Giant cross-magnetic-field steps due to binary collisions between pair particles

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Giant cross-magnetic-field steps can occur as a result of positron-electron collisions. Within a constant magnetic field (e.g., 1 T), a collision between a positron and an electron can result in a correlated drift across the magnetic field for a continuous range of impact parameters. Within this range, drift distances orders of magnitude larger than that associated with like-charge collisions were observed by computer simulation. Outside of this range, the collisional behavior is similar to that for collisions between particles with the same charge. A theoretical analysis of the phenomenon using center-of-mass and relative coordinates provides insights regarding the occurrence of giant cross-magnetic-field steps.

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I. INTRODUCTION

There are various phenomena that are manifestations of the interaction of just two charged particles located within a magnetic field. Examples include magnetized binary Coulomb collisions, which are ubiquitous occurrences within plasmas comprised of free charged particles, quasibound states of antihydrogen [1], magnetobound states of pair particles [2], guiding center drift atoms [3], magnetized positronium [4–6], and protonium within a magnetic field [7]. Consider a classical point charge within a uniform magnetic field that experiences a helical path about the guiding center of its motion. Suppose two such charged particles, which are initially sufficiently far from one another to be unaffected by each other, temporarily approach each other close enough for their trajectories to be affected. As a result of the collision, each particle's guiding center can shift to a new location across the magnetic field. Such guiding center steps are typically smaller than the cyclotron radii associated with the helical paths of the particles prior to their interaction. In the work presented here, a phenomenon that is describable as giant cross-magnetic-field steps is studied. Giant cross-magnetic-field steps can occur in low-energy binary collisions between pair particles, that is, between a particle and its antimatter counterpart. Giant cross-magnetic-field steps are associated with guiding center steps that are much larger than the cyclotron radii associated with the helical paths of the particles prior to their interaction. Here, computer simulations and analytical theory are used for studying the cross-magnetic-field drift phenomenon. Prior related research developed a description of the formation cross section for the two-particle correlated states, which were referred to as magnetobound states [2]. The particle energies and magnetic field strength used for the simulations of giant cross-magnetic-field steps are similar to those used for producing trappable antihydrogen. Collaborations such as ALPHA [8–11], ATRAP [12–14], ASACUSA [15,16], AEgIS [17], and GBAR [18] are developing and improving methods for producing, trapping in some cases, and studying antihydrogen. Despite the difficulties that would be encountered in confining electron-positron plasmas in the laboratory, efforts are underway that may observe some of the unique stability properties that these plasmas are predicted to have [19].

Section II of this paper describes the equations of motion and explains the method used in the simulation to evaluate drift distances for electron-positron collisions. The mean square of the step size is evaluated using a Monte Carlo method. Section III gives a theoretical description of the phenomenon using center-of-mass and relative coordinates. Concluding remarks are in Sec. IV.

II. DRIFT DISTANCE EVALUATIONS

The collision of a positron and an electron is treated classically, considering the electric interaction under the influence of a uniform magnetic field. According to Coulomb's law, the electric force exerted on particle 1 (positron) by 2 (electron) is given by $\boldsymbol{F}_{\text{on1by2}} = k_c q_1 q_2 \boldsymbol{r}_{12}/r_{12}^3$, where k_c is the Coulomb force constant, q_1 and q_2 are the charges, $r_{12} = |\boldsymbol{r}_{12}|$ is the relative distance, and $\boldsymbol{r}_{12} = \boldsymbol{r}_1 - \boldsymbol{r}_2$ is the vector position of particle 1 relative to particle 2. In SI units, $k_c = 1/(4\pi\epsilon_0)$, where ϵ_0 is the permittivity of free space. The electric force exerted on particle 2 by 1 is given by $\boldsymbol{F}_{\text{on2by1}} = k_c q_1 q_2 \boldsymbol{r}_{21}/r_{21}^3$, where $\boldsymbol{r}_{21} = -\boldsymbol{r}_{12}$. The magnetic force on each particle is given by $\boldsymbol{F} = k_L q(\boldsymbol{v} \times \boldsymbol{B})$. Here, k_L is the Lorentz force constant ($k_L = 1$ in SI units), q is the charge, $\boldsymbol{v}$ is the velocity of the particle, and $\boldsymbol{B}(=B\hat{\boldsymbol{k}})$ is a uniform magnetic field parallel to the unit vector $\hat{\boldsymbol{k}}$. The force on particle 1 by the magnetic field is $\boldsymbol{F}_{\text{on1by}B} = k_L q_1 B(v_{1y}\hat{\boldsymbol{\imath}} - v_{1x}\hat{\boldsymbol{\jmath}})$, where $(\hat{\boldsymbol{\imath}},\hat{\boldsymbol{\jmath}},\hat{\boldsymbol{k}})$ are the unit vectors of a Cartesian coordinate system, and v_{1x}, v_{1y}, v_{1z} are velocity components of particle 1. For particle 2, $\boldsymbol{F}_{\text{on2by}B} = k_L q_2 B(v_{2y}\hat{\boldsymbol{\imath}} - v_{2x}\hat{\boldsymbol{\jmath}})$, where v_{2x}, v_{2y}, v_{2z} are velocity components of particle 2.

The classical motion of the particles is governed by Newton's second law. For the positron, $\boldsymbol{F}_{\text{on1by2}} + \boldsymbol{F}_{\text{on1by}B} = m_1\boldsymbol{a}_1$, where m_1 is the mass of the positron and $\boldsymbol{a}_1$ is its acceleration. For the electron, $\boldsymbol{F}_{\text{on2by1}} + \boldsymbol{F}_{\text{on2by}B} = m_2\boldsymbol{a}_2$, where m_2 is the mass of the electron and $\boldsymbol{a}_2$ is its acceleration. Therefore, the acceleration of the positron is given by $k_c q_1 q_2 \boldsymbol{r}_{12}/r_{12}^3 + k_L q_1 B(v_{1y}\hat{\boldsymbol{\imath}} - v_{1x}\hat{\boldsymbol{\jmath}}) = m_1\boldsymbol{a}_1$ and that of the electron is given by $k_c q_1 q_2 \boldsymbol{r}_{21}/r_{21}^3 + k_L q_2 B(v_{2y}\hat{\boldsymbol{\imath}} - v_{2x}\hat{\boldsymbol{\jmath}}) = m_2\boldsymbol{a}_2$. The velocity and position of each particle are functions of time. If the position is written as $\boldsymbol{r}_i(t) = x_i(t)\hat{\boldsymbol{\imath}} + y_i(t)\hat{\boldsymbol{\jmath}} + z_i(t)\hat{\boldsymbol{k}}$ and the time derivative is written as $\boldsymbol{r}_i{}'(t) = x_i'(t)\hat{\boldsymbol{\imath}} + y_i'(t)\hat{\boldsymbol{\jmath}} + z_i'(t)\hat{\boldsymbol{k}}$,

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then the equations of motion for the positron are

$$\frac{k_c q_1 q_2 [x_1(t) - x_2(t)]}{([x_1(t) - x_2(t)]^2 + [y_1(t) - y_2(t)]^2 + [z_1(t) - z_2(t)]^2)^{3/2}} + k_L B q_1 y_1'(t) = m_1 x_1''(t), \tag{1}$$

$$\frac{k_c q_1 q_2 [y_1(t) - y_2(t)]}{([x_1(t) - x_2(t)]^2 + [y_1(t) - y_2(t)]^2 + [z_1(t) - z_2(t)]^2)^{3/2}} - k_L B q_1 x_1'(t) = m_1 y_1''(t), \tag{2}$$

$$\frac{k_c q_1 q_2 [z_1(t) - z_2(t)]}{([x_1(t) - x_2(t)]^2 + [y_1(t) - y_2(t)]^2 + [z_1(t) - z_2(t)]^2)^{3/2}} = m_1 z_1''(t). \tag{3}$$

The equations of motion for the electron are

$$\frac{k_c q_1 q_2 [x_2(t) - x_1(t)]}{([x_1(t) - x_2(t)]^2 + [y_1(t) - y_2(t)]^2 + [z_1(t) - z_2(t)]^2)^{3/2}} + k_L B q_2 y_2'(t) = m_2 x_2''(t), \tag{4}$$

$$\frac{k_c q_1 q_2 [y_2(t) - y_1(t)]}{([x_1(t) - x_2(t)]^2 + [y_1(t) - y_2(t)]^2 + [z_1(t) - z_2(t)]^2)^{3/2}} - k_L B q_2 x_2'(t) = m_2 y_2''(t), \tag{5}$$

$$\frac{k_c q_1 q_2 [z_2(t) - z_1(t)]}{([x_1(t) - x_2(t)]^2 + [y_1(t) - y_2(t)]^2 + [z_1(t) - z_2(t)]^2)^{3/2}} = m_2 z_2''(t). \tag{6}$$

The electron and positron are treated (but not simulated) as having traveled in opposite directions from two sources located at infinite distances from each other and at infinite distances from the coordinate origin. At infinite separation, the electric potential energy is defined to be zero. When the particles are separated by a finite distance at the start of a simulation, conservation of energy requires

$$K_{1\infty} + K_{2\infty} = \frac{1}{2} m_1 \left(v_{x10}^2 + v_{y10}^2 + v_{z10}^2\right) + \frac{1}{2} m_2 \left(v_{x20}^2 + v_{y20}^2 + v_{z20}^2\right) + \frac{k_c q_1 q_2}{r_0}. \tag{7}$$

Here, $v_{x10}, v_{y10}, v_{z10}$ are the components of the initial velocity of the positron, $v_{x20}, v_{y20}, v_{z20}$ are the components of the initial velocity of the electron, the kinetic energies for the two particles at infinite separation are denoted $K_{1\infty}$ and $K_{2\infty}$, respectively, and r_0 is their initial distance of separation.

The initial conditions considered first represent oppositely directed particle beams. A sketch of the initial conditions is shown in Fig. 1. The positron and the electron are considered to be initially positioned at Cartesian coordinates $(x_{10}, y_{10}, z_{10}) = (b/2, 0, \zeta b/2)$ and $(x_{20}, y_{20}, z_{20}) = (-b/2, 0, -\zeta b/2)$, respectively, and $r_0 = b\sqrt{1+\zeta^2}$. The following values are used: $v_{x10} = v_{y10} = v_{x20} = v_{y20} = 0$, $K_{1\infty} = K_{2\infty}$, $v_{z10} = -v_{z20}$, and $m_1 = m_2$. Hereafter, b is referred to as the impact parameter, and ζb is the initial axial separation. Equation (7) is written as

$$2K_{1\infty} = m_1 v_{z10}^2 + \frac{k_c q_1 q_2}{b\sqrt{1+\zeta^2}}. \tag{8}$$

Rearrangement provides an expression for the nonzero component of the initial velocity of the positron

$$v_{z10} = -\sqrt{\frac{2K_{1\infty}}{m_1} - \frac{k_c q_1 q_2}{m_1 b\sqrt{1+\zeta^2}}}. \tag{9}$$

Following a similar procedure for the electron gives

$$v_{z20} = \sqrt{\frac{2K_{2\infty}}{m_2} - \frac{k_c q_1 q_2}{m_2 b\sqrt{1+\zeta^2}}}. \tag{10}$$

A simulation is stopped at a time $t_{\max}$, when the axial separation becomes larger than the initial axial separation. The condition for the simulation to stop is when the inequality

$$|z_1(t_{\max}) - z_2(t_{\max})| > \zeta b \tag{11}$$

is first satisfied. The parameter values used in the simulation are $B = 1$ T, $K_\infty = K_{1\infty} = K_{2\infty} = 6\kappa$, $\zeta = 50$, where κ has the numerical value of Boltzmann's constant and the unit of energy.

An output of the simulation is the cross-magnetic-field drift distance experienced by each particle. For the positron, the cross-magnetic-field drift distance is

$$\Delta d = \sqrt{[x_1(t_{\max}) - x_{10}]^2 + [y_1(t_{\max}) - y_{10}]^2}. \tag{12}$$

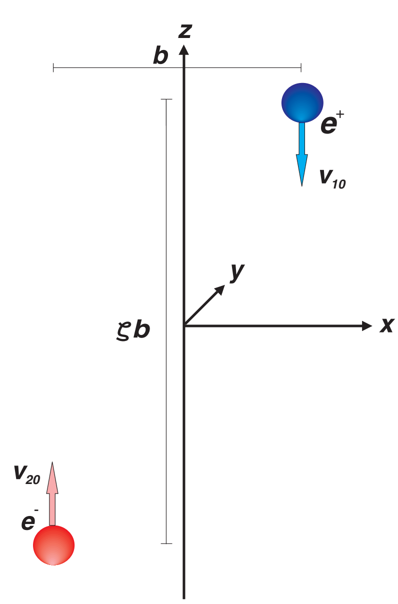

FIG. 1. (Color online) Positron and electron at time $t = 0$.

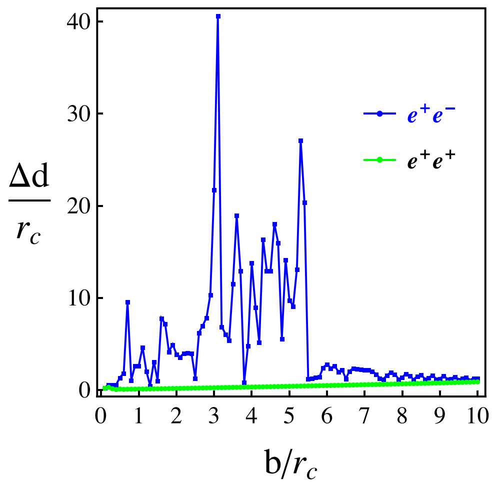

FIG. 2. (Color online) Normalized cross-magnetic-field drift distance versus normalized impact parameter for $B = 1$ T and $K_\infty = 6\kappa$.

The cyclotron radius is useful for normalizing lengths. The cyclotron radius is defined for the present study as the radius of a circular trajectory when one charged particle is under the influence of the magnetic field and has a kinetic energy that is only associated with motion perpendicular to the magnetic field. The cyclotron radius is $r_c = \sqrt{2K_\infty m/(k_L^2 q^2 B^2)}$, where m and K_∞ are the mass and kinetic energy of the particle. For $B = 1$ T and $K_\infty = 6\kappa$ the cyclotron radius is 7.67×10^{-8} m.

The equations of motion and initial velocity conditions have been written in such a way as to also be applicable for simulating a collision between two positrons. Each simulation is run for both electron-positron and positron-positron binary collisions. A monitoring of a constant of the motion, the total energy, was done for some of the trajectories involving long drifts. The relative deviations were typically within the orders 10^{-4} to 10^{-3}. The simulations were carried out using MATHEMATICA. Figures 2 and 3 contrast drift distances for electron-positron and positron-positron collisions. The impact parameter was varied from 0 to $10r_c$ in increments of $0.1r_c$ and then 0 to r_c in increments of $0.01r_c$. For electron-positron collisions, giant drift distances (as compared to positron-positron collisions) occur at values of the impact parameter ranging from $0.41r_c$ to $5.5r_c$. Outside of this range, the electron-positron drift distances drop to values closer to the corresponding drift distances for positron-positron collisions. Although the range of impact parameters within which giant cross-magnetic-field steps occur is well defined, the drift distances are sensitive to small changes in the impact parameter. The chaotic behavior is evident in the Figs. 2 and 3.

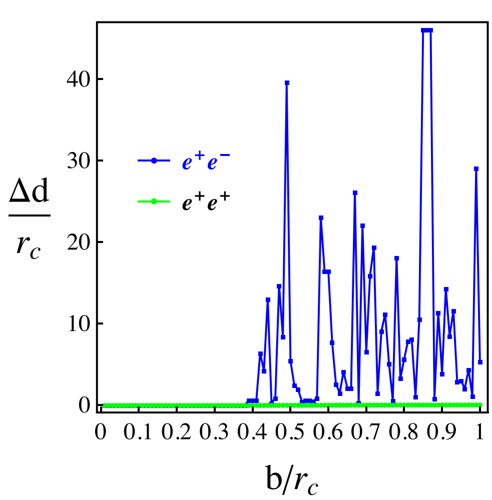

FIG. 3. (Color online) Same as Fig. 2, except with $0 < b < r_c$.

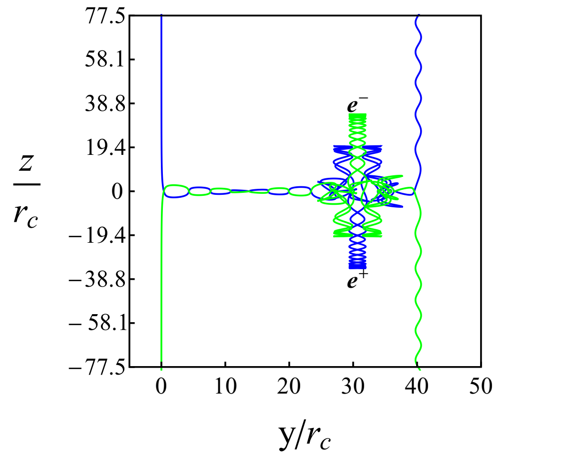

FIG. 4. (Color online) Electron and positron trajectories projected onto the y-z plane for $b = 3.1r_c$, $B = 1$ T, and $K_\infty = 6\kappa$.

Figures 4 and 5 contrast individual trajectories during a collision. It is apparent that the electron and positron drift together as a correlated pair across the magnetic field. Each particle oscillates several times with variable amplitude across the $z = 0$ plane. A drift distance of $40.6r_c$ is observed for the electron-positron collision shown in Fig. 4. The electron-positron collision results in a drift distance that is two order of magnitude larger than that for a positron-positron collision (shown in Fig. 5) having the same impact parameter $b = 3.1r_c$.

The mean square of the steps is evaluated as

$$\langle(\Delta d)^2\rangle_b = \int_0^{b_{\max}} (\Delta d)^2 \, f_b(b) \, db. \tag{13}$$

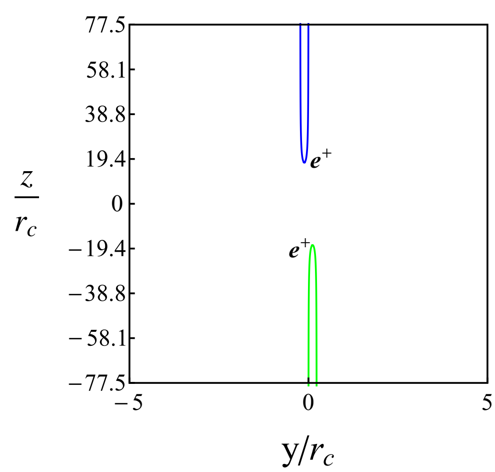

FIG. 5. (Color online) Two positron trajectories projected onto the y-z plane for $b = 3.1r_c$, $B = 1$ T, and $K_\infty = 6\kappa$.

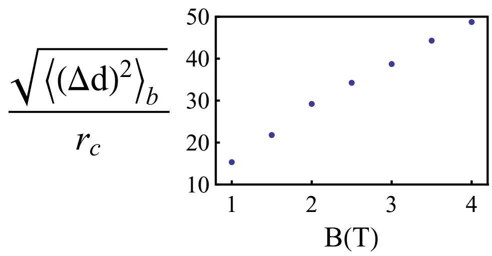

FIG. 6. (Color online) Normalized root-mean-square cross-magnetic-field drift distance versus magnetic field strength with $K_\infty = 6\kappa$.

Here, $f_b(b) = 2b/b_{\max}^2$ is the probability density function for the impact parameter, assuming that the initial guiding centers are uncorrelated. The integral is evaluated using a Monte Carlo method: $\langle(\Delta d)^2\rangle_b = (1/N)\sum_{i=1}^{N}(\Delta d)^2$. A uniform random variate R_i is used to sample the impact parameter as $b_i = b_{\max}\sqrt{R_i}$, where $0 \leqslant R_i \leqslant 1$. For each set of parameters, B and $K_\infty = K_{1\infty} = K_{2\infty}$, the generated trajectories have impact parameters less than $b_{\max}$, where $b_{\max}$ is the maximum impact parameter at which giant cross-magnetic-field steps occur in the simulations.

Figures 6 and 7 show the variation of the normalized root mean square of the steps versus the magnetic field strength and initial kinetic energy. For the evaluation, approximately 20 000 trajectories were simulated for a given set of parameters. For Fig. 6, $K_\infty = 6\kappa$ and the magnetic field was varied from 1 to 4 T in units of 0.5 T. It is observed that the normalized root mean square of the steps varies almost linearly with magnetic field strength. Also, the normalized root mean square of the steps decreases with the initial kinetic energy when the initial kinetic energy is increased and the magnetic field is at 1 T (Fig. 7).

Giant drifts also occur for more general initial velocity orientations. Figure 8 shows the normalized drift for $B = 1$ T, $K_\infty = K_{1\infty} = K_{2\infty} = 6\kappa$, $\zeta = 50$, and $b = 3.1r_c$ when the particles are initially located at $(x_{10}, y_{10}, z_{10}) = (b/2, 0, \zeta b/2)$ and $(x_{20}, y_{20}, z_{20}) = (-b/2, 0, -\zeta b/2)$, respectively. Their initial velocities are oriented making initial pitch angles θ_1 with $-\hat{\boldsymbol{k}}$ and θ_2 with $\hat{\boldsymbol{k}}$: $(v_{x10}, v_{y10}, v_{z10}) = (-\sqrt{2K_\infty/m}\sin\theta_1, 0, -\sqrt{2K_\infty/m}\cos\theta_1)$, $(v_{x20}, v_{y20}, v_{z20}) = (\sqrt{2K_\infty/m}\sin\theta_2, 0, \sqrt{2K_\infty/m}\cos\theta_2)$. Here, $m = m_1 = m_2$, $\theta_1 = 45^\circ$, and θ_2 varies from 0° to 90°. The results indicate the phenomenon reported here can occur within electron-positron plasmas.

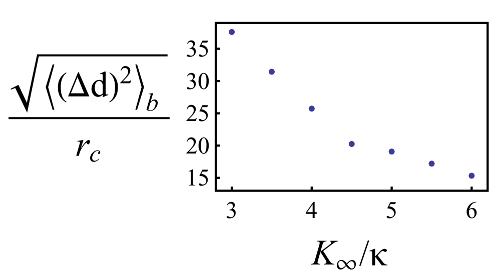

FIG. 7. (Color online) Normalized root-mean-square cross-magnetic-field drift distance versus particle kinetic energy with $B = 1$ T.

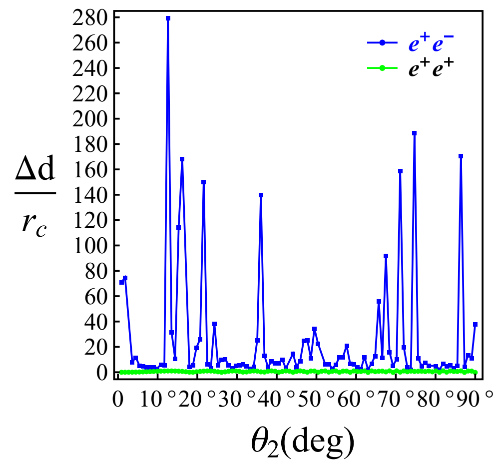

FIG. 8. (Color online) Normalized cross-magnetic-field drift distance versus initial pitch angle.

III. RELATIVE AND CENTER-OF-MASS MOTION

The motion of the center of mass (CM) across the magnetic field in electron-positron collisions is possible due to the coupling between the relative and CM motion. There exists no closed solution, and the presence of the magnetic field opens the possibility of chaotic dynamics [20–22]. In relative coordinates $\boldsymbol{r} = \boldsymbol{r}_1 - \boldsymbol{r}_2 = x\boldsymbol{i} + y\boldsymbol{j} + z\boldsymbol{k}$ and CM coordinates $\boldsymbol{R} = (\boldsymbol{r_1} + \boldsymbol{r_2})/2 = X\boldsymbol{i} + Y\boldsymbol{j} + Z\boldsymbol{k}$, the equations of motion can be written as

$$M\boldsymbol{R}'' = k_L e \boldsymbol{r}' \times \boldsymbol{B}, \tag{14}$$

$$\mu\boldsymbol{r}'' = k_L e \boldsymbol{R}' \times \boldsymbol{B} - \frac{k_c e^2 \boldsymbol{r}}{r^3}. \tag{15}$$

Here, e is the positron charge, $M = 2m = 2m_1 = 2m_2$ is the total mass, $\mu = m/2$ is the reduced mass, and $r = \sqrt{x^2 + y^2 + z^2}$. Equation (14) allows the definition of a conserved quantity, the pseudomomentum ($\mathcal{K}$). Since $\boldsymbol{B}$ is a constant vector

$$M\boldsymbol{R}' - k_L e \boldsymbol{r} \times \boldsymbol{B} = \mathcal{K}. \tag{16}$$

From Fig. 1, the positron and the electron are initially positioned at $\boldsymbol{r}_{10} = (b/2, 0, \zeta b/2)$ and $\boldsymbol{r}_{20} = (-b/2, 0, -\zeta b/2)$, respectively, and $\boldsymbol{v}_{10} = (0, 0, -v_{z20})$ and $\boldsymbol{v}_{20} = (0, 0, v_{z20})$ are their initial velocities. These initial conditions in CM and relative coordinates become $\boldsymbol{R}(0) = (0,0,0)$, $\boldsymbol{R}'(0) = (0,0,0)$, $\boldsymbol{r}(0) = b\hat{\boldsymbol{\imath}} + \zeta b\hat{\boldsymbol{k}}$ and $\boldsymbol{r}'(0) = -2v_{z20}\hat{\boldsymbol{k}}$. Thus, $\mathcal{K} = k_L e b B\hat{\boldsymbol{\jmath}}$ is the pseudomomentum. Replacing the velocity of the CM given by Eq. (16) in the equation for the relative motion Eq. (15) gives

$$\mu\boldsymbol{r}'' = \frac{k_L e}{M}(k_L e \boldsymbol{r} \times \boldsymbol{B} + \mathcal{K}) \times \boldsymbol{B} - \frac{k_c e^2 \boldsymbol{r}}{r^3}. \tag{17}$$

The force associated with the relative motion in components is

$$\boldsymbol{f} = \left[\frac{k_L^2 B^2 e^2 (b - x)}{4\mu} - \frac{e^2 x k_c}{r^3}\right]\hat{\boldsymbol{\imath}} - \left[\frac{k_L^2 B^2 e^2}{4\mu} + \frac{e^2 k_c}{r^3}\right] y\hat{\boldsymbol{\jmath}} - \frac{e^2 z k_c}{r^3}\hat{\boldsymbol{k}}. \tag{18}$$

From Eq. (16), the velocity of the CM is

$$\boldsymbol{R}' = \frac{k_L Bey}{M}\hat{\boldsymbol{\imath}} + \frac{k_L Be(b-x)}{M}\hat{\boldsymbol{\jmath}}. \tag{19}$$

From the dependence of the y relative component of the force in Eq. (18) and the initial conditions, $y = 0$ is found, and the velocity of the CM reduces to

$$\boldsymbol{R}' = \frac{k_L Be(b-x)}{M}\hat{\boldsymbol{\jmath}}. \tag{20}$$

From Eq. (20), the motion of the CM is

$$Y(t) = \frac{k_L Be}{M}\left[bt - \int_0^t x(\tau)\,d\tau\right]. \tag{21}$$

An irregularity in the relative motion along the $\hat{\boldsymbol{\imath}}$ dimension [with $(x \neq b)$] can cause a displacement of the CM in the $\hat{\boldsymbol{\jmath}}$ dimension. This component of the CM is associated with the transverse drift motion of the electron and positron that results in giant cross-magnetic-field steps.

The kinetic energy associated with the CM is

$$K_{\rm CM} = \frac{k_L^2 B^2 e^2 (b-x)^2}{4m}, \tag{22}$$

and the rate of change of CM kinetic energy is

$$\frac{dK_{\rm CM}}{dt} = -\frac{k_L^2 B^2 e^2}{2m}(b-x)x'. \tag{23}$$

If the variation of the x component of the relative position changes harmonically about the impact parameter, the average rate of change of the kinetic energy of the CM $\langle \dot{K}_{\rm CM}\rangle$ is zero. The dynamics of the CM will depend on the dynamics of the transverse relative motion. In the x component of the force in Eq. (18), there are two competing terms. The first term is a harmonic term that tends to keep x near the impact parameter. The second term is the Coulumbian term. When the particles are far from each other, the Coulumbian term is small, the harmonic term dominates, and the x component of the relative position oscillates around b. However, when the Coulumbian term grows larger, the relative motion along the $\hat{\boldsymbol{\imath}}$ dimension can become irregular causing the CM to move.

For positron-positron collisions, the equations of motion in relative and CM coordinate are written as

$$m\boldsymbol{R}'' = k_L e \boldsymbol{R}' \times \boldsymbol{B}, \tag{24}$$

$$m\boldsymbol{r}'' = k_L e \boldsymbol{r}' \times \boldsymbol{B} + \frac{2k_e e^2 \boldsymbol{r}}{r^3}. \tag{25}$$

Since $\boldsymbol{R}(0) = (0,0,0)$, $\boldsymbol{R}'(0) = (0,0,0)$ and the equations of motion for $\boldsymbol{R}$ and $\boldsymbol{r}$ are decoupled, the CM is stationary.

IV. CONCLUSION

In summary, computer simulations predict the existence of a continuous range of binary collision impact parameters within which colliding electron-positron pairs experience giant cross-magnetic-field drift distances. Within this range, drift distances orders of magnitude larger than that associated with corresponding like-charge collisions are possible. In addition a theoretical analysis was conducted that provides insights regarding the behavior of the two-particle correlated states.

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