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Network reciprocity created in prisoner's dilemma games by coupling two mechanisms

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We found that a nontrivial enhancement of network reciprocity for 2×2 prisoner's dilemma games can be achieved by coupling two mechanisms. The first mechanism presumes a larger strategy update neighborhood than the conventional first neighborhood on the underlying network. The second is the strategy-shifting rule. At the initial time step, the averaged cooperation extent is assumed to be 0.5. In the case of strategy shifting, an agent adopts a continuous strategy definition during the initial period of a simulation episode (when the global cooperation fraction decreases from its initial value of 0.5; that is, the enduring period). The agent then switches to a discrete strategy definition in the time period afterwards (when the global cooperation fraction begins to increase again; that is, the expanding period). We explored why this particular enhancement comes about, and to summarize, the continuous strategy during the initial period relaxes the conditions for the survival of relatively cooperative clusters, and the large strategy adaptation neighborhood allows those cooperative clusters to expand easily.

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I. INTRODUCTION

Evolutionary games, such as the prisoner's dilemma (PD), are regarded as good models for solving the mysterious puzzle of why human beings, as well as other animal species, successfully evolve cooperation instead of egocentric defection within their societies. Many papers (for comprehensive reviews, refer to [1–3]) have discussed network reciprocity, which is one of the five fundamental mechanisms that Nowak classified [4] for resolving the dilemma; it attempts to solve the dilemma by adding "ocial viscosity." This network reciprocity continues to receive a lot of attention because, although the central assumption of the model, i.e., "playing with neighbors on an underlying network and copying a strategy from them," is simple, it is nonetheless a very plausible explanation of why cooperation survives in any real context.

Recently, researchers have been concerned with identifying any additional model frameworks that would enhance network reciprocity to levels above those found in the basic spatial prisoner's dilemma game. Among the large number of previous attempts to identify an additional framework that can bring further network reciprocity, we note several outstanding ideas.

One such intriguing idea concerns what happens if a gaming neighborhood, or interaction network (IN) (in other words, the number of agents playing the game with the focal agent in the typical manner), and a strategy adaptation neighborhood, or learning network (LN) (in which the focal agent copies his or her strategy from a number of neighbors), are expanded. In most of the previous studies the IN and LN are consistent, and the first neighborhood is used on the underlying network. Xia *et al.* [5] found that appropriately selecting the size of the IN and LN to be larger than the first neighborhood bolsters network reciprocity.

Another interesting idea, proposed by Kishimoto *et al.* [6], is strategy shifting in the middle course of an evolutionary process. They found that network reciprocity is enhanced if agents are required to obey a continuous strategy, which allows a player to offer a real number defining a middle course that lies between complete defection (0) and complete cooperation (1), during a certain initial period in the evolutionary process. After this initial period, the agents refer to another rule, a discrete strategy, which is the conventional strategy, in which a player offers either complete cooperation (1 or C) or defection (0 or D).

In our recent publications, we have tried to provide a sort of holistic discussion to answer the following question: "What is the central mechanism in bringing about network reciprocity?" [7–9]. The key idea in our discussion is that we should divide an evolutionary path starting from an initially random state and proceeding to a final equilibrium into two periods, as below, and should carefully observe what happens in those two periods separately. Thus, in the present study, following Shigaki *et al.* [7], we use the term *enduring period* (END) to refer to the initial period (because cooperators try to *endure* the defectors' invasion), in which the global cooperation fraction (P_c) decreases from its initial value. As an example, the initial state may have an equal number of cooperators and defectors randomly assigned on the underlying network. Correspondingly, we use the term *expanding period* (EXP) to refer to the period following the END in which P_c increases (see Fig. 1), since cooperators who successfully survive in the END period by forming cooperative clusters (C clusters) *expand* their area by converting defectors into cooperators.

Relying on the concept of the END and EXP periods, we can guess why a LN larger than the first neighborhood enhances network reciprocity as follows (although no explanation was provided by Xia *et al.*). In the EXP period, a large LN could make more defectors that are located close to cooperative clusters switch to being cooperators than in the usual case.

Also, our concepts of the END and EXP periods can also suggest what happens in terms of strategy shifting. The time scale of strategy spreading [10] in the case of continuous strategy definition is unequivocally larger than that in the case of a discrete strategy definition, because the expected

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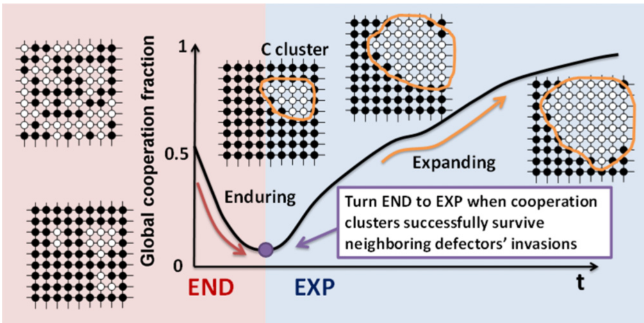

FIG. 1. (Color online) Schematic view of the evolution of cooperation in the spatial prisoner's dilemma game, with the concepts of END and EXP demonstrated. Enduring (END) period: Initial cooperators are rapidly plundered by defectors, allowing only a few cooperators to survive through the formation of compact C clusters. Expanding (EXP) period: C clusters start to expand, as a cooperator on a cluster's border can attract a neighboring defector into the cluster. We presume $P_c^{\text{initial}} = 0.5$ for this visualization.

strategy difference between two randomly selected agents is smaller in the continuous strategy setting. This fact increases the possibility of relatively cooperative clusters surviving until the END period due to a slower expansion of defectors [10], which may help realize a more cooperative equilibrium.

This study is motivated by the question of whether we can gain more significant network reciprocity if these two unique mechanisms are coupled together in one single model. In fact, we find that this bolsters the network reciprocity, and in the discussion, we outline ideas relevant to our holistic question: "What is the central mechanism in bringing about network reciprocity?" using the idea of the END and EXP periods.

This paper is organized as follows. Section II describes our model and the simulation procedure, Sec. III presents and discusses the results, and Sec. IV draws conclusions from the results.

II. MODEL SETUP

At every time step, an agent occupying a vertex on the network plays a prisoner's dilemma game with each of his or her immediate neighbors, and the agent obtains payoffs from all games. As the underlying topology, we use a two-dimensional lattice graph. The total number of agents is set to $N = 10^4$, which has been confirmed to be sufficiently large to yield simulation results that are insensitive to system size. After gaming, each agent synchronously updates his or her strategy by referring to its neighbors, defined by the LN neighborhood explained in the next section.

A. Underlying network

As we reported recently [11], expanding the LN neighborhood to bolster cooperation makes sense only in the case of a homogeneous and regular graphlike lattice. Hence, we adopt the two-dimensional (2D) lattice as our underlying topology. Relying on the so-called Moore neighborhood, the degrees of the first and second neighborhoods are $k = 8$ and 24, respectively. We vary the strategy update neighborhood (the LN) between two Moore neighborhoods: $k = 8$ and 24. It is important that the number of Moore neighborhoods that we compare to each other in this study should be limited. In summary, assuming Moore neighborhoods with $k = 8$ and 24 as well as $k = 4$ (that is, the so-called von Neumann neighborhood) is a bad idea, because the latter topology potentially has different network features. For example, the cluster coefficient of a von Neumann neighborhood with $k = 4$ is 0, which obviously differs from those of our chosen Moore neighborhoods.

B. Game description and strategy definition

In a PD game, a player receives a reward (R) for each mutual cooperation (C) and a punishment (P) for each mutual defection (D). If one player chooses C and the other chooses D, the latter obtains a temptation payoff (T), and the former is labeled a sucker (S). Without losing mathematical generality, we can define a PD game space by presuming $R = 1$ and $P = 0$ as follows:

$$\begin{pmatrix} R & S \\ T & P \end{pmatrix} = \begin{pmatrix} 1 & -D_r \\ 1 + D_g & 0 \end{pmatrix} , \tag{1}$$

where $D_g = T - R$ and $D_r = P - S$ define a chicken-type dilemma and a stag-hunt-type dilemma, respectively [12]. We limit our games to the ranges $0 \leqslant D_g \leqslant 1$ and $0 \leqslant D_r \leqslant 1$.

In a discrete strategy setting, an agent is allowed to offer either C or D. Thus, the payoff can be algebraically derived from Eq. (1).

In a continuous strategy setting, the values of the strategy and offer of an agent i are consistent, and are expressed with a real number s_i in the interval between zero and 1. When agent i plays the game with agent j, (s)he obtains a payoff given by

$$\begin{aligned} \pi(s_i, s_j) &\equiv (S-P)s_i + (T-P)s_j + (P - S - T + R)s_i s_j + P \\ &= -D_r s_i + (1 + D_g)s_j + (-D_g + D_r)s_i s_j . \end{aligned} \tag{2}$$

This might be the simplest and most plausible setup that expands the discrete strategy model but still uses the elementary payoff matrix Eq. (1).

We define the procedure of strategy shifting below. At the beginning of an evolutionary episode, each of the agents on the vertices of a lattice is given a random real number of [0,1] as the agent's strategy s_i, drawn from a uniform distribution. During the END period, in other words, during the period when P_c decreases from its initial value of 0.5, agents keep the continuous strategy system. When P_c begins to increase as the EXP period starts, following the END period, strategy shifting takes place, and all agents shift from the continuous strategy to the discrete strategy system. This implies that an agent shifts strategy system at the next time step just after the END period (in other words, the first time step in the EXP period). The actual procedure of strategy shifting is outlined below. Agents who maintain the same real value as in their continuous strategy (perhaps forming a cluster because they would originate from the same strategy seed) probabilistically take either entire cooperation C (1), or entire defection D (0), according to the continuous strategy value at that time. This is understood by comparing the snapshots of steps 6 and 7 in Fig. 6 which will be discussed later. Some of the gray highlighted clusters in step 6 shift to black clusters (entire

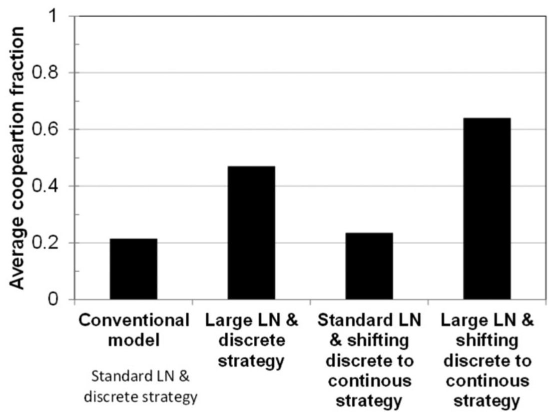

FIG. 2. Averaged cooperation fractions over all PD regions $0 \leqslant D_g \leqslant 1$ and $0 \leqslant D_r \leqslant 1$, for the following four cases: (1) the conventional model (standard IN and LN and a discrete strategy system), (2) a model assuming standard IN, large LN, and a discrete strategy system, (3) a model assuming standard IN and LN and shifting from a continuous to a discrete strategy system, and (4) our proposed model assuming standard IN, large LN, and shifting from a continuous to a discrete strategy system.

cooperation) in step 7, while others shift to white (entire defection).

C. Strategy update

The strategy of an agent, irrespective of whether it is continuous or discrete, is synchronously refreshed for every time step according to the imitation maximum (IM) system, in which the focal player i imitates the strategy with the maximum payoff among all the strategies taken by the focal player and his or her neighbors, as defined by the LN neighborhood. As mentioned above, we vary the LN, i.e., the LN refers to the first (immediate) neighborhood $k = 8$, or the second neighborhood $k = 24$.

D. Simulation procedure

Each simulation is performed as follows. Initially, a real number drawn from a uniform distribution [0,1] is randomly given to each of the N agents allocated to the different vertices of the network. Several simulation time steps, or generations, are run until the frequency of cooperation reached quasiequilibrium. If the cooperation frequency continues to fluctuate, we use the average frequency of cooperation over the last 250 generations of a 10 000-generation run. The results shown below were drawn from 100 runs, i.e., each ensemble average was formed from 100 independent simulations.

III. RESULTS AND DISCUSSION

Figure 2 shows the summarized result, where we compare the averaged cooperation fractions of all PD regions $0 \leqslant D_g \leqslant 1$ and $0 \leqslant D_r \leqslant 1$ in the following four cases: (1) the conventional model (standard IN and LN and a discrete strategy system), (2) a model assuming standard IN, large LN, and a discrete strategy system, (3) a model assuming standard IN and LN and shifting from a continuous to a discrete strategy system, and (4) our proposed model assuming standard IN, large LN, and shifting from a continuous to a discrete strategy system. The proposed model outperforms other settings including the conventional model. Note that the strategy shifting improves network reciprocity by coupling with the enlarged LN.

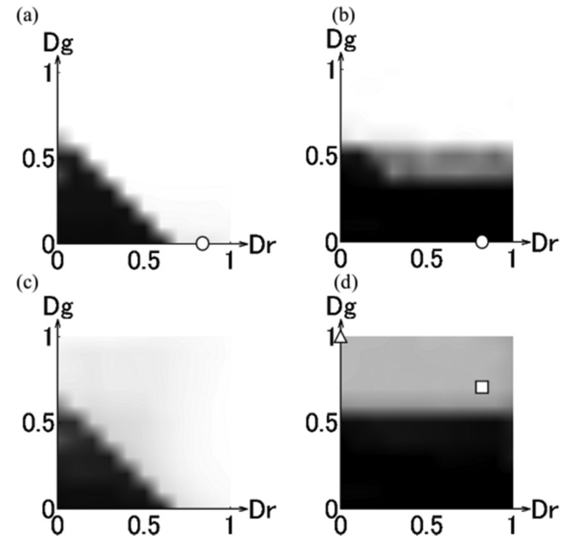

FIG. 3. Ensemble-averaged cooperation fractions over 100 realizations of PD for $0 \leqslant D_g \leqslant 1$ and $0 \leqslant D_r \leqslant 1$: (a) the conventional model (standard IN and LN and a discrete strategy system), (b) a model assuming standard IN, large LN, and a discrete strategy system, (c) a model assuming standard IN and LN and shifting from a continuous to a discrete strategy system, and (d) our proposed model assuming standard IN and large LN and shifting from a continuous to a discrete strategy system. Open circles in (a) and (b) represent $(D_g, D_r) = (0,0.8)$, for which detailed information is shown in Fig. 5. The open square and triangle in (d) respectively represent $(D_g, D_r) = (0.7,0.8)$ and (0,1), shown in more detail in Figs. 6 and 7.

Figure 3 shows the ensemble-averaged cooperation fraction of each of the four settings over 100 realizations of the PD for $0 \leqslant D_g \leqslant 1$ and $0 \leqslant D_r \leqslant 1$. Observing Fig. 3(a), the default setting, we confirm that the region of $D_g + D_r < 2/3$ shows an almost all-cooperators state; otherwise it seems to be an almost all-defectors state. This can be explained as follows. Let us define a term, the "perfect C cluster," meaning a block of nine (3×3) cooperators surrounded by defectors. Suppose we have a situation in which one single perfect C cluster is positioned in a sea of defectors, for which the global cooperation fraction is $9/N = 0.0009$. When we observe Fig. 4(a), we understand that agent $(D-1)$ neighboring the perfect C cluster most effectively exploits the neighboring cooperator and receives a payoff of $3T + 5P$. This is rewritten as $3(1 + D_g)$ by substituting into Eq. (1). The neighbor, cooperative agent $(C-1)$, is exploited by three neighboring defectors and thus earns only $5R + 3S = 5 - 3D_r$. However, one of the neighbors, agent $(C-2)$ at the center of the perfect C cluster, gains a high payoff $8R = 8$. Even if an agent is severely exploited by defecting neighbors, the IM rule compels him or her to remain cooperative as long as there is a cooperative neighbor who obtains a high payoff. For this

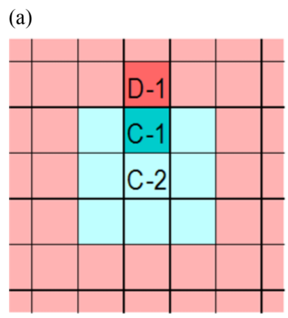

FIG. 4. (Color online) Schematic configuration in which a perfect C cluster is surrounded by defectors: (a) a perfect C cluster surrounded by defectors, (b) a concave configuration of a perfect C cluster and two additional cooperators surrounded by defectors.

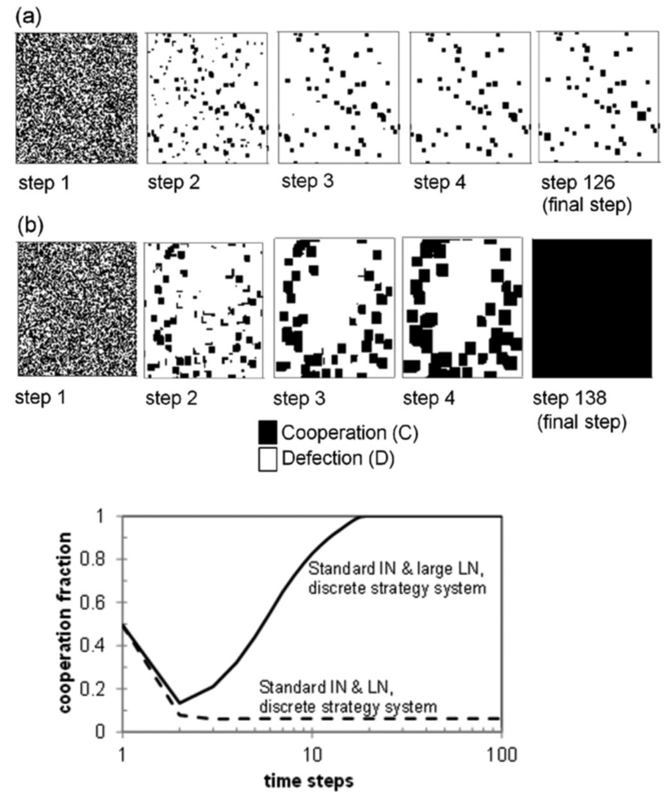

FIG. 5. Snapshots and time series of one of the representative paths of $(D_g, D_r) = (0, 0.8)$ for the model that assumes standard IN, standard LN, and a discrete strategy system (i.e., the conventional case), as shown in Fig. 3(a), and for the model that assumes standard IN, large LN, and a discrete strategy system shown in Fig. 3(b). (a) and (b) show snapshots of the conventional case and the latter case, respectively. (c) shows those time series in terms of the global cooperation fraction.

reason, we can infer that a perfect C cluster, initially placed somewhere in the domain, never perishes, as long as $D_g < 5/3$, and that a perfect C cluster can expand in the domain as long as $D_g + D_r < 2/3$. An initial random assignment of defectors and cooperators on the domain has scarcely any perfect C clusters. This is why the region of $D_g + D_r > 2/3$ in the case of Fig. 3(a) shows an almost all-defectors state. However, as shown in Fig. 3(c), introducing the concept of shifting from the continuous to the discrete strategy system can subtly improve this, thus making the region $D_g + D_r > 2/3$ a little more cooperative than in the conventional model. This is because the implementation of the continuous strategy allows partially cooperative clusters to survive until the END period due to the slower growth of defection (as well as cooperation) brought about by the real number strategy. This helps in the following way: after shifting to the discrete strategy at the beginning of the EXP period, some of the cooperative clusters can survive with higher probability than in the case of the conventional model, but it is still impossible for these clusters to expand in the EXP period. However, this improvement in network reciprocity is not significant as compared with that due to implementation of the larger LN, as shown in Figs. 3(b) and 3(d). The effect of the larger LN is phenomenal, because it can relax the condition described by $D_g + D_r < 2/3$, thus enabling a perfect C clusters to expand. Figure 4(b) suggests that, even if a defector $(D-2)$ efficiently exploits five cooperators, the defector is forced to change to cooperation by copying from a central cooperator $(C-3)$ as long as $D_g < 3/5$. This is not a necessary condition for expansion of cooperation, wherein any episodes in the region of $D_g < 3/5$ can attain an all-cooperators state, but does at least imply that even a defector exploiting five cooperators like $(D-2)$ in Fig. 4(b) may be changed to a cooperator. This is why we can observe a reasonable cooperation level, despite not attaining an almost all-cooperators state, in the region $D_g + D_r > 2/3$ and $D_g < 3/5$ in Fig. 3(b). But in Fig. 3(d), we can observe an almost all-cooperators state in the region $D_g < 3/5$; moreover, the region over that threshold shows a certain level of cooperation. This point is discussed with further insight below.

Let us observe the effect resulting from the larger LN using snapshots. Figure 5 compares a snapshot and its time evolution for one of the 100 episodes of the conventional model (standard IN and LN, with the discrete strategy system), and an alternative model (standard IN and large LN, with the discrete strategy system). The presumed dilemma for these two cases is $(D_g, D_r) = (0, 0.8)$, and the global cooperation fractions are shown in Figs. 3(a) and 3(b), respectively. These two episodes successfully survive the END period, but what happens at particular time steps, e.g., during the EXP period, is evidently different. In the conventional model, the surviving C clusters cannot expand in a sea of defectors because $D_g + D_r > 2/3$. However, in the alternative, larger-LN case, the episode can attain an all-cooperators state by the end of the simulation. This is because, as noted above, the larger LN relaxes the critical condition for cooperators expanding when surrounded by defectors.

Figures 6 and 7 also show snapshots and time series for the proposed alternative model, assuming a standard IN and large LN and shifting from the continuous to the discrete strategy system, when $(D_g, D_r) = (0.7, 0.8)$ and $(0, 1)$ are presumed, respectively. As mentioned before, the global cooperation fractions of the respective cases observed in Fig. 3(d) are both shown to be at a reasonable level, but they nonetheless differ from each other. Although the result of $(D_g, D_r) = (0.7, 0.8)$ is a relatively low level of cooperation, the result of $(D_g, D_r) = (0, 1)$ shows a relatively high cooperation level. This means

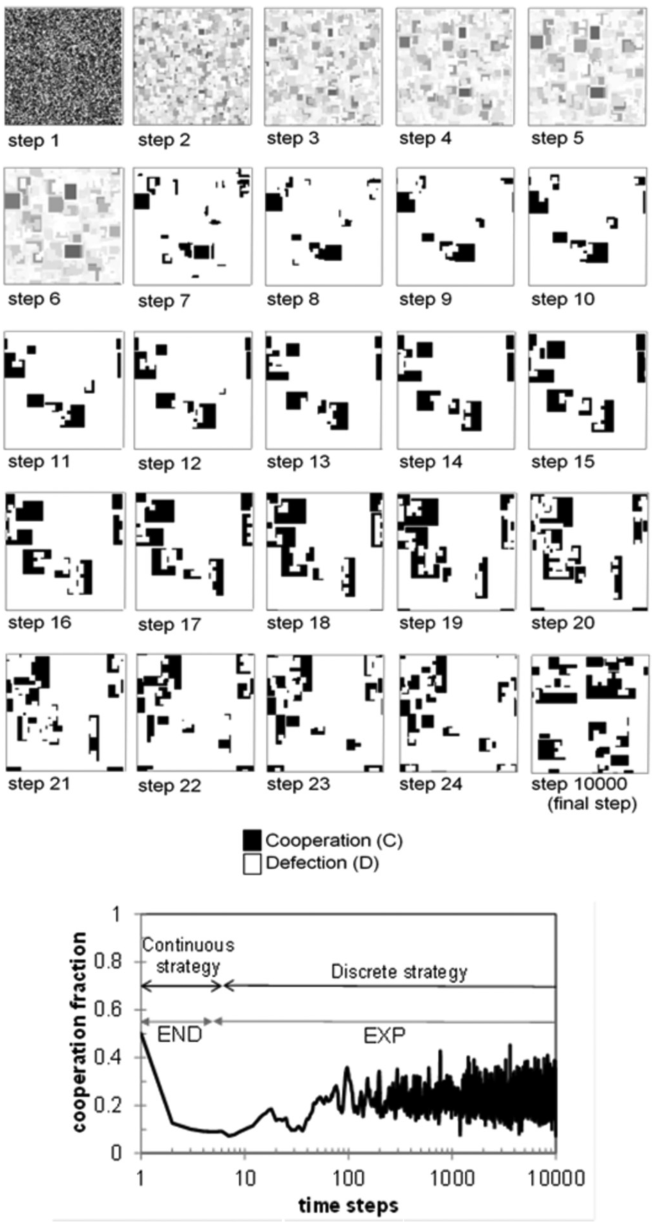

FIG. 6. Snapshots and time series of one of the representative paths of $(D_g, D_r) = (0.7, 0.8)$ by the proposed model assuming standard IN, large LN, and a shift from the continuous to the discrete strategy system, as shown in Fig. 3(d).

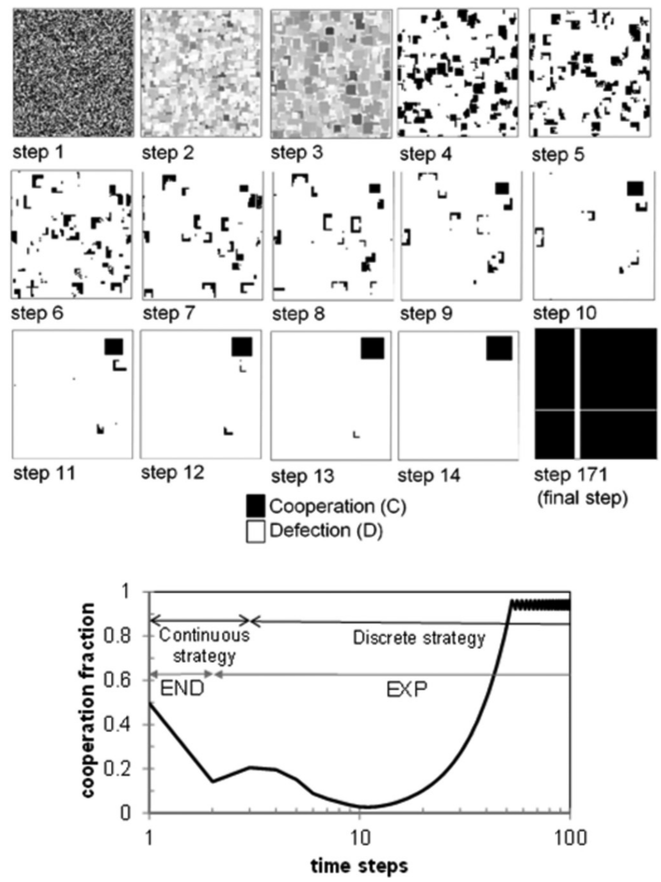

FIG. 7. Snapshots and time series of one of the representative paths of $(D_g, D_r) = (0, 1)$ for the proposed model, assuming standard IN, large LN, and a shift from the continuous to the discrete strategy system, as shown in Fig. 3(d).

that the averaged cooperation fraction shown in the region of $D_g > 3/5$ in Fig. 3(d) results from a so-called bistable (or multistable) feature, where some of the 100 realizations are absorbed by a lower cooperation level while others are attracted by a higher cooperation level. Both cases occur in the regions $D_g + D_r > 2/3$ and $D_g > 3/5$. Moreover, in both cases, fortunately, cooperators survive in the END priod due to a slower strategy-diffusion effect, which we have discussed above, and which results from a characteristic of the continuous strategy system. At the next time step after the change from END to EXP, the strategy system shifts to the discrete strategy system in both cases. This strategy shifting creates inner stress in this dynamical system, which introduces transient dynamics into the system other than the initial transient dynamics we observed at the beginning of the END period. This can be proved by the fact that the global cooperation fractions in the early stage of the EXP period in both cases fluctuate (see the respective time series of Figs. 6 and 7). In Fig. 6 especially, the fluctuation appears phenomenal. During this fluctuation process, C clusters surviving the END period undergo fissions and fusions repeatedly. Also, in the case shown in Fig. 7 alone, those cooperators happen to be merged into one single C cluster at the 14th time step. This is the start of expansion of cooperators in a monotonic way, because defectors are not able to simultaneously exploit many cooperators belonging to different C clusters; thus they are prevented from obtaining a high payoff before the high payoff even becomes possible. Conversely, in the case shown in Fig. 6, this merging of cooperators into one single C cluster does not occur. Instead, they remain spatially segregated clusters, and thus the results do not show a surge of cooperation, even though fluctuation continues for a long time. Whether or not an abrupt increase in emergence of cooperation takes place stochastically, the key point is that the inner stress brought about by the shifting of the strategy system increases fluctuation, letting cooperators merge and dissolve. Just as noise can sometimes work to improve system efficiency through the so-called resonance effect, this perturbation works to give a stochastic resonance effect. This seems analogous to what Qiu *et al.* [13] described as an "annealing" effect.

IV. CONCLUSIONS

We found a nontrivial enhancement of the network reciprocity for 2×2 prisoner's dilemma games when two different mechanisms are dovetailed. The first is to presume a larger strategy update neighborhood than the conventional assumption of the first neighborhood on the underlying network, and the second is the strategy-shifting rule, in which an agent alters from a continuous strategy during the initial period of a simulation episode (when the global cooperation fraction is decreasing from its initial value) to a discrete strategy during a later period (when the global cooperation fraction begins to increase instead).

These settings in our model can be justified using our daily observations of events in the real world, where a human behaves more prudently (by introducing smaller alterations of strategy) during the early stage of an evolutionary episode, which is more transient and dynamically chaotic than later time periods. Furthermore, with better information resources, a human may learn from indirect neighbors as well as direct ones, the latter being their opponents in mutual interactions.

Simulation reveals that our model performs well in enhancing the network reciprocity, and the mechanism behind this is explained using the concepts of the END and EXP periods we previously introduced. To summarize, the continuous strategy adopted in the initial period, i.e., the END period, allows a larger number of relatively cooperative clusters to survive, and the large strategy adaptation neighborhood allows those cooperative clusters to easily expand in the following period, i.e., the EXP period. More importantly, we found that shifting from a continuous to a discrete strategy system works to add a stochastic resonance effect that enables the survival of cooperators in the period by forming a single C cluster through repeated fission and fusion of C clusters, which finally causes a surge in the cooperation level. All our findings related to the working behind this model might help us to understand the important mechanism of network reciprocity and the cause of the emergence of cooperation in real social systems.

Concerning the limitations of the present report, we note them below. It is about underlying topology. There have been very extensive accumulations of the previous works on networks and evolutionary 2×2 games (e.g., [1,14–17]). Since we apply an expanding LN neighborhood as one of the two combined mechanisms, we must be concerned about the homogeneity of the network. Although we have assumed a $k = 8$ lattice series as representative, it might be meaningful to use a ($k = 4$)-lattice series, a ring, a kagome graph, or others. This paper is preliminary and these are directions for future work.

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