

Absence of jamming in ant trails: Feedback control of self-propulsion and noiseDebasish Chaudhuri^{1,*} and Apoorva Nagar^{2,†}¹*Indian Institute of Technology Hyderabad, Yeddumailaram 502205, Telangana, India*²*Indian Institute of Space Science and Technology, Thiruvananthapuram, Kerala, India*

(Received 27 October 2014; published 20 January 2015)

We present a model of ant traffic considering individual ants as self-propelled particles undergoing single-file motion on a one-dimensional trail. Recent experiments on unidirectional ant traffic in well-formed natural trails showed that the collective velocity of ants remains approximately unchanged, leading to the absence of jamming even at very high densities [John *et al.*, *Phys. Rev. Lett.* **102**, 108001 (2009)]. Assuming a feedback control mechanism of self-propulsion force generated by each ant using information about the distance from the ant in front, our model captures all the main features observed in the experiment. The distance headway distribution shows a maximum corresponding to separations within clusters. The position of this maximum remains independent of average number density. We find a non-equilibrium first-order transition, with the formation of an *infinite cluster* at a threshold density where all the ants in the system suddenly become part of a single cluster.

DOI: [10.1103/PhysRevE.91.012706](https://doi.org/10.1103/PhysRevE.91.012706)

PACS number(s): 87.23.Cc, 05.40.Jc, 02.50.Ey, 89.75.Fb

I. INTRODUCTION

The study of collective motion of self-propelled particles—from sub-cellular machines like molecular motors moving on polymeric tracks to unicellular life forms like bacteria, from the coordinated motion of insects as small as ants to large mammals like humans—shows the emergence of rich dynamical behavior and patterns starting with simple rules for the motion of individual units [1–3]. The study of ants, in particular, is fascinating from more than one perspective [4]. From a traffic point of view, the collective motion of ants shows self-organization of flow to maximize efficiency in transport [5,6], and spontaneous formation of lanes in bi-directional traffic [7,8]. Another interesting feature of ant motion is the spontaneous selection of shortest path between the nest and the food source by using only local dynamical rules, without the aid of a global perspective. This has inspired theoretical work on new kinds of optimization algorithms [9,10]. While walking, ants leave chemical trails in the form of pheromone, that later ants follow leading to ant trail formation [4,11–16]. Formation of these trails have been described theoretically in terms of active-walker models. The mechanism is ubiquitous in nature and similar to river basin formation [17], formation of pedestrian trails [18,19], and formation of axon bundles in mammalian sensory neurons [20,21].

A recent experimental study on the collective motion of ants within preformed natural trails in the species *Leptogenys processionalis* showed several intriguing features [22]. It found an absence of jamming in ant-traffic even at very high densities—with only a minor decrease in velocity at higher density, reduction in velocity fluctuations with increasing densities, as well as the formation of clusters of ants within the trail. The flow behavior is in contrast to vehicular traffic where a decrease of flux is observed at high densities, indicating congestion and a tendency to form jams, captured by flux-density plots known as fundamental diagrams of traffic

flow [23]. In this paper, we present a model of self-propelled particles performing single file motion, which captures all the main observations of Ref. [22]. In single-file motion particles constrained to move in one dimension cannot overtake each other, performing sub-diffusive dynamics [24–28].

Earlier theoretical work on ant traffic using asymmetric hopping and a particle exclusion process on discrete lattice showed various interesting features, including the non-monotonic dependence of velocity on density [29,30]. However, these models predict jamming at high densities associated with exclusion interaction in a discrete lattice and random sequential movement of entities governing the dynamics. They fail to capture the absence of jamming in ant traffic as observed in Ref. [22]. Our model takes a different approach. The ants are viewed as particles interacting via nearest-neighbor repulsion. The particles perform continuum dynamics and move together, as opposed to random sequential hopping on a discrete lattice considered in Refs. [29,30]. The biological inputs in the model come through the generation of an active self-propulsion force in the particles, that has a deterministic part and a stochastic noise.

Most ants have extremely limited eyesight, but are still able to efficiently manage traffic by co-operative trail formation. These trails are made by ants depositing pheromones on the ground which act as signals for the trailing ants to follow the same path. Pheromones evaporate with a rate dependent on environmental factors. Ants use differential sensing of pheromones to guide their motion. The sensitivity to concentration gradient decreases at high concentrations. An earlier model of diffusing agents interacting with an external field of pheromone, which itself undergoes addition, evaporation, and diffusion dynamics led to the emergence of trails at reasonable parameter regimes [16]. With time, continuous deposition of pheromones make the signal from a trail strong enough such that all successive ants follow the same path without straying. Our model considers ant motion on preformed trails, thus considering ants as particles moving in one dimension (1D). The motion within this trail could be guided by local sensing—limited vision or antennal touch. We incorporate a feedback mechanism based on inputs

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from these local interactions into the active force generation. Our model captures all the main features of experimental results, showing how this feedback can crucially control ant motion. We present further predictions that are amenable to experimental verification.

II. MODEL AND SIMULATION

We model the motion of ants in a preformed trail, as a 1D system of self-propelled particles (SPP). The dynamics of i -th SPP can be described in terms of the Langevin equations of motion

$$\begin{aligned}\dot{x}_i &= v_i \\ \dot{v}_i &= -\gamma v_i + \eta_i(t) + F_i - \partial_i \sum_{j=i\pm 1} U(x_{ij}),\end{aligned}\quad (1)$$

where $F_i(x_i, x_{i+1})$ is a self-propulsion force, $U(x_{ij})$ denotes a repulsive nearest-neighbor interaction ensuring that particles cannot cross each other in 1D. The viscous dissipation term $-\gamma v_i$, models dissipation in energy, whose origin may lie within the ant's body—in the movement of muscles that it utilizes to walk, or in friction with local environment, like the walking surface. The noise $\eta_i(t)$ is interpreted as a stochastic part of self-propulsion, and thus it has a non-equilibrium origin. We assume that the time-scales associated with the generation of self-propelled force that comes from an internal energy depot is much faster with respect to the mechanical motion of the ants [31]. Thus the stochastic force is assumed to be Gaussian white noise with $\langle \eta_i(t) \rangle = 0$, $\langle \eta_i(t) \eta_j(t') \rangle = 2D_0 \delta(t - t') \delta_{ij}$ where $D_0(x_i, x_{i+1})$ denotes non-equilibrium fluctuations.

The interaction potential between nearest neighbors $U(x_{ij})$ models the impenetrability of the ants, with $x_{ij} = x_j - x_i$ with $j = i \pm 1$. We choose the repulsive part of Lennard-Jonnes potential $U(x_{ij}) = 4\epsilon[(\sigma/x_{ij})^{12} - (\sigma/x_{ij})^6 + 1/4]$, with a cutoff distance set to $r_c = 2^{1/6}\sigma$ such that $U(x_{ij}) = 0$ if $|x_{ij}| > r_c$. Here σ sets the unit of length and is of the order of the average length of one ant, and ϵ sets the unit of energy. We perform molecular dynamics (MD) simulations using the velocity-Verlet scheme, with integration time step $\delta t = 0.01\tau$, where $\tau = \sigma \sqrt{m/\epsilon}$ is the unit of time and corresponds to 1 s. We fix the local temperatures at $D_0(x_i, t)/\gamma k_B$ using Langevin thermostat characterized by an isotropic friction $\gamma = 1/\tau$.

If one uses a constant self-propulsion force F_i , the Langevin model would generate average particle velocity completely independent of density. However, experiments [22] showed a weak but steady decline in velocity with increasing density. This means that the ants sense the local crowding and use a feedback mechanism to control the amount of self-propulsion force generated. Thus we model the self-propulsion force generated by i -th ant as $F_i = \gamma f_0(1 - 1/\rho_f \delta x_i)$, where $\delta x_i = x_{i+1} - x_i$, the separation between i -th ant and the nearest neighbor in front, and ρ_f is a constant. In using the distance headway δx_i to model self-propulsion feedback, we have assumed that the ant senses the position of its leading neighbor using its limited vision or antennal touch, but remains indifferent to the trailing neighbor with regard to self-propulsion force generation. On an average, $\langle \delta x_i \rangle$ is a measure of inverse local density $1/\rho$. Using a fit to

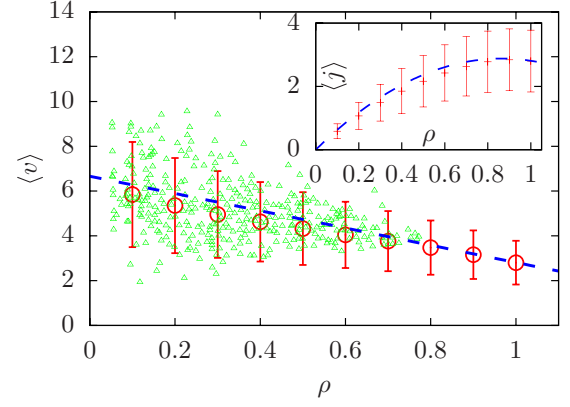


FIG. 1. (Color online) Average velocity with density. The symbols \circ with error bars denote our simulation results, while the small \triangle symbols denote data extracted from Fig. 3 of Ref. [22]. The dashed line is the mean field estimate $\langle v \rangle = f_0(1 - \rho/\rho_f)$. Inset: The fundamental diagram showing current as a function of density. The dashed line is a plot of mean field estimate $\langle j \rangle = \rho \langle v \rangle$.

the experiments on ant trails [22] we choose $f_0 = 6.66$ and $\rho_f = 1.73$ to characterize the force F_i (Fig. 1).

The stochastic force $\eta_i(t)$ helps the ants to explore the area around them, e.g., for food, in the absence of external cue like a well-formed trail, or odorants from food source. While this noise is a good strategy for exploration, it can be a hindrance in traffic flow once a trail is formed, since it can lead to enhanced collisions. In fact, it is well known from the work by Nagel and Schreckenberg [32] that the introduction of noise in lattice models of traffic leads to traffic jams. We model our ants to have a feedback mechanism that reduces noise as the local density increases, leading to reduced collisions and thus reducing the probability of traffic jams. As for the self-propulsion force above, the simplest such choice would be a linear decrease with local density ($\sim 1/\delta x_i$), i.e., $D_0(x_i, t) = \tilde{D}_0(1 - 1/\rho_D \delta x_i)$ with $\tilde{D}_0 = \gamma k_B T$ characterizing an equilibrium-like fluctuation strength, and ρ_D is a constant. However diffusivity D_0 has to be positive for all possible δx_i , a condition that would be violated at high densities if the above-mentioned linear form were chosen. Thus we choose $D_0(x_i, t) = \tilde{D}_0 \exp(-1/\rho_D \delta x_i)$ which captures well the experimentally obtained fluctuations in velocities with $\tilde{D}_0 = 7.66$ and $\rho_D = 0.47$, and remains positive at all densities (Fig. 1).

III. RESULTS AND DISCUSSION

We have chosen our parameters defining the self-propulsion force and fluctuations to fit the data from experiments [22]. As can be seen from Fig. 1, our results for the mean velocity as well as the variance describe the data well. Replacing δx_i by the mean-field value $\langle \delta x_i \rangle = 1/\rho$, the steady state mean velocity obtained from Eq. (1) is $\langle v \rangle = f_0(1 - \rho/\rho_f)$, leading to a mean flux $j = \rho(1 - \rho/\rho_f)$ which agrees with simulation results. Note that our simple assumption for feedback controlled self-propulsion gives an expression of flux j that has the same behavior as the discrete totally asymmetric simple exclusion process (TASEP) [2], however with a ρ_f

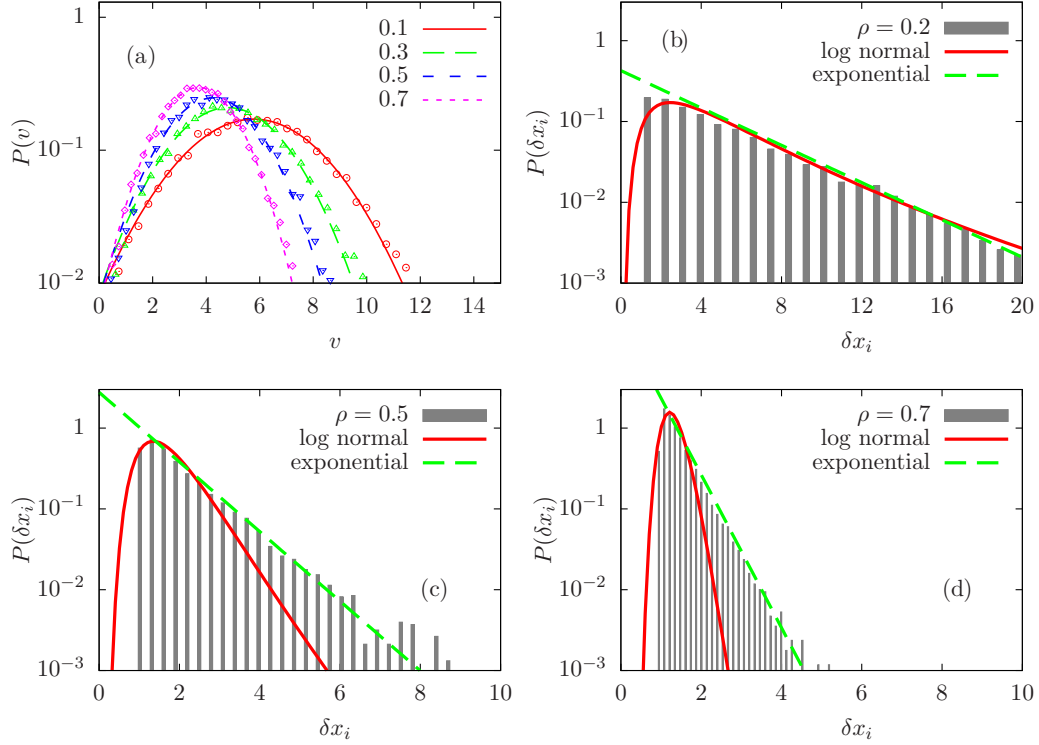


FIG. 2. (Color online) Distribution functions. (a) Probability distribution of velocity of each particle at densities $\rho = 0.1, 0.3, 0.5, 0.7, 0.9$. The points are obtained from simulation, and the lines show expected Gaussian distributions with varying peaks and widths. (b)–(d) Simulation results for probability distributions of distance headways, at densities $\rho = 0.2, 0.5, 0.7$.

that lies at an inaccessibly large value. Thus $\langle v \rangle$ shows a slight decrease with density in the experimentally accessed regime. Unlike the usual traffic model, the current or flow in our system (Fig. 1: inset) does not show a congested branch at high densities thus reflecting the absence of jamming. In Fig. 2(a) we show the probability distribution of velocities of individual particles at various values of mean density. The width of the velocity distribution reduces with increasing density. This happens as the ants reduce the strength of the noise $\eta_i(t)$ in self-propulsion using feedback from the local density. Thus, our model captures the two main features of ant traffic on well-formed trails [22]: absence of jamming even at high densities, and a decrease in velocity fluctuations with an increase in density.

A comparison of our model for ants with Langevin models for traffic [33] shows that the central difference between cars and ants is in the choice of self-propulsion force. In traffic models, self-propulsion is reduced to zero as the distance between two cars vanishes, to avoid collision between cars. Whereas in natural ant traffic, ants may come into touching distances. In our model for ants, the active forces decrease but by a small amount as ants approach each other within touching distances ($F_i > 0$ at $\delta x_i = 1$). The fact that ants collide with each other is not surprising since they are practically blind and navigate essentially through pheromone sensing.

The fluctuations reveal another important aspect of ant traffic. The reduction of velocity fluctuation with density led to our choice for the diffusion constant getting exponentially suppressed when local density increases. This ensures that the *ant fluid* reduces the local *effective temperature* when density

increases, to keep a control over the local pressure. This means that while ants do not completely avoid collisions among themselves, they do make sure that the number of collisions per unit time are kept largely unchanged. The reduction of noise strength D_0 , ensures that at high densities, all ants will generate *almost* exactly the same self-propulsion force, thus everybody may move together although being within touching distances. Note that if the noise were independent of local density, a faster ant would stop because of a collision with a slower ant—but if everyone moves with exactly the same velocity, jamming is avoided.

The other quantity that we compare with experimental data is the distribution of headway distances δx_i . Similar to the experiments in Ref. [22], we find log-normal behavior at short distances $P(\delta x_i) = (1/\sqrt{2\pi\sigma_{\log}^2\delta x_i^2}) \exp[-(\ln(\delta x_i) - \mu)^2/2\sigma_{\log}^2]$ and exponential behavior at long distances $P(\delta x_i) = A \exp(-\delta x_i/\lambda)$ as seen in Fig. 2(b)–2(d). To understand the origin of exponential tail in $P(\delta x_i)$ we consider the single-file motion. In a system of 1D hard rods of length a , the nearest-neighbor distribution at equilibrium is given by $g_{nn}(x, x') = [\rho^2/(1 - \rho a)] \exp[-(|x - x'| - a)/\lambda]$ with $\lambda = (1 - \rho a)/\rho$ [34]. Although in our case the particles are self-propelled, we obtain the same predominantly exponential decay in the distribution of separation between consecutive particles $P(\delta x_i) = A \exp(-\delta x_i/\lambda)$ with decay length $\lambda = (1 - \rho a)/\rho$ that fits well to all the simulation data with $a = 0.96$ [Fig. 2(b)–2(d)]. The origin of the log-normal behavior at small δx_i is in the non-equilibrium self-propulsion. We find that the peak in the headway distribution at $\delta x_i = 1.4$ is independent of the mean system density. This suggests formation of clusters

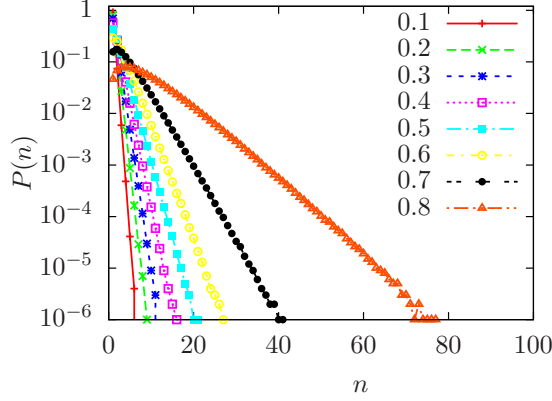


FIG. 3. (Color online) Cluster-size distributions $P(n)$ at various densities ρ denoted in the legend. The largest possible cluster size is $n = N = 4096$. The semi-log plots show exponential tails of the distributions $\exp(-n/n_c)$.

with this typical interparticle separation within a cluster, irrespective of overall density. Similar behavior was observed in the experiment of Ref. [22].

In order to probe this point further, we perform a clustering analysis. A randomly chosen particle is assumed to be part of the first cluster. If its nearest neighbors are separated from this particle by a distance less than 1.4, which is the average headway separation within a cluster, they are also assigned to the same cluster. This procedure is continued until no more particles can be assigned to the first cluster. Then a new random particle which remained unclustered so far is assigned to the next cluster, and the clustering procedure continued in the same way as before until all particles are assigned to a cluster [35]. The size of clusters n may vary from 1 to N , the total number of particles in the system. The resulting cluster size distributions $P(n)$ calculated from the steady-state dynamics of our MD simulation are shown in Fig. 3. In order to obtain better statistics for larger clusters, we performed these simulations using $N = 4096$ particles. The distribution of clusters of ants $P(n) \sim \exp(-n/n_c)$ at all densities $\rho < 1$, with the tail going up to higher fractions n/N signifying increase in typical cluster size. At further higher densities $\rho \geq 0.95$ clusters containing all the ants in the system starts to emerge. In the limit of $\rho = 1$, all the ants belong to the same cluster, as fluctuations of headway distances get completely suppressed.

We obtain the typical cluster sizes n_c at various densities by fitting $P(n)$ to the exponential form $\exp(-n/n_c)$. n_c shows a sharp increase for densities $\rho > \rho_c$ where $\rho_c = 0.9$ (see Fig. 4). This shows a non-equilibrium first-order phase transition towards formation of an *infinite cluster*, containing all the ants available in the system. Note that this infinite cluster formation is unlike the aggregation models of Ref. [36] where the transition was associated with a change in the cluster-size distribution from exponential to power-law.

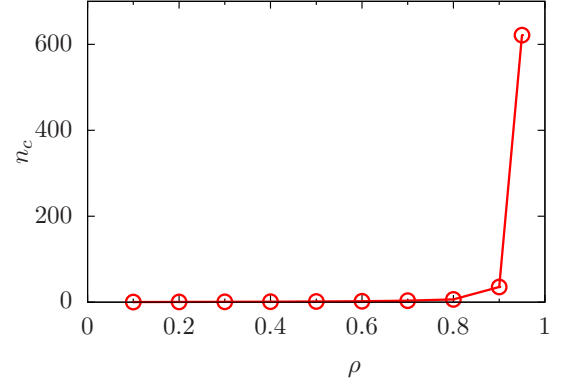


FIG. 4. (Color online) Typical cluster size n_c as a function of overall density ρ , shows sharp increase in n_c at $\rho > 0.9$, indicating a non-equilibrium first-order transition.

IV. SUMMARY AND OUTLOOK

We have presented a model for repulsively interacting self-propelled particles undergoing single-file motion that shows properties in good agreement with the experimental observations on ants presented in Ref. [22]. To describe ant traffic on pre-formed trails, we assumed a generic local crowding dependent feedback control for the deterministic and stochastic parts of self-propulsion force. In agreement with experiments, we find an absence of jamming at all densities. Our model captures the decrease in velocity fluctuations observed in real ants, and shows a peak in the headway distribution which is approximately independent of ant density. We performed a clustering analysis to find an exponential cluster size distribution, independent of mean density. The typical cluster size shows a discontinuous increase at a threshold density indicating a first-order transition. These predictions may be verified from further experiments.

Our model provides a detailed understanding of the dynamics of ants in preformed trails and has implications for technology, e.g., in mechanisms for self-driving cars whose traffic would not jam and robotic swarms that would carry out tasks efficiently and safely like ants. It remains to be seen what patterns emerge from an active walker model with a feedback controlled self-propulsion mechanism proposed in this paper, and whether and to what extent they describe formation of ant trails, in particular, how far they can describe milling or lane-formation in ants [8].

ACKNOWLEDGMENTS

We thank Debashish Chowdhury of IIT-Kanpur for valuable comments, and a critical reading of the manuscript. D.C. thanks MPI-PKS Dresden for hospitality, where a substantial part of this work was done.

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