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Interaction of scroll waves in an excitable medium: Reconnection and repulsion

Nirmali Prabha Das and Sumana Dutta*

Department of Chemistry, Indian Institute of Technology Guwahati, Guwahati 781039, India

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Scroll waves of reentrant activity and their interactions pose a serious threat to cardiac health. In experiments with the Belousov-Zhabotinsky reaction we demonstrate the interaction of scroll waves. We show that depending on their mutual orientation, two scroll rings can push each other away and rupture on touching the system boundary, or they can reconnect to form a single, large ring. Reconnection only occurs when the filaments lie within one core length of each other. The reconnected filament has extended lifetimes, which could have serious implications in systems where they occur. The experimental results are explained on the basis of a simple numerical model.

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Spiral waves occur in systems ranging from biology [1] to astrophysics [2], and fluids [3] to superconductors [4]. Scroll waves are the three-dimensional (3D) counterparts of spiral waves, which rotate around a one-dimensional singularity, known as its filament [5]. Their presence in the cardiac tissues is many times the cause of arrhythmias and tachycardias, which finally lead to heart failure [6,7]. So the interaction of scroll waves may have far-reaching consequences on cardiac activity.

In fluids and liquid crystals, there is evidence of vortex interaction leading to interesting phenomena like filament reconnection [8–10]. If likewise, scroll rings interact and reconnect, then small rings may merge and form large ones that will have enhanced lifetimes. If this happens in heart tissues, it will ensure a long life of the filaments which in turn will have a detrimental effect on cardiac health. The work reported here is motivated by these concerns. Though the study of scroll waves has been ongoing for quite a few decades [11], only a few computational studies of their interactions have been made [12–14]. Some experiments on the interaction of two-dimensional (2D) spiral cores [15,16] and 3D filaments have been carried out more recently [17], but no instances of scroll-wave reconnection have yet been demonstrated.

In this Rapid Communication, we report the experimental evidence of scroll-wave reconnection. Our results demonstrate that when two scroll rings are brought close enough, they can either attract each other, and reconnect to form a large scroll ring, or they can repel so that they rupture on touching the boundaries. We also carry out simple numerical simulations that help explain the filament behavior in our experiments. Both the phenomena will have important consequences on the nature and lifetime of the scroll waves.

For our studies in scroll-wave interaction, we use the three-dimensional Belousov-Zhabotinsky reaction, which is a simple laboratory model where scroll waves can be directly observed [18–23]. Our experimental system was embedded in a 0.8% w/v agar gel matrix. The mixture contained 0.04 M sodium bromate, 0.04 M malonic acid, 0.16 M sulphuric acid, 0.5 mM ferroin, and 0.1 mM SDS (sodium dodecyl sulfate). SDS was added to prevent the formation of CO_2 bubbles during the later stages of the experiment. For the first set of experiments, we initiated two nonrotating semispherical waves some distance apart, touching the tip of a silver wire into a 5-mm-thick Belousov-Zhabotinsky (BZ) gel layer. When the two waves expanded to their required size, and were approximately 1–2 mm apart, another layer of BZ gel was poured over this layer. The semispherical waves curled into the top layer and formed a pair of scroll waves with circular filaments. Both the filaments were coplanar and had a similar sense of rotation. In order to study scroll-wave interactions between filaments having an opposite sense of rotation, we prepared two gel layers, each with a thickness of about 5 mm, in two separate Petri dishes. Two nonrotating half spheres were generated in each one by initiation with a silver wire. When the waves reached desirable dimensions, we placed one Petri dish over the other. The waves curled into the facing gel layer and formed two scroll rings, whose filaments are placed one over the other. We illuminated our experimental system from below, using diffused white light and monitored it from above with a charge coupled device camera (mvBlueFOX 220a) through a blue filter. The images were recorded onto a computer at an interval of 2 s and the data analyzed using MATLAB codes.

In our analysis of the filament interactions, we have assigned a direction along the tangent to the filament at a particular point, via the right-hand rule [14]. This utilizes the curl of the motion of the constituent spirals around the said point on the filament. Thus a unique direction of travel along the ring is defined. The two scroll rings that are formed side by side on the gel, have a clockwise sense of rotation when viewed from the top, while the scroll rings that are formed with their kernels facing each other, have an opposite sense of rotation.

Figure 1 summarizes the results of experiments involving interacting scroll rings with the same sense of rotation. During the scroll initiation process, if the half-spherical waves are too close, on pouring the second gel layer over them the wave fronts merge, and a single scroll wave is formed. When we increase the distance between the wave fronts, two independent scroll rings are generated. However in most cases, the distance between them keeps on increasing, as they shrink under their positive filament tension, known to exist for the filaments in the BZ system. Only within a very specific range of interfilament distance, do the two filaments undergo reconnection. Figure 1 is such an example where scroll-wave reconnection has been observed. Figure 1(a) shows the initiation of two scroll waves formed at a close proximity. The dark region in Fig. 1(e) marks

*sumana@iitg.ernet.in

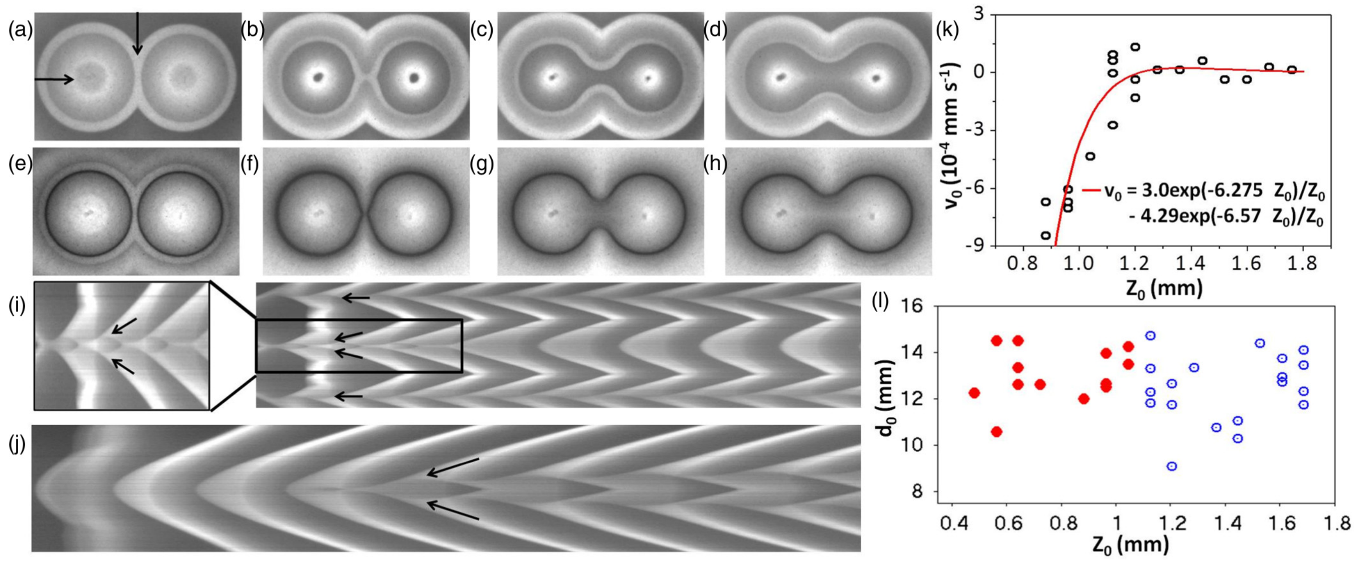

FIG. 1. (Color online) Reconnection of scroll-wave filaments. (a)–(d) Snapshots of a pair of coplanar scroll waves at (a) 7 min, (b) 21 min, (c) 35 min, and (d) 49 min after scroll-wave initiation. Image area of the snapshots are 3.2 cm × 2 cm. (e)–(h) Filaments corresponding to the snapshot above it, calculated over one time period. The dark areas of singularity show the position of the filaments. (i) Time-space plot along the horizontal arrow in frame (a). Magnified area marked by a black box. (j) Time-space plot along the vertical arrow in frame (a). Black arrows in frames (i) and (j) mark the position of filaments. Time evolves left to right. Time-space plots span a time of 52 min. (k) v_0 as a function of Z_0. The curve represents a fitting of the data as a difference of two Yukawa potentials. (l) Reconnection as a function of Z_0 and d_0. Reconnected filaments are marked as full (red) circles and nonreconnected ones as open (blue) circles. Movies available [24].

the position of the filaments at this stage of the experiment. With time, the filaments attract each other at the point of nearest approach [Fig. 1(f)], and a bridge is formed between the two circular wave fronts [Fig. 1(b)]. Eventually this bridge expands and the two filaments evolve into a single one [Figs. 1(c) and 1(g)]. This has been predicted in earlier mathematical studies, and the phenomenon coined *crossover collision* [13,25]. This ensures that after collision, the two filaments do not reconnect as before, reestablishing the previous filaments. Instead they choose to cross over in space and give rise to two new filament sections. In this case, they join to form a single, large filament. The newly formed vortex filament then undergoes curvature-dependent motion, with the bridge expanding at the center, and the two sides shrinking, until it almost forms a hot dog shape. This is in keeping with the curvature-flow model [20]. The filament continues to shrink further until it comes to a point and disappears. The time-space plots clearly show the presence of four filament sections along the horizontal cross section of the snapshots [Fig. 1(i)] at an early stage of the experiment (the first three rotations), while no filaments are encountered along the vertical cross section [Fig. 1(j)] at this time. In the later part of the reaction, only two filament sections are encountered along the horizontal axis, while two new filament sections have now evolved in the vertical cross section. Thus, circular filaments with the same sense of rotation reconnect when they are sufficiently closely spaced.

To get an idea about how closely spaced the filaments should be in order to reconnect, we carried out several sets of experiments, with varying inter-ring distance. The initial velocity of interaction (v_0) has been plotted as a function of the initial interfilament separation, Z_0 [Fig. 1(k)]. It shows an attractive regime for $Z_0 < 1.1$ mm (negative v_0) and then a repulsive region beyond that (positive v_0), which reaches a maxima and then falls off. Beyond a certain distance, the scroll rings had no influence on each other. This could be fitted quite well as a difference of two Yukawa potentials, as has been predicted by earlier theoretical studies [12]. We also wanted to see if the size of the scroll rings played any role in the process. Figure 1(l) shows a phase diagram of the reconnected versus nonreconnected filaments as a function of Z_0, and the average of the initial ring diameters, d_0. The two filaments are initially more or less of similar dimensions (within 1 mm error). Hence the average diameter of the two rings is a good measure of filament size. Our analysis shows that reconnection depends only on the distance separating the filaments, and all filaments that were 1.04 mm apart or lesser, reconnected. This may be because, at this distance the filaments lie within one core length (1.09 mm) of each other [26]. The size of the rings, however, did not have any marked effect on the phenomenon of reconnection.

When scroll rings with an opposite sense of rotation are generated, we observe that at most times, when the scroll rings are appreciably large, one of the circular filaments ruptures (Fig. 2). We may hypothesize safely that they repel each other away, and when one of the filaments touches the system boundary (top or bottom Petri dish), it breaks. When the scroll rings are not so large, filament rupture is not seen, because the positive filament tension makes it disappear before it can touch the boundary. It is evident from the snapshots in Fig. 2 that amongst the two circular filaments in Fig. 2(a), one clearly ruptures after a period of 90 min [Fig. 2(d)]. The corresponding time-space plot [Fig. 2(e)] also depicts the same phenomenon. The enlarged image shown in the left panel of Fig. 2(e) proves the presence of two filaments in that area at the early part of

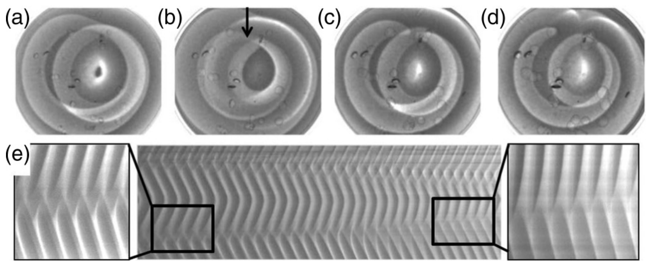

FIG. 2. Repulsion dynamics of vortex filaments with an opposite sense of rotation. (a)–(d) Snapshots of two scroll waves, placed one over the other at (a) 150 min, (b) 180 min, (c) 210 min, and (d) 240 min, after initiation. Each snapshot has an area of 2.5 cm × 2.2 cm. (e) Central panel: Time-space plot along the arrow shown in frame (b). It spans the duration from 140–340 min of the reaction. Time advances from left to right. Left and right panels: magnified areas marked with boxes in the central panel.

the experiment, while the right panel hints at a single filament at a later stage. This area corresponds to the tip of the arrow in Fig. 2(b).

To better understand the experimental results we carried out simple numerical simulations of the two-variable Barkley model, which is frequently employed to model the dynamics of reaction-diffusion systems [27].

$$\frac{\partial u}{\partial t} = \frac{1}{\epsilon}\left\{u(1-u)\left(u - \frac{v+b}{a}\right)\right\} - D_u\nabla^2 u,$$
$$\frac{\partial v}{\partial t} = u - v - D_v\nabla^2 v,$$

where $D_u = 1.0$ and $D_v = 1.0$ are the translational diffusion coefficients of u and v, respectively, and $a = 0.84$, $b = 0.06$, and $\epsilon = 0.02$ are the system parameters. We use Euler's nine-point stencil method for integrating the equations in a finite system size of $350 \times 350 \times 250$ grid points and zero-flux boundary conditions. A time interval $\Delta t = 0.012$ and space step of $\Delta x = 0.35$ are employed. The filament of the scroll is identified at regions having $u = 0.5$ and $v = a/2 - b$.

Figure 3 shows crossover collision of the scroll-wave filaments, leading to reconnection of the two individual scroll rings. The filaments demonstrate that the scroll rings which were in the same plane at the beginning of the reaction, remain so. Time-space plots [Figs. 3(g) and 3(h)] depict the loss of two central filament sections along the horizontal arrow in Fig. 3(a), and the birth of two new filament sections along the vertical arrow. This bears ample proof of the crossover collision occurring at the point of nearest approach of the two filaments. These numerical results have absolute qualitative similarity with our experimental results in Fig. 1.

Figure 4 tries to give us a three-dimensional overview of filament repulsion. The two scroll waves that were placed over each other start repelling at the point of nearest approach. Figure 4(f) shows the tilting of the filaments away from each other due to this mutual repulsion. If the system boundaries are close by, as in the case of our experiments (Fig. 2), the filaments would touch the boundaries and rupture at those points. Towards this end, we also constricted our simulations along the z direction (system size of $350 \times 350 \times 60$ grid points), and obtained similar results [Figs. 4(g) and 4(h)].

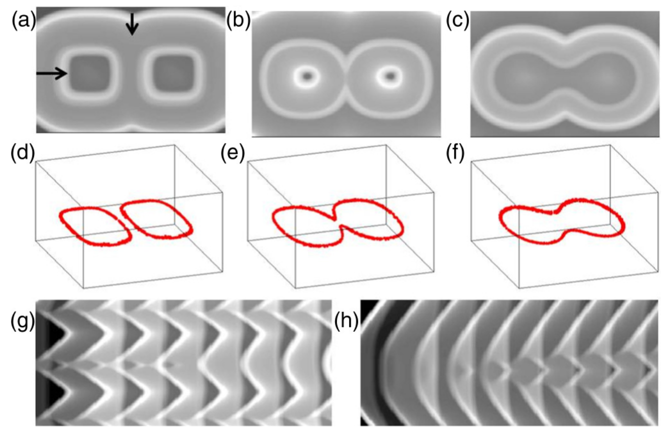

FIG. 3. (Color online) Numerical simulation studies of scroll-wave reconnection. (a)–(c) Two-dimensional snapshots of area 60 space units × 40 space units at (a) 9, (b) 19, and (c) 29 time units. (d)–(f) Filaments for the snapshots placed above. (g),(h) Time-space plots generated along the arrows in frame (a). (g) is along the horizontal direction and (h) along the vertical arrow.

Figure 4(g) shows that the lower filament has tipped so much that it touches the boundary at the bottom and ruptures. Figure 4(h) shows the projection of the filaments onto the xy plane. This has a close resemblance to our experimental results as seen in Fig. 2(d).

The local dynamics of the wave fronts can be explained in view of the motion of the constituent spirals at the point of closest approach. Interactions of 2D spiral cores are found in

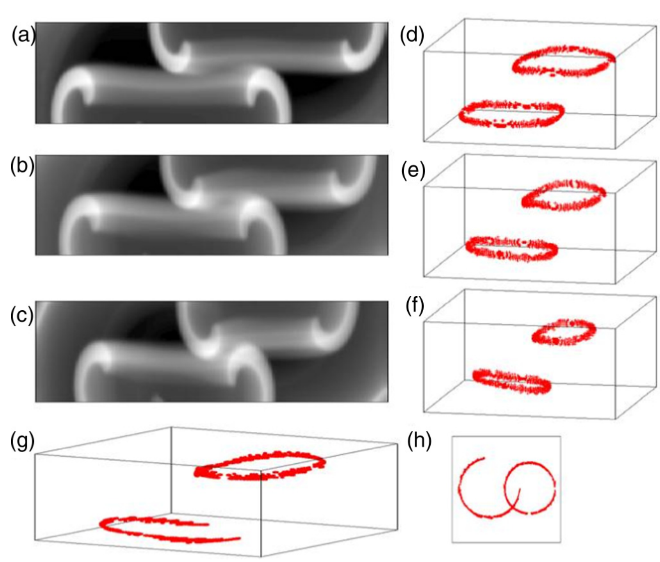

FIG. 4. (Color online) Simulation studies of repulsion of scroll rings, having an opposite sense of rotation. (a)–(c) Side view of a pair of scroll rings placed one over the other at $t = 45$ (a), 130 (b), and 215 (c) time units. Snapshot areas are 350 space units × 100 space units. Frames (d)–(f) depict the position of the filaments of the two scroll rings at the instances corresponding to the adjacent snapshots. (g) Filaments computed in a restricted area of 350 space units × 350 space units × 60 space units. (h) Projection of the filaments shown in frame (g) on the xy plane.

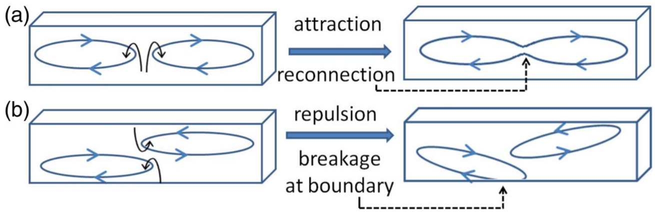

FIG. 5. (Color online) Schematic representation of filament interaction. (a) Reconnection of vortex filaments having same sense of rotation. (b) Repulsion of filaments having an opposite sense of rotation, and possible rupture at the boundary. Small black arrows denote the motion of the constituent spirals around the points of nearest approach of the filaments.

literature [15,28]. It is known from an existing study of the FitzHugh-Nagumo model, that corotating spirals repel each other till they are far enough and become independent [28]. This happens with our filaments having an opposite sense of rotation. As they shrink in size due to their positive filament tension, the constituent spirals of the two scroll rings that gain proximity, are corotating [Fig. 5(b)]. When they are very near, they repel, causing the filaments to tilt away from one another. Again, an earlier experimental study of a pair of spiral waves in 2D has shown that a gradient can be used to move counter-rotating spirals towards each other. The separation between the spiral cores could be reduced to an extent that they annihilated [15]. In a similar fashion, we have seen that vortex filaments of scroll-wave activity with the same sense of rotation, can approach each other. At the point where the filaments are nearest, the constituent spirals are counter-rotating [Fig. 5(a)]. This opposite sense of vorticity allows for them to mutually annihilate and paves the path for reconnection of the filaments. In this case, however, they are not forced to come close by any external gradient, but do so spontaneously. This is because of the interaction potential that acts amongst them and dictates reconnection or repulsion of the filaments depending on the distance that separates them [Fig. 1(k)].

In conclusion, we have demonstrated the phenomenon of reconnection of scroll-wave filaments in an experimental system. When two coplanar scroll rings with the same sense of rotation are brought close enough, the filaments undergo crossover collision and reconnect. It was also shown that the core length is an upper bound to the interfilament distance for reconnection to occur. However, when a scroll ring is placed over another, such that their nearest constituent spirals are corotating, they repel and break at touching the system boundaries. Both these phenomena will immensely effect the dynamics of scroll waves. Future work involving an extensive analysis of the mutual orientation and geometry of the filaments and scroll-ring lifetime could reveal further information. The present study illuminates important physics and paves the way for a better understanding of scroll-wave interaction in other excitable mediums, such as the cardiac tissues.

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[1] O. Steinbock, F. Siegert, S. C. Müller, and C. J. Weijer, Proc. Natl. Acad. Sci. USA **90**, 7332 (1993).
[2] T. Kuroda, T. Takiwaki, and K. Kotake, Phys. Rev. D **89**, 044011 (2014).
[3] T. Fohl and J. S. Turner, Phys. Fluids **18**, 433 (1975).
[4] G. Blatter, M. V. Feigel'man, V. B. Geshkenbein, and A. I. Larkin, Rev. Mod. Phys. **66**, 1125 (1994).
[5] F. X. Witkowski, L. J. Leon, P. A. Penkoske, W. R. Giles, M. L. Spano, W. L. Ditto, and A. T. Winfree, Nature (London) **392**, 78 (1998).
[6] R. A. Gray, J. Jalife, A. V. Panfilov, W. T. Baxter, and C. Cabo, Science **270**, 1222 (1995).
[7] R. A. Gray, J. Jalife, A. Panfilov, W. T. Baxter, C. Cabo, J. M. Davidenko, and A. M. Pertsov, Circulation **91**, 2454 (1995).
[8] Y. Oshima and N. Izutsu, Phys. Fluids **31**, 2401 (1988).
[9] I. Chuang, R. Durrer, N. Turok, and B. Yurke, Science **251**, 1336 (1991).
[10] W. T. Ashurst and D. I. Meiron, Phys. Rev. Lett. **58**, 1632 (1987).
[11] A. T. Winfree and S. H. Strogatz, Physica D **13**, 221 (1984).
[12] M-A. Bray and J. P. Wikswo, Phys. Rev. Lett. **90**, 238303 (2003).
[13] B. Fiedler and R. M. Mantel, Documenta Math. **5**, 695 (2000).
[14] M. Gabbay, E. Ott, and P. N. Guzdar, Phys. Rev. E **58**, 2576 (1998).
[15] J. Schütze, O. Steinbock, and S. C. Müller, Nature (London) **356**, 45 (1992).
[16] S. Dutta and O. Steinbock, Phys. Rev. E **83**, 056213 (2011).
[17] Z. A. Jiménez and O. Steinbock, Phys. Rev. Lett. **109**, 098301 (2012).
[18] A. T. Winfree, Science **175**, 634 (1972).
[19] M. Vinson, S. Mironov, S. Mulvey, and A. Pertsov, Nature (London) **386**, 477 (1997).
[20] S. Dutta and O. Steinbock, Phys. Rev. E **81**, 055202(R) (2010).
[21] T. Amemiya, P. Kettunen, S. Kádár, T. Yamaguchi, and K. Showalter, Chaos **8**, 872 (1998).
[22] A. F. Taylor, B. R. Johnson, and S. K. Scott, Phys. Chem. Chem. Phys. **1**, 807 (1999).
[23] T. Bánsági, Jr. and O. Steinbock, Phys. Rev. Lett. **97**, 198301 (2006).
[24] See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevE.91.030901 for movies of typical experiments (corresponding to Figs. 1 and 2) and simulations (corresponding to Figs. 3 and 4).
[25] A. T. Winfree and S. H. Strogatz, Physica D **9**, 333 (1983).
[26] J. Koplik and H. Levine, Phys. Rev. Lett. **71**, 1375 (1993).
[27] D. Barkley, M. Kness, and L. S. Tuckerman, Phys. Rev. A **42**, 2489 (1990).
[28] E. A. Ermakova, A. M. Pertsov, and E. E. Shnol, Physica D **40**, 185 (1989).