Effects of degree correlations on the explosive synchronization of scale-free networks

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We study the organization of finite-size, large ensembles of phase oscillators networking via scale-free topologies in the presence of a positive correlation between the oscillators' natural frequencies and the network's degrees. Under those circumstances, abrupt transitions to synchronization are known to occur in growing scale-free networks, while the transition has a completely different nature for static random configurations preserving the same structure-dynamics correlation. We show that the further presence of degree-degree correlations in the network structure has important consequences on the nature of the phase transition characterizing the passage from the phase-incoherent to the phase-coherent network state. While high levels of positive and negative mixing consistently induce a second-order phase transition, moderate values of assortative mixing, such as those ubiquitously characterizing social networks in the real world, greatly enhance the irreversible nature of explosive synchronization in scale-free networks. The latter effect corresponds to a maximization of the area and of the width of the hysteretic loop that differentiates the forward and backward transitions to synchronization.

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I. INTRODUCTION

During the last 15 years, network theory has successfully portrayed the interaction among the constituents of a variety of natural and man-made systems [1,2]. It was shown that the complexity of most real-world networks (RWNs) can be reproduced, in fact, by a growth process that eventually shapes a highly heterogeneous [scale-free (SF)] topology in the graph's connectivity pattern [3]. Furthermore, such a SF degree distribution affects, in its turn, in a non-negligible way almost all the dynamical processes taking place over RWNs [1].

Actually, and distinct from the degree distribution, many other important properties account for the fine details of the structure of any RWN, mostly due to particular forms of correlation (or mixing) among the network vertices [4]. The simplest case is the degree correlation [5], in which the network constituents tend to interact according to their respective degrees. Remarkably, nontrivial forms of degree correlation have been (experimentally and ubiquitously) detected in RWNs, with social networks displaying typically an assortative mixing (i.e., a situation in which each network's unit is more likely to connect to other nodes with approximately the same degree), while technological and biological networks exhibiting a disassortative mixing (which takes place when connections are more frequent between vertices of fairly different degrees). Both the assortative and disassortative mixing properties are known to considerably affect the organization of the network into collective dynamics, such as synchronization [6–8], epidemic spreading [9], and network controllability [10].

Possibly the most studied emerging collective dynamics in SF networks is synchronization [1,11], as such a state plays a crucial role in many relevant phenomena like, for instance, the emergence of coherent global behaviors in both normal and abnormal brain functions [12], the food web dynamics in ecological systems [13], or the stable operation of electric power grids [14–16]. In particular, it has been recently shown that the transition to the graph's synchronous evolution may have either a reversible or an irreversible discontinuous nature. The former case is what is traditionally investigated in coupled oscillators, where a second-order phase transition characterizes the continuous passage from the incoherent to the coherent state of the network [17,18]. The latter, instead, corresponds to a discontinuous transition, called explosive synchronization (ES) [19,20]. ES based on Kuramoto oscillators has rapidly become a subject of enormous interest [19–28]. While originally it was suggested that ES was due to a positive correlation between the natural frequencies of oscillators and the degrees of nodes [20], more recent studies have proposed the unifying framework of a mean field, where the effective couplings are conveniently weighted [22,27,29]. Yet only preliminary studies exist on the effect of degree mixing on ES [30–33], and their evidence is still not conclusive and sometimes also conflicting.

In this paper, we focus on ES of coupled phase oscillators in two different models of SF networks, in the presence of a positive correlation between the node's degree and its associated oscillator's natural frequency, and show that the degree mixing has important effects on the nature of the phase transition characterizing the passage from the phase-incoherent to the phase-coherent network state. In particular, we will first show that growing and static SF networks having the same degree distribution display in fact different explosive

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transitions and, second, we hypothesize that this dissimilarity is due to the presence of some sort of degree mixing. Actually, our evidence is that there is an optimal level of assortativity for which the width and area of the hysteretic region associated with ES is maximal, thus magnifying the irreversible nature of that transition.

II. THE MODEL

To this purpose, let us start by considering a network of N coupled phase oscillators whose phases θ_i $(i = 1,\dots,N)$ evolve according to the Kuramoto model [17]:

$$\frac{d\theta_i}{dt} = \omega_i + \sigma \sum_{i=1}^{N} a_{ij} \sin(\theta_j - \theta_i), \qquad (1)$$

where ω_i is the natural frequency of the ith oscillator. Oscillators interact through the sine of their phase difference, and are coupled according to the elements of the network's adjacency matrix a_{ij}, with $a_{ij} = 1$ if oscillators i and j are coupled, and $a_{ij} = 0$ otherwise. The strength of the coupling is controlled by the parameter σ, by increasing which one eventually (i.e., above a critical value of the coupling) promotes the transition to the coherent state, where all phases evolve in a synchronous way [17,34].

Following the changes in the level of synchronization among oscillators as the coupling strength increases is tantamount to monitoring the classical order parameter $s(t) = \frac{1}{N}|\sum_{j=1}^{N} e^{i\theta_j(t)}|$ [17]. Indeed, the time average of $s(t)$, $S = \langle s(t)\rangle_T$, over a large time span T assumes values ranging from $S \sim 0$ (when all phases evolve independently) to $S \sim 1$ (when oscillators are phase synchronized).

Typically, Eq. (1) give rise to a second-order phase transition from $S \simeq 0$ to $S \simeq 1$ for a unimodal and even frequency distribution $g(\omega)$, with a critical coupling at $\sigma_c = 2/[\pi g(\omega=0)]$ for the case of all-to-all connected oscillators [35], and $\sigma_c' = \sigma_c \frac{\langle k\rangle}{\langle k^2\rangle}$ for the case of a complex network with first and second moments of the degree distribution $\langle k\rangle$ and $\langle k^2\rangle$, respectively [11]. However, in the last few years it was pointed out that a different scenario (ES) can arise, featuring an abrupt, first-order-like transition to synchronization, and associated with the presence of a hysteretic loop [19–23]. In this latter case, the forward (from $S \simeq 0$ to $S \simeq 1$) and backward (from $S \simeq 1$ to $S \simeq 0$) transitions occur in a discontinuous way and for different values of the coupling strength, in this way marking an irreversible character of the phase transition, which is of particular interest at the moment of engineering (or controlling) magneticlike states of synchronization [23].

III. EXPLOSIVE SYNCHRONIZATION DEPENDENCE ON THE SF MODEL

We concentrate on ES in growing and static SF networks, when a microscopic relationship between the structure and the dynamical properties of the system is imposed. In particular, and following the approach of Ref. [20], we will choose a direct proportionality between the frequency and the degree distribution $[g(\omega) = P(k)]$, implying that each network's oscillator is assigned a natural frequency equal to its degree, $\omega_i = k_i$, where $k_i = \sum_j a_{ij}$ is the number of neighbors of the oscillator i in the network.

As for the stipulations followed in our simulations, SF growing networks are constructed following the procedure introduced in Ref. [36]. Such a technique, indeed, allows construction of graphs with the same average connectivity $\langle k\rangle$, and grants one the option of continuously interpolating from Erdős-Rènyi (ER) [37] to Barabàsi-Albert (BA) [3] SF topologies, by tuning a single parameter $0 \leqslant \alpha \leqslant 1$. With this method, networks are grown from an initial small clique of size $N_0 > m$, by sequentially adding nodes, up to the desired graph size N. Each newly added node then establishes m new links, having a probability α of forming them randomly with already existing vertices, and a probability $1 - \alpha$ of following a *preferential attachment* (PA) rule for the selection of its connection. When the latter happens, a generalization of the original PA rule [3] is used that includes an initial and constant attractiveness A for each of the network's sites, so that the attractiveness of node i (the probability that such a node has to receive a connection) is $A_i = A + k_i$ [38]. The result of the above procedure is that the limit $\alpha = 1$ induces an ER configuration, whereas the limit $\alpha = 0$ corresponds to a SF network with degree distribution $P(k) \sim k^{-\gamma}$, with $\gamma = 2 + A/m$ (when $A = m$, $\gamma = 3$, and the BA model is recovered).

With the aim of inspecting whether ES depends on the chosen SF network model, we further comparatively consider ensembles of networks displaying the very same SF distributions as those obtained with the PA algorithm described above ($\alpha = 0$) but this time we construct the SF topology by means of the so called *configuration model* (CM) [39,40], a randomized realization of a given network where the node degree distribution remains intact. In both cases, we set $N = 10^3$, $\langle k\rangle = 6$, and distribute the oscillators' frequencies so as to determine a direct correlation with the node degree ($\omega_i = k_i$) and, therefore, both network models display identical frequency distributions.

The results are shown in Fig. 1, and reveal a dramatic dependence of the ES behavior on the underlying SF network model used, despite both having exactly the same $P(k)$. In particular, the top row of Fig. 1 reports the order parameter S when σ is gradually increased in steps of $\delta\sigma$ (forward tuning, solid line), and also in the reverse way, i.e., departing from a network state where $S = 1$ and gradually decreasing the coupling by $\delta\sigma$ at each step (backward tuning, dashed line). The different areas of hysteresis displayed by the PA (left panel) and CM (right panel) networks seem to indicate that a crucial condition to obtain a strong irreversibility in ES is having an underlying growth process through which the SF topology is shaped. In order to statistically characterize such critical behavior, the bottom panels of Fig. 1 account for the probability density functions of the area of hysteresis (left panel) and the largest difference in $S(\sigma)$ (right panel) for both SF models. Averages are obtained from the simulation of 400 PA networks (and their corresponding CM network realizations). The values of the hysteretic areas are much larger for PA than for CM networks up to the point that the ES for the latter is almost absent (the most likely values for the hysteresis area and $\Delta S_{\max}$ are very small).

As a first conclusion, we can affirm that, despite having the same $P(k)$ and therefore the same $g(\omega)$, the two classes of

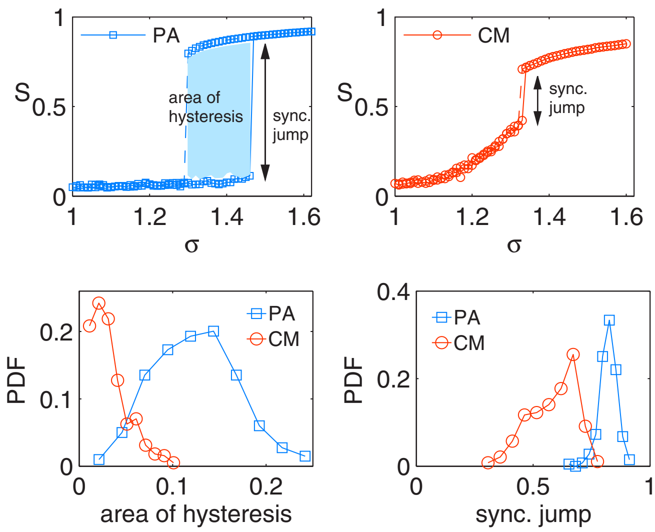

FIG. 1. (Color online) Comparative results on ES between SF networks belonging to two different ensembles: preferential attachment (PA) and configuration model (CM). Top row: Forward (solid lines) and backward (dashed lines) synchronization curves for PA (left panel) and CM (right panel) SF networks with exactly the same degree distribution. The area of the hysteresis is depicted as a blue shaded area in the PA-SF network while the synchronization jumps are marked in both cases. Bottom row: Probability density functions of the area of hysteresis (left panel) and of the synchronization jumps (right panel) for 400 PA (CM) network realizations. In all cases, $N = 10^3$, $\langle k \rangle = 6$, $\gamma = 2.4$, and natural frequencies $\omega_i = k_i$. The CM network realizations are constructed using the degree sequences of the PA networks.

SF networks exhibit a different explosive behavior. Supported by the evidence provided in Fig. 1, we hypothesize that this difference is related to the presence of two-point degree correlations $P(k,k')$. One customary way to quantify the amount of degree correlation with a single parameter is by using the Pearson correlation coefficient r between the degrees of all nodes at either ends of a link, which can be calculated as in Ref. [4]:

$$r = \frac{L^{-1}\sum_i j_i k_i - \left[L^{-1}\sum_i \frac{1}{2}(j_i + k_i)\right]^2}{L^{-1}\sum_i \frac{1}{2}\left(j_i^2 + k_i^2\right) - \left[L^{-1}\sum_i \frac{1}{2}(j_i + k_i)\right]^2},$$

where j_i and k_i are the degrees of the nodes at the ends of the ith link, with $i = 1, \ldots, L$. Actually, one has that $-1 \leqslant r \leqslant 1$, with positive (negative) values of r quantifying the level of assortative (disassortative) mixing of the network. We recall here that the BA model does not exhibit any form of mixing in the thermodynamic limit [$r \to 0$ as $(\log^2 N)/N$ for $N \to \infty$ [4,41] and that a random CM produces networks that are highly disassortative.

IV. THE EFFECT OF THE DEGREE MIXING

In the following, we study the impact of increasing or decreasing the assortativity mixing on a network with a given degree sequence $\{k_i\}$ from a power-law distribution $k^{-\gamma}$. In order to generate SF networks with given and tunable levels of degree mixing, we follow an adjusted version of the degree-preserving [42] rewiring algorithm of Xulvi-Brunet and Sokolov [43], but similar and fully consistent results are obtained using other procedures to impose degree mixing, such as simulated annealing [44] or with prescribed correlations [4]. For each one of the growing and static SF networks, we choose a pair of links at random and monitor the degrees of the four nodes at the ends of such links. The links are then rewired in such a way that the two largest- and the two smallest-degree nodes become connected provided that none of those links already exist in the network (in which case the rewiring step is aborted and a new pair of links is selected). Repeating such a procedure iteratively results in progressively increasing the assortativity of the network, in that more and more connected nodes of the network will display a similar degree. Conversely, if the rewiring is operated in a way to determine that the largest- (second-largest-) and the smallest- (second-smallest-) degree nodes are connected, the resulting network becomes progressively dissassortative.

In this way, we first generate a PA network of size $N = 5 \times 10^3$ with a given mean degree $\langle k \rangle = 2m$ and slope γ as previously described, and produce the corresponding random CM network realization. Then we check whether those networks are uncorrelated, that is, whether $r = 0$. If not (which is always the case due to finite-size effects), we perform the link rewiring procedure until the networks are neutral (i.e., with no degree correlations). Finally, we take these resulting configurations as our PA and CM reference networks, and further perform on them the link rewiring procedure in order to produce an ensemble of networks, all of them having the same degree distribution, but different values of the assortativity coefficient r.

Figure 2 illustrates the effect of the imposed degree mixing on ES. Extensive numerical simulations of Eq. (1) were performed at various values of r, and for SF networks with different slopes γ ranging from 2.4 to 3.0, and the same mean degree $\langle k \rangle = 6$. The most relevant result is that, regardless of the specific SF network model, the hysteresis of the phase transition is highly enhanced (weakened) for positive (negative) values of the assortative mixing parameter, and that there is an optimal positive value of r where the irreversibility of the phase transition is maximum. Notice that the enhancement is far more pronounced in PA (left panel) than in CM (right panel) netoworks. Moreover, as the slope of the power law of the degree distribution becomes steeper (large values of γ), the enhancement produced by a positive degree mixing gradually vanishes and the optimum point slightly shifts to higher values of r. Notice, finally, that null values of the hysteretic area indicate that the transition has lost its irreversible character, so that degree mixing can turn an explosive irreversible phase transition into a second-order, reversible one.

The nonmonotonic behavior of the area of hysteresis is further exemplified by looking at the synchronization curves S shown in the bottom panels of Fig. 2. The left (right) panel shows the forward and backward synchronization transitions for three values of the assortative (disassortative) mixing. From the results shown in the right panel it is evident that an increasing level of disassortative mixing reduces the threshold of the forward transition (which is consistent with the general claims of Refs. [6–8] that disassortativity favors the network's synchronizability), but it progressively reduces

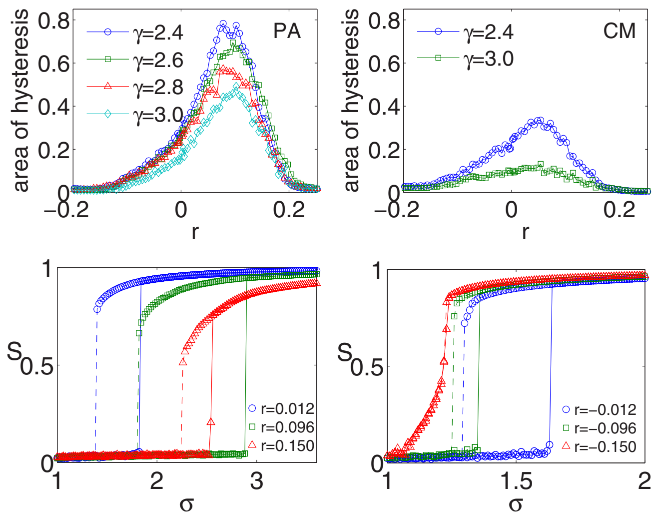

FIG. 2. (Color online) ES as a function of the degree mixing. Top row: Area of the hysteretic region delimited by the forward and backward synchronization curves vs the Pearson correlation coefficient r for PA (left panel) and CM (right panel) networks with different values (reported in the legend) of the exponent γ of the degree distribution $P(k) \sim k^{-\gamma}$. Each point is an average over ten different simulations. Bottom row: Forward (solid lines) and backward (dashed lines) synchronization curves for PA networks ($\gamma = 2.4$) displaying different levels of assortative (left panel) and disassortative (right panel) mixing. Curves are coded accordingly to the specific value of the parameter r (reported in the legend of each panel). In all cases, networks are SF with $N = 5 \times 10^3$, $\langle k \rangle = 6$, and $\omega_i = k_i$.

the hysteretic area associated with ES, until eventually a second-order reversible transition is recovered. In contrast, the effects of assortativity (left panel) are seemingly nontrivial: the threshold for the forward synchronization has an increasing trend with $r > 0$, but the area of hysteresis appears to widen for intermediate values of r.

Further information can be gathered from inspection of Fig. 3, where we illustrate the effect of varying the mean

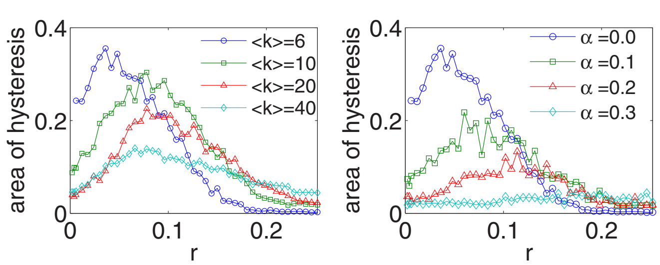

FIG. 3. (Color online) Area of hysteresis as a function of r for PA networks with different amounts of heterogeneity. In the left panel, the different curves correspond to different values of the mean degree $\langle k \rangle$ (reported in the legend), while in the right panel $\langle k \rangle = 6$ and the network heterogeneity is varied by means of increasing the parameter α (see the legend for the color and symbol code of the different reported curves), from pure SF ($\alpha = 0$) to $\alpha = 0.3$ ($\alpha = 1$ corresponds to a pure ER network). In all cases, $N = 10^3$, $\gamma = 2.4$, and $\omega_i = k_i$, and each point is an average of ten simulations.

degree $\langle k \rangle$ (left panel), and the level of heterogeneity α (right panel) in PA networks of smaller size ($N = 10^3$). In the left panel it is observed that already at $r = 0$ (uncorrelated networks), increasing the mean degree results in narrowing the area of hysteresis, with the consequence that the phase transition becomes smoother and smoother, until eventually ES is lost. For generic values of r, as $\langle k \rangle$ increases, the curves of the area of hysteresis are attenuated and shifted to higher values of r. Regarding the effect of the heterogeneity in the network's connectivity (right panel), moving from pure PA networks ($\alpha = 0$) to slightly larger values of α causes rapid deterioration of the enhancement of hysteresis. However, a positive degree mixing can still turn a second-order phase transition (for $r = 0$) into an abrupt and irreversible one at a value of $r \sim 0.1$ when $\alpha = 0.2$.

V. DISCUSSION

From the results reported in the top row of Fig. 2 one clearly sees how growing PA networks present a larger hysteresis area than static CM networks for any value of r, although both classes of network models display an enhancement of irreversibility in connection with an increase in the degree-degree correlation. Figure 4 provides further evidence of how ES is achieved for a particular value of γ in terms of the maximum gap in the order parameter $\Delta S_{\max}$ (left panel) and the critical coupling strengths σ_c^{fw} and σ_c^{bk} (right panel) marking, respectively, the forward (solid symbols) and backward (hollow symbols) transition points. As a function of the degree mixing, the curves obtained for growing PA (red circles) and static CM (blue triangles) SF networks have slightly different trends. This figure makes it more apparent that the critical coupling for the forward transition decreases almost linearly as the mixing increases in the region $r < 0$ [6–8], while for $r > 0$, the dependence of σ_c^{fw} on r is clearly nonlinear. This suggests that specific pronounced topological mesoscales arise at those values of r which influence the forward transition, having the effect of obstructing the otherwise increasing trend

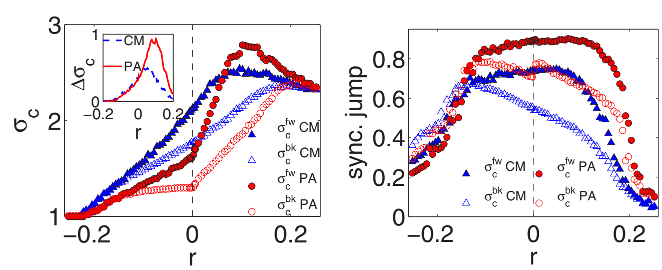

FIG. 4. (Color online) Behavior of the critical parameters characterizing the synchronization transition. Critical coupling strengths (left panel) and synchronization jumps $\Delta S_{\max}$ of the order parameter (right panel) at the synchronization transitions during the forward (solid symbols) and backward (hollow symbols) continuations for growing PA (○) and static CM (△) SF networks as a function of the degree mixing r. In all cases, $N = 5 \times 10^3$, $\langle k \rangle = 6$, and $\gamma = 2.4$. The vertical dashed line marks $r = 0$. The inset of the left panel reports the corresponding width of the hysteresis curves, calculated as $\Delta\sigma_c = |\sigma_c^{fw} - \sigma_c^{bk}|$ for PA (○) and CM (△) SF networks, with "fw" and "bk" indicating "forward" and "backward," respectively.

of σ_c^{fw}. In contrast, in the backward continuations (hollow symbols) the relationship between σ_c and r exhibits a change of slope only at $r = 0$. The inset in the left panel of Fig. 4 reports the corresponding widths of the hysteresis curves, calculated as $\Delta\sigma_c = |\sigma_c^{fw} - \sigma_c^{bk}|$, and underlines once again the existence of a maximum for $\Delta\sigma_c(r)$, in correspondence with the maximum in the area of the hysteresis reported in Fig. 2. Regarding the behavior of the maximum gap of the order parameter at the forward transition (right panel of Fig. 4), it displays a plateau at large values within the interval of r where the ES still holds, and the abruptness of the transition deteriorates for large values of $|r|$.

Figure 4 then clarifies that the enhancement of the hysteresis is associated with a moderate increase in the degree-degree correlation, recovering a second-order transition for large values of positive and negative r. This nontrivial effect can be understood by examining the inner mechanism of the frequency-degree correlation. Explosive transitions result from a frustration in the path to synchronization [45]. In the case of ER networks, where the path to synchronization starts from multiple seeds homogeneously distributed in the network, this frustration can be induced by imposing a gap in the frequency differences of each pair of nodes. The larger is the gap frequency, the higher is the frustration (explosivity) of the system, which shows a positive correlation between the explosive character of the system and the width of the hysteresis [23]. In the case of general SF networks, this path starts from the hubs, leading to the synchronization of the system by progressively recruiting nodes [46]. However, under positive frequency-degree correlations this frustration is induced by an emergent frequency gap existing between hubs and their neighbors. Therefore, frustrating the path to synchronization in SF networks is tantamount to isolating the influence of the hubs in the system. In this way, the more connected the network is through the hubs, the more explosive the transition becomes once the hubs are isolated.

The above arguments can be quantified by evaluating the node betweenness centrality, which computes the fraction of all shortest paths passing through each node of the network. Figure 5 shows the mean betweenness (left panel) for the core made of the set of the first three higher-degree nodes (for the remaining nodes the betweenness does not change significantly) and the mean distance from the hub to the rest of the network (right panel), for both PA (red and solid symbols) and CM (blue and hollow symbols) networks. PA networks are comparatively more connected through the hubs than CM static networks within the region of degree-degree correlation where explosive behavior is observed. Therefore, in the case of the CM there are more paths connecting the network that do not necessarily pass through the hubs, allowing progressive local synchronization and thus reducing the explosive character of the transition and the associated hysteresis width. This is of course due to the specific characteristics of the hubs in each network model. While a growing PA network starts from an all-to-all connected seed, for the static CM network the hubs are randomly distributed in the network, as their natural degree-degree correlations reveal: $r \simeq 0$ for growing PA networks and $r = -0.19$ for the CM networks.

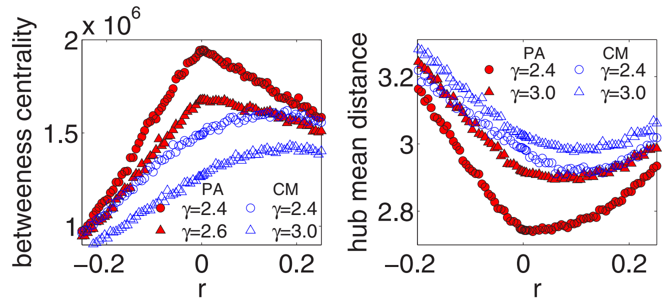

FIG. 5. (Color online) Dependence of the network structural properties on the degree mixing. Betweenness centrality of the first three higher-degree hubs (left panel) and mean network distance of the hub of the network (right panel) as a function of the Pearson correlation coefficient r for PA (red and solid symbols) and CM (blue and hollow symbols) SF networks with different values of the exponent γ of the degree distribution $P(k) \sim k^{-\gamma}$ as indicated in the legend. Each point is the average of ten network realizations with $N = 5 \times 10^3$ and $\langle k \rangle = 6$.

According to Ref. [25], σ_c increases with the degree of the main hub for uncorrelated SF networks in the limit of small mean degree networks, where the role of the hubs is certainly dominant. Therefore, we suggest that a small increase in the degree-degree correlation promotes the connectivity of the hubs, leading to the emergence of a core with a larger effective degree, which increases the hysteresis width accordingly. However, further increase of the assortativity or dissassortativity enhances the modularity of the network, thus breaking the dominant role of hubs by over- or underconnecting them. This is reflected by the decrease of the core's betweenness for large positive and negative values of r for both growing and static networks (see again Fig. 5).

VI. SUMMARY

In summary, we have reported simulations of the dynamics of networked ensembles of phase oscillators whose interactions are mediated by a scale-free topology of connections, and for which a positive correlation exists between each oscillator's natural frequency and the corresponding node degree. Our results allow us to conclude that the further presence of degree-degree mixing in the network structure has crucial consequences for the nature of the phase transition accompanying the emergence of the phase-coherent state of the network. In particular, we have shown that high levels of both positive and negative mixings consistently produce a second-order phase transition, whereas moderate values of assortative mixing magnify the irreversible nature of ES in both static and growing SF networks. When related to the fact that nontrivial forms of degree correlation indeed ubiquitously characterize the structure of real-world SF networks, our results may be of relevance for understanding why real-world biological and technological networks organize themselves on topological structures that tend to avoid explosive synchronization phenomena (which are there usually associated with pathological states of the networks), whereas social network topologies actually favor the explosive and irreversible emergence of synchronous states.

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