Effective time-independent analysis for quantum kicked systems

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We present a mapping of potentially chaotic time-dependent quantum kicked systems to an equivalent approximate effective time-independent scenario, whereby the system is rendered integrable. The time evolution is factorized into an initial kick, followed by an evolution dictated by a time-independent Hamiltonian and a final kick. This method is applied to the kicked top model. The effective time-independent Hamiltonian thus obtained does not suffer from spurious divergences encountered if the traditional Baker-Cambell-Hausdorff treatment is used. The quasienergy spectrum of the Floquet operator is found to be in excellent agreement with the energy levels of the effective Hamiltonian for a wide range of system parameters. The density of states for the effective system exhibits sharp peaklike features, pointing towards quantum criticality. The dynamics in the classical limit of the integrable effective Hamiltonian shows remarkable agreement with the nonintegrable map corresponding to the actual time-dependent system in the nonchaotic regime. This suggests that the effective Hamiltonian serves as a substitute for the actual system in the nonchaotic regime at both the quantum and classical level.

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I. INTRODUCTION

Quantum systems driven periodically in time are known to undergo remarkable alterations in their long-time dynamical evolution [1–7]. This is an area of extensive study. External driving is known to create nontrivial gauge structures [8,9] and topological effects [10–14]. Modulated driving systems have the potential to fabricate new materials and phases of matter [10,11,15–21]. In this scheme, the modulation of the driving system is used to recreate an effective static Hamiltonian which is thereby investigated for interesting features of time-independent systems [6,7,22,23].

Kicked Hamiltonian systems are widely studied as a prototype model for classical and quantum chaos [24,25]. The traditional approach to such periodic systems uses the Floquet analysis to obtain the quasienergies of the systems. Earlier works [1,5,26] use the formulation based on the Cambell-Baker-Hausdorff (CBH) expansion to provide an effective time-independent Hamiltonian [23]. This method has been used to explore nonchaotic regimes and study critical quasienergy states [26]. A qualitative change in the wave function of the ground state on changing one or more parameters of the Hamiltonian of large systems characterizes a quantum phase transition (QPT) [27]. However, nonanalytic behavior of excited states has also been studied [26,28–30]. The clustering together of a whole branch of states implies that the average local level spacing Δ becomes vanishingly small leading to a divergence of local density of states (Δ^{-1}). This is a characteristic feature of excited state quantum phase transition (ESQPT). The corresponding semiclassical Hamiltonian in these cases indicate dynamical instability.

We claim that the traditional CBH approach to study such a time-dependent problem has intrinsic flaws and an alternative formulation [6,7] is more robust and accurate for the study of kicked systems. In this paper, we have applied this alternative formulation on quantum kicked systems where the time dependence of the Hamiltonian is in the form of a series of Dirac-δ pulses. We have studied the specific case of the kicked top model [24]. We show that this alternative formulation allows the effective static Hamiltonian to accurately mimic the exact evolution for a significantly large range of parameters and the eigenvalues of the effective Hamiltonian faithfully trace the exact quasienergies without any spurious divergences (as observed in [26]) that are pathologies of the CBH based formulation. In the absence of such singularities, the effective Hamiltonian obtained in our method can be used to study divergences in the density of states as a signature of ESQPT for a much wider range of parameter values. We have also shown a remarkable match between the phase-space dynamics of the classical limits of the exact quantum map [24] and symplectic evolution governed by the effective static Hamiltonian in the nonchaotic regime.

II. FORMALISM

Generic time-dependent problems where $\widehat{H}(t) = \widehat{H}_0 + \widehat{V}(t)$ are tackled using the Floquet theory when $\widehat{V}(t) = \widehat{V}(t + T)$ is time-periodic with period T. The Floquet operator $\widehat{\mathcal{F}}(t)$ corresponds to the time-evolution unitary operator $\widehat{U}(t)$ after one time period of the driving potential and has eigenvalues of the form $\exp(-i\,\phi\,T)$ where ϕ are referred to as the quasienergies of the system. In the traditional approach to find an effective Hamiltonian for kicked systems where

$$\widehat{V}(t) = \widehat{V} \sum_{n=-\infty}^{\infty} \delta(t - nT) \tag{1}$$

the Floquet operator is factorized as

$$\widehat{\mathcal{F}} = \exp(-i\,\widehat{V})\exp(-i\,\widehat{H}_0 T) = \exp(-i\,\widehat{H}_{\rm eff} T). \tag{2}$$

In the formalism described in [23] and used in [26], the effective Hamiltonian $\widehat{H}_{\rm eff}$ is extracted using the CBH expansion and truncated up to a certain order in $T \sim 1/\omega$. In the absence of any unique prescription for splitting the Hamiltonian, the Trotter-CBH method is sensitive to the initial phase of the driving potential. It is evident that any shift of the initial

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time t_i will lead to spurious artifact in $\widehat{H}_{\rm eff}$. This ambiguity has been discussed in the case of square two-phase modulation of a static Hamiltonian [7]. The effective Hamiltonian obtained for the kicked top system is also seen to exhibit unphysical singularities for a class of parameters [26]. In view of such criticisms of the earlier works that use the CBH expansion, to find the effective static Hamiltonians, we adopt the alternative formulation [6,7] to study Floquet systems.

The periodic potential in Eq. (1) may be expanded in a Fourier series as

$$\widehat{V}(t) = \widehat{V}_0 + \sum_{n=1}^{\infty} (\widehat{V}_n e^{in\omega t} + \widehat{V}_{-n} e^{-in\omega t}). \tag{3}$$

The method expresses the evolution operator $\widehat{U}(t_i \to t_f)$ between initial time instant t_i and final time instant $t_f = t_i + T$, as a sequence of an initial kick followed by an evolution under a static Hamiltonian and a final "micromotion" [7]

$$\widehat{U}(t_i \to t_f) = \widehat{\mathcal{U}}^{\dagger}(t_f) e^{-i\widehat{H}_{\rm eff}T}\, \widehat{\mathcal{U}}(t_i), \tag{4}$$

where $\widehat{\mathcal{U}}(t) = e^{i\widehat{F}(t)}$ such that $\widehat{F}(t) = \widehat{F}(t+T)$ with zero average over one time period. In our analysis with $t_i = 0$ and $t_f = T$, we have $\widehat{F}(t_i) = \widehat{F}(t_f)$. Thus the effect of $\widehat{\mathcal{U}}(t_i) = \widehat{\mathcal{U}}(t_f)$ is that of similarity transformation on $\exp(-i\widehat{H}_{\rm eff})$. The eigenvalues of $\widehat{H}_{\rm eff}$ shall hence mimic the quasienergies obtained from the Floquet operator. The operators $\widehat{H}_{\rm eff}$ and $\widehat{F}(t)$ are expanded in a perturbation series in powers of $1/\omega$ as

$$\widehat{H}_{\rm eff} = \sum_{n=0}^{\infty} \frac{1}{\omega^n} \widehat{H}_{\rm eff}^{(n)}, \quad \widehat{F}(t) = \sum_{n=1}^{\infty} \frac{1}{\omega^n} \widehat{F}^{(n)}. \tag{5}$$

Comparing this with Eq. (4), the perturbation series can be obtained up to any desired accuracy. At each order of perturbation, the averaged time-independent component is retained in $\widehat{H}_{\rm eff}$ and all time dependence is pushed into the operator $\widehat{F}(t)$. This yields up to $\mathcal{O}(1/\omega^2)$ for the Hamiltonian $\widehat{H}(t)$ [7]:

$$\begin{aligned}
\widehat{H}_{\rm eff} &= \widehat{H}_0 + \widehat{V}_0 + \frac{1}{\omega}\sum_{n=1}^{\infty}\frac{1}{n}[\widehat{V}_n, \widehat{V}_{-n}] + \frac{1}{2\omega^2}\sum_{n=1}^{\infty}\frac{1}{n^2}([[\widehat{V}_n, \widehat{H}_0], \widehat{V}_{-n}] + \text{H.c.}) \\
&\quad + \frac{1}{3\omega^2}\sum_{n,m=1}^{\infty}\frac{1}{nm}([\widehat{V}_n, [\widehat{V}_m, \widehat{V}_{-n-m}]] - 2[\widehat{V}_n, [\widehat{V}_{-m}, \widehat{V}_{m-n}]] + \text{H.c.}), \\
\widehat{F}(t) &= \frac{1}{i\omega}\sum_{n=1}^{\infty}\frac{1}{n}(\widehat{V}_n e^{in\omega t} - \widehat{V}_{-n} e^{-in\omega t}) + \frac{1}{i\omega^2}\sum_{n=1}^{\infty}\frac{1}{n^2}([\widehat{V}_n, \widehat{H}_0 + \widehat{V}_0]e^{in\omega t} - \text{H.c.}) \\
&\quad + \frac{1}{2i\omega^2}\sum_{n,m=1}^{\infty}\frac{1}{n(n+m)}([\widehat{V}_n, \widehat{V}_m]e^{i(n+m)\omega t} - \text{H.c.}) + \frac{1}{2i\omega^2}\sum_{n\neq m=1}^{\infty}\frac{1}{n(n-m)}([\widehat{V}_n, \widehat{V}_{-m}]e^{i(n-m)\omega t} - \text{H.c.}).
\end{aligned} \tag{6}$$

For general quantum kicked systems with Dirac-δ forcing, $\widehat{V}_n = \widehat{V}/T$ for all $n = 0, \pm1, \pm2, \ldots, \pm\infty$. Therefore, the above expression can be simplified as

$$\begin{aligned}
\widehat{H}_{\rm eff} &= \widehat{H}_0 + \frac{\widehat{V}}{T} + \frac{1}{\omega^2 T^2}[[\widehat{V}, H_0], \widehat{V}]\left(\sum_{n=1}^{\infty}\frac{1}{n^2}\right) = \widehat{H}_0 + \frac{\widehat{V}}{T} + \frac{1}{24}[[\widehat{V}, H_0], \widehat{V}], \\
\widehat{F}(t) &= \frac{2\widehat{V}}{\omega T}\sum_{n=1}^{\infty}\frac{\sin(n\omega t)}{n} + \frac{2}{i\omega^2 T}[\widehat{V}, \widehat{H}_0]\sum_{n=1}^{\infty}\frac{\cos(n\omega t)}{n^2} = \frac{\widehat{V}}{\pi}\sum_{n=1}^{\infty}\frac{\sin(n\omega t)}{n} - i\,\frac{T}{2\pi^2}\,[\widehat{V}, \widehat{H}_0]\sum_{n=1}^{\infty}\frac{\cos(n\omega t)}{n^2}.
\end{aligned} \tag{7}$$

It is evident that in the partitioning of the time-evolution operator as in Eq. (4) the system is assumed to undergo an initial kick $\exp[i\widehat{F}(t_i)]$ which is sensitive to the launching time t_i. This therefore has a long term bearing on the dynamical evolution, though the evolution after the initial kick is essentially dictated by the static effective Hamiltonian.

III. KICKED TOP MODEL

The kicked top model is representative of a host of such kicked systems and manifests chaotic dynamics [31–33]. This model is also known to have closed bearings with condensed matter systems like the metal-topological insulator [34,35] and has also been studied in the context of quantum critical transition [26,36]. The study of the kicked top is also motivated by recent experiments [37,38].

We consider the Hamiltonian for the kicked top

$$\widehat{H}(t) = \frac{\alpha}{2jT}\widehat{J}_z^2 + \beta\widehat{J}_x\sum_{n=-\infty}^{\infty}\delta(t - nT), \tag{8}$$

where $\widehat{J}_i$ denotes the components of the angular momentum, $j(j+1)$ is the eigenvalue of $\widehat{J}^2$. The operators $\widehat{J}_z$ and $\widehat{J}_x$ are the z and x components of the angular momentum operator, respectively. Three components of the angular momentum operator satisfy the standard commutation relation $[\widehat{J}_i, \widehat{J}_j] = i\epsilon_{ijk}\widehat{J}_k$. The Floquet operator for the above Hamiltonian is

given by

$$\widehat{\mathcal{F}} = \exp(-i\beta\widehat{J}_x)\exp\left(-i\frac{\alpha\widehat{J}_z^2}{2jT}\right). \tag{9}$$

The first factor in $\widehat{\mathcal{F}}$ describes a rotation operator around the x axis by the angle β. The second factor corresponds to a nonuniform rotation or torsion around the z axis. This term consists of a rotation angle which is itself proportional to the angular momentum component J_z. Therefore, the parameter α measures the torsional strength.

The perturbation expansion is tracked using the tracking parameter $1/\omega$. Evaluating the commutators in Eq. (6), we have the expressions for the truncated $\widehat{H}_{\rm eff}$ and $\widehat{F}$ up to $\mathcal{O}(1/\omega^2)$ given by

$$\begin{aligned}\widehat{H}_{\rm eff} &= \frac{\alpha}{2j}\widehat{J}_z^2 + \beta\widehat{J}_x - \frac{\alpha\beta^2}{24j}\left(\widehat{J}_z^2 - \widehat{J}_y^2\right),\\ \widehat{F}(t_i) &= \widehat{F}(t_f) = -\frac{\alpha\beta}{24j}(\widehat{J}_y\widehat{J}_z + \widehat{J}_z\widehat{J}_y).\end{aligned} \tag{10}$$

Here we have truncated the perturbation series at the second order and subsequently used the units of time such that $\omega = 2\pi$.

We note that $\widehat{H}_{\rm eff}$ does not show any singularities for any values of the parameters α and β. This feature differs crucially from the results obtained earlier [26] where the effective quasienergies showed divergence when $\alpha(2m+1) = 4jl\pi$ where $l \in \mathbb{Z}$ and m denotes the eigenvalues of $\widehat{J}_z$. We claim that these singularities are an artifact of the naive use of the CBH formula.

A. Quasienergy spectrum

Figure 1 shows the numerically obtained eigenvalues of the Floquet operator $\widehat{\mathcal{F}}$ and the effective Hamiltonian $\widehat{H}_{\rm eff}$ for $\beta = 0.1$. We note that the former is nonintegrable whereas the later is static, approximate, and integrable. The figure shows the eigenvalues ϵ of $\widehat{H}_{\rm eff}$ mapped into the first Brillouin zone. A remarkable match of ϵ with the quasienergies ϕ is obtained for broad range of values of α. The agreement of the approximate eigenvalues with the exact quasienergies at a very high level of precision is noticed even in the domain of the parameters α for which the system approaches a chaotic regime. This is different from the results obtained in an earlier work [26] where such agreement is noticed for a very small range of values of α.

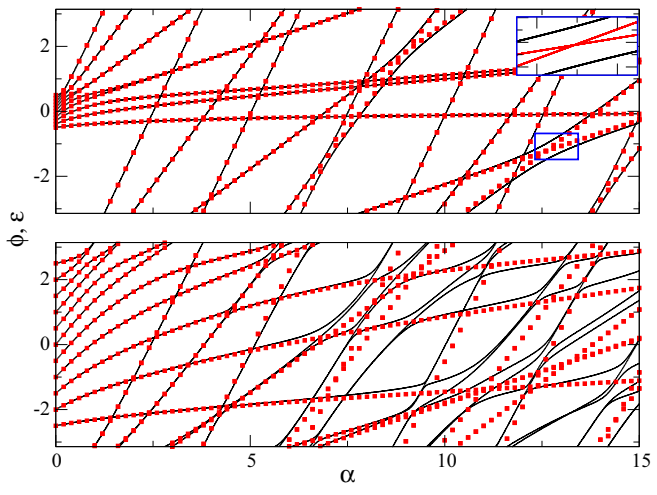

FIG. 1. (Color online) Quasienergy spectrum (solid line) of the Floquet operator $\widehat{\mathcal{F}}$ and the energy eigenvalues (solid square) of the effective Hamiltonian $\widehat{H}_{\rm eff}$ are compared as a function of α. Upper window shows the result for $\beta = 0.1$, and the lower window shows the same for $\beta = 0.5$. Whereas the integrable $\widehat{H}_{\rm eff}$ shows level crossing, the quasienergy spectrum indicates level repulsion (see inset).

The alternative formulation yields physical insight regarding the use of approximate methods to deal with time-dependent systems. The splitting of the time-evolution operator into an initial kick, a final kick, and an intermediate time evolution dictated by a stationary Hamiltonian is expected to give better results if the original time-dependent Hamiltonian itself is comprised of periodic kick pulses as in the present case of the kicked top. The idea has been to shift the effects of these kicks to the initial and final moments of time evolution by means of a unitary transformation. The success of this formulation implies that the method should be used for the generic class of kicked systems as against the CBH based method.

The fundamental departure in the approximate analysis adopted here occurs around regions where the exact quasienergy spectrum shows level repulsion which is characteristic of nonintegrable systems. In the eigenspectrum of $\widehat{H}_{\rm eff}$ which is an integrable system there is manifestation of degeneracy. The actual quasienergy spectrum avoids such crossings [39]. The situation gets worse for higher values of β where more of such spurious crossings appear.

To investigate the nature of critical quasienergy states for a much wider range of parameters in the Hamiltonian we compare the density of states (DOS) for the quasienergy spectrum and that of the eigenspectrum of $\widehat{H}_{\rm eff}$. The parameter values for which the energy spectrum indicates a tendency of clustering is also expected to manifest as divergence in the density of states,

$$\rho(\mathcal{E}) = \frac{1}{2j+1}\sum_r \delta(\mathcal{E} - \mathcal{E}_r), \tag{11}$$

where $\mathcal{E} = \phi$ and $\mathcal{E} = \epsilon$ for the quasienergy spectrum and the eigenvalues of $\widehat{H}_{\rm eff}$, respectively. This can alternatively be written as

$$\rho(\mathcal{E}) = \frac{1}{2\pi} + \frac{1}{\pi(2j+1)}\mathrm{Re}\left\{\sum_{n=1}^{\infty}\xi_n e^{in\mathcal{E}}\right\}, \tag{12}$$

where $\xi_n = \sum_r \exp(-in\mathcal{E}_r)$. We note that Eq. (12) uses the Fourier representation of Dirac δs in Eq. (11).

The DOS given in (11) or (12), for a discrete spectrum would yield a series of jagged vertical lines corresponding to the Dirac δs. Binning these discrete distributions would yield a histogram. We have numerically obtained the smoothed DOS by considering the Gaussian representation of the Dirac δs and obtaining a continuous curve by summing these Gaussian functions centered at different values of $\mathcal{E}_r$. The widths of the normalized Gaussians are kept at $\sim$10% of the mean spacing to ensure smooth and continuous DOS. The same numerics are used to obtain both the DOS for exact quasienergy spectrum and the eigenvalues of $\widehat{H}_{\rm eff}$. The parameter of the Hamiltonian

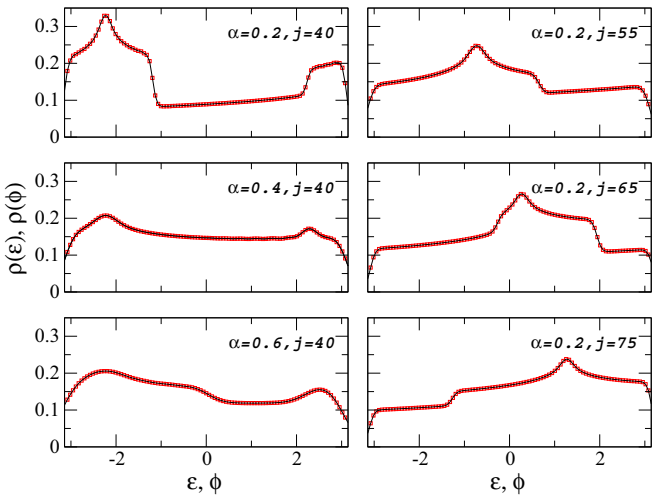

FIG. 2. (Color online) Density of states of the exact quasienergy spectrum of $\widehat{\mathcal{F}}$ and of the energy spectrum of $\widehat{H}_{\rm eff}$ are compared. Here the parameter $\beta = 0.1$.

for which there is a clustering tendency of the levels shall also reflect as the critical transition point where the DOS is likely to diverge.

Figure 2 compares the DOS for the exact quasienergy spectrum and the energy spectrum of $\widehat{H}_{\rm eff}$. The strong resemblance of these spectra (see Fig. 1) indicates that their DOS should also match. We find that the two are indeed found to match to a high degree of accuracy for different values of the parameter α, and also for different spin. The sharp peak feature that appears in the DOS for $\alpha = 0.2$ for spin $j = 40$ indicates quantum criticality as suggested in an earlier work [26]. At the critical quasienergy, a clustering of states occurs leading to the tendency of DOS to diverge.

The feature however is sensitive to α and flattens out at larger values of α. The position of the peak is seen to shift as j increases. This is quite expected since the DOS is closely related to the degeneracies of the energy levels which in turn depends on j. The claim for the indication of quantum criticality as reflected in logarithmic divergence of the DOS is noted in an earlier work [26]. This result is vindicated by our present analysis which uses an effective Hamiltonian intrinsically differing from the one used earlier but however does not suffer from mismatches in the spectrum at larger values of α.

B. Classical limit

Using the Heisenberg equation of motion, we find the following quantum dynamical map for the angular momentum operator:

$$\widehat{\mathbf{J}}_{n+1} = \widehat{\mathcal{F}}^{\dagger}\,\widehat{\mathbf{J}}_n\,\widehat{\mathcal{F}}. \tag{13}$$

The classical limit of this map can be achieved by first rescaling the operator $\widehat{\mathbf{J}}$ as $\mathbf{X} = \widehat{\mathbf{J}}/j$, i.e., $\{X,Y,Z\} = \{\widehat{J}_x,\widehat{J}_y,\widehat{J}_z\}/j$, where the commutators of the different components of the rescaled angular momentum operators take the form $[X,Y] = iZ/j$, and so on. This shows that, in $j \to \infty$ limit, components of this rescaled angular momentum operator will commute and become classical c-number variables, and we get the classical limit of the quantum dynamical map as

$$\begin{aligned} X_{n+1} &= \widetilde{X}\cos\alpha\widetilde{Z} - \widetilde{Y}\sin\alpha\widetilde{Z},\\ Y_{n+1} &= \widetilde{X}\sin\alpha\widetilde{Z} + \widetilde{Y}\cos\alpha\widetilde{Z},\\ Z_{n+1} &= \widetilde{Z}, \quad \text{where}\\ \widetilde{X} &= X,\\ \widetilde{Y} &= Y\cos\beta - Z\sin\beta,\\ \widetilde{Z} &= Y\sin\beta + Z\cos\beta. \end{aligned} \tag{14}$$

The above map satisfies the condition $X^2 + Y^2 + Z^2 = 1$. This suggests that the classical phase-space dynamics of the kicked top lies on the surface of a unit sphere, and each point on that surface is represented by two canonically conjugate dynamical variables $Z = \cos\theta$ and $\psi = \tan^{-1}(Y/X)$.

This classical map is to be compared with the dynamical solution in the phase space corresponding to the classical limit of the effective Hamiltonian $\widehat{H}_{\rm eff}$ in Eq. (10). We use the following prescription to find this classical limit designated by $H_{\rm cl}$:

$$H_{\rm cl} = \lim_{j\to\infty} \frac{\langle\gamma|\widehat{H}_{\rm eff}|\gamma\rangle}{j}, \tag{15}$$

where $|\gamma\rangle$ is the spin coherent state [24]. From Eq. (10), this yields

$$\begin{aligned} H_{\rm cl} = {}& \frac{\alpha Z^2}{2} + \beta\sqrt{1-Z^2}\cos\psi + \frac{\alpha\beta^2}{24}\sin^2\psi\\ & - \frac{\alpha\beta^2}{24}(1+\sin^2\psi)Z^2. \end{aligned} \tag{16}$$

This classical Hamiltonian represents the effective integrable model corresponding to the original Floquet system. The form of this Hamiltonian shows that it is nonseparable. The terms contain the canonical variables (Z,ψ) in a manner which does not allow a separation of $H_{\rm cl}$ into a purely position or momentum dependent components. Hamilton's canonical equations are given as follows:

$$\begin{aligned} \dot{Z} &= -\frac{\partial H_{\rm cl}}{\partial\psi} = \beta\sqrt{1-Z^2}\sin\psi - \frac{\alpha\beta^2}{24}(1-Z^2)\sin 2\psi,\\ \dot{\psi} &= \frac{\partial H_{\rm cl}}{\partial Z} = \alpha Z - \frac{\beta Z}{\sqrt{1-Z^2}}\cos\psi - \frac{\alpha\beta^2}{12}Z(1+\sin^2\psi). \end{aligned} \tag{17}$$

The symplectic evolution for this dynamical system mimics the map in Eq. (14) despite the fundamental difference between the two situations. The former represents conservative evolution satisfying Liouville's theorem without any signature of chaos for any values of the parameters α and β. The latter, on the contrary, is a well studied candidate of quantum and classical chaos for large $\alpha,\beta \gg 1$.

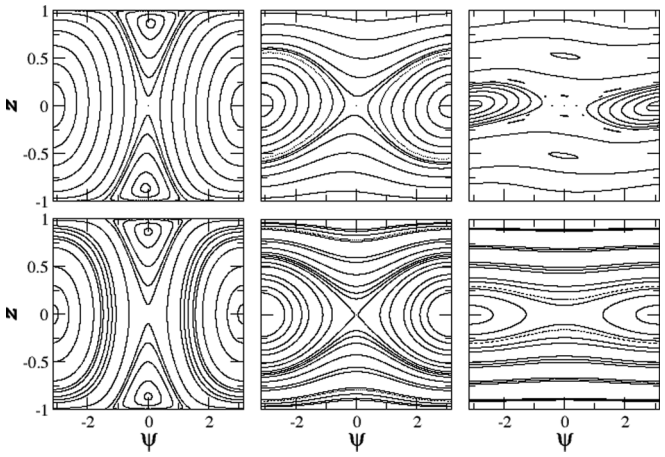

FIG. 3. The upper panel shows the phase space corresponding to the nonintegrable evolution governed by the map given in Eq. (14) for parameter $\beta = 0.1$. The lower panel shows the corresponding phase for the effective time-independent integrable Hamiltonian in Eq. (18). The parameter α is increasing from left to right as $\alpha = 0.2$ (left column), 1.0 (middle column), and 6.0 (right column). From left to right, the upper panel shows the system evolving towards chaotic regime. There is no such indication for the integrable system depicted in the lower panel.

Equation (17) can be also recast in terms of the variables $\{X, Y, Z\}$ as

$$\dot{X} = -\left(\alpha - \frac{\alpha\beta^2}{6}\right) YZ,$$
$$\dot{Y} = \left(\alpha - \frac{\alpha\beta^2}{12}\right) XZ - \beta Z, \tag{18}$$
$$\dot{Z} = \beta Y - \frac{\alpha\beta^2}{12} XY.$$

We note that this equation can be obtained as the large j limit of the quantum Hamiltonian

$$\widehat{H}/j = (\alpha/2)\widehat{Z}^2 + \beta\widehat{X} + (\alpha\beta^2/24)(\widehat{Z}^2 - \widehat{Y}^2) \tag{19}$$

using Heisenberg's equation of motion for the operators $\{\widehat{X}, \widehat{Y}, \widehat{Z}\}$, and subsequently replacing the quantum commutators by classical Poisson brackets.

Figure 3 shows the phase space for the dynamical systems represented by Eq. (17) and the classical map obtained for the actual time-dependent Floquet system described in Eq. (14). In the predominantly nonchaotic regular regime for small α, β values, the phase space trajectories agree to a very high degree and the effective time-independent classical Hamiltonian mimics the actual system for most purposes pertaining to dynamics. The departure begins to show up as the actual time-dependent system approaches the chaotic regime, and the trajectories, despite reflecting the same generic form, do not quite follow the same path. As the actual nonintegrable system approaches the chaotic regime the phase space shows the formation of islands which eventually break into further substructures. Expectedly, this feature is completely missing in the dynamics of the effective integrable Hamiltonian, where the phase space always remains regular and thereby cannot represent the actual time-dependent system for large parameter values. We note that there are studies indicating the disappearance of islandlike features in certain time-independent nonintegrable systems when they are approximated by integrable models [40].

IV. CONCLUSION

We conclude by noting that introducing a unitary transformation that pushes the time dependence of a time-periodic system to the initial time and final time instants in the form of kicks leads to an effective time-independent Hamiltonian that governs evolution for the bulk of the time. The energy spectrum of this effective Hamiltonian matches the exact quasienergies of the actual Floquet operator in the near integrable regime with deviations due to avoided crossings. We also find that the classical limit mimics the phase-space dynamics of the actual time-dependent nonintegrable Hamiltonian in the nonchaotic regime.

The CBH based method has been used extensively in the scientific literature without any concerns about its validity in a given context. The present work illustrates the fact that there are situations where the indiscriminate use may lead to incorrect conclusions. We also establish that the formulation adopted by us is probably a more appropriate approximate analysis of classical and quantum systems involving short duration pulse driving.

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