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Using trading strategies to detect phase transitions in financial markets

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We show that the log-periodic power law singularity model (LPPLS), a mathematical embodiment of positive feedbacks between agents and of their hierarchical dynamical organization, has a significant predictive power in financial markets. We find that LPPLS-based strategies significantly outperform the randomized ones and that they are robust with respect to a large selection of assets and time periods. The dynamics of prices thus markedly deviate from randomness in certain pockets of predictability that can be associated with bubble market regimes. Our hybrid approach, marrying finance with the trading strategies, and critical phenomena with LPPLS, demonstrates that targeting information related to phase transitions enables the forecast of financial bubbles and crashes punctuating the dynamics of prices.

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I. INTRODUCTION

Complex systems often exhibit nonlinear dynamics due to the interactions among their constituents. In particular, phase transitions are a very common phenomenon that emerge due to coupling, leading to positive and negative feedback loops. Examples of such systems can be found in a broad range of fields such as biology with the firing of neurons in the brain [1], ecology with the turbidity of lakes [2], sociology with the Arab Spring [3], or, as we propose, the economy with stock market bubbles and crashes [4].

The stock market is a complex system whose constituents are economic agents. They are heterogenous in size and preference (financial institutions, individual traders, etc.) and interact through a complex network topology [5]. As such, statistical stationarity and complete unpredictability of price dynamics as postulated by classical economic theory seem unlikely, as exemplified by the emergence and run-up of financial bubbles until the Global Financial Crisis of 2008 [6]. In fact, many models have been put forward to describe bubbles [7–11] or diagnose their occurrence [12–14], but quantifying their explanatory and predictive power remains an outstanding problem [15].

We propose that bubbles and their ensuing crashes can be seen as phase transitions where the behavior of the economic agents becomes synchronized through positive feedback loops, building up the bubble and eventually leading to its collapse. The crash is not the result of a new piece of information becoming available to market participants; instead, it is the result of a system close to criticality, where even a tiny perturbation is enough to reveal the large susceptibility associated with the approach to the phase transition. These concepts are captured by the log-periodic power law singularity model (LPPLS) [16,17], which has been usefully applied in the description of other physical phenomena such as earthquakes [18] or the rupture of materials [19]. In the case of the stock market, the basic mechanism generating the positive feedback loops is herding, both technical and behavioral: during a bubble regime, the action of buying the asset pushes its price up, which itself leads paradoxically to an increased demand in the asset. This process is unsustainable and inevitably leads to a change of regime, which often results in a financial crash [16].

We extend previous LPPLS-related studies [16,17], not by determining how well LPPLS can predict the time of the regime change according to some definition of a crash, but by developing a hybrid methodology combining physics and finance based on the outcome of trading strategies constructed on the log-periodic power law singularity model. Usually the province of finance, trading strategies are here proposed as genuine nonlinear transformations mapping an input time series (here a price) onto an output profit-and-loss time series that, when coupled with physical mechanism(s), may reveal novel properties of the studied system [20–22].

II. THEORY

More formally, the positive feedback process at the core of LPPLS, on which the trading strategies are built, can be described by the simple differential equation (1) capturing the market impact of excess demand fueled by growing prices:

$$\frac{dp(t)}{dt} \propto p(t)^{\delta}, \tag{1}$$

with p the price. When the exponent δ is greater than 1, it represents positive feedback of the price on the instantaneous rate of return. In this case, the solution to Eq. (1) becomes

$$p(t) \propto (t_c - t)^{-m}, \tag{2}$$

where $m = \frac{1}{\delta - 1}$. This solution is quite remarkable because of the emergence of the hyperbolic power law describing a superexponential regime ending in a finite-time singularity occurring at the movable time t_c determined by initial conditions, beyond which Eq. (2) has no solution. This can be interpreted as a change of regime, where the price dynamics go from a superexponential to something different, a crash for example. By abandoning the need to describe the full process of the bubble, followed by a crash, and the subsequent market recovery and evolution, we gain the insight that the information on the end of the bubble regime, embodied in the critical time t_c, is contained in the price dynamics itself during the bubble.

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In addition to the exuberant growth dynamics of the bubble, one can observe the existence of medium-term volatility dynamics decorating the superexponential price growth. Several mechanisms have been found to be at the origin of these structures [23], the most notable being the hierarchical organization of the network of traders [24], leading to dynamics obeying the symmetry of discrete scale invariance [25].

Combining these mechanisms with the positive feedback process and imposing that the price should remain finite leads to the so-called log-periodic power law singularity specification for the deterministic component (or expected logarithmic price) of the price dynamics [26]:

$$\ln[p(t)] = A + B(t_c - t)^m + C(t_c - t)^m \cos[\omega \ln(t_c - t) - \phi], \tag{3}$$

where $A = \ln[p(t_c)]$ is the log price at t_c, B ($B < 0$ for positive bubbles) gives the amplitude of the bubble, $0 < m < 1$, and C determines the amplitude of the oscillations. ω is the angular log frequency determining the scaling ratio of accelerating oscillations, and ϕ is a phase embodying a characteristic time scale.

III. METHODOLOGY

Because empirical time series such as prices contain many complex unknown features, using trading strategies to extract properties requires a robust null reference, which we take as random strategies, i.e., decisions to buy or sell at random. Because the random strategies are exposed to the same complex patterns as the supposedly LPPLS-informed trading strategies, they are exposed to the same bias and same idiosyncratic features. Hence, any significant performance of the LPPLS-informed strategies over the random ones signal a causal relationship between LPPLS and the price formation process. This idea can be formulated mathematically as follows: if ϕ_{LPPLS} and ϕ_{random} are, respectively, an LPPLS-based and random strategies, then the following statements are true in expectation:

$$r(\phi_{\text{LPPLS}}(M)) = r(M) \quad if\ M\ is \text{ random}, \tag{4}$$

$$r(\phi_{\text{LPPLS}}(M)) = \omega \neq r(M) \quad if\ M\ \sim \text{LPPLS}, \tag{5}$$

$$r(\phi_{\text{random}}(M)) = r(M)\ \forall M, \tag{6}$$

where M is the time series of prices of the market and r the annualized return "operator." Equations (4) and (5) follow from the martingale condition (no free lunch), and Eq. (6) translates the fact that random strategies have no skill and do not use any pattern that might be existing in the financial market time series. From Eqs. (5) and (6), it follows that if LPPLS strategies outperform random ones, the market has some structure connected to the LPPLS pattern. Trading strategies are thus used as the analogs of "spectrometers" that probe the market, where deviations from the performance of random strategies reveal the existence of information [20]. Thus only the relative performances of the LPPLS strategies are relevant within this context; their absolute performance is secondary for our purpose. (It is obviously important for would-be investors.) The strength of this methodology lies in the fact that our tests do not depend on the definition of bubbles, of crash, or of market phase transitions, or on any assumption about the underlying process. It should also be noted that the impact of any trading activity on the market dynamics falls outside of the scope of our methodology for the following reason: If a significant number of agents would start applying LPPLS-based strategies, they would possibly influence the market dynamics and it is an open question as to whether this would modify the LPPLS patterns that we aim at probing.

In order to implement LPPLS-based strategies, we first need to create a signal aggregating the information contained in the calibration of the LPPLS model to financial prices at different time scales. Not knowing *a priori* which time scales capture a potential bubble dynamic, for any given day t, we fit Eq. (3) for every interval $[t - \Delta, t]$ with $\Delta \in [20, 21,..., 500]$ days, corresponding to one business month to two business years. For each day of observation t, we fit $500 - 20 = 480$ intervals, each of them representing a different time scale. Naturally, not all the fits are relevant, especially outside of a superexponential regime. In order to distinguish those that are phenomenologically compatible with bubble regimes, we filter them according to theoretically and empirically motivated criteria. For instance, we want $m \in [0,1]$ [in Eq. (3)]: $m > 1$ would lead to a decelerating price, inconsistent with the concept of a superexponential regime, whereas $m < 0$ would lead to diverging prices, which is unrealistic. Similarly, we need $B < 0$ to filter for increasing price and $Bm > C\omega$ to ensure that the probability of a crash remains positive in the rational expectations framework [27]. The remaining criteria are based on empirical observations and are $\omega \in [6,13]$ and the *number of oscillations until t* should be larger than 2.5 [28]. The selected range of ω is motivated by general theoretical arguments that its associated scaling factor $\lambda := \exp[\frac{2\pi}{\omega}]$ should be of the order of 2 [29]. This range is also supported empirically by studying 20 well-known bubbles and crashes and using a Lomb periodogram to determine the relevant range of ω values [28]. Concerning the number of oscillations, Ref. [30] showed that the most probable number of oscillations of a geometric random walk is 1.5. Thus we impose a larger number so as to decrease the likelihood that the oscillations may be spurious. We then define our signal s_t simply as $s_t = \frac{\text{number of qualified fits}}{\text{number total fits}}$, the fraction of qualified fits, which represents the equally weighted vote among all the time scales of the strength of the superexponential regime. Figure 1 shows the signal average over the SP500 constituents (as of 2003) together with a plot of the SPY index.

Based on the signal, we build simple trading strategies and compare them with two different benchmarks consisting of different ways of randomizing our strategies.

The LPPLS-based strategies were constructed as follows: When the LPPLS signal s_t is above a threshold $\bar{s}$, we enter a short position as shown in Fig. 2. This means that we expect the price to decrease, as the LPPLS signal is detecting a strong superexponential regime that, due to its unsustainable nature, indicates an imminent correction. We close the position according to some predefined exit gain or loss ($\bar{g}$, $\bar{l}$) and minimum or maximum holding times ($h_{\min}$, $h_{\max}$). We then wait $\omega_{\min}$ days to open a new position. In practice, we explored all possible combinations of $\bar{s} \in [0.01, 0.025, 0.05, 0.075, 0.1]$,

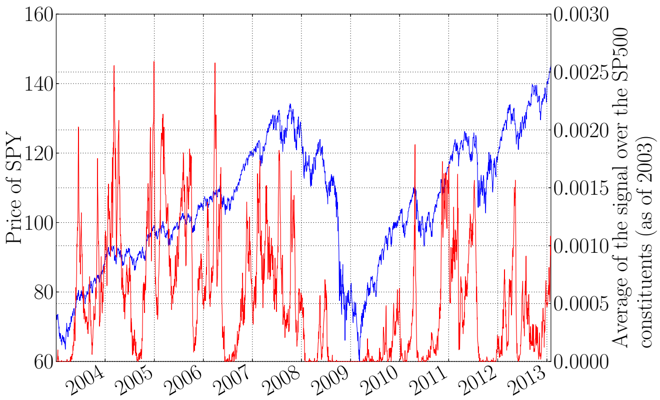

FIG. 1. (Color online) Price trajectory of the SPY (blue and left scale) together with the average of the signal over the SP500 constituents (as of 2003) (red and right scale) as a function of time.

$\bar{g} \in [1\%, 5\%, 15\%, 50\%]$, $\bar{l} \in [-1\%, -5\%, -15\%, -50\%]$, $h_{\min} \in [1,5,10]$ trading days, $h_{\max} \in [20,100,\infty]$ trading days, and $\omega_{\min} \in [1,5,10]$ trading days, i.e., 2160 strategies. The parameters were allowed to vary within broad bands to show that the difference in the outcome of the LPPLS-based versus random strategies is robust.

The random strategies were constructed in two different ways. In the first one, for each of the 2160 LPPLS strategies, the positions taken based on the LPPLS signal were shuffled.

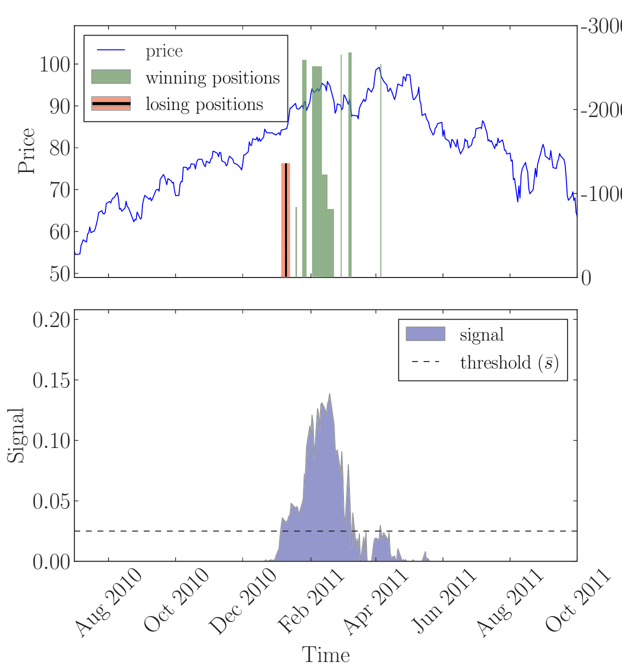

FIG. 2. (Color online) Example of a strategy applied on Deere and Co. (DE). We see a successful example of going short with a signal threshold $\bar{s}$ of 0.025, as manifested by the predominance of green bars on the upper figure. The size of the positions changes due to reallocation of resources on the other assets of the portfolio. Notice that most green bars coincide with the end of a strong price appreciation that the LPPLS signal qualifies as a mature bubble ready to burst.

In other words, for each LPPLS position (open-close pair), the entry time of the random position was picked arbitrarily and its duration was set to be the same as the LPPLS one. The upside of this method was that the duration and the number of positions were the same in both the LPPLS and random cases. Its downside was that the exit strategies were ignored, since the duration of the positions was enforced. The second randomization process consisted in shuffling the LPPLS signal (an example of which is shown on the lower panel of Fig. 2), effectively destroying its structure and applying the strategies as in the LPPLS case but on the shuffled signal. While not suffering from the downside of the first method, the number and lengths of the positions were not conserved. The two processes are complementary: shuffling the signals ensures that any difference between LPPLS and "shuffling the positions" is not solely due to the exit strategies $(\bar{g}, \bar{l}, h_{\min}, h_{\max}, \omega_{\min})$, while shuffling the positions ensures that any difference between LPPLS and "shuffling the signal" is not solely due to the difference in the number and duration of the positions between the two processes.

IV. RESULTS

Figure 3 shows the comparison between LPPLS strategies and their two random counterparts in the five-year period starting on 1 January 2008, and ending on 31 December 2012. The strategies were not applied on a single asset, but rather on a portfolio of 50 assets chosen randomly among the SP500 constituents. We see a clear difference between the performances of the LPPLS-based strategies and the random ones, in that the annualized returns of the LPPLS-based strategies are greater than those of their random counterparts, the vast majority of the points on Fig. 3 lying above the $x = y$

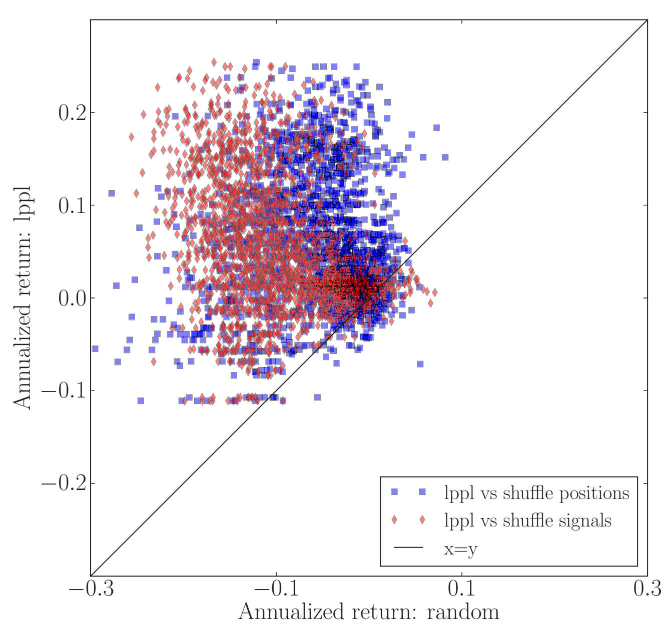

FIG. 3. (Color online) Annualized returns of the LPPLS-based strategies vs the random ones on a basket of 50 assets between 1 January 2008 and 31 December 2012. Each point represents the annualized return of a single strategy.

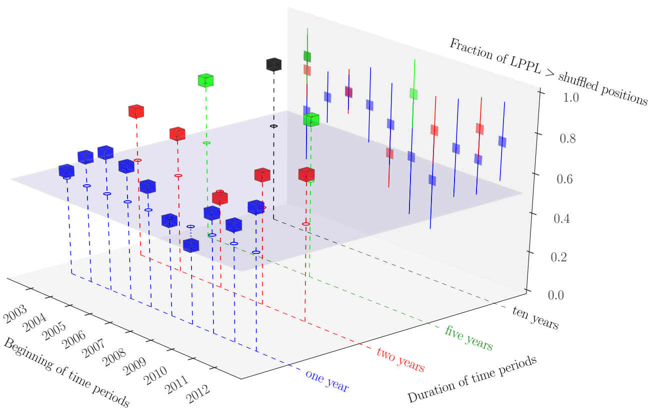

FIG. 4. (Color online) Each polygon represents the fraction of LPPLS > shuffle positions of a cloud plot of Fig. 3. The vertical bars on the yz projections represent the standard deviation of the fraction of LPPLS > shuffle positions over ten different choices of assets. In the vast majority of cases, LPPLS is above the 50% plane and thus significantly differs from shuffle positions. Although LPPLS vs shuffle signals is not shown here, the results are qualitatively the same.

line representing equal performance. Indeed, the LPPLS-based strategies outperform the outcome of the shuffled positions and shuffled signal process 92% and 96% of the time, respectively. In other words, the portfolio performance metrics of LPPLS strategies are significantly and consistently different from those of random strategies in this time period, strongly suggesting that LPPLS contains information.

Proving that LPPLS-based strategies outperform their random counterpart in a given time period is not enough to make a general statement. In fact, asset price dynamics can be very different depending on the time period chosen. For instance, during the 2008 financial crisis, the stock market went down as opposed to the 2003–2006 period. This motivated us to compare the LPPLS-based results with the random ones in different time periods. Concretely, we chose the nonoverlapping ten yearly periods, five 2-year periods, two 5-year periods, and the 10-year period from 1 January 2003 until 31 December 2012. Moreover, in order to show that the deviation from randomness was robust with respect to the choice of the 50 assets portfolio on which the 2160 strategies were applied, for each time period we ran the strategies on ten different portfolios of 50 assets chosen randomly among the SP500 constituents.

Figure 4 shows the extension of the procedure reported in Fig. 3 to all the time periods and portfolios of 50 assets described previously. The cubes represent the fraction of strategies that performed better in the LPPLS case than in the shuffled position ones in a given time period. The error bars on the yz projection result from repeating the comparison on ten different portfolios of 50 assets. We can see that LPPLS shorting clearly outperforms the outcome of the two randomized processes in most of the time periods and choices of assets, confirming LPPLS's predictive nature. However, contrary to naive expectations, the few time periods in which the LPPLS strategies perform similarly to their random counterparts is between 2007 and 2009, i.e., during the financial crash and the ensuing recession. This seemingly unintuitive behavior has a straightforward explanation. By construction, the signal is built to detect positive bubbles. However, superexponential regimes of positive price growth are by definition rare during a financial crash. As such, it is not surprising to see our signal losing its relevance during such a period. There are ways to take into account the so-called negative bubbles [31], but that is beyond the scope of this paper.

V. CONCLUSION

The statistically significant skills of our LPPLS-based strategies support the hypothesis that the dynamics of financial market prices exhibits transient regimes characterized by the approach to phase transitions revealed by the power law singularities. The present work adds to the existing sum of evidence concerning the relevance of the log-periodic power law singularity model for describing stock market bubbles [4,26,28,32–37] by providing the most robust and broadest test available until now. In our approach discussed above, we refrained from optimization of any kind and just launched a wide web of parameter sets applied to arbitrary chosen assets in a simple, fair horse race with the random strategies. While one can never prove that a model or a theory is right [38], we argue that the evidence of the present paper significantly increases our trust further in the relevance of the power law singularity view of financial markets, a hallmark of phase transitions.

In summary, we have applied a hybrid methodology to financial markets, marrying trading strategies with the log-periodic power law singularity model, which takes its roots in critical phenomena. Our results have clearly demonstrated that the outcome of LPPLS strategies persistently outperforms that of the random ones, in other words, they have predictive skills. This work supports a view in which financial markets are inherently unstable, out of equilibrium, a view dramatically opposed to the consensus in classical finance and economics.

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