Quantum phase transitions exposed by rotating the spins

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(Received 10 June 2014; revised manuscript received 22 November 2014; published 16 January 2015)

Using the exactly solvable XY spin chain as an example, we consider nonadiabatic variation of the Hamiltonian along an isospectral trajectory. We suggest that quantum phase transitions (QPTs) can be revealed by the nonadiabatic geometric or Aharonov-Anandan phase, accumulated in a cycle of the state that starts from a particular initial state. On the other hand, starting as the ground state of the instantaneous Hamiltonian, the state does not return to the initial state if the variation of the Hamiltonian is not adiabatic, but the survival probability can indicate QPTs and display revival phenomena.

DOI: 10.1103/PhysRevE.91.012129 PACS number(s): 05.30.Rt, 03.65.Vf, 75.10.Pq

I. INTRODUCTION

In recent years, various properties of quantum states, such as the Berry phase [1], fidelity [2], the Loschmidt echo [3], survival probability (SP) [4], and the quantum geometric tensor [5], have been used as indicators of quantum phase transitions (QPTs) in many-body systems.

As the Berry phase is the geometric part of the phase accumulated in an adiabatic cycle in the parameter space [6,7], its experimental measurement requires the cancellation of the dynamical part of the phase [8]. Moreover, in order for the adiabatic condition to be satisfied, the evolution time, which increases with the system size [9], needs to be long enough, conflicting with the finiteness of coherence time due to the coupling with the environment, and the fact that the nonadiabatic correction increases with the system size [10]. These problems led us to consider nonadiabatic variation of the Hamiltonian in the present context.

In this paper, considering an XY spin chain as an example, we propose to use nonadiabatic variation of the Hamiltonian to expose QPTs. It is noted that QPTs can be related to the nonadiabatic geometric phase, i.e., the Aharonov-Anandan (AA) phase [11], through its relation with the Berry phase starting with a different initial state. The AA phase can be obtained if the initial state is specifically chosen. On the other hand, if the initial state is the ground state of the instantaneous Hamiltonian, then nonadiabaticity destroys the cycle of the state. We show that QPTs can be exposed in the behavior of the SP, that is, the probability for the initial ground state to survive in the final state after a nonadiabatic cycle. We also study the the time-dependent SP for the evolution of an arbitrary time.

The rest of the paper is organized as follows. In Sec. II, we describe the evolution of the quantum state under the cyclic Hamiltonian. In Sec. III, we discuss the AA phase. In Sec. IV, we discuss the SP for one cycle of the Hamiltonian. In Sec. V, we discuss the SP defined for an arbitrary time. Finally we summarize the paper in Sec. VI.

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II. HAMILTONIAN AND THE STATE

Consider the Hamiltonian for the transverse-field XY spin chain,

$$H_\lambda = -\frac{1}{2}\sum_{j=-M}^{M}\left(\frac{1+\gamma}{2}\sigma_j^x\sigma_{j+1}^x + \frac{1-\gamma}{2}\sigma_j^y\sigma_{j+1}^y + \lambda\sigma_j^z\right), \tag{1}$$

where σ_j^α (with $\alpha = x,y,z$) is the Pauli operator at lattice site j, $\gamma \in [0,1]$ represents the degree of anisotropy, λ is the field strength, $N = 2M+1$ is the number of lattice sites assumed to be odd, and the periodic boundary condition is assumed. Exactly solvable with a rich structure, this model is often used in proposing new concepts. It belongs to the Ising universality class if $\gamma \neq 0$ and to the XX universality class if $\gamma = 0$ [12].

By using the Jordan-Wigner transformation $\sigma_j^\dagger = \prod_{l<j}(1-2c_l^\dagger c_l)c_j$, $\sigma_z^j = 1 - 2c_j^\dagger c_j$, where c_j is a fermion annihilation operator, the Fourier transformation $c_j = \frac{1}{\sqrt{N}}\sum_{k=-M}^{M} c_k e^{i2\pi jk/N}$ and the Bogoliubov transformation $\Gamma_{k,\lambda} = \cos\frac{\theta_\lambda^k}{2}c_k - i\sin\frac{\theta_\lambda^k}{2}c_{-k}^\dagger$, with $\cos\frac{\theta_\lambda^k}{2} = (\lambda - \cos\frac{2\pi k}{N})/\Lambda_\lambda^k$, $\Lambda_\lambda^k = \sqrt{(\lambda-\cos\frac{2\pi k}{N})^2 + \gamma^2\sin^2\frac{2\pi k}{N}}$, H_λ is diagonalized as

$$H_\lambda = \sum_{k=-M}^{M}\Lambda_\lambda^k\left(N_{k,\lambda} - \frac{1}{2}\right), \tag{2}$$

where $N_{k,\lambda} \equiv \Gamma_{k,\lambda}^\dagger\Gamma_{k,\lambda}$. The ground state of H_λ is

$$|g_\lambda\rangle = \prod_{k=1}^{M}\left(\cos\frac{\theta_\lambda^k}{2}|0\rangle_k|0\rangle_{-k} + i\sin\frac{\theta_\lambda^k}{2}|1\rangle_k|1\rangle_{-k}\right), \tag{3}$$

where $|0\rangle_k$ and $|1\rangle_k$ denote the vacuum and single excitation of the kth mode corresponding to c_k. The ground-state energy is $E_\lambda = -\sum_{k=-M}^{M}\Lambda_\lambda^k/2$.

Now we consider rotating each spin for a time-dependent angle ϕ around z [1]. The Hamiltonian becomes a time-dependent one,

$$\mathcal{H}_\lambda[\phi(t)] = U[\phi(t)]H_\lambda U^\dagger[\phi(t)], \tag{4}$$

where $U(\phi) \equiv \prod_{j=-M}^{M} e^{-i\phi\sigma_j^z/2}$. The ground state of $\mathcal{H}_\lambda[\phi(t)]$ is

$$|g_{\lambda,\phi}\rangle = U(\phi)|g_\lambda\rangle. \quad (5)$$

If ϕ is adiabatically varied from 0 to π, there accumulates a Berry phase,

$$\beta_B = -i\int_0^\pi \langle g_{\lambda,\phi}|\partial_\phi|g_{\lambda,\phi}\rangle d\phi = -\pi\sum_{k=1}^{M}\left(1-\cos\theta_\lambda^k\right), \quad (6)$$

which can also be written as $-\pi M + \pi M_z$, where $M_z = \sum_{k=1}^{M}\cos\theta_\lambda^k$ is the magnetization in the original ground state $|g_\lambda\rangle$.

Here we consider $\phi = \Omega t$. The Schrödinger equation is

$$i\partial_t|\psi(t)\rangle = \mathcal{H}_\lambda[\phi(t)]|\psi(t)\rangle. \quad (7)$$

It can be obtained that

$$|\psi(t)\rangle = U[\phi(t)]e^{-iH_{\lambda+\Omega}t}|\psi(0)\rangle, \quad (8)$$

where $H_{\lambda+\Omega}$ is a *time-independent effective* Hamiltonian, with field strength $\lambda+\Omega$.

In general, suppose the initial state is the ground state of H_χ with field strength χ, i.e., $|\psi(0)\rangle = |g_\chi\rangle$. By using the idempotency $N_{k,\lambda}^2 = N_{k,\lambda}$, we obtain the identity

$$e^{-it\Lambda_{\lambda+\Omega}^k\Gamma_{k,\lambda+\Omega}^\dagger\Gamma_{k,\lambda+\Omega}} = 1 + (e^{-it\Lambda_{\lambda+\Omega}^k}-1)\Gamma_{k,\lambda+\Omega}^\dagger\Gamma_{k,\lambda+\Omega},$$

with which we obtain

$$e^{-iH_{\lambda+\Omega}t}|g_\chi\rangle = e^{-iE_{\lambda+\Omega}t}\prod_{k=1}^{M}[A_k(t)|0\rangle_k|0\rangle_{-k} + iB_k(t)|1\rangle_k|1\rangle_{-k}], \quad (9)$$

where $A_k(t) \equiv \cos\frac{\theta_\chi^k}{2} + (e^{-2it\Lambda_{\lambda+\Omega}^k}-1)\sin\frac{\theta_{\lambda+\Omega}^k}{2}\sin\frac{\theta_{\lambda+\Omega}^k-\theta_\chi^k}{2}$, $B_k(t) \equiv \sin\frac{\theta_\chi^k}{2} + (e^{-2it\Lambda_{\lambda+\Omega}^k}-1)\cos\frac{\theta_{\lambda+\Omega}^k}{2}\sin\frac{\theta_\chi^k-\theta_{\lambda+\Omega}^k}{2}$.

Therefore, under $H_\lambda[\phi(t)]$, the evolution of the state starting from $|g_\chi\rangle$ is

$$|\psi(t)\rangle = e^{-it(E_{\lambda+\Omega}+\frac{N\Omega}{2})}\prod_{k=1}^{M}[A_k(t)|0\rangle_k|0\rangle_{-k} + ie^{2i\Omega t}B_k(t)|1\rangle_k|1\rangle_{-k}]. \quad (10)$$

III. AA PHASE FOR A CYCLE

Now suppose the initial state is specifically prepared to be an eigenstate of $H_{\lambda+\Omega}$, that is,

$$|\psi(0)\rangle = |g_{\lambda+\Omega}\rangle. \quad (11)$$

Note that the Hamiltonian is $\mathcal{H}_\lambda[\phi(t)]$. According to (8), $|\psi(t)\rangle$ must be cyclic in the ray space with the period $T_0 = \pi/\Omega$.

Using the definition of the AA phase, $\beta_{AA} = \arg[\langle\psi(0)|\psi(T_0)\rangle] + i\int_0^{T_0} dt\langle\psi(t)|\dot\psi(t)\rangle$ [11], the AA phase after the Hamiltonian undergoes a cycle can be found to be

$$\beta_{AA}(\lambda) = -\pi\sum_{k=1}^{M}\left(1-\cos\theta_{\lambda+\Omega}^k\right), \quad (12)$$

which is the same as the Berry phase. $\beta_B(\lambda+\Omega)$ denotes the Berry phase of the Hamiltonian $H_{\lambda+\Omega}$ with field strength $\lambda+\Omega$, according to (6).

Therefore, because of this relation between the Berry phase and the AA phase [8], one can obtain the Berry phase of $H_{\lambda+\Omega}$ by measuring the AA phase of H_λ, which does not require adiabaticity but only requires the initial state to be prepared as the ground state of $H_{\lambda+\Omega}$. Therefore, the AA phase can be used for studying QPTs through its relation with the Berry phase, which has been known to be related to QPTs.

IV. SP AND QPTS

The preceding section noted that, governed by $\mathcal{H}_\lambda[\phi(t)]$, the state itself cycles if the initial state is specifically prepared to be the ground state of $H_{\lambda+\Omega}$. Now we consider a different initial state, which is supposed to be the instantaneous ground state of the time-dependent Hamiltonian itself. At $t=0$, it is merely the ground state $|g_\lambda\rangle$ of the original Hamiltonian H_λ. Then the state is not cyclic in general. It is cyclic only if the cycle of the Hamiltonian is adiabatic.

In this situation, we address the following question instead. With $|\psi(0)\rangle = |g_\lambda\rangle$, how close is the final state $|\psi(\pi/\Omega)\rangle$ to the initial state $|g_\lambda\rangle$?

This can be quantified as the SP, which is the fidelity between the final and initial states,

$$S \equiv \left|\left\langle g_\lambda|\psi\left(\frac{\pi}{\Omega}\right)\right\rangle\right|^2 = \prod_{k=1}^{M}\left\{1-\left[\frac{\Omega}{\Lambda_\lambda^k}\sin\theta_\lambda^k\sin\left(\frac{\pi}{\Omega}\Lambda_{\lambda+\Omega}^k\right)\right]^2\right\}, \quad (13)$$

according to which we have numerically calculated S, as shown in Figs. 1–4. The three-dimensional (3D) plot in Fig. 1 shows the dependence of SP on λ and γ, while the 2D plots in Fig. 2 give the dependence of SP on γ for various values of λ, according to (13).

To better understand these results, we give some approximated formulas for some limiting cases. First, in the thermodynamic limit $N\to\infty$, the summation over the momentum

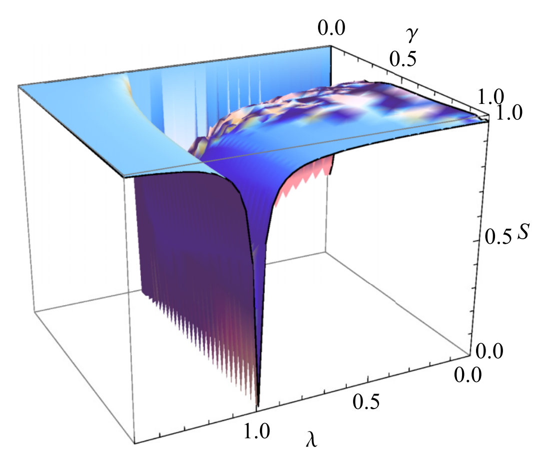

FIG. 1. (Color online) SP as a function of the parameters λ and γ, for $\Omega = 0.01$ and $N = 2001$.

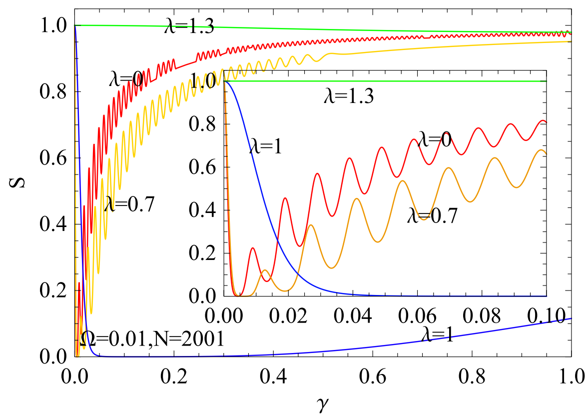

FIG. 2. (Color online) SP as a function of γ for various values of λ. $\Omega = 0.01$, $N = 2001$.

modes in (13) can be replaced by an integral, therefore

$$S = \exp\left[\frac{N}{2\pi}\int_0^{\pi}\ln\left\{1-\left[\frac{\Omega\gamma\sin k}{\Lambda_\lambda^k\Lambda_{\lambda+\Omega}^k}\sin\left(\frac{\pi}{\Omega}\Lambda_{\lambda+\Omega}^k\right)\right]^2\right\}dk\right]. \tag{14}$$

Moreover, we consider the adiabatic condition that Ω is much smaller than the energy gap $\min_k(\Lambda_k)$. Then,

$$S \approx \exp\left[-\frac{N\Omega^2\gamma^2}{2\pi}\int_0^{\pi}\frac{[\sin k\sin(\frac{\pi}{\Omega}\Lambda_\lambda^k)]^2}{(\Lambda_\lambda^k)^4}dk\right]. \tag{15}$$

First we investigate the regime near $\gamma_c = 0$ (XX model). As observed in Figs. 1 and 2, for $\lambda < \lambda_c = 1$, when γ increases from $\gamma_c = 0$, S first drops quickly from 1 to 0 and then increases. However, when λ increases toward $\lambda_c = 1$, the valley of drop and increase becomes less steep. On the other hand, for $\lambda > \lambda_c = 1$, SP decreases much more slowly with the increase of γ. These features suggest that SP can expose the existence of QPT at $\lambda_c = 1$.

For $\lambda = 0$, which is far from $\lambda_c = 1$, SP can be written as

$$S(\lambda = 0,\gamma) \approx \exp\left[\frac{N\pi\gamma^2}{\gamma^2-1}\int_{\frac{\pi\gamma}{\Omega}}^{\frac{\pi}{\Omega}} f(x)dx\right], \tag{16}$$

where $f(x) \equiv \sqrt{\frac{x^2-(\frac{\pi}{\Omega})^2}{(\frac{\pi\gamma}{\Omega})^2-x^2}\frac{\sin^2 x}{x^3}}$. $S(\lambda = 0,\gamma)$ as a function of γ is displayed in Fig. 3. Obviously at $\gamma = 0$, $S(\lambda = \gamma = 0) \approx 1$, as the Hamiltonian with $\gamma = 0$ is time-dependent. Away from $\gamma = 0$, $S(\lambda = 0,\gamma)$ drops sharply, as can be noted from the rapid decay of the above integration with the increase γ, and it can be clearly seen in the inset of Fig. 3 based on numerical calculation of the exact formula (13).

Now consider the case of $\gamma = 1$ (Ising model). We obtain

$$S(\lambda,\gamma = 1) \approx \exp\left[-\frac{A(\lambda,\Omega)N\Omega}{2\lambda}\int_{\frac{\pi|1-\lambda|}{\Omega}}^{\frac{\pi|1+\lambda|}{\Omega}}\frac{\sin^2 x}{x^2}dx\right], \tag{17}$$

where $0 < A(\lambda,\Omega) < 1$ is some function of λ and Ω. Because $\frac{\sin^2 x}{x^2}$ is a damped vibration function, the sensitivity of the integral to the lower limit suggests that $\lambda_c = 1$ is a singular point. This is consistent with the plot of $S(\lambda,\gamma = 1)$ as a function of λ according to the exact formula (13), as shown in Fig. 4, which indicates criticality at $\lambda = 1$.

As depicted in Figs. 3 and 4, the minimal points of SP vary with Ω, and they approach the critical points of H_λ when $\Omega \to 0$. For a given N, the valley of SP becomes steeper with the decrease of Ω, in agreement with the fact that the minimal points at $\Omega = 0$ correspond to the critical points of H_λ. Moreover, for a given Ω, a lattice that is too large diminishes the efficacy of SP in detecting the QPTs, as the adiabatic condition becomes stronger.

To conclude this section, even for a finite value of Ω, one can still observe that $\lambda = 1$ and $\gamma = 0$ are critical values of QPTs of the original Hamiltonian H_λ.

V. TIME-DEPENDENT SURVIVAL PROBABILITY (TDSP)

Finally we extend the use of SP to the state $|\psi(t)\rangle$ at an arbitrary time t evolving from the initial state $|g_\lambda\rangle$. Based on the exact solution $|\psi(t)\rangle$, we can obtain the TDSP at time t,

$$S_t \equiv |\langle g_\lambda|\psi(t)\rangle|^2 = \prod_{k=1}^{M} f^k(t), \tag{18}$$

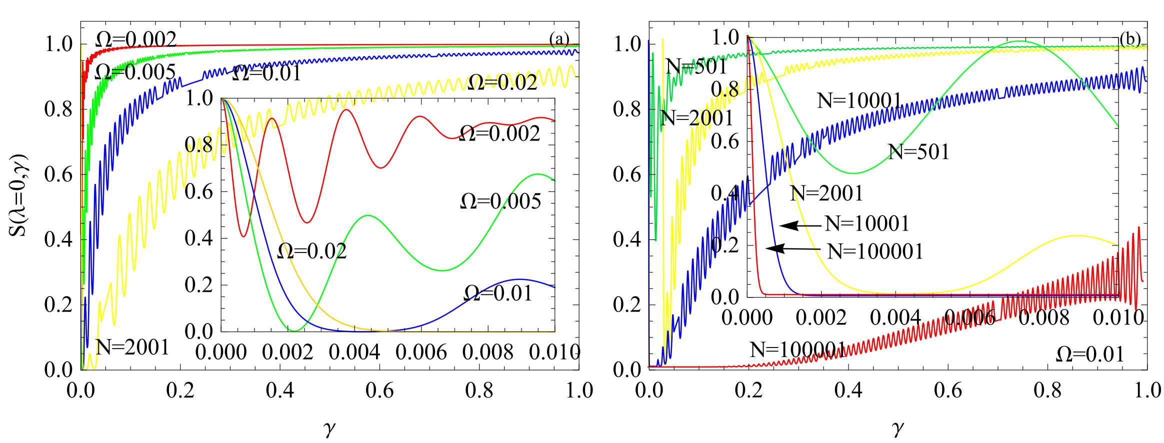

FIG. 3. (Color online) SP for $\lambda = 0$. (a) γ dependence of the SP for $N = 2001$ and $\Omega = 0.002, 0.005, 0.01, 0.02$. (b) γ dependence of the SP for $\Omega = 0.01$ and $N = 100001, 10001, 2001, 501$. The insets enlarge the regimes close to the quantum critical points.

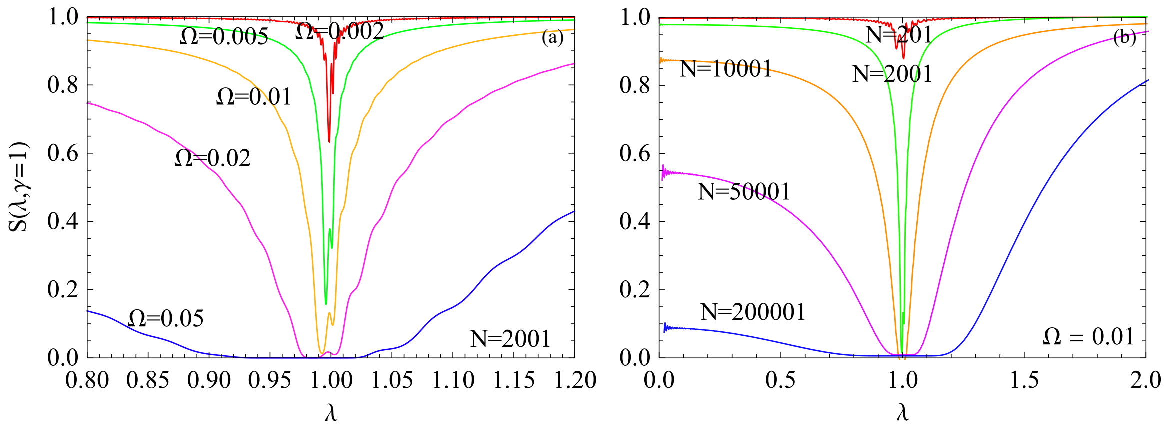

FIG. 4. (Color online) Ising criticality (with $\gamma = 1$). (a) λ dependence of SP for $N = 2001$ and $\Omega = 0.002, 0.005, 0.01, 0.02, 0.05$. (b) λ dependence of SP for $\Omega = 0.01$ and $N = 200001, 50001, 10001, 2001, 201$.

where $f^k(t) = \cos^2(t\Omega)[1 - \sin^2(\theta_\lambda^k - \theta_{\lambda+\Omega}^k)\sin^2(t\Lambda_{\lambda+\Omega}^k)] + \sin^2(t\Omega)[\cos^2(\theta_\lambda^k)\cos^2(t\Lambda_{\lambda+\Omega}^k) + \cos^2(\theta_{\lambda+\Omega}^k)\sin^2(t\Lambda_{\lambda+\Omega}^k)] + \frac{1}{2}\sin(2t\Omega)\sin(\theta_\lambda^k)\sin(\theta_\lambda^k - \theta_{\lambda+\Omega}^k)\sin(2t\Lambda_{\lambda+\Omega}^k)$.

Now we look at several limiting cases. First of all, if $\gamma = 0$, $\mathcal{H}_\lambda[\phi(t)] = H_\lambda$, hence $S(t) = 1$, as can be confirmed from $A_k = \cos\frac{\theta_\lambda^k}{2}$ and $B_k = 0$.

For a short time $t << 1/\Omega$, we have $S(t) \approx \prod_{k=1}^{M}[1 - \sin^2(\theta_\lambda^k - \theta_{\lambda+\Omega}^k)\sin^2(t\Lambda_{\lambda+\Omega}^k)]$. This is the same as the Loschmidt echo between two states studied previously [3], whose various consequences as a witness of QPTs can thus be applied to the short-time behavior of $S(t)$ here.

Now we examine the long-time behavior, which exhibits clear revival behavior, referring to the deviation from the average value of an observable [13]. If $|\gamma| \ll 1$ while $|\lambda + \Omega| > 1$, then $\theta_{\lambda+\Omega}^k \approx 0$ for all k, hence $S(t) \approx \prod_{k=1}^{M}\{1 - \sin^2(\theta_\lambda^k)\sin^2[(\Omega - \Lambda_{\lambda+\Omega}^k)t]\}$. A further condition $|\gamma| \ll |\lambda|$ results in $\sin\theta_\lambda^k \approx 0$ unless $k \neq k_0 \equiv (N/2\pi)\arccos\lambda$. Consequently, $S(t) \approx 1 - \sin^2(\theta_\lambda^{k_0})\sin^2[t(\Omega - \Lambda_{\lambda+\Omega}^{k_0})]$, whose periodicity gives the revival time $T_{\rm rev} \approx \pi/|\Omega - \Lambda_{\lambda+\Omega}^{k_0}|$, as shown in Fig. 5(a). The small oscillation in the enlarged inset of Fig. 5(a) can be suppressed by increasing Ω. The inset in Fig. 5(a) shows that $\sin^2(\theta_\lambda^{k_0})$ has the maximum at $k_0 = 41$, hence the above formula gives $T_{\rm rev} \approx 211.4$, which is very close to the numerical value $T_{\rm rev} = 210.1$.

Similarly, if $|\gamma| << 1$ while $|\lambda| > 1$, then $\theta_\lambda^k \approx 0$ for all k, consequently $S(t) \approx \prod_{k=1}^{M}[1 - \sin^2(\theta_{\lambda+\Omega}^k)\sin^2(t\Lambda_{\lambda+\Omega}^k)]$. A further condition $|\gamma| \ll |\lambda|$ leads to the revival time $T_{\rm rev} \approx \pi/|\Lambda_{\lambda+\Omega}^{k_0}|$, where $k_0 = (N/2\pi)\arccos(\lambda + \Omega)$.

On the other hand, if $\Omega << 1$, then $\theta_\lambda^k \approx \theta_{\lambda+\Omega}^k$, and we have $S(t) \approx \prod_{k=1}^{M}[1 - \sin^2(\theta_\lambda^k)\sin^2(t\Omega)]$, which exhibits revival behavior with frequency Ω, as shown for $\Omega = 0.02$ in Fig. 5(b). If $\Omega \gg 1$, then $\theta_{\lambda+\Omega}^k \approx 0$, and we have $S(t) \approx \prod_{k=1}^{M}\{1 - \sin^2(\theta_\lambda^k)\sin^2[t(\Omega - \Lambda_{\lambda+\Omega}^k)]\}$. As $\Omega - \Lambda_{\lambda+\Omega}^k \approx \cos(2\pi k/N) - \lambda$, $S(t)$ is independent of Ω. The revival time of $S(t)$ can be given by $N/2v_{\max}$, with $v_{\max} \equiv |\partial\Lambda_{\lambda+\Omega}^k/\partial(2\pi k/N)|_{\max} = 1$ [13], hence $T_{\rm rev} \approx N/2$, as confirmed for $\Omega = 100$ in Fig. 5(b).

Finally, we also study the time evolution and revival of the magnetization per spin [13], $m_z(t) \equiv \langle\psi(t)|\sigma^z|\psi(t)\rangle = 1 - \frac{1}{N}\sum_{k=1}^{M}[2 - \cos(\theta_\lambda^k) - \cos(\theta_\lambda^k - 2\theta_{\lambda+\Omega}^k) + 2\sin(\theta_{\lambda+\Omega}^k)\sin(\theta_\lambda^k - \theta_{\lambda+\Omega}^k)\cos(2t\Lambda_{\lambda+\Omega}^k)]$, which shows revival behavior with $T_{\rm rev} \approx N/2$, which is different from $T_{\rm rev}$ of $S(t)$, as shown in the inset in Fig. 5(b). If Ω is too large or too small, $m_z(t)$ approaches the constant $\sum_{k=1}^{M}\frac{\cos\theta_\lambda^k}{M}$.

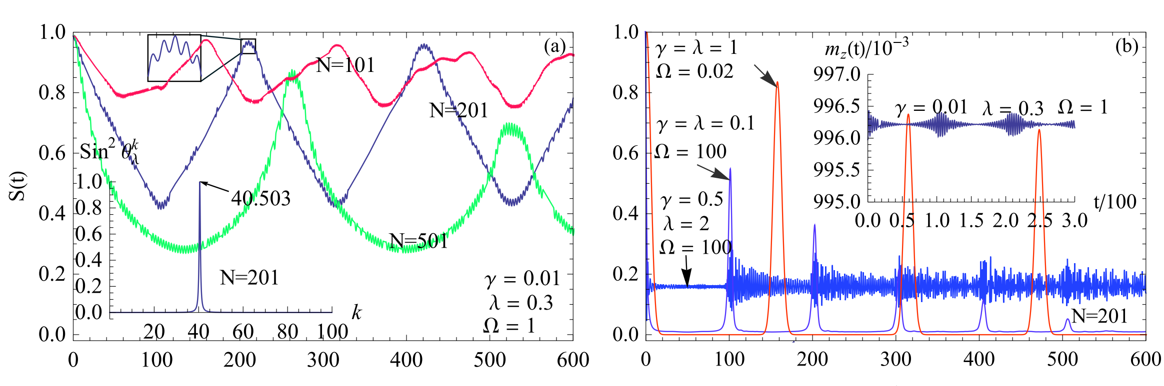

FIG. 5. (Color online) Three kinds of revival phenomena of the SP with different revival time. (a) $T_{\rm rev} \approx \pi/|\Omega - \Lambda_{\lambda+\Omega}^{k_0}|$. (b) $T_{\rm rev} \approx \pi/\Omega$ for $\Omega = 0.02$, which is close to adiabaticity. $T_{\rm rev} \approx N/2$ for $\Omega = 100$. The inset in (b) shows the revival of the average magnetization $m_z(t)$ with $T_{\rm rev} \approx N/2$ for $N = 201$. The revival of $S(t)$ under the same parameter values as for $m_z(t)$ is shown in (a).

All the results in this section indicate that in a nonadiabatic cycle of the Hamiltonian, the revival time of $S(t)$ or $m_z(t)$ is different from the periodicity of Hamiltonian π/Ω when Ω is large enough.

VI. SUMMARY

To summarize, we have considered nonadiabatic rotation of the spins of an XY spin chain to expose QPTs of the original time-independent Hamiltonian. The Hamiltonian is a time-dependent one, $\mathcal{H}_\lambda[\phi(t)]$, with $\mathcal{H}_\lambda(t=0)=H_\lambda$. Starting with the initial state prepared as the ground state of $H_{\lambda+\Omega}$, the state can cycle with an AA phase $\beta_{AA}(\lambda)$, which equals the Berry phase $\beta_B(\lambda+\Omega)$ for $H_{\lambda+\Omega}$, which is already known to contain information on QPTs. On the other hand, starting as the ground state $|g_\lambda\rangle$ of H_λ, which cycles in the parameter space, the quantum state does not return to the ground state $|g_\lambda\rangle$ because of nonadiabaticity. We show, however, that SP can indicate the QPTs if the rotation frequency Ω is small enough. Note that even under the adiabatic condition, there is a nonadiabaticity effect, hence $S(t)<1$. We have also discussed the general TDSP, which exhibits interesting survival phenomena, with revival time different from the periodicity of the Hamiltonian if Ω is large enough.

Experimentally, measuring the SP or AA phase does not require adiabaticity or cancellation of the dynamical phase, as in the case of measuring the Berry phase [8]. We hope our investigations will bring some new theoretical and experimental perspectives on geometric phases in many-body physics.

ACKNOWLEDGMENTS

We thank Wen-ge Wang for discussions. This work was supported by the National Science Foundation of China (Grant No. 11374060), and by the Special Fund of Theoretical Physics (Grant No. 11447103).

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