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Ion cooling in collisionless plasma expansion

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The ion cooling in collisionless plasma expansion is revisited. It is shown that, in the case of an initial Maxwellian ion distribution, the ion cooling is much slower than predicted by an adiabatic law linking the ion temperature to the ion density. The origin of this behavior is a strong distortion of the ion distribution function resulting in a large ion heat flow (not predicted by a simple water-bag model). Also noticeable is the increase of the electron heat flux in the unperturbed plasma compared to the zero ion temperature case.

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I. INTRODUCTION

The problem of the expansion of a collisionless plasma into a vacuum has been widely studied since the pioneering work of Gurevich, Pariiskaya, and Pitaevskii [1], in particular in recent years, in the context of the interaction of an ultraintense laser pulse with a solid target and of the resulting generation of high energy ion beams [2–9]. For instance, recent theoretical works address the charge separation effects and the maximum ion velocity [10–12], the electron cooling [13–21], and the effect of a two-temperature electron distribution function [20–23].

In most of these works the ions are assumed to be cold throughout the expansion, or at least to cool down very quickly, so that the ion temperature can be neglected. However, in some specific cases, the ion cooling can be treated exactly, as in the case of the expansion of a Gaussian plasma, in the quasineutral limit, and when the electron and ion distribution functions are self-similar [14].

In this paper we revisit the ion cooling in the expansion of a one-dimensional semi-infinite collisionless plasma, in the quasineutral limit. The analysis is relevant to the cases when the plasma heating is such that the initial ion temperature is comparable to the electron temperature and when some source of energy maintains the electron temperature at its initial value. In typical laser-plasma experiments, this corresponds to the case of a laser pulse in the hundreds of picoseconds or in the nanosecond range impinging on a thick foil, in contrast with the case of a femtosecond range laser impinging on a thin foil, for which the ion temperature can be neglected (temperature balance between ions and electrons has not enough time to be reached) and for which the electron cooling occurs on the same time scale as the expansion.

Though the ion cooling in the expansion of a one-dimensional semi-infinite collisionless plasma was tackled in the original paper of Gurevich *et al.* [1], all of its characteristics have not been revealed. We show in particular that when the initial ion distribution function is a Maxwellian distribution, the strong distortion of the ion distribution function during the expansion leads to an ion cooling that is much slower than predicted by the often accepted law stating that the ion temperature varies as the square of the ion density in such a one-dimensional system, and we show that the very large ion heat flux is responsible for this behavior. The comparison of the cases of initial ion distribution functions in the form of a water bag and in the form of a Maxwellian is particularly illustrative in this respect. A significant increase in the electron heat flow in the unperturbed plasmas is also evidenced, about 38% for equal ion and electron temperature and singly ionized ions.

II. MODEL AND FUNDAMENTAL EQUATIONS

We consider the expansion into a vacuum of a one-dimensional collisionless semi-infinite plasma. At time $t = 0$, the plasma is assumed to occupy the half-space $x < 0$. The ions have initially a density $n_i = n_{i0}$ for $x < 0$ and $n_i = 0$ for $x > 0$ with a sharp boundary, with no mean velocity, $v_i = 0$. The ion charge number is Z and the ion mass is m_i. The ions are described by a distribution function $f(x,v,t)$ that evolves according to the Vlasov equation

$$\frac{\partial f}{\partial t} + v\frac{\partial f}{\partial x} - \frac{Ze}{m_i}\frac{\partial \Phi}{\partial x}\frac{\partial f}{\partial v} = 0, \tag{1}$$

where e is the elementary charge and $\Phi(x,t)$ is the electrostatic potential. The ion density n_i, mean velocity v_i, and temperature T_i are easily deduced from f:

$$n_i = \int f\, dv, \tag{2}$$

$$n_i v_i = \int f v\, dv, \tag{3}$$

and

$$n_i k_B T_i = m_i \int f (v - v_i)^2\, dv. \tag{4}$$

The electrons are in equilibrium with the electrostatic potential and are assumed to correspond to a Boltzmann distribution

$$n_e = n_{e0} \exp\left(\frac{e\Phi}{k_B T_e}\right), \tag{5}$$

where $n_{e0} = Z n_{i0}$ is the electron density in the unperturbed plasma (i.e., for $x = -\infty$) and T_e is the electron temperature. We are interested here in the late time expansion, when the characteristic density scale length $c_{s0}t$, where $c_{s0} = (Z k_B T_e/m_i)^{1/2}$ is the ion-acoustic velocity, is much larger than the initial Debye length $\lambda_{D0} = (\epsilon_0 k_B T_e/n_{e0}e^2)^{1/2}$. This corresponds to the condition $\omega_{pi} t \gg 1$, where $\omega_{pi} = (n_{e0} Z e^2/m_i \epsilon_0)^{1/2}$ is the ion plasma frequency. In this limit,

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the plasma can be considered quasineutral, with $n_e = Zn_i$, and

$$\Phi = \frac{k_B T_e}{e}\ \ln n_i = \frac{k_B T_e}{e}\ \ln \int f\, dv, \tag{6}$$

where the ion density has been normalized to n_{i0}.

In such a condition the motion is self-similar [1], $f = f(\xi, v)$ with $\xi = x/t$, and the Vlasov equation reads as

$$(v - \xi)\frac{\partial f}{\partial \xi} - \frac{Ze}{m_i}\frac{d\Phi}{d\xi}\frac{\partial f}{\partial v} = 0. \tag{7}$$

If we normalize the velocities (i.e., v and ξ) to c_{s0} and use Eq. (6), it can be written

$$(v - \xi)\frac{\partial f}{\partial \xi} + E(\xi)\frac{\partial f}{\partial v} = 0, \tag{8}$$

where $E(\xi)$ is the dimensionless electric field (i.e., normalized to $E_{ss} = m_i c_{s0}/Zet$, which is the electric field of the self-similar solution in the $T_{i0} = 0$ case, where T_{i0} is the ion temperature in the unperturbed plasma),

$$E(\xi) = -\frac{d}{d\xi}\left(\ln \int f\, dv\right). \tag{9}$$

The characteristics of Eq. (8) satisfy

$$\frac{dv}{d\xi} = \frac{E(\xi)}{v - \xi}. \tag{10}$$

Note that the limits of the integrals over velocity are normally $-\infty$ and $+\infty$. However, as explained in Ref. [1], f vanishes for $v < \xi$, so that the lower limit of the integral can be taken equal to ξ.

III. CASE OF A WATER-BAG ION DISTRIBUTION

In this section we assume that the ions are described by a top-hat or water-bag model, i.e.,

$$f(\xi, v) = \begin{cases} f_0 & \text{if } \mathrm{v}_-(\xi) < \mathrm{v} < \mathrm{v}_+(\xi), \\ 0 & \text{otherwise}, \end{cases} \tag{11}$$

where the functions $v_\pm(\xi)$ are equal to $\pm v_0$ in the unperturbed plasma (for ξ sufficiently negative), and where $f_0 = 1/2v_0$, so that the normalized ion density n_i and mean velocity v_i are simply

$$n_i = \frac{v_+ - v_-}{2v_0}, \tag{12}$$

$$v_i = \frac{1}{2}(v_+ + v_-), \tag{13}$$

the dimensionless electric field E is

$$E(\xi) = -\frac{d}{d\xi}\ln(v_+ - v_-), \tag{14}$$

the ion temperature T_i is given by

$$\frac{T_i}{ZT_e}(\xi) = \frac{1}{3}n_i^2 v_0^2, \tag{15}$$

and the ion heat flux is equal to zero.

To perform the integration of Eq. (10), one needs to know the position of the boundary ξ_0 between the expanding plasma and the unperturbed plasma, and the value of the dimensionless electric field at this point, i.e., $E(\xi_0)$. Both quantities can be determined by the following analysis.

Applying Eq. (10) to the characteristics $v_-(\xi)$ and $v_+(\xi)$, one obtains

$$\frac{dv_\pm}{d\xi} = \frac{E(\xi)}{v_\pm - \xi} = -\frac{1}{v_\pm - \xi}\frac{d}{d\xi}\ln(v_+ - v_-). \tag{16}$$

Combining the two equations for $v_\pm$, one obtains the following condition in the expanding plasma (where the electric field does not vanish):

$$(v_+ - \xi)(v_- - \xi) = 1. \tag{17}$$

This condition is valid at the unperturbed plasma boundary, so that

$$(v_0 - \xi_0)(v_0 + \xi_0) = -1, \tag{18}$$

or

$$\xi_0 = -\sqrt{1 + v_0^2}, \tag{19}$$

for an expansion of a semi-infinite plasma toward the right and a rarefaction wave going to the left with an absolute velocity (in physical units)

$$v_{s0} = \left[\frac{k_B(ZT_e + 3T_{i0})}{m_i}\right]^{1/2}. \tag{20}$$

Combining Eqs. (12), (13), and (17), one can write

$$v_i - \xi = v_s(\xi) = \sqrt{1 + n_i^2 v_0^2}, \tag{21}$$

where $v_s(\xi)$ is the local normalized sound velocity.

It is convenient to introduce the auxiliary quantities

$$u_\pm = v_\pm - \xi, \tag{22}$$

with

$$u_+ u_- = 1, \tag{23}$$

$$n_i = \frac{1}{2v_0}\frac{u_+^2 - 1}{u_+}, \tag{24}$$

and

$$v_i - \xi = \frac{u_+^2 + 1}{2u_+}, \tag{25}$$

where we have eliminated u_-. Using Eq. (16), one gets

$$\frac{du_+}{d\xi} = -\frac{u_+^2(u_+^2 - 1)}{u_+^4 + 1} \tag{26}$$

and

$$E(\xi) = \frac{u_+(u_+^2 + 1)}{u_+^4 + 1}. \tag{27}$$

Equation (26) can be integrated to give

$$\xi - \xi_0 = g(u_+) - g(u_0), \tag{28}$$

where

$$g(u) = \ln\left(\frac{u + 1}{u - 1}\right) - \frac{u^2 + 1}{u} \tag{29}$$

and

$$u_0 = u_+(\xi_0) = v_0 + \sqrt{1 + v_0^2}. \qquad (30)$$

Using Eqs. (27) and (30), one can express the electric field in ξ_0 as

$$E(\xi_0) = \frac{\sqrt{1 + v_0^2}}{1 + 2v_0^2}. \qquad (31)$$

Note that g can also be expressed as a function of v_i or n_i via Eq. (21) and

$$g(u_+) = \frac{1}{2} \ln\left(\frac{v_s + 1}{v_s - 1}\right) - 2v_s. \qquad (32)$$

Equivalent results were obtained by Medvedev [24] by solving directly the fluid equations with zero heat flux, i.e., by using the closure relation $T_i/n_i^2 = T_{i0}/n_{i0}^2$.

According to Eq. (28), u_+ is decreasing from u_0 for $\xi = \xi_0$ to 1 for $\xi \to \infty$. The asymptotic behavior for $\xi \to \infty$ is easily obtained as

$$g(u_+ \to 1) \simeq \ln 2 - 2 - \ln(u_+ - 1). \qquad (33)$$

In the same limit, one has

$$n_i \simeq \frac{u_+ - 1}{v_0} \qquad (34)$$

and

$$v_i - \xi \simeq 1. \qquad (35)$$

Combining Eqs. (28), (33), and (34), one gets in the $\xi \to \infty$ limit

$$n_i \simeq C_0 \exp(-\xi), \qquad (36)$$

with

$$C_0 = \frac{2}{v_0}\left(\frac{\sqrt{1 + v_0^2} - 1}{\sqrt{1 + v_0^2} + 1}\right)^{1/2} \exp\left(\sqrt{1 + v_0^2} - 2\right). \qquad (37)$$

For $v_0 = 0$ one recovers $C_0 = \exp(-1)$.

The case $T_{i0} = ZT_e$ is of particular interest as it corresponds to a plasma with singly charged ions and equal ion and electron temperatures in the unperturbed plasma. It corresponds to $v_0 = \sqrt{3}$, $\xi_0 = -2$, and $u_0 = 2 + \sqrt{3}$, with $E(\xi_0) = 2/7$ and $C_0 = 2/3$.

Figure 1 shows the quantities v_-, v_+, and v_i as functions of ξ for the case $v_0 = \sqrt{3}$. Figure 2 shows the corresponding normalized electric field and Fig. 3 shows the ion density. In Figs. 2 and 3 the dashed lines correspond to the zero ion temperature case, $v_0 = 0$.

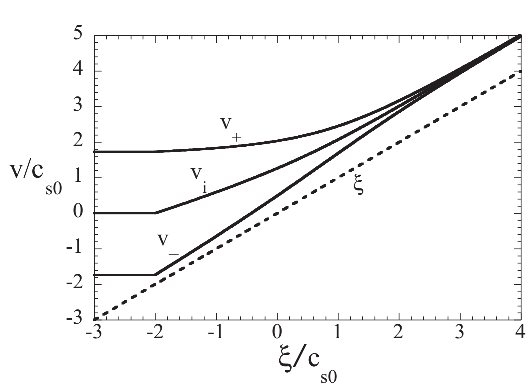

FIG. 1. Water-bag case. Quantities v_-, v_+, and v_i as functions of ξ for $v_0 = \sqrt{3}$, i.e., $T_{i0} = ZT_e$.

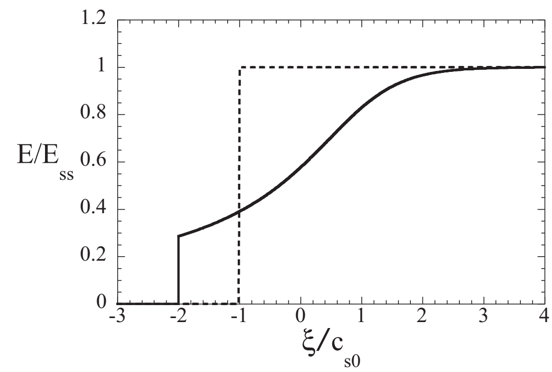

FIG. 2. Water-bag case. Normalized electric field as a function of ξ for $v_0 = \sqrt{3}$, i.e., $T_{i0} = ZT_e$. The dashed line corresponds to the zero ion temperature case ($v_0 = 0$).

IV. CASE OF AN INITIAL MAXWELLIAN ION DISTRIBUTION

The case where the initial ion distribution is a Maxwellian one is more realistic and presents more interesting features. In contrast with the water-bag case, the analytic approach is limited and one has to resort to the numerical integration of Eq. (10) to determine the solution of Eq. (8), taking into account the constancy of f along the characteristics. To do so, one has to use self-consistently Eq. (9) to determine the electric field.

The ion distribution function in the unperturbed plasma is given by

$$f(\xi \to -\infty, v) = \frac{1}{\sqrt{2\pi}\, v_{ti0}} \exp\left(-\frac{v^2}{2v_{ti0}^2}\right), \qquad (38)$$

where $v_{ti0} = (T_{i0}/ZT_e)^{1/2}$ in the normalized units used here.

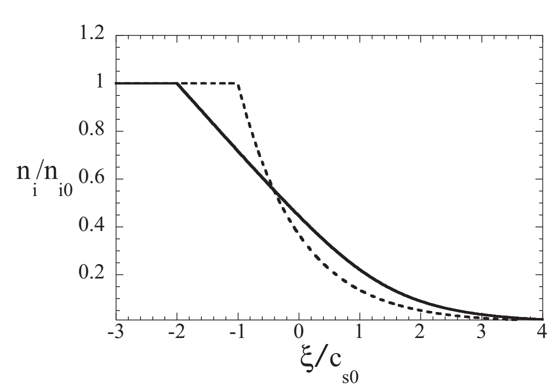

FIG. 3. Water-bag case. Density as a function of ξ for $v_0 = \sqrt{3}$, i.e., $T_{i0} = ZT_e$. The dashed line corresponds to the zero ion temperature case ($v_0 = 0$).

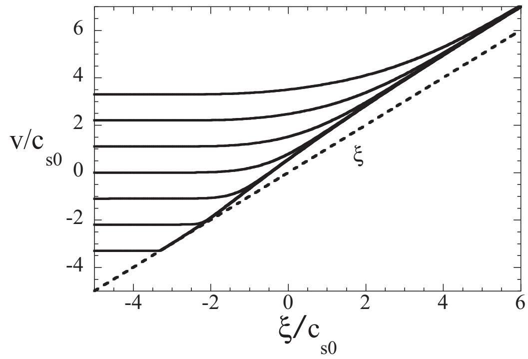

FIG. 4. Case of an initial Maxwellian ion distribution. Characteristics as functions of ξ for $v_{ti0} = 1$, i.e., $T_{i0} = ZT_e$.

Equation (10) is solved for a sufficiently large number of characteristics (typically 1×10^3) from the left to the right, beginning at an initial position ξ_1 (typically $\xi_1 = -6$ for $v_{ti0} = 1$). The spatial mesh is on the order of $d\xi = 1 \times 10^{-3}$.

Let us suppose that the solution is known up to a point ξ_n. Between the two successive mesh points, ξ_n and ξ_{n+1}, with $\xi_{n+1} - \xi_n = d\xi$, Eq. (10) is solved analytically, assuming a constant electric field $E_{n+1/2}$ (as it is in fact not initially known, $E_{n-1/2}$ is used as a first guess value for $E_{n+1/2}$ in a first iteration), i.e.,

$$v_{j,n+1} - \xi_{n+1} - E_{n+1/2} = (v_{j,n} - \xi_n - E_{n+1/2}) \times \exp\left(-\frac{v_{j,n+1} - v_{j,n}}{E_{n+1/2}}\right). \quad (39)$$

This implicit equation is in fact solved iteratively by a Newton method (two iterations are sufficient). In Eq. (39) $v_{j,n}$ corresponds to the value of the velocity of a given characteristic (labeled j) at the position ξ_n and $E_{n+1/2}$ is the electric field taken at the intermediate position $\xi_{n+1/2} = (\xi_n + \xi_{n+1})/2$. Once all the $v_{j,n+1}$ are known, the distribution function is determined at the position ξ_{n+1}, the ion density is evaluated, and the electric field $E_{n+1/2}$ is then determined by the discretized version of Eq. (9). The solving of Eq. (39) is then iterated with this more accurate value of $E_{n+1/2}$, and so on until convergence is obtained.

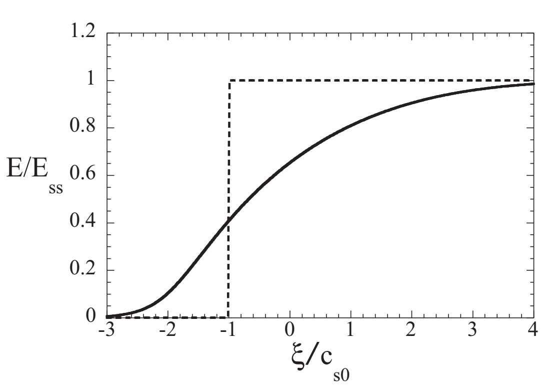

FIG. 5. Case of an initial Maxwellian ion distribution. Normalized electric field as a function of ξ for $v_{ti0} = 1$, i.e., $T_{i0} = ZT_e$. The dashed line corresponds to the zero ion temperature case ($v_{ti0} = 0$).

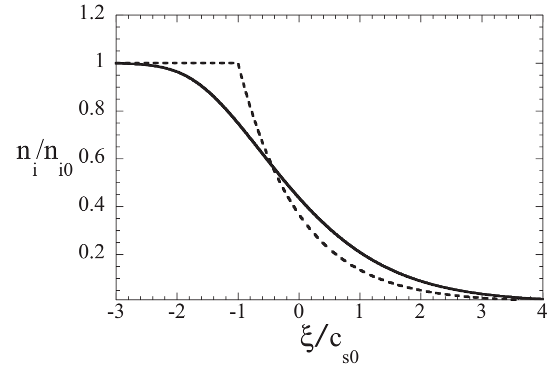

FIG. 6. Case of an initial Maxwellian ion distribution. Density as a function of ξ for $v_{ti0} = 1$, i.e., $T_{i0} = ZT_e$. The dashed line corresponds to the zero ion temperature case ($v_{ti0} = 0$).

Figure 4 shows a selected number of characteristics corresponding to the case $v_{ti0} = 1$, i.e., $T_{i0} = ZT_e$, while Figs. 5 and 6 show the corresponding normalized electric field and the ion density, respectively. Note that the ion velocity behaves as $v_i \simeq \xi + 1$ and that the ion density behaves as $n_i \simeq C_0 \exp(-\xi)$ in the $\xi \to \infty$ limit, with $C_0 \approx 0.726\,65$ (Ref. [1] gives $C_0 = 0.70$ with a far less precise numerical scheme). The quantity C_0 is a function of the ion temperature which can be obtained numerically. It is shown as a function of T_{i0}/ZT_e in Fig. 7.

The case of an initial Maxwellian ion distribution differs significantly from the water-bag case when one studies the ion temperature. In Fig. 8 one shows the ion temperature and the ion density as functions of ξ. It appears that the ion temperature does not decrease as n_i^2 as in the water-bag case. On the contrary, its decreasing is much slower. This is more apparent in Fig. 9, where one plots the ratio T_i/n_i^2 as a function of ξ. As a matter of fact, the $T_i \propto n_i^2$ behavior appears for quite large values of ξ ($\xi \geqslant 8$), with $T_i \simeq 193.7\, n_i^2$, that is, more than two orders of magnitude larger than predicted by the water-bag model.

This result may be related to the distorted form of the ion distribution function. This distortion is illustrated in Fig. 10,

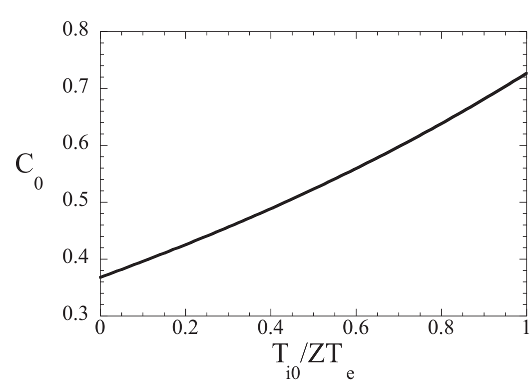

FIG. 7. Case of an initial Maxwellian ion distribution. The quantity C_0 is shown as a function of the ion temperature, where $C_0 = \lim_{\xi \to \infty} n_i \exp(\xi)$.

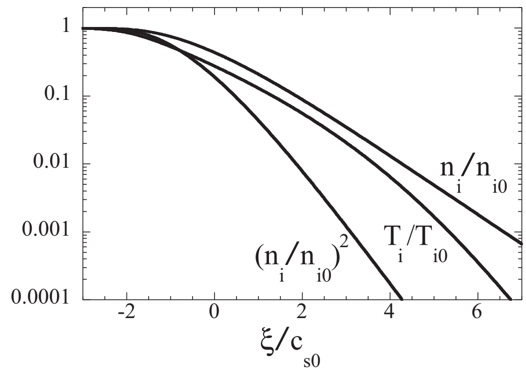

FIG. 8. Case of an initial Maxwellian ion distribution. Temperature and density as functions of ξ for $v_{ti0} = 1$, i.e., $T_{i0} = ZT_e$. Also shown is the square of the density.

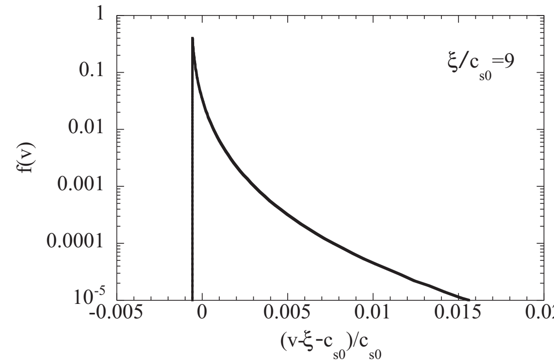

FIG. 10. Case of an initial Maxwellian ion distribution with $v_{ti0} = 1$, i.e., $T_{i0} = ZT_e$. Ion distribution function at the position $\xi = 9$.

which shows the ion distribution function at the position $\xi = 9$, where the mean velocity is almost equal to its asymptotic expression, $v_i \simeq \xi + 1$, and where $v_{ti} \approx 1.2 \times 10^{-3}$. One notes the strongly asymmetric shape of the ion distribution function. This asymmetry also results in a large ion heat flow, as shown in Fig. 11, on which is plotted the ion heat flow, normalized to the ion free-streaming value $n_i m_i v_{ti}^3$, as a function of ξ. For $\xi \to \infty$, one has $q_i \simeq 5.64\, n_i m_i v_{ti}^3$.

It is interesting to compare the ion heat flow to the electron heat flow. To calculate the electron heat flow, one has to resort to the equation of conservation of energy of the electrons in the expansion,

$$P_e \frac{\partial v_e}{\partial x} = -\frac{\partial q_e}{\partial x}, \tag{40}$$

where $P_e = n_e k_B T_e$ is the electron pressure and v_e is the electron mean velocity, and where one has taken into account the fact that the electron temperature is assumed to be constant in the expansion. In the self-similar variable, Eq. (40) reads

$$P_e \frac{dv_e}{d\xi} = -\frac{dq_e}{d\xi}. \tag{41}$$

In the quasineutral approximation $P_e = n_i m_i c_{s0}^2$ and $v_e = v_i$, so that one has

$$q_e(\xi) = m_i c_{s0}^2 \int_\xi^\infty n_i \frac{dv_i}{d\xi} d\xi = -m_i c_{s0}^2 \int_\xi^\infty (v_i - \xi) \frac{dn_i}{d\xi} d\xi, \tag{42}$$

where one has used the continuity equation and the fact that the electron heat flux vanishes for $x \to \infty$. Knowing $v_i(\xi)$ and $n_i(\xi)$, it is easy to numerically integrate Eq. (42) to obtain the electron heat flow. The result is shown in Fig. 12, on which one has plotted the electron heat flow q_e, the ion heat flow q_i, and the total heat flow q, normalized to $n_i m_i c_{s0}^3$, as functions of ξ. One observes that the ion heat flux, though noticeable, stays smaller than the electron heat flux. One also notes that the electron heat flow is larger than in the zero ion temperature case (for which it is equal to $n_i m_i c_{s0}^3$). In particular, in the case $T_{i0} = ZT_e$, one has $q_e \approx 1.3776\, n_{i0} m_i c_{s0}^3$ in the unperturbed plasma, instead of $n_{i0} m_i c_{s0}^3$ in the $T_{i0} = 0$ case.

One can easily verify, from the global energy conservation law, that the electron heat flux in the unperturbed plasmas is also given by

$$q_e(-\infty) = \frac{1}{2} \int_{-\infty}^\infty n_i \left[k_B (T_i - T_{i0}) + m_i v_i^2\right] d\xi. \tag{43}$$

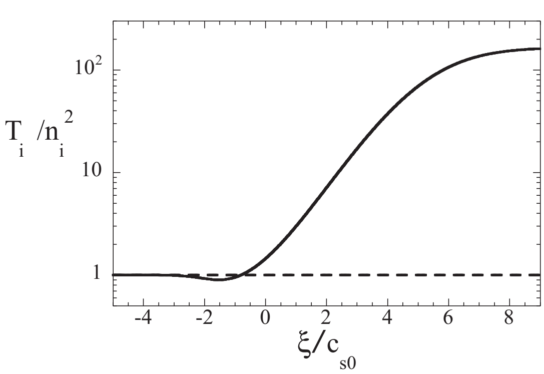

FIG. 9. Case of an initial Maxwellian ion distribution. Ratio T_i/n_i^2, normalized to T_{i0}/n_{i0}^2, as a function of ξ for $v_{ti0} = 1$, i.e., $T_{i0} = ZT_e$. The dashed line corresponds to the water-bag case.

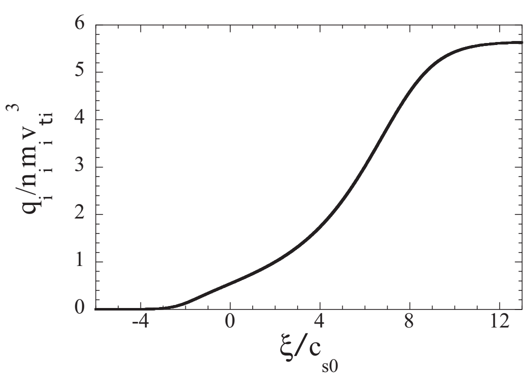

FIG. 11. Case of an initial Maxwellian ion distribution with $v_{ti0} = 1$, i.e., $T_{i0} = ZT_e$. Ion heat flow, normalized to the free-streaming value $n_i m_i v_{ti}^3$, as a function of ξ.

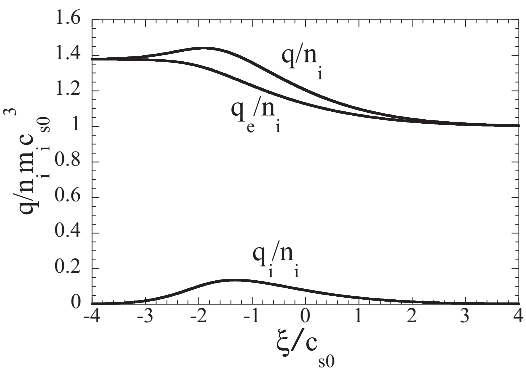

FIG. 12. Case of an initial Maxwellian ion distribution with $v_{ti0} = 1$, i.e., $T_{i0} = ZT_e$. Electron heat flow, ion heat flow, and total heat flow, normalized to $n_i m_i c_{s0}^3$, as functions of ξ.

V. DISCUSSION AND CONCLUSION

In this paper we have limited the analysis to the self-similar solution, obtained when $\omega_{pi}t \gg 1$, a condition also written $c_{s0}t \gg \lambda_{D0}$. However, this self-similar solution becomes in any case invalid for large ξ, when the local Debye length $\lambda_D = \lambda_{D0}(n_{e0}/n_e)^{1/2} = \lambda_{D0}\exp(\xi/2c_s)/\sqrt{C_0}$ becomes equal to the density scale length, $c_{s0}t$. At that point the self-similar solution predicts a velocity $v_i = c_s \ln[C_0 e(\omega_{pi}t)^2]$ (where e denotes the numerical constant 2.718 28 . . .), which is expected to be approximately the cutoff of the ion velocity spectrum.

In conclusion, we have revisited the expansion of a one-dimensional semi-infinite collisionless plasma, in the quasineutral limit, taking into account a finite ion temperature in the unperturbed plasma. We have shown that, in the case where the initial ion distribution is a Maxwellian one, the ion cooling is much slower than expected, ending with a temperature dependence with self-similar parameter that is more than two orders of magnitude larger than expected according to the adiabatic law, for $T_{i0} = ZT_e$. This behavior is due to a very large heat flow, which goes up to more than five times the ion free-streaming value $n_i m_i v_{ti}^3$. Also noticeable is the increase of the electron heat flux in the unperturbed plasma (nearly 38% for $T_{i0} = ZT_e$) compared to the zero ion temperature case.

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