Discrete model of gas-free spin combustion of a powder mixture

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We propose a discrete model of gas-free combustion of a cylindrical sample which reproduces in detail a spin combustion mode. It is shown that a spin combustion, in its classical sense as a continuous spiral motion of heat release zones on the surface of the sample, does not exist. Such a concept has arisen due to the misinterpretation of the experimental data. This study shows that in fact a spinlike combustion is realized, at which two energy release zones appear on the lateral surface of the sample and propagate circumferentially in the opposite directions. After some time two new heat release zones are formed on the next layer of the cylinder surface and make the same counter-circular motion. This process continues periodically and from a certain angle it looks like a spiral movement of the luminous zone along the lateral surface of the sample. The model shows that on approaching the combustion limit the process becomes more complicated and the spinlike combustion mode shifts to a more complex mode with multiple zones of heat release moving in different directions along the lateral surface. It is shown that the spin combustion mode appears due to asymmetry of initial conditions and always transforms into a layer-by-layer combustion mode with time.

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I. INTRODUCTION

Gas-free combustion of powder mixtures is a subject of intensive studies, both experimental and theoretical. According to experimental data, stationary layer-by-layer gas-free combustion loses its stability under certain conditions and shifts to more complex nonstationary modes (pulsating, spin, chaotic, etc.) [1–8]. In particular, under certain conditions so-called spin combustion is observed. This mode is implemented on approaching the extinction limit and represents a motion of an isolated zone of energy release (hot spot, glowing area) along the surface of the cylindrical sample. There is a concept of spin combustion based on experimental observations assuming that the hot spot moves along a continuous spiral path on the lateral surface of the sample [2]. We note that there are other systems that exhibit spin modes. For example, frontal polymerization under certain conditions can propagate in the spin modes [9].

Spin combustion has been a subject of intensive study over the past 40 years. The majority of studies are limited only to analysis of the stability of a stationary flat combustion wave, and are shown that under certain conditions bifurcations occur that result in new transient combustion modes, which may include spin combustion. To date, a number of models [10–14] have been developed in order to explain and describe the spin combustion of gas-free systems. In rare cases [12] it is possible to reproduce numerically the appearance of a heat release zone on the lateral surface of the sample and its a short-term motion across the surface, after which the destruction of this hot spot occurs and the new hot spots arise on the lateral surface of the sample elsewhere. One can state that, despite numerous attempts, up to this day there is no spin combustion model that would be able to describe the experimentally observed spiral motion of a hot spot on the lateral surface of the sample. Furthermore, even a qualitative explanation of such unusual behavior of a combustion wave does not exist, because the propagation of combustion in a single direction on the lateral surface of the cylinder is inconsistent with the symmetry of the sample: Why does a hot spot "rotate" only in one direction on the lateral surface of the sample; what prevents it from motion in the opposite direction?

In this paper we describe a model of gas-free spin combustion of a cylindrical sample which allows obtaining results similar to those obtained in experiments and investigate the actual combustion wave topology in this combustion mode. We use the discrete combustion wave model [15–17] as a basis.

II. DISCRETE COMBUSTION WAVE ON CYLINDER

A. Symmetric initial conditions

Recently, it became clear [1,8] that for the gas-free systems, representing a powder mixture, the model of a discrete combustion wave describes the structure of a combustion wave and the processes accompanying combustion more accurately and correctly, from a physical point of view, than conventional continuous reaction-diffusion models. Let us consider the discrete three-dimensional combustion model [15–17]. The system is assumed to consist of point heat sources (particles), distributed in heat-conductive inactive environment. The particles are thermally inertialess and their temperature always equal to the temperature of the environment in corresponding points. When the temperature in point $\mathbf{r}_i$ reaches T_{ign}, which is considered a constant characteristic of the system, the corresponding particle i ignites and burns out instantaneously releasing heat energy Q_i. This causes nonuniform and nonstationary heating of the system, so other particles may reach the ignition temperature T_{ign} and ignite. Thus, an ignition wave (discrete combustion wave) spreads over the system. The temperature distribution in such a system is described by the heat equation

$$\frac{\partial T}{\partial t} = \kappa \nabla^2 T + \frac{1}{c\rho} \sum_i Q_i \delta(t - t_i)\delta(\mathbf{r} - \mathbf{r}_i), \tag{1}$$

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where $\kappa = \lambda/\rho c$ is thermal diffusivity of the system; δ is the delta function; t_i is the moment of ignition of the particle i; λ, ρ, and c are thermal conductivity, density, and heat capacity of the system.

An exact solution of the Eq. (1) is as follows:

$$T(t,\mathbf{r}) = T_{in} + \frac{1}{c\rho}\sum_{k(t)}\frac{Q_k}{[4\pi\kappa(t-t_k)]^{3/2}}\exp\left[-\frac{|\mathbf{r}-\mathbf{r}_k|^2}{4\kappa(t-t_k)}\right], \tag{2}$$

where T_{in} is the initial temperature of the system; the sum includes all the particles that have ignited by the time t. In the following, we will consider all the particles to be identical: $Q_k = Q$ for every k.

Let us introduce the nondimensional parameters,

$$\mathbf{q} = \mathbf{r}/l, \quad \tau = t\kappa/l^2, \quad \theta = \frac{T - T_{in}}{T_{ad} - T_{in}}, \tag{3}$$

where l is the characteristic spatial scale; $T_{ad} = T_{in} + \frac{Q}{c\rho l^3}$ is the adiabatic temperature of the system.

The solution (2) in nondimensional variables (3) looks like

$$\theta(\tau,\mathbf{q}) = \sum_{k(\tau)}\frac{1}{[4\pi(\tau-\tau_k)]^{3/2}}\exp\left[-\frac{|\mathbf{q}-\mathbf{q}_k|^2}{4(\tau-\tau_k)}\right]. \tag{4}$$

The ignition moment of the particle i is defined by the equation

$$\varepsilon = \sum_{k(\tau)}\frac{1}{[4\pi(\tau_i-\tau_k)]^{3/2}}\exp\left[-\frac{|\mathbf{q}_i-\mathbf{q}_k|^2}{4(\tau_i-\tau_k)}\right], \tag{5}$$

where

$$\varepsilon = \frac{T_{ign} - T_{in}}{T_{ad} - T_{in}} \tag{6}$$

is the nondimensional ignition temperature of a particle.

For a given location of the particles in the system, given ε and given initial conditions (initially ignited particles), the time points of ignition for all particles are defined by numerical solution of Eq. (5). In some particular cases, one can find an analytical solution of Eq. (5). For one-dimensional systems such solutions are considered and discussed in [15,16].

In this paper we consider the combustion of a cylindrical layer of particles on the assumption that the particles are located periodically on the surface of a cylinder with the radius R. Such a system represents a set of parallel circular layers where l is the distance (period) between two neighboring particles in a layer and ld is the distance between two neighboring layers. The case of $d = 1$ corresponds to an isotropic system. We choose l as the characteristic spatial scale of the system. Thus, the spatial period in a circular layer calculated in terms of nondimensional variables is equal to 1, and the period between two layers is equal to d. It is more convenient to use the number of particles in a circular layer N instead of R; $2\pi r = N$, where $r = R/l$ is the nondimensional radius of the cylinder. In the (x,y,z) coordinate system where z axis coincides with the axis of the cylinder the coordinates of particles are calculated as follows:

$$x_{mn} = R\sin(2\pi m/N); \;\; y_{mn} = R(2\pi m/N), \; z_{mn} = nld,$$

where $n = 1,2,\ldots$ is the layer number; $m = 1,\ldots,N$ is the number of a particles in a layer.

Then Eq. (5) can be transformed into

$$\varepsilon = \sum_{k(\tau)}\frac{1}{[4\pi(\tau_i-\tau_k)]^{3/2}} \times \exp\left(-\frac{2r^2\{1-\cos[2\pi(m_i-m_k)/N]\}+(i-k)^2d^2}{4(\tau_i-\tau_k)}\right). \tag{7}$$

Let us consider the stationary layer-by-layer combustion of a cylindrical sample.

We assume all the particles belonging to a particular circular layer n to ignite simultaneously. In case of the stationary combustion wave, the interval between the ignitions of two neighboring circular layers τ_0 is the same for any couple of layers.

Hence $\tau_n = n\tau_0$. Then Eq. (7) takes the form of

$$\varepsilon = \sum_{m=1}^{N}\sum_{j=1}^{\infty}\frac{1}{(4\pi j\tau_0)^{3/2}} \times \exp\left\{-\frac{j^2d^2+2r^2[1-\cos(2\pi m/N)]}{4j\tau_0}\right\}, \tag{8}$$

where $r = N/2\pi$. The nondimensional burning rate for the system under consideration can be defined as $\omega = 1/\tau_0$. The expression (8) represents an exact solution to Eq. (5), which defines the dependence of nondimensional burning rate on nondimensional ignition temperature of particles $\omega(\varepsilon)$.

Figure 1 shows the dependences $\omega(\varepsilon)$ calculated by the expression (8) for various N and d.

Alongside with this, we solved numerically the complete equation (7). For this purpose we took a number of layers consisting of initially ignited particles as initial conditions. Then the ignition time points of other particles were defined by the numerical solution of Eq. (7). In the case of symmetrical initial conditions (i.e., all the initially ignited particles in a layer were ignited simultaneously regardless of the time of the layer ignition) the layer-by-layer combustion occurs: All the particles in each next layer are ignited simultaneously. The intervals between the ignition of neighboring layers are changed and tend to the stationary value corresponding to the solution (8). The numerical solution of Eq. (7) shows that the combustion of such a system under symmetrical initial conditions is only possible provided that $\varepsilon \leqslant \varepsilon_{cr}$. For an isotropic system ($d = 1$) we obtained in calculations $\varepsilon_{cr} = 0.12$. If $\varepsilon > \varepsilon_{cr}$, the combustion stops regardless of the number of initially ignited particles. This is manifested in that some layers do not ignite and the cooling of the system occurs monotonically due to thermal conductivity. The more the number of initially ignited layers, the more the layers are able to ignite under $\varepsilon > \varepsilon_{cr}$ before the combustion process stops completely. This is explained by the initial amount of heat energy in the system.

The analysis of one-dimensional stability of the system under consideration (similar to that in [15]) has shown that the layer-by-layer combustion mode is absolutely stable. This is confirmed by the numerical solution of Eq. (7): On approaching the combustion limit $\varepsilon = \varepsilon_{cr}$ no oscillatory modes

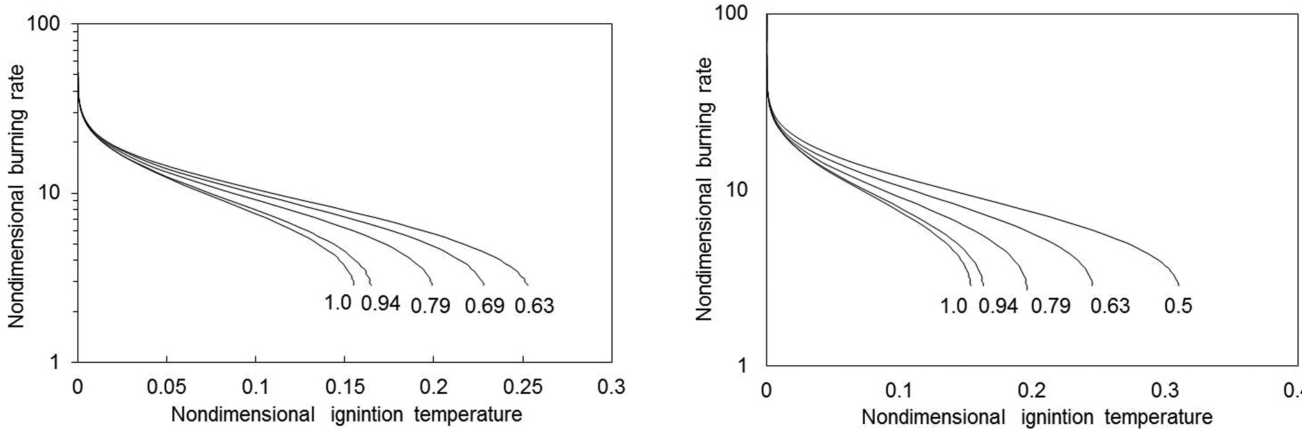

FIG. 1. Exact solutions of Eq. (8) for $N = 20$ (left) and $N = 40$ (right) depending on the distance between circular particle layers d. Values of d are shown in the graph.

were observed and the combustion process was always steady state. At transition to $\varepsilon > \varepsilon_{cr}$ the process stopped immediately. This constitutes the key difference between a symmetric combustion wave on the surface of a cylinder and a one-dimensional discrete combustion wave [15–17]. The latter one shows the instability near the combustion limit: The stationary mode is changed to more complex oscillatory combustion modes complicating with the increase of parameter ε. The transition to a more complex nonstationary mode occurs in the form of doubling period bifurcation. The stability of the layer-by-layer combustion of a cylindrical system may be explained by heat dissipation in the environment associated with thermal conductivity, which does not take place in case of a one-dimensional model [15–17].

This is also the reason for the substantial difference in the critical value of parameter ε for a three-dimensional cylindrical system ($\varepsilon_{cr} = 0.12$) as compared to a one-dimensional system ($\varepsilon_{cr} = 0.45 - 0.54$ [15–17]).

B. Asymmetric initial conditions

Let us consider the impact of asymmetry in the initial conditions. We assume that at the starting moment all the particles have been ignited simultaneously in the layers from 1 to n_0, while the layer $n = n_0 + 1$ holds only one ignited particle. These initial conditions were used for numerical solution of Eq. (7). The ignition moments for the rest of the particles in the system were calculated from this equation. The calculations show that, similarly, self-sustaining combustion is only possible under the condition $\varepsilon \leqslant \varepsilon_{cr}$. However, $\varepsilon_{cr} = 0.16$ for the asymmetric initial conditions, which is slightly higher than that for the symmetric conditions.

This implies that introducing asymmetry into the initial conditions extends the range of the parameters of the system enabling self-sustaining combustion. The calculations show that only nonstationary combustion takes place near the $\varepsilon \approx \varepsilon_{cr}$ limit.

The analysis has revealed that a discrete combustion wave propagates differently depending on ε and N parameter values. The classical spin combustion takes place under the condition $\varepsilon < \varepsilon_1$; otherwise there are two more combustion modes in the system. If $\varepsilon_1 < \varepsilon < \varepsilon_{cr}$, a more complex combustion mode is implemented; we called it a "leapfrog" mode. Immediately near the combustion limit ($\varepsilon \approx \varepsilon_{cr}$) some of the calculations showed the presence of an oscillatory spin combustion mode which we called a "swing" mode. Here we consider these modes in detail on the basis of isotropic systems ($d = 1$) containing different numbers of particles in the circular layers: $N = 20$, 40, and 60.

1. Spin combustion mode

In spin combustion mode, a discrete combustion wave propagates along the circular layer of particles and simultaneously the combustion spreads from one layer to another. The combustion wave propagates from the ignited particle to the neighboring unignited one in a relay-race mode. The velocity of propagation of a discrete combustion wave inside the layer is significantly higher than that between circular layers. In other words, the intervals between the ignitions of two neighboring particles within a single layer are always significantly smaller than the ones between the ignitions of two neighboring particles in different layers. It is convenient to describe the propagation of the combustion wave along the lateral surface of the cylinder in terms of cylindrical coordinates (n,ϕ), where n is the number of circular layer of particles on the lateral surface of the cylinder and $\varphi = [0,2\pi]$ is the angular coordinate of a particle within the layer n. The coordinates $\varphi = 0$ and $\varphi = 2\pi$ correspond to the same particle located on the same generatrix of the cylinder as the "seed" source of spin within the layer $(n_0 + 1)$. The propagation of the discrete combustion wave in the system is described by the functions $\phi(\tau,n)$ and $n(\tau)$. The first one is responsible for the propagation within the layer n and the second one for the propagation between the layers. For each layer n, the origin point $\tau = 0$ corresponds to the ignition moment of the first particle in the layer.

Figures 2–4 show the results of calculations for various N and ε.

Analysis of Figs. 2–4 allows the following conclusions. If the values of ε are relatively small, the discrete combustion wave propagates in a layer very nonuniformly; the more particles the layer contains, the more considerable this nonuniformity is. This happens due to acceleration of the

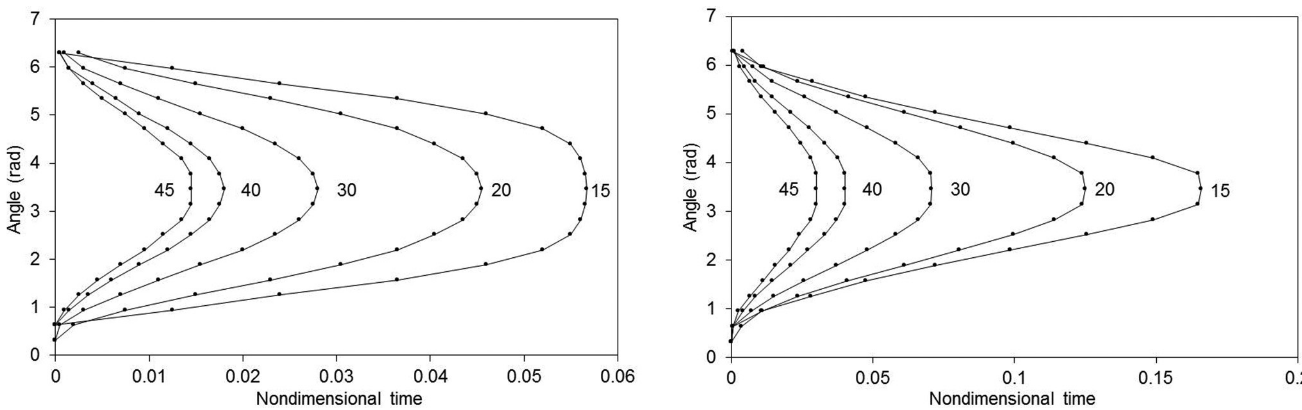

FIG. 2. Spin combustion mode. The propagation of the discrete combustion wave along a circular layer $\phi(\tau)$. Values of n are shown in the graph. The number of particles in a layer is $N = 20$; nondimensional ignition temperature of the particles is $\boldsymbol{\varepsilon} = 0.120$ (left) and $\boldsymbol{\varepsilon} = 0.130$ (right), $n_0 = 10$.

discrete combustion wave within the layer, and for large N a significant number of particles in the layer are ignited almost simultaneously. With increase of ε the propagation of the discrete combustion wave in circular layers becomes more uniform until it reaches almost constant velocity near the limit of spin mode ε_1. The combustion wave propagation in the spin mode is characterized by two parameters: the spin period T_{spin} (the interval between the ignition of the first particle in a layer and complete combustion of this layer) and layer-by-layer ignition period $T_{l\text{-}l}$ (the interval between the ignitions of first particles in a pair of neighboring layers). Figures 2–4 show that the lower the nondimensional ignition temperature ε, the less the T_{spin}, and therefore the faster the discrete combustion wave propagates along the layer. There is an inverse relation between the layer number n and T_{spin}, which means that the spin rotation accelerates with the increase of n.

These conclusions are confirmed by Fig. 5 (left) which shows the dependence of T_{spin} on the layer number for various ε. Figure 5 (right) shows the analogous dependence for $T_{l\text{-}l}$. One can notice that the period of layer-by-layer ignition is independent of the layer number and remains approximately the same during the combustion process. The variation of $T_{l\text{-}l}$ from layer to layer in the case when ε is close to the critical value ε_1 (Fig. 5, on the right) is caused by the loss of stability of the relay-race combustion mode.

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Thus, asymptotic degeneration of spin takes place and at $n \to \infty$ this process degenerates into the layer-by-layer combustion with the constant velocity $\omega = 1/T_{l\text{-}l}$. The degeneration time τ_t depends on the number of particles in a circular layer and on the value of ε. If the observation time τ is relatively small ($\tau \ll \tau_t$, which is typical of the experiments), one might get an impression that the spin combustion mode is stationary and can last infinitely.

Obviously, if $T_{\text{spin}} > T_{l\text{-}l}$, the next layer will ignite before the current layer burns out completely. In this case a multilayer

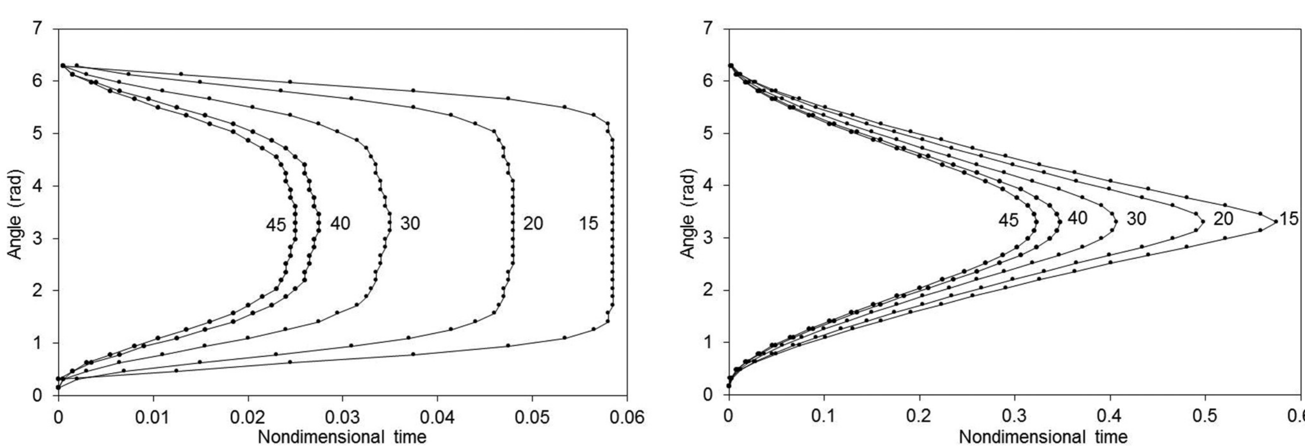

FIG. 3. Spin combustion mode. The propagation of the discrete combustion wave along a circular layer $\phi(\tau)$. Values of n are shown in the graph. The number of particles in a layer is $N = 40$; nondimensional ignition temperature of the particles is $\boldsymbol{\varepsilon} = 0.120$ (left) and $\boldsymbol{\varepsilon} = 0.130$ (right), $n_0 = 10$.

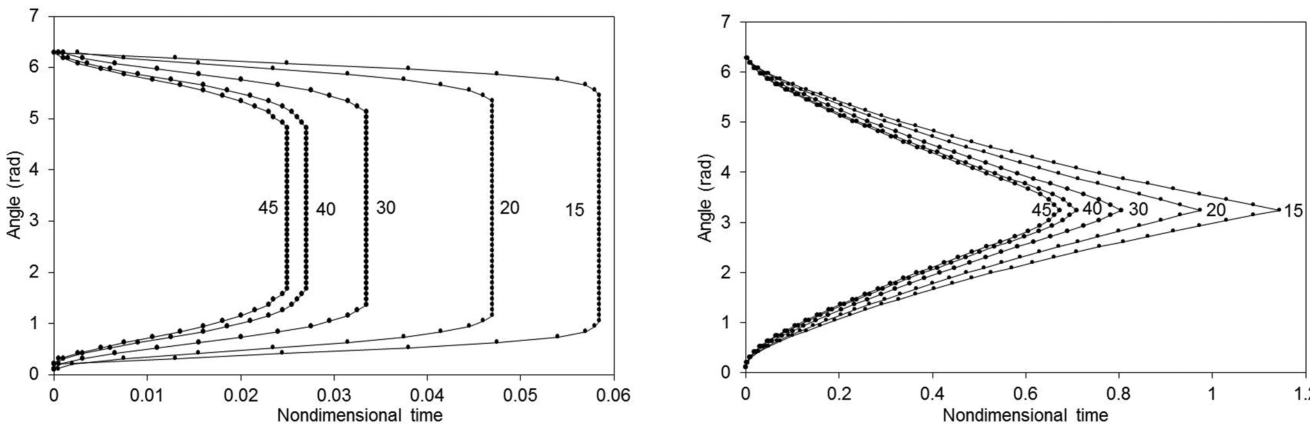

FIG. 4. Spin combustion mode. The propagation of the discrete combustion wave along a circular layer $\phi(\tau)$. Values of n are shown in the graph. The number of particles in a layer is $N = 60$; nondimensional ignition temperature of the particles is $\boldsymbol{\varepsilon} = 0.120$ (left) and $\boldsymbol{\varepsilon} = 0.130$ (right), $n_0 = 10$.

spin combustion mode takes place (in the literature this phenomenon is known as "multihead" spin [2]) at which several circular layers ignited at different instants burn simultaneously in relay-race mode. The number of layers burning at the same time is defined by the expression

$$k = [T_{\text{spin}}/T_{l\text{-}l}] + 1,$$

and depends on N and ε, where $[\cdots]$ stands for the integral part of a number.

The calculations revealed from one to six layers within a spin.

Figure 6 shows the dependence of ratio $T_{\text{spin}}/T_{l\text{-}l}$ on the layer number n for various N and ε.

As shown in Fig. 6, the multilayer spin degenerates with time: the number of simultaneously burning layers decreases monotonically, which leads to the asymptotic degeneration of spin from a multilayer down to a monolayer one. A degradation of spin can be observed if the length of the sample is big enough.

For visualization of the results of the calculations we used a special computer program which allows showing the combustion process in dynamics [18]. Figures 7 and 8 show the characteristic frames of combustion in multilayer spin mode. For clarity, the particles are depicted as unit spheres (in nondimensional variables) although they are considered as points in the model.

Looking at these pictures (Figs. 7 and 8), one may conclude that a discrete combustion wave performs a spiral motion along the surface of the sample, which corresponds to the traditional concept of spin combustion [2–14]. However, thorough analysis shows that it is not exactly true. Figure 9 illustrates the same process as in Fig. 8 from a different angle. One can see that in fact there is no spiral motion of the spin. The process actually goes as follows: After the ignition of the first particle in a layer, the two discrete combustion waves arise which propagate along the layer in the opposite directions from the first ignited particle. After a while, a particle belonging to the next layer ignites and the process repeats in the same way from layer to layer. Looking from a certain angle, an illusion of spiral motion of hot spots on the surface of the sample may really arise.

2. "Leapfrog" mode

The calculations show that at $\varepsilon = \varepsilon_1$ the relay-race spin mode becomes unstable and at $\varepsilon > \varepsilon_1$ it changes by a more

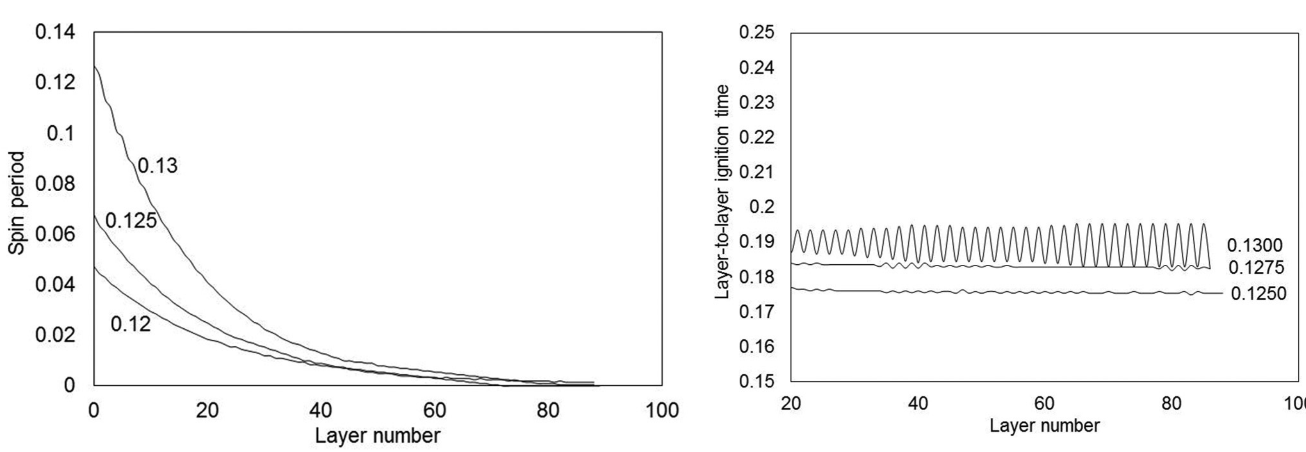

FIG. 5. Dependence of spin period T_{spin} (left) and layer-to-layer ignition period $T_{l\text{-}l}$ (right) on layer number for $N = 20$ and various ε. Values of ε are shown in the graph; $n_0 = 10$.

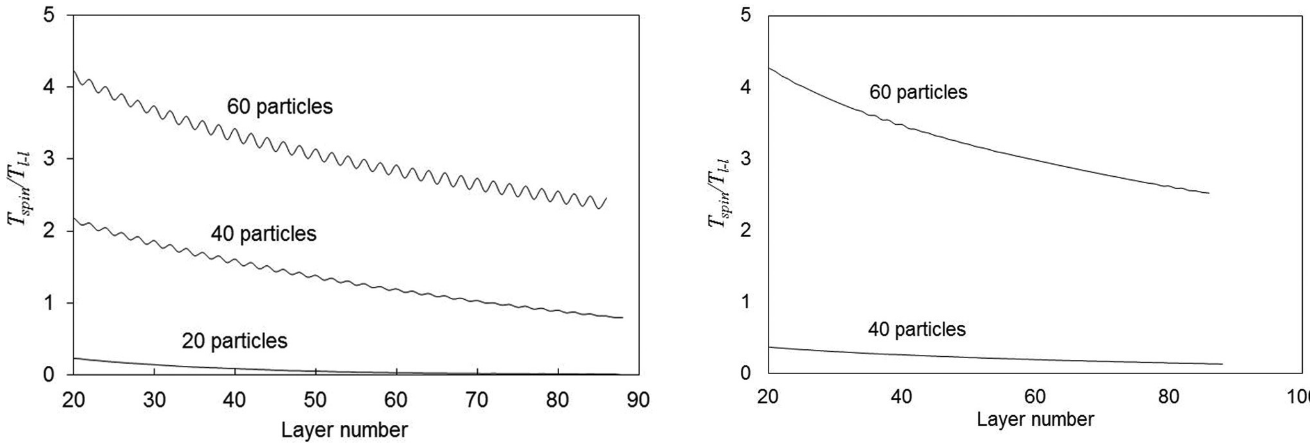

FIG. 6. Dependence of the ratio $T_{\mathrm{spin}}/T_{l\text{-}l}$ on the layer number for various N; $\varepsilon = 0.13$ (left) and $\varepsilon = 0.1275$ (right); $n_0 = 10$.

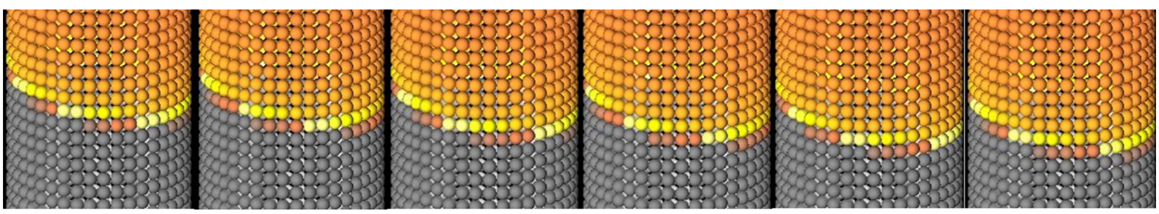

FIG. 7. (Color online) Propagation of a discrete combustion wave in multilayer spin mode (light particles are ignited ones; dark particles are nonignited ones); $N = 40$, $\varepsilon = 0.130$, $n_0 = 10$. Combustion spreads downward; the spin is moved along the surface from right to left.

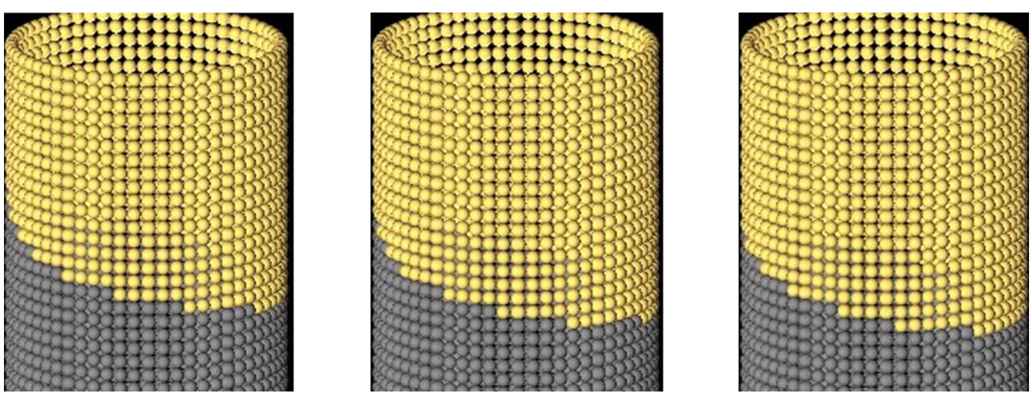

FIG. 8. (Color online) Propagation of a discrete combustion wave in multilayer spin mode (light particles are ignited ones; dark particles are nonignited ones). $N = 60$, $\varepsilon = 0.130$, $n_0 = 10$. Combustion spreads downward, the spin is moved along the surface from right to left.

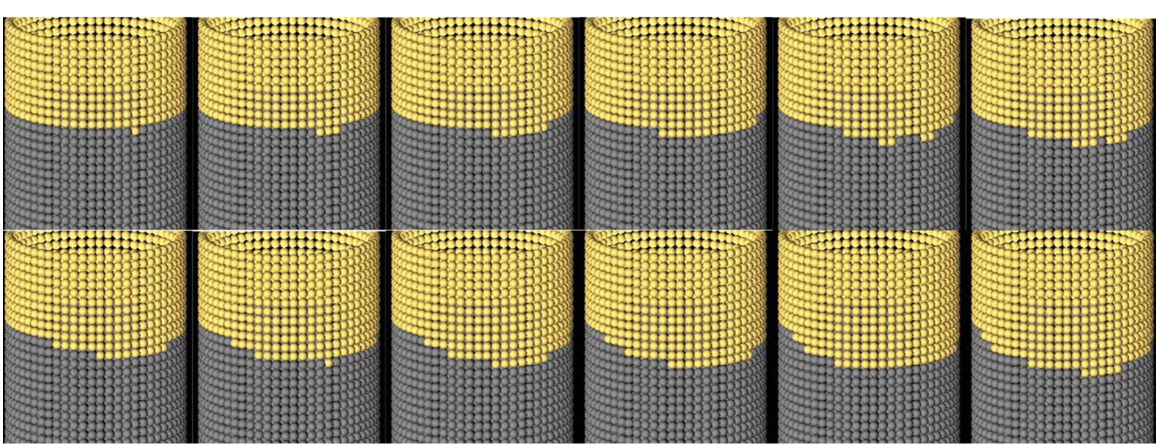

FIG. 9. (Color online) Propagation of discrete combustion wave in multilayer spin mode (light particles are ignited ones; dark particles are nonignited ones). $N = 60$, $\varepsilon = 0.130$, $n_0 = 10$ (side view).

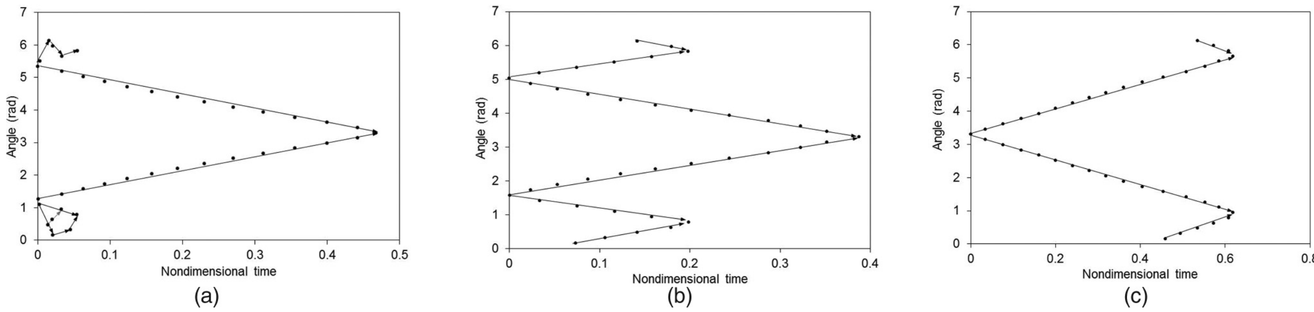

FIG. 10. "Leapfrog" mode. Propagation of discrete combustion wave in a circular layer $\phi(\tau)$. The number of particles in a layer is $N = 40$; nondimensional ignition temperatures of the particles are $\varepsilon = 0.145$ (a), $\varepsilon = 0.150$ (b), $\varepsilon = 0.155$ (c); $n_0 = 10$. The arrows show the direction of movement of the "combustion front."

complex combustion mode. The critical value $\varepsilon_1 \approx 0.13$ obtained in calculations turned out to be independent of the number of particles N within circular layers.

The loss of stability of the relay-race spin mode is manifested in the fact that ignition of particles within a circular layer occurs inconsequently. As a result, a "leapfrog" of hot spots occurs, that is, the igniting of particles as if "jumping" over each other. Sometimes several particles within a layer can ignite simultaneously while some unignited particles remain between them. Ignited particles become sources of a discrete combustion wave propagating in different directions. The combustion wave can spontaneously change its direction of propagation in a layer.

This can be explained in terms of a phenomenon similar to the doubling period bifurcation observed in a one-dimensional system [15,16]. As the system under consideration is actually two dimensional, it can be represented as a superposition of two one-dimensional systems: One of them consists of chains of particles located along the generatrices of the cylinder while the second one consists of circular layers. Under certain conditions the doubling period bifurcations can occur in each of the chains. A superposition of these bifurcations leads to the "leapfrog." Figure 10 shows the examples of propagation of a discrete combustion wave in the "leapfrog" mode.

At the changing of system parameters (N and ε) the "leapfrog" mode shifts to a new, almost periodical process with several initially ignited hot spots in every circular layer. In this case the ignition of a new circular layer takes place not at a single point but at several initial points located almost equidistantly from each other. The number of initial sources is the same in all layers, but their location may be different for different circular layers, for example, between the initial sources of the previous layer (see Fig. 11). In this case a sort of a periodical wave pattern can be observed. In Fig. 11(a), the even particles are initially ignited on the first layer while odd particles are ignited on the second one, etc.

3. "Swing" mode

On approaching the combustion limit ε_{cr} this periodic process becomes more and more regular until it turns into an oscillatory spin combustion near ε_{cr}. This mode was called a "swing" mode. Figure 12 shows the characteristic frames of this mode, and Fig. 13 illustrates the dependence $\phi(\tau)$ for a few circular layers.

In "swing" combustion mode, a discrete combustion wave emerges in a particular layer at the point $\phi = 0$ ($\phi = 2\pi$) and spreads from it in opposite directions along the layer. The wave propagates in the relay-race mode. When the wave reaches point $\phi = \pi$, the circular layer burns out completely. After that, the first particle on the next layer ignites. In contrast to regular spin mode, in "swing" mode ignition of the first particle in the next layer occurs in point $\phi = \pi$. This point becomes a source of a discrete combustion wave. The process is periodical. Thus, at "swing" mode the initial ignition in neighboring layers takes place in diametrically opposite points, and the combustion waves originating from these points move in opposite directions towards each other. The next layer ignites only after the previous one has burned out completely. As a result, the oscillatory mode of spin combustion wave

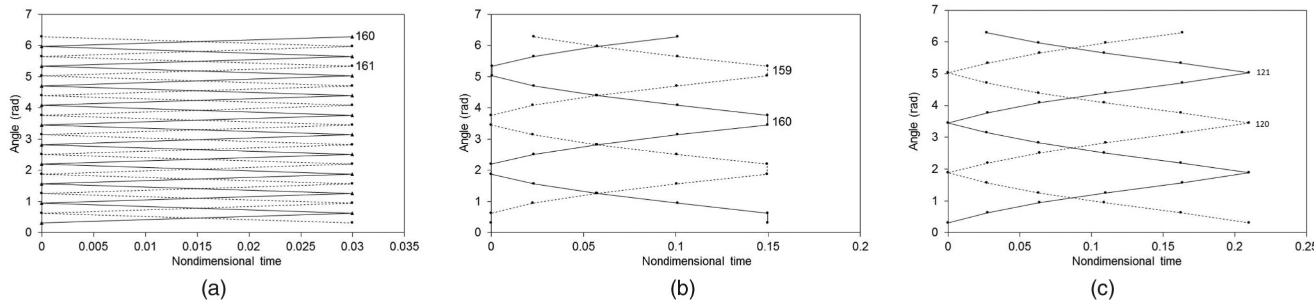

FIG. 11. "Leapfrog" mode. Propagation of discrete combustion wave in a circular layer $\phi(\tau)$. Values of n are shown in the graph. The number of particles in a layer is $N = 20$, nondimensional ignition temperatures of the particles are $\varepsilon = 0.145$ (a), $\varepsilon = 0.155$ (b), $\varepsilon = 0.160$ (c); $n_0 = 10$.

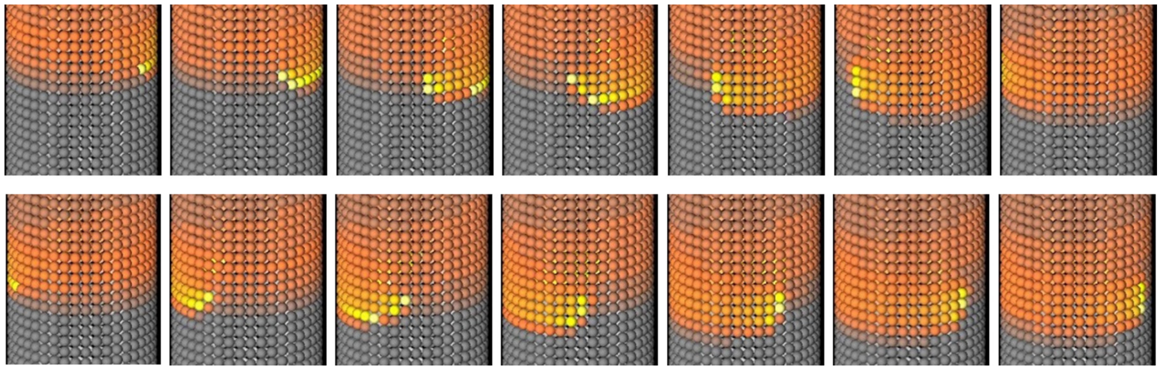

FIG. 12. (Color online) Combustion wave propagation in the "swing" mode (light particles are ignited ones; dark particles are nonignited ones). $N = 40$, $\varepsilon = 0.160$; $n_0 = 10$. Combustion spreads from top downward.

propagation arises at which the combustion wave periodically changes its direction of propagation to the opposite.

III. DISCUSSION

Let us briefly discuss the assumptions of the model considered.

In this work, calculations were performed for a small number of particles in the surface layer of the sample ($N \leqslant 60$). This is due, primarily, to the computer's performance. Nevertheless, the results show the general trend that occurs when changing the number of particles in a cylindrical layer: Aa spin combustion mode is stored with increasing the number of particles N. However, if we assume that the hot spot is not a single particle but a cluster of primary particles which has a characteristic size, e.g., the order of 0.5 mm, the cylinder with $N = 60$ particles will has a diameter of about 1 cm, which is comparable with the diameter of the real samples used in the experiments.

Let us explain why we consider only the cylindrical layer of particles, although the actual samples are continuous. The actual samples are produced by compaction of the powders in a cylindrical shell. Due to friction with the shell during compaction, the properties of the outer layer of the particles can be different from the properties of the inner layers of the sample: The outer layers may have a different concentration of active particles; the hot spots in the outer layer may have a different effective ignition temperature and a different adiabatic burning temperature, etc. In our opinion, precisely the difference in properties of the outer and inner layers of the sample results in the appearance of such an effect as spin combustion near combustion limits.

Let us assume, for example, that in the process of the sample compaction, the outer layer of the sample acquired the properties that promote combustion (in our model, it can be described, e.g., by decreasing of nondimensional ignition temperature ε). In this case, it may turn out that at the combustion limit, the internal layers are not capable of self-sustained combustion ($\varepsilon > \varepsilon_{cr}$ for them), while the outer layer is still capable of self-sustained combustion ($\varepsilon < \varepsilon_{cr}$ for it). Then only the outer tubular layer will burn, while the inner layers will play the role of an inert heat-conducting medium. This is completely consistent with the proposed model. However, a heat generated in the outer cylindrical layer will heat the inner layers, and near the combustion limit, the inner layers will also be capable of combustion, but only in the forced mode: Their own heat release is not enough for self-sustained combustion, but a missing amount of heat will come from the burning outer cylindrical layer. As a result, combustion will occur throughout the sample volume; however, the combustion front of the inner layers will lag behind the combustion front of the outer layer, and hence, combustion of the inner layers will have no effect on the combustion of the outer layer. We plan to study this problem in detail in the future.

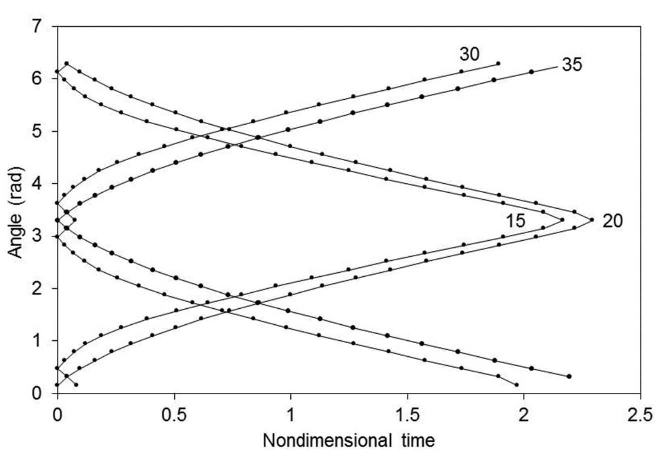

FIG. 13. Dependence $\phi(\tau)$ for combustion wave propagation in the "swing" mode for different layers. Values of n are shown in the graph. The number of particles in a layer is $N = 40$; nondimensional ignition temperature of the particles is $\varepsilon = 0.160$; $n_0 = 10$.

IV. CONCLUDING REMARKS

In this paper, we have reproduced numerically the "spin" propagation mode of the gas-free combustion wave. Detailed study of this process has shown that the traditional concept of the spin combustion as a continuous spiral motion of the energy release area along the surface of the sample is incorrect. The existing concept might have arisen due to the misinterpretation of the experimental data. This study shows that actually a spinlike combustion mode takes place, with two energy release areas appearing on the lateral surface of the sample and spreading circumferentially in the opposite directions; their movement stops when they meet. After some time two new heat release zones are formed on the next layer of the cylinder surface committing the same counter-circular motion. This process continues periodically, and from a certain angle it looks like spiral motion of a hot spot on the lateral surface of the sample.

A spinlike mode can only take place near the combustion limit in a narrow range of parameters of combustible mixture. On approaching the combustion limit the process becomes more complicated and spinlike combustion shifts to a more complex mode with multiple hot spots moving in different directions along the lateral surface. This is likely to be the reason why it is not always possible to detect "spin combustion mode" experimentally. According to the theory, spin combustion occurs due to asymmetric initial conditions and transforms into a layer-by-layer combustion mode with time. In order to observe the degeneration of spin combustion mode, the sample needs to be long enough. In the experiments on gas-free combustion, the relatively short samples are usually used (the length-to-diameter ratio is less than 10), so the degeneration of spin mode cannot be observed experimentally. This produces an illusion of stationarity of spin combustion.

A movie demonstrating the described results is provided in the Supplemental Material [18].

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[18] See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevE.91.012805 for the movies demonstrating the described results.