Separated spin-up and spin-down quantum hydrodynamics of degenerated electrons: Spin-electron acoustic wave appearance

Pavel A. Andreev*

Faculty of Physics, Lomonosov Moscow State University, Moscow 119991, Russian Federation

(Received 23 October 2014; revised manuscript received 4 February 2015; published 24 March 2015)

The quantum hydrodynamic (QHD) model of charged spin-1/2 particles contains physical quantities defined for all particles of a species including particles with spin-up and with spin-down. Different populations of states with different spin directions are included in the spin density (the magnetization). In this paper I derive a QHD model, which separately describes spin-up electrons and spin-down electrons. Hence electrons with different projections of spins on the preferable direction are considered as two different species of particles. It is shown that the numbers of particles with different spin directions do not conserve. Hence the continuity equations contain sources of particles. These sources are caused by the interactions of the spins with the magnetic field. Terms of similar nature arise in the Euler equation. The z projection of the spin density is no longer an independent variable. It is proportional to the difference between the concentrations of the electrons with spin-up and the electrons with spin-down. The propagation of waves in the magnetized plasmas of degenerate electrons is considered. Two regimes for the ion dynamics, the motionless ions and the motion of the degenerate ions as the single species with no account of the spin dynamics, are considered. It is shown that this form of the QHD equations gives all solutions obtained from the traditional form of QHD equations with no distinction of spin-up and spin-down states. But it also reveals a soundlike solution called the spin-electron acoustic wave. Coincidence of most solutions is expected since this derivation was started with the same basic equation: the Pauli equation. Solutions arise due to the different Fermi pressures for the spin-up electrons and the spin-down electrons in the magnetic field. The results are applied to degenerate electron gas of paramagnetic and ferromagnetic metals in the external magnetic field. The dispersion of the spin-electron acoustic waves in the partially spin-polarized degenerate neutron matter are also considered.

DOI: 10.1103/PhysRevE.91.033111 PACS number(s): 52.30.Ex

I. INTRODUCTION

Considering the quantum plasmas of spinning particles we apply equations of the quantum hydrodynamics (QHD) or the quantum kinetics. Different methods of the derivation of QHD equations were presented in Refs. [1–4], they have also been applied to the quantum plasmas of spinning particles [4–6]. These equations contain the particle concentration $n(\mathbf{r},t)$, the momentum density $\mathbf{j}(\mathbf{r},t)$, the velocity field $\mathbf{v}(\mathbf{r},t)$, and the distribution function $f(\mathbf{r},\mathbf{p},t)$ describing all particles of a species independently of their spin directions. The difference between the numbers of particles in the different spin states is included in the spin density $\mathbf{S}(\mathbf{r},t)$ or magnetization $\mathbf{M}(\mathbf{r},t) = \gamma\mathbf{S}(\mathbf{r},t)$, where γ is the gyromagnetic ratio. These models do not contain any explicit difference between the spin-up and the spin-down states of particles.

Basic equations of the many-particle quantum hydrodynamic of spin-1/2 particles were developed in Refs. [5–7]. Further development of the method can be found in Refs. [8–13]. This includes the explicit consideration of the spin-current [8] and the spin-orbit [10] interactions. Derivation of the energy evolution equation [1,6,8] and the spin-current (the magnetization flux) evolution equation [11] were performed. The exchange interaction was considered in Refs. [1,7,12]. The QHD model for particles with the electric dipole moment was developed in Ref. [13]. All these developments were performed in terms of one method: the method of many-particle quantum hydrodynamics suggested in Refs. [1,6]. Comprehensive analysis of quantum hydrodynamic equations for a single spin-1/2 particle in an external field had been performed by Takabayasi [14–20].

In the single-fluid model of electrons with different spins the dynamic of spins is governed by the generalization of Bloch equation [14]

$$n(\partial_t + \mathbf{v}\nabla)\boldsymbol{\mu} - \frac{\hbar}{2m\gamma}\partial^\beta[n\boldsymbol{\mu},\partial^\beta\boldsymbol{\mu}] = \frac{2\gamma}{\hbar}n[\boldsymbol{\mu},\mathbf{B}], \quad (1)$$

where $\boldsymbol{\mu}$ is the reduced magnetization $\mathbf{M}(\mathbf{r},t) = n\boldsymbol{\mu}$, $[\mathbf{a},\mathbf{b}]$ is the vector product of vectors **a** and **b**. The first group of terms on the left-hand side of Eq. (1) is the substantial derivative of the reduced magnetization. The second term is the quantum Bohm potential for the Bloch equation. On the right-hand side of Eq. (1) we have the torque caused by the interaction with the external magnetic field and the interparticle spin-spin interactions. In the two-fluid model the z projection of magnetization M_z is no longer an independent variable. It is proportional to the difference of concentrations of the spin-up electrons and the spin-down electrons. Other projections of the magnetization M_x and M_y appear in two-fluid model as independent variables, but they do not wear indexes "up" or "down" being related to both species of electrons. This happens because the definitions of M_x and M_y contain the wave functions of the spin-up and spin-down electrons.

For the development of the field of quantum plasma [10,21–23] it is interesting to derive a set of QHD equations for the degenerate electrons considering two different spin states (spin-up and spin-down) as two different species of particles. In

*andreevpa@physics.msu.ru

this paper we perform a derivation of QHD equation explicitly distinguishing the spin-up states and the spin-down states.

An attempt to consider the hydrodynamic model of quantum particles with the separated spin-up and spin down evolution has been made [24]. This model has found a number of applications [25–29]. Unfortunately, this model does not have any justification, and it is in contradiction with the model we directly derive, in this paper, from the Pauli equation.

Different linear and nonlinear excitations were considered in the quantum plasmas [21,22]. In this paper we also focus our attention on the linear excitations in the magnetized quantum plasmas of the degenerate electrons in terms of new form of the QHD model. Basic linear phenomena in the spin-1/2 quantum plasmas were considered in Refs. [9,10,30–33], where the contribution of spins in the dispersion of plasma waves was found and the existence of spin-plasma waves was demonstrated. Electrons were considered as a single fluid in these papers. We are going to find out that changes at application of the spin-separated QHD.

This paper is organized as follows. In Sec. II we derive the QHD model considering the spin-up electrons and the spin-down electrons as different species. In Sec. III we consider the propagation of waves parallel to the external magnetic field as an illustration of derived equations. This problem has been solved in literature in term of usual QHD. We compare results of two different methods of fluidization of the Pauli equation. In Sec. IV we consider the contribution of the ion motion into the properties of the spin-electron acoustic waves (SEAWs) discovered in Sec. III. In Sec. V we study the spin-electron acoustic waves in the neutron matter. In Sec. VI a brief summary of obtained results is presented.

II. MODEL

In this section we are going to derive the set of QHD equations for the degenerate electrons considering the spin-up states and the spin-down states as two different species. This derivation can be performed in terms of the many-particle quantum hydrodynamics [5–13]. However, for simplicity of presentation, we consider the Pauli equation for a single particle in an external electromagnetic field following papers of Takabayasi [14–20]. We should also notice that the set of basic QHD equations for charged spinning particles considered in the self-consistent field approximation almost coincide with the single-particle one [34]. This coincidence has been actively used over last decade (see, for instance, Refs. [22,30,31,34]).

Recently Krishnaswami *et al.* [35] and Melrose and Weise [36] wisely pointed out that the theory of quantum plasmas should be grounded on the many-particle methods instead of being recaptured from the evolution of separated particles. In the last case one drops the statistical many-particle effects such as the Fermi pressure and properties of symmetry of the full N particle wave function. In single particle picture these effects can be partially restored by hand as it was done in Refs. [30,31]. Thus we should stress attention of readers that we follow the many-particle formalism called the many-particle QHD [5–13]. And only reason why we apply, in this paper, the single-particle Pauli equation is the simplification of presentation.

Thus we start with the Pauli equation

$$\imath\hbar\partial_t\psi = \left(\frac{\left(\frac{\hbar}{\imath}\nabla - \frac{q_e}{c}\mathbf{A}\right)^2}{2m} + q_e\varphi - \gamma_e\widehat{\boldsymbol{\sigma}}\mathbf{B}\right)\psi \tag{2}$$

governing the evolution of the spinor wave function $\psi(\mathbf{r},t)$. In Eq. (2) $\varphi = \varphi_{\text{ext}}$, $\mathbf{A} = \mathbf{A}_{\text{ext}}$ are the scalar and vector potentials of the external electromagnetic fields, $\mathbf{B} = \mathbf{B}_{\text{ext}}$ is the external magnetic field, $q_e = -e$ is the charge of electron, m is the mass of the particle under consideration, γ_e is the gyromagnetic ratio of electron, ∇ is the gradient operator, $\boldsymbol{\sigma}$ is the vector of Pauli matrices, $\hbar$ is the reduced Planck constant, and c is the speed of light.

Let us present the explicit form of the Pauli matrices

$$\widehat{\sigma}_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \widehat{\sigma}_y = \begin{pmatrix} 0 & -\imath \\ \imath & 0 \end{pmatrix}, \quad \widehat{\sigma}_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{3}$$

The commutation relation for spin-1/2 matrices is

$$[\widehat{\sigma}^\alpha,\widehat{\sigma}^\beta] = 2\imath\varepsilon^{\alpha\beta\gamma}\widehat{\sigma}^\gamma. \tag{4}$$

$\rho = \psi^+\psi$ is the probability density to find the particle in a point $\mathbf{r}$ regardless its spin, where ψ^+ is the Hermitian conjugated wave function.

The spinor wave function ψ can be presented as

$$\psi = \begin{pmatrix} \psi_\uparrow \\ \psi_\downarrow \end{pmatrix}. \tag{5}$$

Applying the wave functions describing spin-up $\psi_\uparrow$ and spin-down $\psi_\downarrow$ states we can write the probability density to find the particle in a point $\mathbf{r}$ with spin-up $\rho_\uparrow = |\psi_\uparrow|^2$ or spin-down $\rho_\downarrow = |\psi_\downarrow|^2$. We also see $\rho = \rho_\uparrow + \rho_\downarrow$. Directions up $\uparrow$ (down $\downarrow$) corresponds to spins having same (opposite) direction as (to) the external magnetic field. While the magnetic moments have the direction opposite to the spin directions.

In many-particle systems we have the concentration of particles $n(\mathbf{r},t)$, which is proportional to the probability density to find each particle in the point $\mathbf{r}$, hence we have $n_\uparrow = \langle\rho_\uparrow\rangle$ and $n_\downarrow = \langle\rho_\downarrow\rangle$. Full concentration of particles in the sum of the particle concentrations with spin-up and spin-down $n = n_\uparrow + n_\downarrow$. The spin density S_z of electrons is the difference between the concentrations of electrons with different projection of spin $S_z = n_\uparrow - n_\downarrow$. Its definition is $S_z = \psi^+\sigma_z\psi$. We have that the z projection of the spin density S_z is not an independent variable in this representation of the quantum hydrodynamics.

We can derive equations for $\rho_\uparrow$, and $\rho_\downarrow$. They are analogous to the continuity equations, but the number of particles with the different spin projection (or corresponding probability for a single particle) are not constants.

Let us rewrite the Pauli Eq. (2) in more explicit form

$$\imath\hbar\partial_t\psi_\uparrow = \left(\frac{\left(\frac{\hbar}{\imath}\nabla - \frac{q_e}{c}\mathbf{A}\right)^2}{2m} + q_e\varphi - \gamma_e B_z\right)\psi_\uparrow - \gamma_e(B_x - \imath B_y)\psi_\downarrow, \tag{6}$$

and

$$\imath\hbar\partial_t\psi_\downarrow = \left(\frac{\left(\frac{\hbar}{\imath}\nabla - \frac{q_e}{c}\mathbf{A}\right)^2}{2m} + q_e\varphi + \gamma_e B_z\right)\psi_\downarrow - \gamma_e(B_x + \imath B_y)\psi_\uparrow. \tag{7}$$

The spin-up and spin-down directions are related to a preferable direction in space. If we have an uniform external magnetic field its direction can be taken as preferable direction. In this case only the z projection of the magnetic field B_z enters the Pauli equation for a single particle in the external magnetic field. However we are going to apply corresponding QHD equations for plasma description, where motion of charges and spin evolution create B_x and B_y. An example of the existence of nonzero B_x and B_y for a single particle in external field is the presence of a weak electromagnetic wave propagating parallel to the z direction.

Considering the time evolution of the probability densities $\rho_\uparrow$ and $\rho_\downarrow$ we derive the continuity equations

$$\partial_t n_\uparrow + \nabla(n_\uparrow \mathbf{v}_\uparrow) = \frac{\gamma_e}{\hbar}(B_y S_x - B_x S_y), \tag{8}$$

and

$$\partial_t n_\downarrow + \nabla(n_\downarrow \mathbf{v}_\downarrow) = \frac{\gamma_e}{\hbar}(B_x S_y - B_y S_x), \tag{9}$$

where we have applied S_x and S_y for mixed combinations of $\psi_\uparrow$ and $\psi_\downarrow$. Their explicit form is presented and discussed below.

Usually the continuity equation shows the conservation of the particle number. If we consider the spin-up electrons and the spin-down electrons separately, we find that the particle numbers change due to the interaction. The total number of electrons $N = N_\uparrow + N_\downarrow$ conserves only.

Particle current appears in the continuity equation in usual form $\mathbf{j}_s = \frac{1}{2m}(\psi_s^* \mathbf{D}\psi_s + \text{c.c.})$, where $s = \uparrow$ or $\downarrow$, and $\mathbf{D} = \widehat{\mathbf{p}} - \frac{q_e}{c}\mathbf{A}$. We have introduced the velocity fields $\mathbf{v}_s$ via the particle currents $\mathbf{j}_s \equiv n_s \mathbf{v}_s$, with the following explicit form of the velocities $\mathbf{v}_s = \frac{\hbar}{m}\nabla\phi_s - \frac{q_e}{mc}\mathbf{A}$. Here we have applied the phase of wave function $\psi_s = a_s e^{\imath\phi_s}$.

Considering the time evolution of the particle currents for each projection of spin $\mathbf{j}_\uparrow$ and $\mathbf{j}_\downarrow$ we can derive corresponding Euler equations

$$\begin{aligned} m n_\uparrow(\partial_t + \mathbf{v}_\uparrow\nabla)\mathbf{v}_\uparrow + \nabla p_\uparrow - \frac{\hbar^2}{4m} n_\uparrow \nabla\left(\frac{\Delta n_\uparrow}{n_\uparrow} - \frac{(\nabla n_\uparrow)^2}{2n_\uparrow^2}\right) \\ = q_e n_\uparrow\left(\mathbf{E} + \frac{1}{c}[\mathbf{v}_\uparrow,\mathbf{B}]\right) + \gamma_e n_\uparrow \nabla B_z \\ + \frac{\gamma_e}{2}(S_x\nabla B_x + S_y \nabla B_y) \\ + \frac{m\gamma_e}{\hbar}(\mathbf{J}_{(M)x}B_y - \mathbf{J}_{(M)y}B_x) - m\mathbf{v}_\uparrow\frac{\gamma_e}{\hbar}(B_y S_x - B_x S_y), \end{aligned} \tag{10}$$

and

$$\begin{aligned} m n_\downarrow(\partial_t + \mathbf{v}_\downarrow\nabla)\mathbf{v}_\downarrow + \nabla p_\downarrow - \frac{\hbar^2}{4m} n_\downarrow \nabla\left(\frac{\Delta n_\downarrow}{n_\downarrow} - \frac{(\nabla n_\downarrow)^2}{2n_\downarrow^2}\right) \\ = q_e n_\downarrow\left(\mathbf{E} + \frac{1}{c}[\mathbf{v}_\downarrow,\mathbf{B}]\right) - \gamma_e n_\downarrow \nabla B_z \\ + \frac{\gamma_e}{2}(S_x\nabla B_x + S_y \nabla B_y) \\ + \frac{m\gamma_e}{\hbar}(\mathbf{J}_{(M)y}B_x - \mathbf{J}_{(M)x}B_y) - m\mathbf{v}_\downarrow\frac{\gamma_e}{\hbar}(B_x S_y - B_y S_x), \end{aligned} \tag{11}$$

with the following explicit form of the spin current

$$\mathbf{J}_{(M)x} = \frac{1}{2}(\mathbf{v}_\uparrow + \mathbf{v}_\downarrow)S_x - \frac{\hbar}{4m}\left(\frac{\nabla n_\uparrow}{n_\uparrow} - \frac{\nabla n_\downarrow}{n_\downarrow}\right)S_y, \tag{12}$$

and

$$\mathbf{J}_{(M)y} = \frac{1}{2}(\mathbf{v}_\uparrow + \mathbf{v}_\downarrow)S_y + \frac{\hbar}{4m}\left(\frac{\nabla n_\uparrow}{n_\uparrow} - \frac{\nabla n_\downarrow}{n_\downarrow}\right)S_x, \tag{13}$$

where $q_e = -e$, $\gamma_e = -g\frac{e\hbar}{2mc}$ is the gyromagnetic ratio for electrons, and $g = 1 + \alpha/(2\pi) = 1.00116$, where $\alpha = 1/137$ is the fine structure constant, which takes into account the anomalous magnetic moment of electron. $\mathbf{J}_{(M)x}$ and $\mathbf{J}_{(M)y}$ are elements of the spin current tensor $J^{\alpha\beta}$.

Most of terms in the Euler equations (10) and (11) have traditional meaning. The first group of terms on the left-hand sides of the Euler equations are the substantial time derivatives of the velocity fields $\mathbf{v}_\uparrow$ and $\mathbf{v}_\downarrow$. The second terms are the gradients of the thermal pressures or the Fermi pressures. They do not appear from the single-particle Pauli equation, but we have included it assuming that the many-particle QHD gives this effect [1,6,8]. The next group of terms, proportional to the square of the Plank constant, are the contributions of the quantum Bohm potential.

The right-hand sides of the Euler equations present the interaction force fields. The first groups of terms on the right-hand side are the Lorentz forces. Since we consider two species of electrons these forces have the same structure, with no explicit dependence on the spin direction. The implicit dependence is presented via the subindexes of the concentrations and velocity fields. The second terms describe the action of the z projection of magnetic field on the magnetic moments (spins) of particles. Dependence on the spin projection reveals in different signs before these terms. The third groups of terms in the Euler equations contain a part of well-known force field $\mathbf{F}_S = M^\beta \nabla B^\beta$ describing the action of the magnetic field on the magnetic moments [5,14]. Part of this force field has been presented by previous terms $\mathbf{F}_{S(z)} = \pm\gamma_e n_{\uparrow,\downarrow}\nabla B_z$. The second part of the force field is $\mathbf{F}_{S(x,y)} = \gamma_e(S_x\nabla B_x + S_y\nabla B_y)$. The half of this force field enters each of the Euler equations. The last two groups of terms is related to nonconservation of particle number with different spin-projection. This nonconservation gives extra mechanism for change of the momentum density revealing in the extra force fields. The last term in Eq. (10) [Eq. (11)] appears due to application of the continuity Eq. (8) [Eq. (9)] to the following terms $\partial_t(mnv^\alpha) + \partial^\beta(mnv^\alpha v^\beta)$ arising at the derivation of hydrodynamic equations.

Here we describe an explicit form of the spin density projections on x and y axes. We have used notations S_x and S_y in Eqs. (8)–(11). These quantities appear as follows $S_x = \psi^*\sigma_x\psi = \psi_\downarrow^*\psi_\uparrow + \psi_\uparrow^*\psi_\downarrow = 2a_\uparrow a_\downarrow\cos\Delta\phi$, $S_y = \psi^*\sigma_y\psi = \imath(\psi_\downarrow^*\psi_\uparrow - \psi_\uparrow^*\psi_\downarrow) = -2a_\uparrow a_\downarrow \sin\Delta\phi$, where $\Delta\phi = \phi_\uparrow - \phi_\downarrow$. S_x and S_y appear as mixed combinations of $\psi_\uparrow$ and $\psi_\downarrow$. These quantities do not related to different species of electrons having different spin direction. S_x and S_y describe the simultaneous evolution of both species.

S_x and S_y enter Eqs. (8)–(11). We need to derive equations for these quantities to get a closed set of QHD equations. Differentiating the explicit forms of S_x and S_y and applying the Pauli equation (6) and (7) for the time derivatives of the wave functions $\psi_\uparrow$ and $\psi_\downarrow$ we obtain the following equations

$$\partial_t S_x + \frac{1}{2}\nabla[S_x(\mathbf{v}_\uparrow + \mathbf{v}_\downarrow)] - \frac{\hbar}{4m}\nabla\left(S_y\left(\frac{\nabla n_\uparrow}{n_\uparrow} - \frac{\nabla n_\downarrow}{n_\downarrow}\right)\right) = \frac{2\gamma_e}{\hbar}(B_z S_y - B_y(n_\uparrow - n_\downarrow)), \quad (14)$$

and

$$\partial_t S_y + \frac{1}{2}\nabla[S_y(\mathbf{v}_\uparrow + \mathbf{v}_\downarrow)] + \frac{\hbar}{4m}\nabla\left(S_x\left(\frac{\nabla n_\uparrow}{n_\uparrow} - \frac{\nabla n_\downarrow}{n_\downarrow}\right)\right) = \frac{2\gamma_e}{\hbar}(B_x(n_\uparrow - n_\downarrow) - B_z S_x). \quad (15)$$

The first term in Eq. (14) [Eq. (15)] is the time derivative of S_x (S_y). The second terms in these equations are gradients of the spin fluxes. The third terms are the quantum Bohm potential revealing the quantum part of the gradients of the spin fluxes. The right-hand sides of Eqs. (14) and (15) contain the torque caused by the interaction of the magnetic moments with the magnetic field. The right-hand sides of these equations correspond to the traditional form. For instance, let us consider the torque in equation for S_x, which is $T_x = \frac{2\gamma_e}{\hbar}(S_y B_z - S_z B_y) = \frac{2\gamma_e}{\hbar}(S_y B_z - (n_\uparrow - n_\downarrow)B_y)$, that coincides with the right-hand side of Eq. (14).

Let us mention that S_x and S_y do not wear subindexes $\uparrow$ and $\downarrow$. As we can see from the definitions of S_x and S_y they are related to both projections of electron spin simultaneously: spin-up $\psi_\uparrow$ and spin-down $\psi_\downarrow$.

Electromagnetic fields in the QHD equations presented above obey the Maxwell equations

$$\nabla\mathbf{E} = 4\pi(en_i - en_{e\uparrow} - en_{e\downarrow}), \quad (16)$$

$$\nabla\mathbf{B} = 0, \quad (17)$$

$$\nabla\times\mathbf{E} = -\frac{1}{c}\partial_t\mathbf{B}, \quad (18)$$

and

$$\nabla\times\mathbf{B} = \frac{1}{c}\partial_t\mathbf{E} + \frac{4\pi}{c}\sum_{a=e,i}(q_a n_{a\uparrow}\mathbf{v}_{a\uparrow} + q_a n_{a\downarrow}\mathbf{v}_{a\downarrow}) + 4\pi\sum_{a=e,i}\nabla\times\mathbf{M}_a, \quad (19)$$

where $\mathbf{M}_a = \{\gamma_a S_{ax}, \gamma_a S_{ay}, \gamma_a(n_{a\uparrow} - n_{a\downarrow})\}$ is the magnetization of electrons in terms of hydrodynamic variables.

A. Equation of state

We need to get a closed set of equations, so we should use an equation of state for the pressure of spin-up $p_\uparrow$ and spin-down $p_\downarrow$ electrons. We consider the degenerate electrons. Hence, in the nonrelativistic case, we have

$$p_s = \frac{(6\pi^2)^{2/3}}{5}\frac{\hbar^2}{m}n_s^{5/3}. \quad (20)$$

From this equation of state we find $\frac{\partial p_s}{\partial n_s} = \frac{(6\pi^2)^{2/3}}{3}\frac{\hbar^2}{m}n_s^{2/3}$ giving the contribution in the Euler equation via $\nabla p_s = \frac{\partial p_s}{\partial n_s}\nabla n_s$. Here we see that the equations of state for the spin-up electrons and the spin-down electrons are different due to the presence of the external magnetic field, which changes an equilibrium concentration of each species $n_{0\uparrow} \neq n_{0\downarrow}$. We have included that only one particle with a chosen spin direction can occupy one quantum state. As a consequence we have $(6\pi^2)^{2/3}$ instead of $(3\pi^2)^{2/3}$ appearing in the Fermi pressure. At the derivation of the Fermi pressure one assumes that two particles with different spin directions could occupy a quantum state, but we now consider spin-up and spin-down electrons as different species.

We show below that the difference between $p_\uparrow$ and $p_\downarrow$ existing due to the difference of $n_\uparrow$ and $n_\downarrow$ leads to effects in quantum plasmas. One of these effects is the appearance of a wave, which we call the spin-electron acoustic wave.

Interactions of magnetosonic waves in spin-1/2 degenerate quantum plasmas have been recently considered in Ref. [37] in terms of quantum magnetohydrodynamics. In Ref. [38] the quantum magnetohydrodynamics was applied as well. Let us mention that the magnetohydrodynamics is a very useful tool, where electron-ion plasmas are considered as a single liquid, whereas we move in the opposite direction developing a many-liquid model for electrons.

Considering the quantum spin-1/2 plasmas, researchers usually apply the equation of state for unpolarized electrons

$$p_{\text{unpol}} = \frac{(3\pi^2)^{\frac{2}{3}}}{5}\frac{\hbar^2}{m}n^{\frac{5}{3}}, \quad (21)$$

see for instance Refs. [39,40], however, this does not include the effect of the external magnetic field on the equation of state. In Ref. [28] the authors use another equation of state, but they give no change in the problem under consideration.

In Ref. [29] the author presented an attempt to consider a two-fluid model of electrons, which treats the spin-up and spin-down populations relative to the magnetic field as different species, to obtain the permittivity tensor of plasmas, which appears to be incomplete. Moreover, the equation of state for unpolarized single-liquid electrons (21) was used there. In Ref. [29], the author followed the model presented in Ref. [24], which does not fit equations we have directly derived from the Pauli equation.

Obtained hydrodynamic equations (8)–(19), together with the equations of state (20), are aimed to consider degenerate electron gas of paramagnetic and ferromagnetic metals. Simple modification can also allow us to consider the two-dimensional electron gas, or the electron-hole gas of semiconductors.

Considering the low-frequency limit we deal with the frequencies near the ion Langmuir frequency. In this region the ion-acoustic wave transforms into the ion-plasma wave. Both these waves are parts of the one branch of the dispersion dependence, but they have different mechanisms of propagation. While the frequencies of the ion-acoustic wave lie below the ion Langmuir frequency, the frequencies of the ion-plasma wave are above the ion Langmuir frequency [41].

Regarding the exchange of momentum between two systems of electrons at the collisions of electrons we need to mention the following: Scattering of the electrons with the same spin projection has zero amplitude of scattering in a system of degenerate electrons since all states with momentum between 0 and $(6\pi^2 n_{0s})^{\frac{1}{3}}\hbar$ are occupied.

The Fermi step of the spin-up electrons is shorter than the Fermi step of the spin-down electrons. In spite this fact we do not have any contribution of outer-species collisions. If we have two electrons with energies lower than the Fermi energy of the spin-up electrons our conclusion is obvious. Let us illustrate our conclusion for the collision of the spin-down electron with energy in the interval between the Fermi energies of spin-up electrons and the Fermi energy of spin-down electrons, while energy of the spin-up electron is below than the Fermi energy of the spin-up electrons. This collision can increase the energy of the spin-up electron, and we have many unoccupied states with energies $E > E_{\mathrm{Fe}\uparrow}$, where $E_{\mathrm{Fe}\uparrow}$ is the Fermi energy for the spin-up electrons, $E_{\mathrm{Fe},s} = (6\pi^2)^{\frac{2}{3}}\hbar^2 n_s^{\frac{2}{3}}/2m$. However, in this process the spin-down electron would decrease its energy, while all spin-down states with the lower energies are occupied. Hence this channel of collisions is also blocked. Therefore we conclude that we do not have the exchange of the momentum between two systems of electrons with different spin projection during collisions.

The collision process can play a role in the momentum exchange between electron species at the larger temperatures comparable with the Fermi temperatures. However, we do not consider this regime in this paper.

Vranjes *et al.* [42,43] expressed concerns about agreement between the areas of application of the quantum hydrodynamic equations and the areas where they were applied. This is a reasonable remark for a large number of papers. Therefore, let us describe the area of application of our results. In this paper we consider degenerate electrons in metals at room temperature and below $T \in (10,300)$ K, since at temperatures below 10 K metals demonstrate the superconductivity of electrons. Concentrations of electrons in metals are about $10^{21} \div 10^{23}$ cm^{-3}. Corresponding Fermi temperatures $T_{\mathrm{Fe}} = k_B(3\pi^2 n_e)^{\frac{2}{3}}\hbar^2/(2m_e)$ are about $3.6 \div 77.6\ 10^3$ K, where k_B is the Boltzmann constant. Hence we have electrons well below the degeneracy limit $T \ll T_{\mathrm{Fe}}$. The low-temperature limit minimizes deformation of the Fermi-step distribution due to finite temperatures, while the external magnetic field leads to additional change of occupation of the quantum states by the spin-up and spin-down electrons.

We consider quantum plasmas of degenerate electrons, where the concentration is small enough to drop the contribution of the quantum Bohm potential. Meanwhile, the notion of quantum plasmas is frequently used in the narrower meaning of the study of the contribution of the quantum Bohm potential.

The hydrodynamic structure of equations for the quantum collective variables directly follows from the many-particle Schrödinger or Pauli equations with no particular limits of the space or time scale [1,6,13,44]. Nevertheless, in our paper, we apply the self-consistent field approximation, which corresponds to the long-range interaction between particles. The range of the Coulomb interaction in plasmas is restricted by the Debye screening. In the classic plasmas the potential of a chosen motionless charge q surrounded by plasmas decays as $\varphi(r) = qe^{-r/r_D}/r$, where $r_D = v_{\mathrm{Te}}/\omega_{\mathrm{Le}}$ is the Debye radius. The Debye radius characterizes the area of action of the charge. The Debye radius should be much larger than the average interparticle distance $1/\sqrt[3]{n_0}$ to bring the collective effects.

In the degenerate plasmas, on the other hand, we do not have the fast exponential decay of the potential. Calculation of the static response of the degenerate plasmas on the electric field created by the small charge q presented in literature (see for instance the Landau textbook [45], Sec. 40, formula 40.23) gives

$$\varphi(r) = \frac{q\alpha\hbar^2}{2\beta^2\tilde{p}_{\mathrm{Fe}}^2}\frac{\cos\left(\frac{2\tilde{p}_{\mathrm{Fe}}r}{\hbar}\right)}{r^3}, \tag{22}$$

where $\tilde{p}_{\mathrm{Fe}} = (3\pi^2 n_0)^{\frac{1}{3}}\hbar$ is the Fermi momentum, $\alpha = me^2/(2\pi\hbar\tilde{p}_{\mathrm{Fe}})$, $\beta = 1+\alpha$. Formula (22) shows the slow decay of the potential as $1/r^3$ combined by the space oscillation. It reveals a rather large area of interaction of the charge with the surrounding charges and applicability of the self-consistent field approximation.

III. PERTURBATION EVOLUTION

Interest in the spin contribution in the properties of plasmas has appeared since the many-particle quantum hydrodynamics of spin-1/2 particles was derived [5,46]. Many results have been obtained (see review papers [21,22,47]), but we should especially mention Refs. [9,10,32,33], where some interesting effects were found in the linear regime on the small perturbations in the magnetized plasmas. It was shown that the spin evolution leads to the existence of new wave solutions. There are two types of spin excitations in quantum plasmas propagating by means of the perturbations of the electric field [32,33], and by means of the perturbations of the magnetic field with no electric field involved in it (the quasimagnetostatic regime) [9,10,32].

Some recent research reveals new linear wave solutions. Most of them are related to spin evolution [10,32,33]. In addition, a longitudinal solution, which is called the positron sound wave, was found in Ref. [48]. In this section we present a longitudinal wave in degenerate electrons moving on a background of motionless ions, which we call the spin-electron acoustic wave.

Here we consider the propagation of waves parallel to the external field. This includes the consideration of spin-plasma waves propagating by means perturbations of the electric field [32,33].

Equilibrium condition is described by the nonzero concentrations $n_{0\uparrow}$, $n_{0\downarrow}$, $n_0 = n_{0\uparrow} + n_{0\downarrow}$, and the external magnetic field $\mathbf{B}_{\mathrm{ext}} = B_0\mathbf{e}_z$. Other quantities equal to zero $\mathbf{v}_{0\uparrow} = \mathbf{v}_{0\downarrow} = 0$, $\mathbf{E}_0 = 0$, $S_{0x} = S_{0y} = 0$. Assuming that the perturbations are monochromatic

$$\begin{pmatrix}\delta n_\uparrow\\ \delta n_\downarrow\\ \delta\mathbf{v}_\uparrow\\ \delta\mathbf{v}_\downarrow\\ \delta\mathbf{E}\\ \delta\mathbf{B}\\ \delta S_x\\ \delta S_y\end{pmatrix} = \begin{pmatrix}N_{A\uparrow}\\ N_{A\downarrow}\\ \mathbf{V}_{A\uparrow}\\ \mathbf{V}_{A\downarrow}\\ \mathbf{E}_A\\ \mathbf{B}_A\\ S_{Ax}\\ S_{Ay}\end{pmatrix} e^{-\imath\omega t+\imath\mathbf{kr}}, \tag{23}$$

we get a set of linear algebraic equations relatively to $N_{A\uparrow}$, $N_{A\downarrow}$, $V_{A\uparrow}$, $V_{A\downarrow}$, $\mathbf{E}_A$, $\mathbf{B}_A$, S_{Ax}, and S_{Ay}. Condition of the existence of nonzero solutions for amplitudes of perturbations gives us a dispersion equation.

The difference of spin-up and spin-down concentrations of electrons $\Delta n = n_{0\uparrow} - n_{0\downarrow}$ is caused by the external magnetic field. Since electrons are negative their spins get the preferable direction opposite to the external magnetic field $\frac{\Delta n}{n_0} = \tanh(\frac{\gamma_e B_0}{T_e}) = -\tanh(\frac{|\gamma_e| B_0}{T_e})$. Here, as always we consider the temperature in units of energy, so we do not write the Boltzmann constant.

We consider plasmas in the uniform constant external magnetic field. We see that in the linear approach numbers of electrons of each species are conserved.

After some straightforward calculations we find the following dispersion equations for the longitudinal

$$1 - \frac{\omega_{\mathrm{Le}\uparrow}^2}{\omega^2 - u_\uparrow^2 k^2} - \frac{\omega_{\mathrm{Le}\downarrow}^2}{\omega^2 - u_\downarrow^2 k^2} = 0, \tag{24}$$

and the transverse

$$k^2c^2 - \omega^2 + \omega_{\mathrm{Le}}^2 \frac{\omega}{\omega \pm |\Omega|} - 4\pi\gamma k^2 c^2 \frac{2\gamma}{\hbar} \frac{n_{0\uparrow} - n_{0\downarrow}}{\omega \pm g|\Omega|} = 0, \tag{25}$$

waves, where

$$\omega_{\mathrm{Le}(s)}^2 = \frac{4\pi e^2 n_{0s}}{m} \tag{26}$$

is the Langmuir frequency for species $s = \uparrow, \downarrow$ of electrons, $\omega_{\mathrm{Le}}^2 = \omega_{\mathrm{Le},\uparrow}^2 + \omega_{\mathrm{Le},\downarrow}^2$ is the full Langmuir frequency, $u_s^2 = \frac{2^{2/3}}{3} v_{\mathrm{Fe}}^2 + \frac{\hbar^2 k^2}{4m^2}$.

Keeping in mind that $n_0 = n_{0\uparrow} + n_{0\downarrow}$ and $M_{ez} = \gamma_e(n_{e0\uparrow} - n_{e0\downarrow}) = \chi_e B_0$, where χ_e is the ratio between the equilibrium magnetic susceptibility and the magnetic permeability of electrons, we find no crucial difference between Eq. (25) and results of usual QHD applied in Refs. [10,32,33]. However a great difference appears for the longitudinal waves presented by Eq. (24). Let us mention that two different signs in formula (25) correspond to left- and right-circular polarized waves.

Now we focus our attention on Eq. (24). If equilibrium concentrations approximately equal $n_{0\uparrow} \approx n_{0\downarrow}$, it is possible in the small magnetic field, Eq. (24) gives spectrum of the Langmuir waves

$$\omega^2 = \omega_{\mathrm{Le}}^2 + \frac{1}{3} v_{\mathrm{Fe}}^2 k^2 + \frac{\hbar^2 k^4}{4m^2}, \tag{27}$$

where $v_{\mathrm{Fe}} = (3\pi^2 n_0)^{1/3}\hbar/m$ is the Fermi velocity. It has the well-known structure.

If we can not neglect difference between $n_{0\uparrow}$ and $n_{0\downarrow}$, which increases with the increase of the external magnetic field we have the following dispersion equation

$$\omega^4 - \omega^2\left[(u_\uparrow^2 + u_\downarrow^2)k^2 + \omega_{\mathrm{Le}\uparrow}^2 + \omega_{\mathrm{Le}\downarrow}^2\right] + \left(u_\uparrow^2 \omega_{\mathrm{Le}\downarrow}^2 + u_\downarrow^2 \omega_{\mathrm{Le}\uparrow}^2\right)k^2 + u_\uparrow^2 u_\downarrow^2 k^4 = 0. \tag{28}$$

The general solution of the dispersion equation for the longitudinal waves appears as a couple of solutions

$$\omega^2 = \tfrac{1}{2}\left[(u_\uparrow^2 + u_\downarrow^2)k^2 + \omega_{\mathrm{Le}\uparrow}^2 + \omega_{\mathrm{Le}\downarrow}^2 \pm \sqrt{(u_\uparrow^2 - u_\downarrow^2)^2 k^4 + \left(\omega_{\mathrm{Le}\uparrow}^2 + \omega_{\mathrm{Le}\downarrow}^2\right)^2 + 2(u_\uparrow^2 - u_\downarrow^2)\left(\omega_{\mathrm{Le}\uparrow}^2 - \omega_{\mathrm{Le}\downarrow}^2\right)k^2}\right]. \tag{29}$$

We now describe some limit cases of these formulas.

As the first step we consider the limit of small magnetic fields and, consequently, we have a small, but non-neglectable, difference between $n_{0\uparrow}$ and $n_{0\downarrow}$. In this limit we obtain

$$\omega_+^2 = \omega_{\mathrm{Le}}^2 + \frac{1}{2}(u_\uparrow^2 + u_\downarrow^2)k^2 + (u_\uparrow^2 - u_\downarrow^2)k^2 \frac{(u_\uparrow^2 - u_\downarrow^2)k^2 + 2\left(\omega_{\mathrm{Le}\uparrow}^2 - \omega_{\mathrm{Le}\downarrow}^2\right)}{4\left(\omega_{\mathrm{Le}\uparrow}^2 + \omega_{\mathrm{Le}\downarrow}^2\right)}, \tag{30}$$

and

$$\omega_-^2 = \frac{1}{2}(u_\uparrow^2 + u_\downarrow^2)k^2 - (u_\uparrow^2 - u_\downarrow^2)k^2 \frac{(u_\uparrow^2 - u_\downarrow^2)k^2 + 2\left(\omega_{\mathrm{Le}\uparrow}^2 - \omega_{\mathrm{Le}\downarrow}^2\right)}{4\left(\omega_{\mathrm{Le}\uparrow}^2 + \omega_{\mathrm{Le}\downarrow}^2\right)}. \tag{31}$$

ω_- presents a soundlike solution existing in the electron gas due to the different equilibrium distribution of spin-up and spin-down electrons.

Formula (30) presents the Langmuir wave dispersion. However, the coefficient in front of k^2 has more complicate form instead of the usual contribution of the Fermi pressure $\frac{1}{3}v_{\mathrm{Fe}}^2$. The equilibrium distribution of spinning particles being in the external magnetic field differs from the distribution in absence of the magnetic field. This difference reveals a more complicated form of the equation of state. A suitable equation of state can be applied even in the single-fluid model of electron motion [12,46]

$$p_{sf} = \frac{(6\pi^2)^{\frac{2}{3}}}{5}\frac{\hbar^2}{m}\left(n_{(av)} + \frac{\Delta n}{2}\right)^{\frac{5}{3}} + \frac{(6\pi^2)^{\frac{2}{3}}}{5}\frac{\hbar^2}{m}\left(n_{(av)} - \frac{\Delta n}{2}\right)^{\frac{5}{3}}. \tag{32}$$

However, this effect was not included in Refs. [10,30–33,40] at the consideration of spectrum of magnetized plasmas of spinning particles. Now we consider the spin-up electrons and the spin-down electrons separately having different equations of state for each of them. Hence it hard to miss this effect. So let us describe its contribution in the spectrum of the Langmuir waves.

In the small external magnetic field we can make the expansion of $n_\uparrow$ and $n_\downarrow$ in series on the small deviation of spin-up and spin-down concentrations from the average one $n_{(av)} \equiv n_0/2$, with $n_\uparrow = n_{(av)} - \Delta n/2$ and $n_\downarrow = n_{(av)} + \Delta n/2$. Thus we have more a explicit form of solutions

$$\omega_+^2 = \omega_{\rm Le}^2 + \frac{1}{3}v_{\rm Fe}^2k^2\left[1 - \frac{1}{9}\left(\frac{\Delta n}{n_0}\right)^2\right] + \frac{\hbar^2k^2}{4m^2} + \left(\frac{\Delta n}{n_0}\right)^2\frac{v_{\rm Fe}^2k^2}{9\omega_{\rm Le}^2}\left(\frac{1}{9}v_{\rm Fe}^2k^2 + \omega_{\rm Le}^2\right), \quad (33)$$

and

$$\omega_-^2 = \frac{1}{3}v_{\rm Fe}^2k^2\left[1 - \frac{1}{9}\left(\frac{\Delta n}{n_0}\right)^2\right] + \frac{\hbar^2k^2}{4m^2} - \left(\frac{\Delta n}{n_0}\right)^2\frac{v_{\rm Fe}^2k^2}{9\omega_{\rm Le}^2}\left(\frac{1}{9}v_{\rm Fe}^2k^2 + \omega_{\rm Le}^2\right). \quad (34)$$

In this limit the external magnetic field gives an extra term in the Langmuir wave dispersion dependence.

For the first step on the path of estimations we consider $n_0 = 10^{22}$ cm^{-3}, $k \sim 10^7$ cm^{-1}, $\Delta n/n_0 \sim 10^{-2}$, what corresponds to short-wavelength dynamics in metals. In this case we can simplify formulas (33) and (34)

$$\omega_+^2 = \omega_{\rm Le}^2 + \frac{1}{3}v_{\rm Fe}^2k^2\left[1 + \frac{2}{9}\left(\frac{\Delta n}{n_0}\right)^2\right], \quad (35)$$

and

$$\omega_-^2 = \frac{1}{3}v_{\rm Fe}^2k^2\left[1 - \frac{4}{9}\left(\frac{\Delta n}{n_0}\right)^2\right]. \quad (36)$$

At the parameters under consideration we find that the shift of the Fermi pressure prevails the quantum Bohm potential. We see that the dependence of dispersion on $\Delta n/n_0$ is quadratic at the small magnetization. Formula (36) shows the linear dependence of frequency on the wave vector, so we call this solution the spin-electron acoustic wave.

Let us mention that in absence of the spin we do not have dependence of the frequency on the magnetic field for the Langmuir waves propagating parallel to the external magnetic field.

Spins are highly polarized at the large external magnetic fields. In this limit we can neglect the concentration of spin-up electrons and consider $n_0 \approx n_\downarrow$, so all spins are antiparallel to the external magnetic field. Taking into account the small amount of spin-up particles we introduce the following variables $n_\downarrow = n_0 - \delta$, $n_\uparrow = \delta$, $\Delta n = n_0 - 2\delta$, $\delta \ll n_\downarrow$, $\delta \ll n_0$, $\delta \ll \Delta n$. In this limit the general dispersion dependence (29) simplifies to

$$\omega_+^2 = \omega_{\rm Le}^2 + \frac{1}{3}2^{2/3}v_{\rm Fe}^2k^2\left(1 - \frac{2}{3}\frac{\delta}{n_0}\right) + \frac{\hbar^2k^2}{4m^2} - \omega_{\rm Le}^2\frac{\delta}{n_0}\frac{\frac{1}{3}2^{2/3}v_{\rm Fe}^2k^2}{\omega_{\rm Le}^2 + \frac{1}{3}2^{2/3}v_{\rm Fe}^2k^2}, \quad (37)$$

and

$$\omega_-^2 = \frac{1}{3}2^{2/3}v_{\rm Fe}^2\left(\frac{\delta}{n_0}\right)^{2/3}k^2 + \omega_{\rm Le}^2\frac{\delta}{n_0}\frac{\frac{1}{3}2^{2/3}v_{\rm Fe}^2k^2}{\omega_{\rm Le}^2 + \frac{1}{3}2^{2/3}v_{\rm Fe}^2k^2}. \quad (38)$$

If we neglect $\Delta n/n_0$ in formula (37) ($\omega_+^2 = \omega_{\rm Le}^2 + \sqrt[3]{2}\frac{1}{3}v_{\rm Fe}^2k^2 + \frac{\hbar^2k^4}{4m^2}$) we find the increase of the Fermi pressure contribution in $\sqrt[3]{2}$ times in comparison with the Fermi pressure of unpolarized systems usually applied in literature [10,30–33].

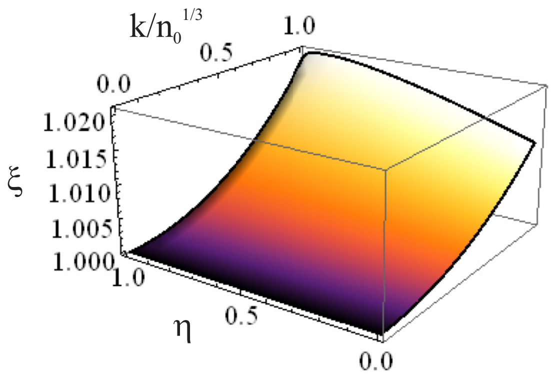
Formulaes (37) and (38) are obtained for the large magnetization. Formula (37) shows the linear dependence of ω_+^2 on $\Delta n/n_0$. Spin-electron acoustic wave dispersion $\omega_-^2(k)$ has two terms with different dependence on $\Delta n/n_0$. One of them has linear dependence and another one is proportional to $(\Delta n/n_0)^{\frac{2}{3}}$.

The spin-electron acoustic wave is a low-frequency solution. Consequently its properties might be affected by the ion motion. This problem will be considered during further development and application of the spin-separated QHD model developed in this paper.

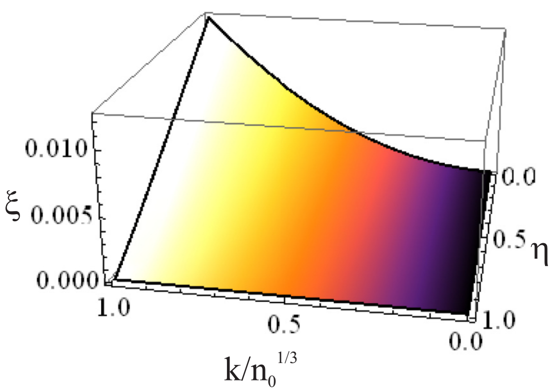
Now we move to the description of Eq. (25). The first two terms in Eq. (25) describe the propagation of the light in vacuum. The third term presents the contribution of the medium of charged particles moving in the external magnetic field. The last term presents the medium of spinning particles. The last term exists even for neutral particles. Each of the last two terms increase the degree of the dispersion equation on one in comparison with the mediumless case. A simultaneous account of these two terms increases the degree of the dispersion equation on two due to the difference of denominators of these terms. The difference of denominators is caused by the anomalous magnetic moment of electrons. If we neglect the anomalous magnetic moment of electrons we find that the account of the electron spin does not change the degree of dispersion equation. This gives contribution in coefficients of the equation only. Corresponding spin-plasma waves are described in Refs. [10] and [33]. The quantum Bohm potential in the spin evolution equation [14] [see also the second term in Eq. (1) of this paper] of the single-fluid QHD model of electrons gives the shift of the cyclotron frequency of magnetic moment rotation. Thus, it leads to the appearance of the spin-plasma wave along with the anomalous part of the magnetic moment [23,49] (see also Ref. [50]).

We have presented formulas for the limit cases of solution (29). Let us now present the numerical analysis of solutions in the area of intermediate polarizations with the relevant plasma parameters.

Presenting the spectrum (29) via the dimensionless variables $\xi = \omega^2/\omega_{\rm Le}^2$, $k/\sqrt[3]{n_0}$, and $\eta = \Delta n/n_0$ we get an extra



(a)



(b)

FIG. 1. (Color online) Dependence of dimensionless frequency square in units of the square of Langmuir frequency $\xi = \omega^2/\omega_{\rm Le}^2$ on the dimensionless wave vector $k/\sqrt[3]{n_0}$ and the spin polarization $\eta = \Delta n/n_0$ for two longitudinal waves existing in the spin-up–spin-down degenerate electron quantum plasmas. The figure is obtained for the waves propagating parallel to the external magnetic field. Ions are assumed to be motionless. (a) shows the dispersion of the Langmuir wave, (b) presents the dispersion properties of the spin-electron acoustic wave. This figure is obtained for $n_0 = 10^{21}$ cm^{-3}. Hence the Langmuir frequency $\omega_{\rm Le}$, in our case, is equal to 1.5×10^{15} radians per second. Estimation for maximal wave vector is $k < 10^7$ cm^{-1}.

parameter $\Lambda = (\hbar^2/me^2)\sqrt[3]{n_0}$ depending on the fundamental physical constants $\hbar$, m, e and the equilibrium particle concentration. For numerical estimations we need to choose the equilibrium concentration n_0 and obtain Λ. Next we can find $\xi(k/\sqrt[3]{n_0})$ at different spin polarization η. Results of these calculations are presented in Fig. 1. Figure 1 is obtained for $n_0 = 10^{21}$ cm^{-3}. Figure 1(a) [1(b)] shows the upper (lower) branch, which is the Langmuir (new) wave.

In Fig. 1(a) we see the growth of the frequency square ξ with the increase of the wave vector. This happens due to the presence of the Fermi pressure. This growth speeds up with the increase of polarization η in agreement with approximate formulas (33) and (35). This effect is related to the modification of equation of state (32). In different notations this effect is included in Ref. [12] at consideration of electrons as a single fluid.

Figure 1(b) shows the new longitudinal wave (the spin-electron acoustic wave), which has been found due to consideration of electrons as two different species: spin-up electrons and spin-down electrons. It is essential for the existence of this wave that the occupation numbers of the spin-up and spin-down degenerate electrons are different. Hence, the equilibrium concentrations are different $n_{0\uparrow} \neq n_{0\downarrow}$ and the contributions of pressure of degenerate spin-up and spin-down electrons are different. Figure 1(b) shows the increase of the frequency square of the spin-electron acoustic wave with the increase of the wave vector. We also see that this growth slows down with the increase of the spin polarization. This is in agreement with approximate formulas (34) and (36). We see that the frequencies of the spin-electron acoustic wave $\sim\sqrt{\xi}$ is about 1% of the Langmuir frequencies $0.01\omega_{\rm Le}$ at intermediate wave vectors $k \sim 10^6$ cm^{-1}.

IV. CONTRIBUTION OF THE ION MOTION

As we have shown above the frequency of the SEAWs becomes rather small at the large spin polarization. Hence the ion motion is relevant in this regime. Therefore we apply Eqs. (8)–(16) along with the hydrodynamic equations for ions. We consider ions as a single fluid without the spin separation. It leads to the following dispersion equation

$$1 - \frac{\omega_{\rm Le\uparrow}^2}{\omega^2 - u_\uparrow^2 k^2} - \frac{\omega_{\rm Le\downarrow}^2}{\omega^2 - u_\downarrow^2 k^2} - \frac{\omega_{Li}^2}{\omega^2 - v_{\rm Fi}^2 k^2/3} = 0. \quad (39)$$

Equation (39) corresponds to Eq. (24), but it contains an extra term describing the ion contribution. This equation allows to study the ion Langmuir wave [41]. We analyze this equation numerically.

Figure 2 demonstrates that at large spin polarization up to $\eta = 0.98$ the frequency of the spin-electron acoustic waves are

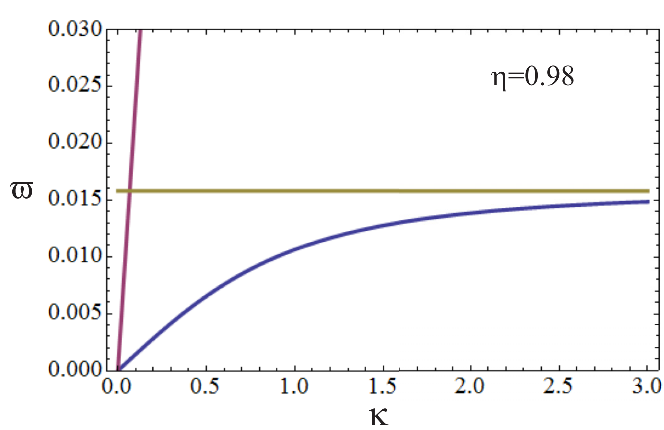


FIG. 2. (Color online) The dispersion of the ion acoustic wave and the spin-electron acoustic wave at rather large spin polarization $\eta = 0.98$. The branch with the largest phase velocity ω/k (the red branch) presents the spin-electron acoustic wave. Lower curve presents the ion acoustic wave. Here and in other figures below we draw the horizontal line presenting the ion Langmuir frequency. The frequency is presented in units of the electron Langmuir frequency $\varpi = \omega/\omega_{\rm Le}$, and the wave vector is presented in units of the inverse electron Debye radius $\kappa = kr_{\rm De}$, with $r_{\rm De} = v_{\rm Fe}/\omega_{\rm Le}$.

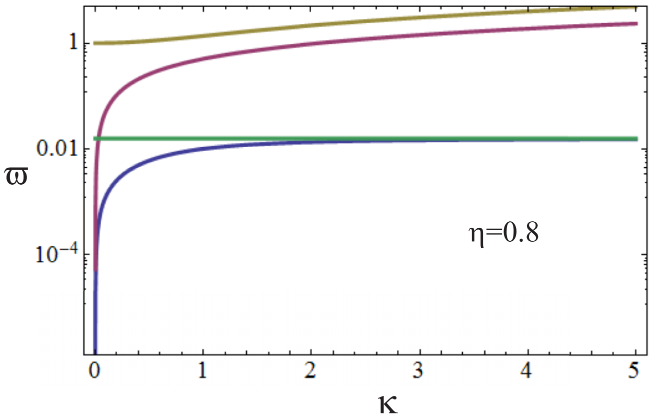


FIG. 3. (Color online) We present all three longitudinal waves propagating parallel to the external magnetic field at $\eta = 0.8$. We have the following waves in order of the frequency decrease: the Langmuir wave, the spin-electron acoustic wave, and the ion-acoustic wave.

much larger than frequencies of the ion-acoustic waves for all wave vectors.

Position of the dispersion branch of the spin-electron acoustic wave relatively to the Langmuir wave and the ion-acoustic wave is demonstrated in Figs. 3 and 4 for different spin polarizations. Figures 3 and 4 show the decrease of the frequencies of the spin-electron acoustic wave with the increase of the polarization.

At very small concentration of the spin-up electrons $n_{0u}/n_0 = 10^{-6}$ and $1 - \eta = 2 \times 10^{-6}$ the dispersion dependencies of the ion-acoustic wave and the spin-electron acoustic wave get close to each other (see upper picture in Fig. 5).

The approach of two branches creates a resonance point, where two branches could cross each other. Therefore we have the area of interaction of oscillations with close values of ω and k, related to two different branches of the spectrum of the dissipationless systems. Presence of the interaction between branches changes the behavior of the branches. We obtain real functions $\omega(k)$ and $k(\omega)$ in this area. Hence, systems are stable. Nevertheless we see that new hybrid branches have the following structure. The wave with the larger frequencies coincides with the ion-acoustic wave at small wave vectors

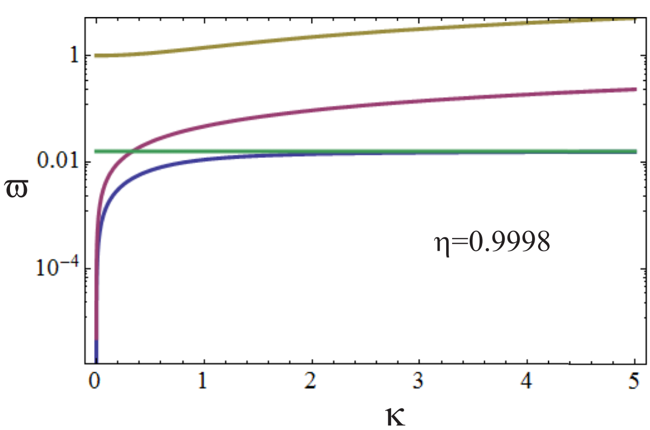


FIG. 4. (Color online) Comparison of this figure with Fig. 3 shows the decrease of the frequency of the spin-electron acoustic wave with the increase of spin polarization up to $\eta = 0.9998$. Figures 3 and 4 show this decrease in compare with the frequency of the ion acoustic wave. Distribution of the dispersion branches in this figure corresponds to their distribution in Fig. 3.

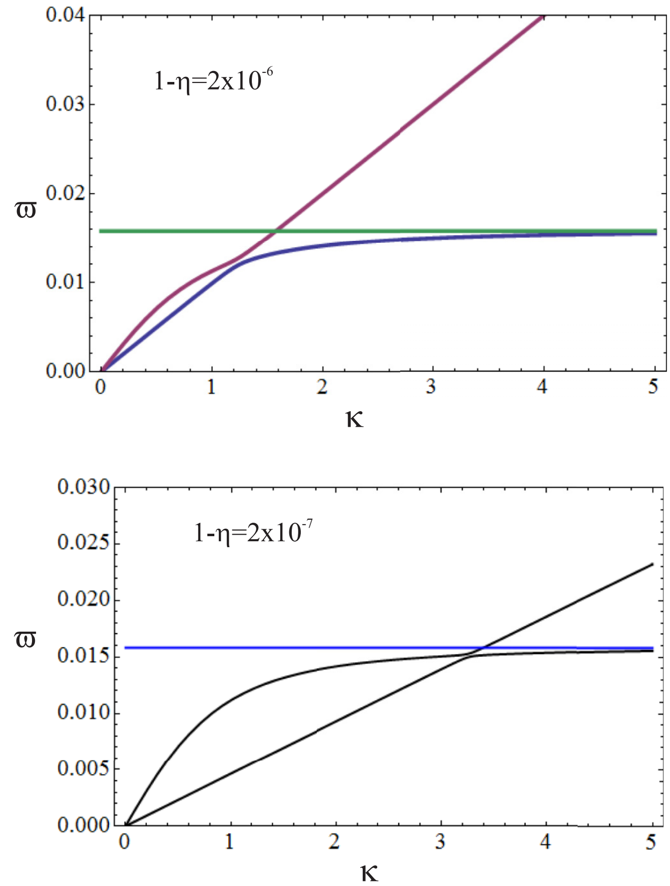


FIG. 5. (Color online) Continuation of the comparison of the dispersion branch positions presented in Figs. 3 and 4. Increasing the spin polarization we decrease the frequency of the spin-electron acoustic waves. At $\eta = 999998$ and $\eta = 9999998$ the frequencies of the spin-electron acoustic wave and the ion-acoustic wave are comparable.

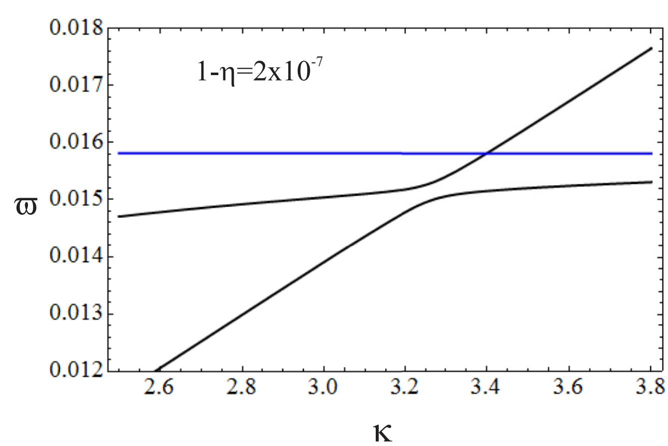


FIG. 6. (Color online) Figure 5 shows hybridization of the ion-acoustic and spin-electron acoustic waves and area of splitting of the hybrid waves. Detailed picture of the splitting of the hybrid waves is presented in this figure.

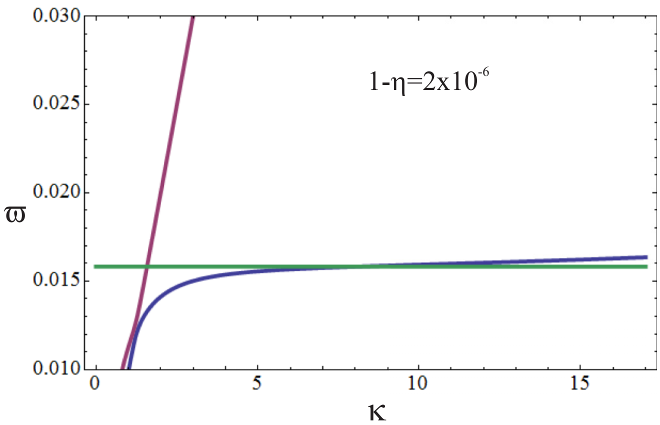


FIG. 7. (Color online) Comparison of dispersion dependence of the spin-electron acoustic wave and the ion-plasmas wave at large spin polarization of electrons $n_{0u}/n_0 = 10^{-6}$, $\eta = 0.999998$, when the frequency of the spin-electron acoustic wave is rather small.

and turns into the spin-electron acoustic wave after the area of interaction of branches. The hybrid wave with the lower frequencies coincides with the spin-electron acoustic wave at the small wave vectors and turns into the ion-acoustic wave at the large wave vectors.

Further increase of the polarization decreases the frequency of the spin-electron acoustic wave, so the dispersion branches cross each other at the larger wave vectors k (see Fig. 5, bottom). Figure 6 presents a detailed picture of the splitting of the hybrid dispersion branches.

Figure 7 shows that in spite of the hybridization of the dispersion branches at low frequencies, the high-frequency part of the spin-electron acoustic wave remains unchanged. Figure 8 repeats the results of Figs. 5 and 6 for larger spin polarization, when area of interaction of branches shifts to the larger wave vectors, where the ion-acoustic wave transforms into the ion Langmuir wave.

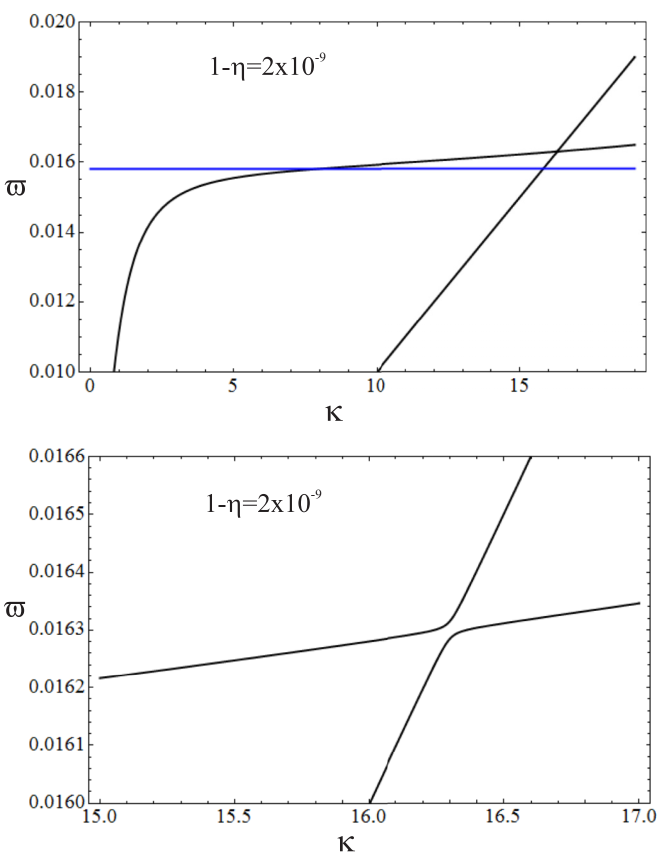


FIG. 8. (Color online) Comparison of dispersion dependence of the spin-electron acoustic wave and the ion-plasmas wave at larger spin polarization of electrons $n_{0u}/n_0 = 10^{-9}$, $\eta = 0.999999998$ than in Fig. 7. The upper figure shows dispersion branches in a wide range of the wave vectors. The lower figure shows area of splitting of the hybrid branches. Comparing this figure with Fig. 6 we see that at larger spin polarization the splitting of the branches becomes smaller.

V. SPIN-ELECTRON ACOUSTIC WAVES IN NEUTRON MATTER: REGIME OF PARTIAL SPIN POLARIZATION OF DEGENERATE NEUTRONS

The appearance of the spin-electron acoustic waves is not related to the charge of particles. However, the properties of plasmas seriously affect the properties of the spin-electron acoustic waves, as we have demonstrated in Secs. III and IV.

It is interesting to consider the spin-electron acoustic waves in the magnetized neutron stars [51,52], where the spin-electron acoustic waves appear as the splitting of the acoustic waves existing due to the Fermi pressure in nonpolarized matter. Acoustic waves and spin-electron acoustic waves exist in the neutron matter along with the spin wave on the cyclotron frequency.

In electron-ion plasmas the spin-electron acoustic wave can be found at application of the Poisson equation, since it is a longitudinal wave. The Poisson equation gives us the contribution of the mixture of species: spin-up electrons, spin-down electrons, and ions.

Due to neutrality of the neutron matter we have a different picture. Considering propagation of waves parallel to the external magnetic field $\mathbf{B}_0 = B_0\mathbf{e}_z$ we obtain $\delta B_z = 0$ from $\nabla\mathbf{B} = 0$. As a consequence the right-hand sides of the Euler equations (10) and (11) are equal to zero. It gives us two independent dispersion equations, which can be written as follows:

$$\omega_s^2 = u_s^2 k^2 + \frac{\hbar^2 k^4}{4m^2}. \quad (40)$$

Formula (40) shows that the sound wave in the magnetized neutron matter splitting on two acoustic branches. This appears due to the difference of the Fermi pressures for the spin-up and spin-down electrons. At zero spin polarization these branches coincide giving the usual sound wave.

VI. CONCLUSIONS

We have derived QHD equations for charged spin-1/2 particles considering evolution of electrons with spin-up and spin-down separately. These equations appear as a generalization of usual quantum hydrodynamics, where physical quantities appear via the contribution of all particles together, with the spin-up and the spin-down. This generalization reveals the existence of a wave solution and the possibility to find more solutions.

We have studied the propagation of waves parallel to the external magnetic field. We have found the contribution of the magnetic field in the Langmuir wave dispersion via the difference of occupation of the spin-up and spin-down states. We have considered the limits of small and large magnetic fields, which reveal the small and large spin polarizations $\Delta n/n_0$ and the contribution of $\Delta n/n_0$ in the dispersion dependence. Similarly we have described a solution. It appears as a soundlike solution, which we call the spin-electron acoustic wave. We have a general form of this solution and considered its limits for the small and large magnetization.

We have paid special attention to the ion motion contribution in the spectrum at the large spin polarization of electrons, when the frequency of the spin-electron acoustic wave is rather small. We have also considered the dispersion of waves in the partially polarized neutron matter, studying the consequences of the spin separation and the difference in occupation of the spin-up and spin-down states. We have obtained splitting of the sound wave on two branches.

ACKNOWLEDGMENT

The author thanks Professor L. S. Kuz'menkov for fruitful discussions.

[1] L. S. Kuz'menkov and S. G. Maksimov, Teor. i Mat. Fiz. **118**, 287 (1999) [Theor. Math. Phys. **118**, 227 (1999)].
[2] F. Haas, G. Manfredi, and M. Feix, Phys. Rev. E **62**, 2763 (2000).
[3] G. Manfredi and F. Haas, Phys. Rev. B **64**, 075316 (2001).
[4] T. Koide, Phys. Rev. C **87**, 034902 (2013).
[5] L. S. Kuz'menkov, S. G. Maksimov, and V. V. Fedoseev, Russian Phys. J. **43**, 718 (2000).
[6] L. S. Kuz'menkov, S. G. Maksimov, and V. V. Fedoseev, Theor. Math. Fiz. **126**, 136 (2001) [Theor. Math. Phys. **126**, 110 (2001)].
[7] L. S. Kuz'menkov, S. G. Maksimov, and V. V. Fedoseev, Theor. Math. Fiz. **126**, 258 (2001) [Theor. Math. Phys. **126**, 212 (2001)].
[8] P. A. Andreev and L. S. Kuz'menkov, Russian Phys. Jour. **50**, 1251 (2007).
[9] P. A. Andreev and L. S. Kuz'menkov, Phys. Atomic Nuclei **71**, 1724 (2008).
[10] P. A. Andreev and L. S. Kuz'menkov, Int. J. Mod. Phys. B **26**, 1250186 (2012).
[11] P. A. Andreev and L. S. Kuz'menkov, arXiv:1210.1090 [Int. J. Mod. Phys. (to be published)].
[12] P. A. Andreev, Ann. Phys. (NY) **350**, 198 (2014).
[13] P. A. Andreev, L. S. Kuzmenkov, and M. I. Trukhanova, Phys. Rev. B **84**, 245401 (2011).
[14] T. Takabayasi, Prog. Theor. Phys. **14**, 283 (1955).
[15] T. Takabayasi, Prog. Theor. Phys. **12**, 810 (1954).
[16] T. Takabayasi, Prog. Theor. Phys. **13**, 222 (1955).
[17] T. Takabayasi, Phys. Rev. **102**, 297 (1956).
[18] T. Takabayasi, Nuovo Cimento **3**, 233 (1956).
[19] T. Takabayasi, Prog. Theor. Phys. Suppl. **4**, 2 (1957).
[20] T. Takabayasi, Prog. Theor. Phys. **70**, 1 (1983).
[21] P. K. Shukla and B. Eliasson, Phys. Usp. **53**, 51 (2010).
[22] P. K. Shukla and B. Eliasson, Rev. Mod. Phys. **83**, 885 (2011).
[23] P. A. Andreev, F. A. Asenjo, and S. M. Mahajan, arXiv:1304.5780.
[24] G. Brodin, A. P. Misra, and M. Marklund, Phys. Rev. Lett. **105**, 105004 (2010).
[25] M. Stefan, J. Zamanian, G. Brodin, A. P. Misra, and M. Marklund, Phys. Rev. E **83**, 036410 (2011).
[26] A. P. Misra, G. Brodin, M. Marklund, and P. K. Shukla, Phys. Rev. E **82**, 056406 (2010).
[27] A. P. Misra, G. Brodin, M. Marklund, and P. K. Shukla, Phys. Plasmas **17**, 122306 (2010).
[28] M. Shahid and G. Murtaza, Phys. Plasmas **20**, 082124 (2013).
[29] M. Iqbal, J. Plasma Physics **79**, 19 (2013).
[30] M. Marklund and G. Brodin, Phys. Rev. Lett. **98**, 025001 (2007).
[31] G. Brodin and M. Marklund, New J. Phys. **9**, 277 (2007).
[32] P. A. Andreev and L. S. Kuz'menkov, Moscow Univ. Phys. Bull. **62**, 271 (2007).
[33] A. P. Misra, G. Brodin, M. Marklund, and P. K. Shukla, J. Plasma Phys. **76**, 857 (2010).
[34] S. Rand, Phys. Fluids **7**, 64 (1964).
[35] G. S. Krishnaswami, R. Nityananda, A. Sen, and A. Thyagaraja, Phys. Rev. Lett. **112**, 129501 (2014).
[36] D. B. Melrose and J. I. Weise, J. Phys. A **45**, 395501 (2012).
[37] Sheng-Chang Li and Jiu-Ning Han, Phys. Plasmas **21**, 032105 (2014).
[38] V. Bychkov, M. Modestov, and M. Marklund, Phys. Plasmas **17**, 112107 (2010).
[39] L. Landau and E. M. Lifshitz, *Statistical Physics* (Pergamon, New York, 1980), p. 167.
[40] F. A. Asenjo, Phys. Lett. A **376**, 2496 (2012).
[41] J. Vranjes and M. Kono, Phys. Plasmas **21**, 042104 (2014).
[42] J. Vranjes, B. P. Pandey, and S. Poedts, Eur. Phys. Lett. **99**, 25001 (2012).
[43] J. Vranjes, B. P. Pandey, and S. Poedts, Eur. Phys. Lett. **99**, 65002 (2012).
[44] P. A. Andreev and L. S. Kuzmenkov, Phys. Rev. A **78**, 053624 (2008).
[45] L. Landau and E. M. Lifshitz, *Statistical Physics, part II* (Pergamon, New York, 1980).
[46] L. S. Kuz'menkov, S. G. Maksimov, and V. V. Fedoseev, Vestn. Mosk. Univ., Ser. 3: Fiz., Astron. **5**, 3 (2000) [Moscow Univ. Phys. Bull. **5**, 1 (2000)].
[47] D. A. Uzdensky and S. Rightley, Reports on Progress in Physics **77**, 036902 (2014).
[48] N. L. Tsintsadze, L. N. Tsintsadze, A. Hussain, and G. Murtaza, Eur. Phys. J. D **64**, 447 (2011).
[49] M. I. Trukhanova, Prog. Theor. Exp. Phys. **2013**, 111I01 (2013).
[50] P. A. Andreev, arXiv:1404.4899.
[51] A. K. Harding and D. Lai, Rep. Progr. Phys. **69**, 2631 (2006).
[52] S. M. Mahajan and F. A. Asenjo, Phys. Lett. A **377**, 1430 (2013).