the conduction electron density and the scalar potential (top to bottom panels) for $H = 0.233$ (left column) and $H = 0.074$ (right column) with velocity $v_0 = 0$.

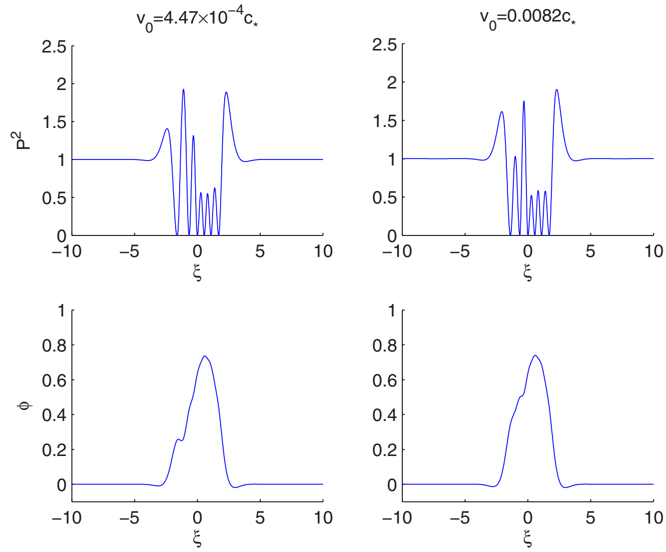

FIG. 2. (Color online) The spatial profiles of the conduction electron density and the scalar potential (top to bottom panels) for $v_0 = 4.47 \times 10^{-4} c_*$ (left column) and $v_0 = 0.008\,2 c_*$ (right column) with the quantum parameter $H = 0.233$.

representation $\Psi = P(\xi)\exp[i\theta(\xi)]$, where P and θ are real valued. Then the coupled system of equations is derived as

$$\frac{d^2 P}{d\xi^2} + \frac{\gamma_0^4}{H^2}\left[(\phi+1)^2 - \frac{\beta^2}{P^4} - \frac{1}{\gamma_0^2}\right]P = 0, \tag{5}$$

$$\frac{d^2\phi}{d\xi^2} + \gamma_0^2[1 - (\phi+1)P^2] = 0, \tag{6}$$

where $\gamma_0 = 1/\sqrt{1-\beta^2}$ and $\beta = v_0/c_*$. The parameter $H = \hbar\omega_{pe}/m_* c_*^2$ stands for quantum tunneling effects, where plasma frequency is $\omega_{pe} = \sqrt{n_0 e^2/m_*\varepsilon}$. For convenience, the coupled system (5) and (6) was normalized as follows: $e\phi/m_* c_*^2 \to \phi$, $P/\sqrt{n_0} \to P$. The time and space coordinates were also normalized as $\xi\omega_{pe}/c_* \to \xi$, $t\omega_{pe} \to t$, respectively. We solved the system (5)–(6) as a nonlinear boundary value problem with the boundary conditions $P = -1$ at the left boundary $\xi = -10$, and $P = 1$ at the right boundary $\xi = 10$. The potential ϕ is set to 0 at the two boundaries. The spatial domain is numerically resolved with 2000 intervals, and the second derivatives in the system (5)–(6) are approximated by centered second-order approximations. The resulting nonlinear system of equations is then solved numerically by Newton's method. The typical parameters for semiconductor InSb plasma are chosen as [24] $E_g = 3.7 \times 10^{-20}$ J, $m_* = m_e/74$, $\varepsilon = 16\varepsilon_0$, $c_* = c/253$, $n_0 \sim 10^{20} - 10^{23}$ m^{-3}. The numerical solutions are displayed in Fig. 1. As seen in Fig. 1, the local depletions of the conduction electron densities with a multipeaked dark soliton are associated with a localized positive potential ϕ for $v_0 = 0$. With the increasing of quantum effects, the width of the multipeaked dark soliton also increases and the number of peaks of the dark soliton decreased from eight to six. But the dark number is always five. In order to investigate the moving dark soliton, it is illustrated in Fig. 2 for $v_0 = 4.47 \times 10^{-4} c_*$ and $v_0 = 0.0082 c_*$. One can see that the profile of the moving dark soliton is quite different from a static soliton. The profile of the multipeaked dark soliton has no symmetry compared to the static soliton. The electrostatic potential of the moving soliton becomes single peak. The electron density goes to zero at the center of both moving solitons and a static soliton, due to the choice of boundary conditions where the conduction electron wave function has a phase shift and changes sign at the center of the solitons.

In order to assess the dynamics and stability of the dark solitons, the time-dependent systems of Eqs. (2) and (4) are solved numerically in one dimension. Here the system of Eqs. (2) and (4) is rewritten in new dimensionless

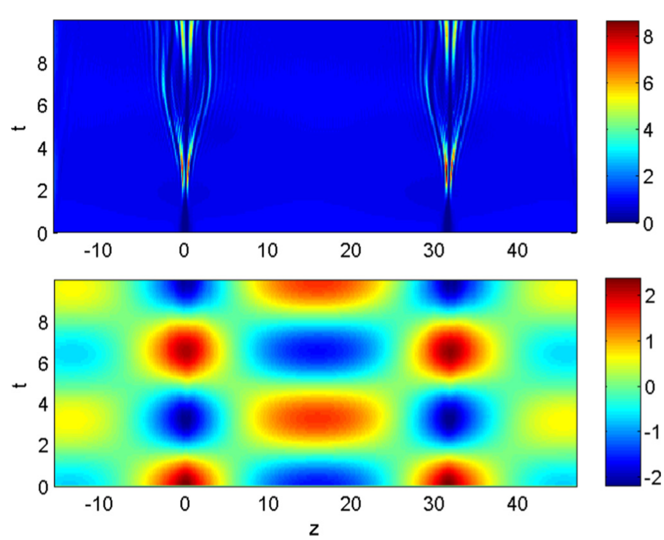

FIG. 3. (Color online) The dynamics of the dark soliton of the conduction electron quantum plasma $|\Psi|^2$ (top panels) and the scalar potential ϕ (bottom panel) for the quantum parameter $H = 0.074$.

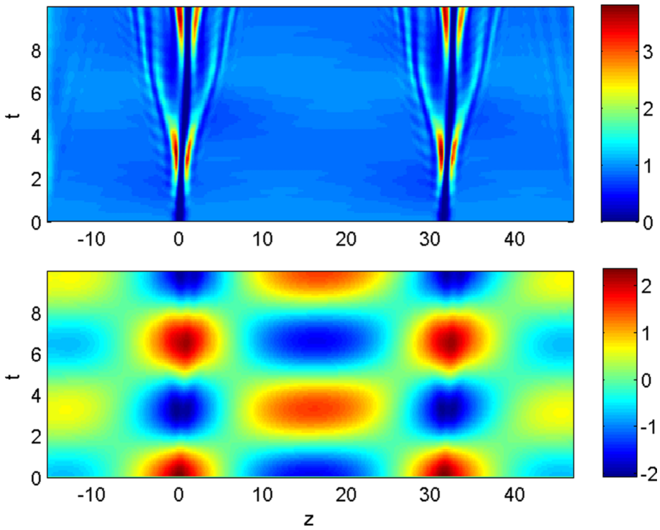

FIG. 4. (Color online) The dynamics of the dark soliton of conduction electron quantum plasma $|\Psi|^2$ (top panels) and the scalar potential ϕ (bottom panel) for the quantum parameter $H = 0.233$.

form as

$$\left(iH\frac{\partial}{\partial t} + \phi + 1\right)\Psi = W, \tag{7}$$

$$\left(iH\frac{\partial}{\partial t} + \phi + 1\right)W + H^2\frac{\partial^2\Psi}{\partial z^2} - \Psi = 0, \tag{8}$$

$$\frac{\partial^2\phi}{\partial z^2} = \frac{1}{2}(\Psi^* W + \Psi W^*) - 1. \tag{9}$$

Here in order to solve Eqs. (2) and (4) conveniently, the new wave function $\Psi(z,t)$ is introduced again by the transformations $\psi(\mathbf{r},t) = \Psi(z,t)\exp(-im_* c_*^2 t/\hbar)$, and a new quantity is introduced as $W = (iH\partial/\partial t + \phi + 1)\Psi$. For convenience, the coupled system (7)–(9) is normalized as $e\phi/m_* c_*^2 \to \phi$, $\Psi/\sqrt{n_0} \to \Psi$, and $W/\sqrt{n_0} \to W$. The time and space coordinates are also normalized as $z\omega_{pe}/c_* \to z$, $t\omega_{pe} \to t$, respectively. A pseudospectral method is used for calculating the spatial derivatives with periodic boundary conditions, and the standard fourth-order Runge-Kutta method is used to advance the solution in time. The spatial domain is from $z = -5\pi$ to $z = +15\pi$ with 1024 intervals in space. The simulation is from time $t = 0$ to $t = 10$ with the time step being $\Delta t = 0.000\,01$. The initial conditions are $\Psi = \tanh[20\sin(x/10)]$, which is consistent with the periodic boundary conditions used in the simulations. In Fig. 3, one can find that the dark soliton will become a multipeaked dark soliton after some time for the quantum parameter $H = 0.074$. The multipeaked dark soliton will appear at time $t = 3$, as the initial soliton is single dark soliton. In Fig. 4, the dynamics is illustrated for quantum parameter $H = 0.233$, where the multipeaked dark soliton will appear at time $t = 5$. The width of the dark soliton in the case of $H = 0.233$ is slightly larger than that in the case of $H = 0.074$, which is consistent with the steady soliton solution as illustrated in Fig. 1. The conduction electron plasma wake oscillations are found in both Figs. 3 and 4, as illustrated by the electrostatic potential $e\phi/m_* c_*^2$ in the lower panel of Figs. 3 and 4.

IV. SUMMARY AND CONCLUSIONS

In conclusion, the nonlinear quantum electrostatic waves in pseudorelativistic quantum semiconductor plasma are investigated by using Kane's dispersion relation of n-doped narrow-gap semiconductors, where the nonlinearity comes from the nonparabolicity of conduction electrons. By introducing the energy operator and momentum operator into the Kane's dispersion relation, the KGE is derived for an ensemble of conduction electrons standing for the combined effects of quantum and pseudorelativity, which is closed by Poisson's equation for our system. The typical parameters of semiconductor InSb are chosen for the numerical solution. The steady static and moving dark soliton are obtained by the numerical method, which shows that the multipeaked dark soliton has a symmetric profile for the static soliton, and the moving dark soliton has no symmetric properties and a more complex profile. In order to assess the dynamics and stability of the dark solitons, Eqs. (2) and (4) are numerically solved by using a single dark soliton as the initial value. The results show that a single dark soliton will become a multipeaked dark soliton after a short time and can withstand perturbations and turbulence during a considerable time. The profile of a dark soliton in dynamic simulation is consistent with the steady soliton. The present results may be useful in understanding the nonlinear properties of semiconductor plasmas on an ultrafast time scale, as femtosecond pump-probe spectroscopy developed quickly.

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