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Propagation of economic shocks in input-output networks: A cross-country analysis

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This paper investigates how economic shocks propagate and amplify through the input-output network connecting industrial sectors in developed economies. We study alternative models of diffusion on networks and we calibrate them using input-output data on real-world inter-sectoral dependencies for several European countries before the Great Depression. We show that the impact of economic shocks strongly depends on the nature of the shock and country size. Shocks that impact on final demand without changing production and the technological relationships between sectors have on average a large but very homogeneous impact on the economy. Conversely, when shocks change also the magnitudes of input-output across-sector interdependencies (and possibly sector production), the economy is subject to predominantly large but more heterogeneous avalanche sizes. In this case, we also find that (i) the more a sector is globally central in the country network, the larger its impact; (ii) the largest European countries, such as those constituting the core of the European Union's economy, typically experience the largest avalanches, signaling their intrinsic higher vulnerability to economic shocks.

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I. INTRODUCTION

Studying the mechanisms through which shocks diffuse in economic systems is today a fundamental issue in both theoretical and applied economics, for both its positive and normative implications. A better knowledge of such mechanisms can indeed serve as a basis to devise predictive tools and policy measures that can help regulators to dampen aggregate fluctuations and reduce the likelihood of systemic crises [1,2]. In fact, as the recent financial and economic crisis has clearly shown, shocks can quickly percolate among countries and through their industrial sectors, turning country-specific shocks originated in the financial sectors into worldwide recessions hitting the real side of the economy as well [3].

Although a large number of contributions have analyzed the mechanisms of contagion in banking and financial networks [4,5], much less is known about how the network structure of interdependencies between the sectors of an economy shapes the process of diffusion of exogenous shocks. From an empirical perspective, a handful of studies have characterized the structure of input-output (IO) networks to better understand the topology of intersectoral dependencies and their repercussions at the macroeconomic level [6–8]. Moreover, from a theoretical perspective, a few studies have highlighted that the topology of IO linkages [9] between sectors can amplify small productivity shocks. For example, Acemoglu *et al.* [10] compute the centrality of sectors in order to capture intersectoral linkages in a multisectoral macroeconomic model, and measure the impact of sectoral idiosyncratic shocks on aggregate volatility using USA data. Acemoglu *et al.* [11] further show how those idiosyncratic shocks can propagate over the input-output linkages across sectors creating large economic downturns. However, the propagation mechanism used in these studies remains limited by the macroeconomic models employed and does not exploit the advantages of network analysis. Therefore, shock propagation and the emergence of avalanches in IO network structures that mimic the real-world structure of industrial interlinkages is still poorly understood [12].

This work begins to fill this gap by blending together economically meaningful shock-diffusion models and Eurostat data on IO tables for European Union (EU) countries for the year 2005. We are interested in analyzing how the economies react to different types of shocks and how avalanches emerge according to the IO structure of an economy [13]. We use an IO table to build a weighted-directed IO network for each country in isolation and compare results across countries. Each IO network describes the structure of dependencies between sectors in a national economy. We employ the observed IO networks to calibrate and simulate three network-diffusion models (see Sec. II).

In the last years, shock propagation in economic networks has been mostly explored using models borrowed from the literature studying propagation of infectious diseases. Examples include applications to shock diffusion in financial or trade networks using susceptible-infected-susceptible (SIS) [14] or susceptible-infected-recovered (SIR) diffusion models [14,15]. We depart from such literature to study simple but economically meaningful diffusion models that differ as to their assumptions about where a shock comes from and how it is locally diffused in the economy [16,17]. In particular, we explore three basic diffusion models, capturing two main dimensions: (i) the nature of the shock and its impact on IO linkages; and (ii) the possibility that after a shock hits a sector, also its production levels are accordingly adjusted. By doing so, we try to disentangle the roles played by the nature of the shock, the structure of sectoral interdependencies, and the dynamics of production during the propagation. We explore, for each diffusion model and each country, the likelihood that shocks generate avalanches (e.g., cascades) in

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the system and the final distribution of their extent (avalanche size distribution) [18]. Furthermore, we ask whether these outcomes depend on the type of shock (e.g., impacting final demand vs affecting technological interlinkages), the size of the economy, the sector where the shock has originated, and the topological properties of the underlying IO networks.

Our main result is that precrisis impact of economic shocks on European countries strongly depends on the nature of the shock and country size. Shocks that impact on final demand without changing, during diffusion, production and the technological relationships between sectors have on average a larger but very homogeneous impact on the economy. Instead, shocks that can change input-output interdependencies (and possibly sector production) as they percolate through the economy engender predominantly large but more heterogeneous avalanche sizes. Typically, the more a sector is globally central in a country IO network, the larger its impact on the economy when it is hit by a shock. We also find that countries constituting the core of the European Union economy typically experience the largest avalanches. This signals their intrinsic higher vulnerability to economic shocks.

The rest of the paper is organized as follows. Section II briefly describes the models and the data employed to calibrate them [19]. Results are reported in Sec. III. Finally, Sec. IV concludes.

II. DATA AND METHODS

Consider a closed economy composed of S industrial sectors linked via a set of input-output relations described as a weighted directed graph with self-loops. A node in the graph is a sector and a weighted directed edge represents an economic transaction conducted between sectors to buy or sell inputs used in the production process [7,20,21]. The weighted adjacency matrix Z has entries $z_{ij} > 0$, $i \neq j$, proportional to the value of the intersectoral flows from sector i to sector j, i.e., the output of sector i to be used as input in sector j's production process. If a flow between i and j is zero, then the two sectors are not connected. Strictly positive self-loops $z_{ii} > 0$ capture the idea of a sector using its own product as input. Since the network is directed, in general we have $z_{ij} \neq z_{ji}$.

A. Diffusion models

We use IO networks as the backbone over which shocks are propagated from a sector to a neighboring one along production chains. We are interested in understanding how shocks initially originating in a certain area of the technological space of a given country can possibly propagate across the entire structure of the economic system, i.e., how local shocks can have global effects [10]. We employ three different shock-diffusion models, which we explain in the next sections. We focus on *progressive* diffusion processes [22], where once a sector has been hit by a shock it cannot be hit again [23].

Model 1

The first diffusion model we use is a standard input-output model where shocks hit the final demand of a sector [9,24]. In the model, sectoral output (or production) linearly depends on the input requirements from all sectors in the economy and the final demand from households, government, exports, and capital investment:

$$\mathbf{x} = \mathbf{Z1} + \mathbf{d}, \tag{1}$$

where $\mathbf{x}$ is the $S \times 1$ output vector, $\mathbf{Z}$ is the intersectoral input-output matrix defined above, $\mathbf{1}$ is a column vector of 1s, and $\mathbf{d}$ is the $S \times 1$ column vector of final demand [25]. Simple algebra [19] allows one to get sectoral production as a function of final demand and the matrix of technical coefficients $\Theta = \{\theta_{ij}\} = \{z_{ij}/x_j\}$:

$$\mathbf{x} = (\mathbf{I} - \Theta)^{-\mathbf{1}}\mathbf{d} = \mathbf{Ld}, \tag{2}$$

where $\mathbf{L} = (\mathbf{I} - \Theta)^{-1} = [l_{ij}]$ is an $S \times S$ matrix known as the Leontief inverse or the total requirements matrix.

The effect on output of sectoral shocks on final demand $\mathbf{d}$ can be easily modeled. Suppose that final demand of sector s is hit by a shock that results in new levels equal to $d_s + \epsilon_s$. Then the change in the output needed to compensate the change on final demand is defined as follows:

$$\Delta\mathbf{x} = \mathbf{Ld}^* - \mathbf{Ld} = \mathbf{L}\Delta\mathbf{d} = \mathbf{L}\epsilon, \tag{3}$$

where $\epsilon = (\epsilon_1, \ldots, \epsilon_S)$. This implies that the impact of a shock that reduces final demand by fraction $0 < f < 1$ originating from sector s will be equal to $-fd\mathbf{L}^{(\mathbf{s})}$, where $\mathbf{L}^{(\mathbf{s})}$ is the sth column of the Leontief matrix $\mathbf{L}$. Sector j is part of an avalanche triggered by a shock on final demand of sector i if $\Delta x_j < 0$.

Let A^s be the total number of sectors affected by a propagating shock for which $\Delta\mathbf{x} < 0$, i.e., the avalanche size. By repeating this exercise for all sectors s, one can therefore characterize and study the avalanche size distribution $\{A^s, s = 1, \ldots, S\}$. Notice that our definition of avalanche and avalanche size is not affected by the size of the sector-specific shock.

Model 2

The input-output diffusion model described above assumes an exogenous shock on final demand and computes the impact on sectoral production keeping fixed, during the diffusion process, the magnitudes of intersectoral linkages and sectoral production. In the second diffusion model we allow the magnitudes of economic input-output transactions to change during the propagation of the shock. The propagation is a discrete process in which the shock spreads step by step following production chains. As in model 1, in this first simple diffusion model sectoral production only changes at the end of the diffusion process. In model 3 we introduce the possibility that production changes during the diffusion. Then, in the results section we compare the results between the simple and the more realistic mechanism.

Borrowing from Refs. [16,26], suppose each node s has a *capacity* equal to its production x_s. Assume that both final demand and production are fixed during the diffusion and only change at the end of the process. We also assume that when a (negative) exogenous shock hits sector s the supply and demand of inputs decrease by fraction $0 < f < 1$. This shock affects a sector as a whole, therefore, all the firms in the sector modify their supply and demand behaviors and change the sector input-output linkages. This will affect all sectors that are linked to s as buyers or suppliers of inputs (all $j \in N_s$). If the decrease of the supply and demand of inputs of these sectors connected to s decrease above a certain capacity threshold, these sectors will be hit by the shock too. When they are hit by the shock, the magnitude of their intersectoral connections decreases too. This affects the supply and demand of inputs of the sectors connected to all j's (all $k \in N_j$ for all j), and so on. The reaction chain stops when all sectors in the economy have evaluated the decrease in their supply and demand of inputs with respect to their capacity threshold. At the end of the process, production is updated and we evaluate the size of the avalanche by counting the sectors eventually hit after the initial shock to sector s.

More precisely, suppose that, after the negative shock, output supply and input demand by sector s is symmetrically decreased by fraction $0 < f < 1$. Consequently, the new link weights between sector s and its neighbors become $z^*_{sj} = (1-f)z_{sj}$ and $z^*_{is} = (1-f)z_{is}$, where j is any sector that uses output of s as input and i is any sector from which s buys inputs. In the next stage, every sector $h \neq s$, which is a neighbor of s, evaluates the change in its total node strength:

$$\Delta\sigma_h = \sum_k (z^*_{hk} + z^*_{kh}) - \sum_k (z_{hk} + z_{kh}), \tag{4}$$

i.e., the change in the sum of all its incoming and outgoing link weights. If such a change exceeds a given threshold $0 < c < 1$ of its capacity x_h, then the sector is hit by the shock too. It will therefore decrease its incoming and outgoing link weights by the fraction f and propagate the shock farther away.

Using the definitions of z^*_{hk} and z^*_{kh} in Eq. (4), one gets that the condition for a sector being hit by the initial shock becomes

$$\frac{\sigma_h}{x_s} = \frac{\sigma_h^{\text{in}}}{x_s} + \frac{\sigma_h^{\text{out}}}{x_s} > \frac{c}{f} = \alpha, \tag{5}$$

where σ_h^{in} and σ_h^{out} are node in and out strength, i.e., the total value of the inputs bought by sector h and the total value of sector s's output used as input in the production processes by all other sectors respectively.

Equation (5) implies that a shock is transmitted to a neighboring node only when this sector is too connected (relative to its capacity) to input-output relationships. Second, the shock propagation only depends on α and not on c and f separately, as already discussed by Ref. [16]. For larger α, a sector is more likely to absorb the shock. This is because a large α translates into high resilience to shocks. Therefore, we interpret α as a global measure of network resilience. Again, we are interested in the avalanche size distribution $\{A^s, s = 1, \ldots, S\}$, resulting from the diffusion process starting from shocks hitting in any sector.

Model 3

Model 2 is a simple process of the propagation of shocks. However, in reality the firms in each sector may adjust to the new conditions after a shock. A sector that is hit by a shock has fewer inputs to produce and supplies fewer inputs to other sectors, thereby changing sectoral production. Model 3 incorporates this idea by introducing a second step in the diffusion process. Indeed, in the second model above, after a sector s gets hit by a shock, the magnitude of its economic transactions decreases by fraction $0 < f < 1$. This means that, as the diffusion process unfolds, the matrix $\mathbf{Z}$ keeps changing, but this does not have any effect on sectoral production, which remains constant. Comparably, model 3 defines two steps in each diffusion stage. First a sector is hit by a shock which decreases the supply and demand of inputs. Second, production is updated to these new conditions according to Eq. (1). Final demand remains fixed and the condition for the shock to spread to other sectors is the same as in model 2, only now production updates, changing the capacity threshold for each sector.

Formally, assume that at some stage τ of the diffusion process, the system is characterized by the intersectoral weight matrix $\mathbf{Z}(\tau)$ and production vector $\mathbf{x}(\tau)$. At this point, assume that sector h is hit by the shock. This results in the new weight matrix $\mathbf{Z}(\tau+1)$. This matrix differs from $\mathbf{Z}(\tau)$ because its hth row and column has been updated according to the rules $z_{hj}(\tau+1) = (1-f)z_{hj}(\tau)$ and $z_{ih}(\tau+1) = (1-f)z_{ih}(\tau)$. In a second step we use Eq. (2) and define the new production vector as

$$\mathbf{x}(\tau+1) = [1 - \Theta(\tau+1)]^{-1}\mathbf{d} = \mathbf{L}(\tau+\mathbf{1})\mathbf{d}, \tag{6}$$

where $\Theta(\tau+1)$ is the new technological coefficients matrix, whose generic entry reads $z_{ij}(\tau+1)/x_j(\tau)$. This mechanism can be viewed as a self-fulfilling process where feedbacks arise and effects are reinforced. In this self-fulfilling process each update is incorporating previous updates. Updated production levels are then employed to evaluate if a shock hits a sector using Eqs. (4) and (5).

B. Data

We calibrate the foregoing three models for EU countries using IO data tables provided by Eurostat [27]. See the Supplemental Material for more details [19]. Tables give information on the economic transactions that sectors made by buying and supplying inputs in million Euros using a two-digit (division-level) NACE Rev. 1 classification. We employ the year 2005 because this is the latest year where the largest number of countries have a complete input-output table. Using 2005 data has the advantage of providing a picture of the precrisis conditions over which the propagation of shocks from financial to the other sectors unfold. Only four countries (Bulgaria, Cyprus, Latvia, and Malta) have been left out from the analysis due to absence of data. This leaves us with 22 European countries for the analysis [28].

We employ the data on intersectoral IO flows to build $c = 1, \ldots, 22$ an IO weighted-network matrix $\mathbf{Z}^c$ for each country. We use these data together with data on final demand to compute production and the Leontief inverse matrix as in Eq. (1).

The topological properties of country-specific IO networks have been already studied in Refs. [6–8], from both a binary and weighted perspective. In the Supplemental Material, we report a short overview of the statistical features of the IO networks in our database [19]. We highlight in this section that the European IO networks we analyze have a highly asymmetrical structure. This property will be important in determining the propagation of shocks [29–34]. In the next section we show the results where we report the avalanche size distribution of each country and then compare the different distributions across countries according to the three diffusion models.

III. RESULTS

Model 1

The diffusion in model 1 triggers homogeneous and large avalanche sizes in each country independently of the size of the shock on final demand. This can be seen in Fig. 1, where for each country we plot the coefficient of variation (CoV) of the country avalanche size distribution $\{A^s, s = 1, \ldots, S\}$, defined as the ratio between standard deviation and mean, vs the density of the correspondent IO country network. To appreciate the extent of avalanches triggered by final-demand shocks, as well as the dependence on economic size, we draw a ball for each country where the color is proportional to the log of country GDP and the size is proportional to the largest avalanche size, i.e., $\max_s\{A^s\}$. We can see that the CoV are all very small and homogeneous across all countries [35]. These results indicate that all avalanche-size distributions are very concentrated on high values. All countries experienced avalanches that reached the maximum attainable value for avalanche size covering the entire economy (see ball colors in the plot).

The homogeneity in avalanche sizes is a result of the linear and fixed framework defined in the input-output model. Due to these properties, we also notice that most of the avalanches were triggered by similar *primary* sectors in all countries [36], independently on the size of the shock.

The fact that model 1 generated homogeneous and very large avalanche sizes for all countries also implies the absence of any clear relationship between the size of the largest avalanche and country characteristics, such as the density of its IO network, GDP per capita, or their size (see Fig. 2).

To overcome the homogeneity implied by the limitations of model 1 to study the impact of sectoral shocks, we apply network diffusion models.

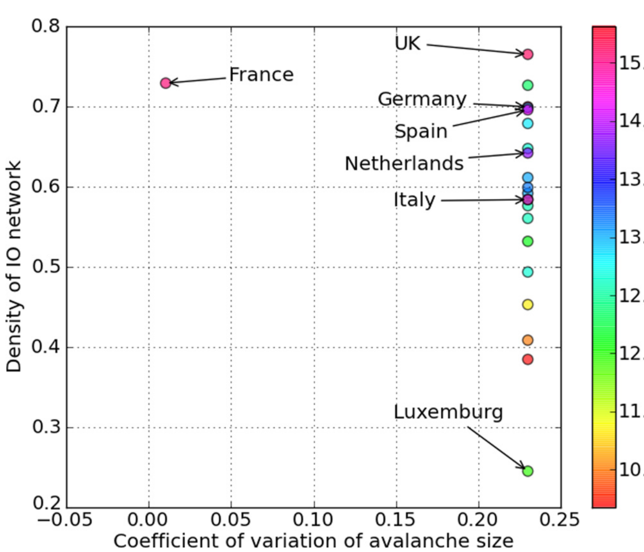

FIG. 1. (Color online) Model 1: Density of the input-output network (y axis), coefficient of variation (ratio of sample standard deviation to sample mean) of avalanche size distribution (x axis), logs of country GDP (ball color, see color map), largest avalanche size (size of balls).

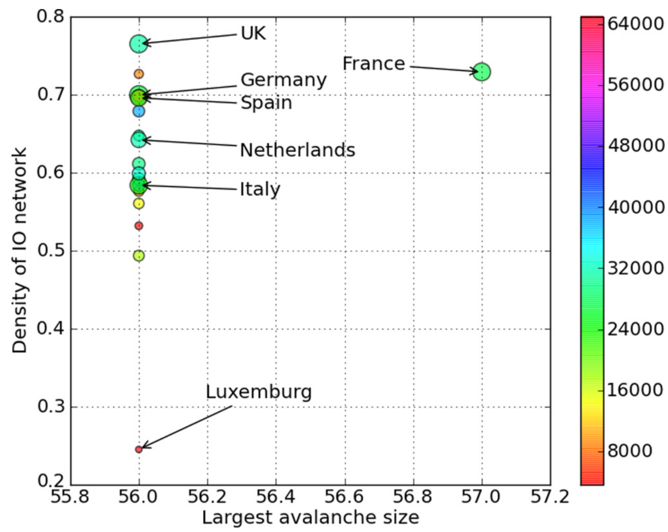

FIG. 2. (Color online) Model 1: Density of the input-output network (y axis), largest avalanche size (x axis), logs of country GDP (size of balls), country GDP per capita (ball color, see color map).

Model 2

We now report results for model 2 where a shock changes the flow of inputs as the propagation unfolds after a sector is hit by a negative shock. We study the diffusion according to this model using two extreme scenarios, i.e., high or small network resilience $\alpha = c/f$. In the first scenario we set $c = 0.4$ and $f = 0.6$, whereas in the second one we set $c = 0.1$ and $f = 0.7$ (similar results hold also for other similar parameter-value combinations of c and f). For values of f/c too small ($f < c$), that is for a shock too small and a capacity threshold too high, European countries experienced no avalanches (avalanches are of size zero).

In contrast to model 1, a shock that changes the flows of inputs directly generates very heterogeneous avalanche size distributions in each country. The size of the avalanches are concentrated on medium values that represent half of the number of sectors in the economy. Moreover, the distributions are different across countries (see Figs. 3 and 4). Note that a lower system resilience induces broader avalanche-size distributions, with more likely high-impact avalanches.

The heterogeneity in avalanche-size distributions introduces interesting correlation patterns with country characteristics. The countries with more interconnected input-output networks are more likely to experience stronger crises. These countries are likely to be big in terms of GDP, although not necessarily the richest ones in terms of GDP per capita. These results are shown in Figs. 5 and 6 for the high-resilience case, where a high connectivity of the IO network translates into larger avalanches. This result gives evidence that a high development of the economic system also comes with a higher vulnerability. Additionally, countries that experience larger avalanche sizes also show smaller CoV of the avalanche size distribution. The largest European countries in terms of their GDP typically experience the largest avalanches (with the exception of Italy), whereas there seems to be only a slightly

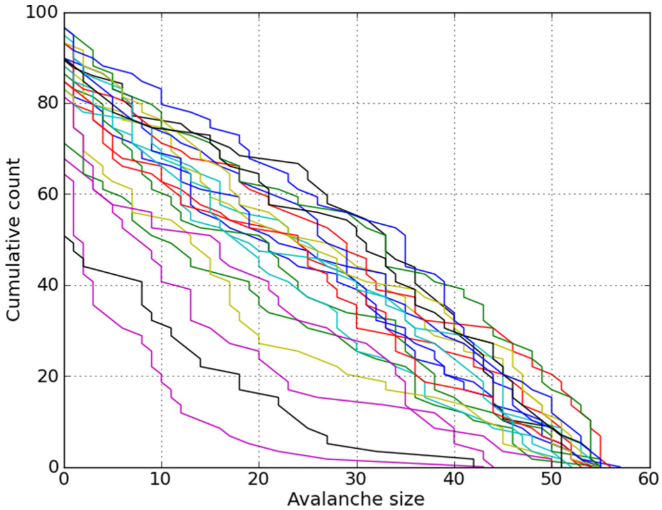

FIG. 3. (Color online) Model 2: Counter cumulative avalanche-size distributions for EU countries. High resilience ($f = 0.6$ and $c = 0.4$).

negative relationship with country GDP. Similar results hold for the low-resilience scenario too.

Therefore, what counts to induce larger avalanches is the development of the IO structure in terms of connectivity and not country income. However, small countries that have a less connected IO structure experience lower but more diverse avalanche sizes.

Heterogeneity in avalanche sizes also allows us to identify the sectors that are more likely to trigger the largest avalanches (see Supplemental Material for more details [19]). Generally, we find that sectors that are more globally central in the IO networks are also those triggering the largest avalanche sizes. To get a better feel for this result, Fig. 7 plots, for the high-resilience scenario, cross-country averages of hubs and authority centrality scores (in log scale) [37] against cross-country averages of largest avalanche sizes. A strong

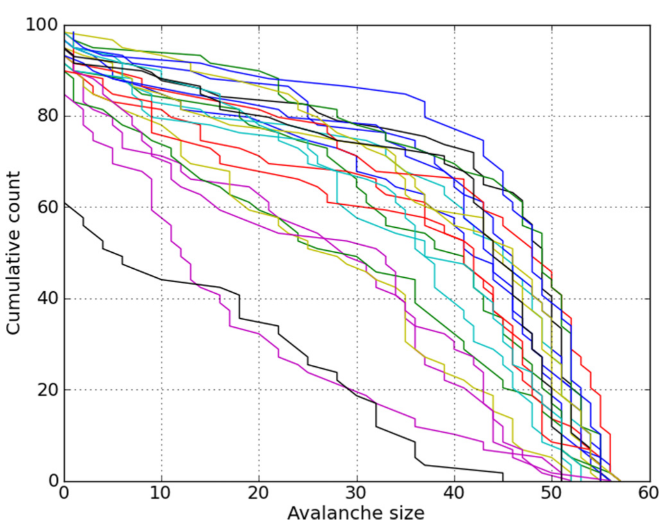

FIG. 4. (Color online) Model 2: Counter cumulative avalanche-size distributions for EU countries. Low resilience ($f = 0.7$ and $c = 0.1$).

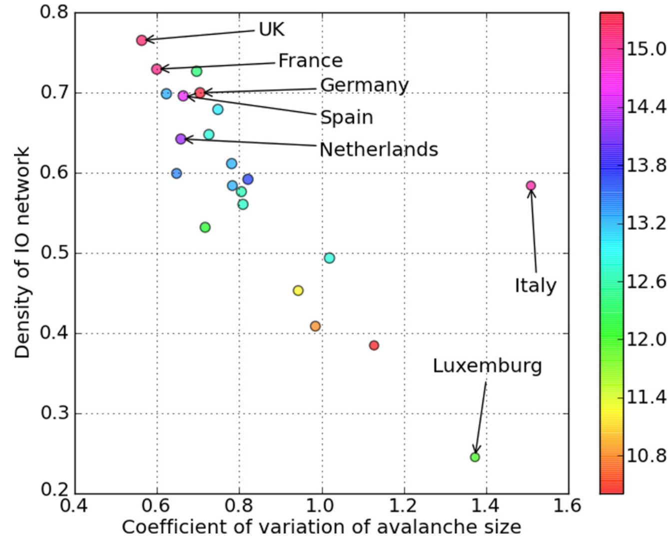

FIG. 5. (Color online) Model 2, high-resilience scenario: Density of the input-output network (y axis), coefficient of variation (ratio of sample standard deviation to sample mean) of avalanche size distribution (x axis), logs of country GDP (ball color, see color map), largest avalanche size (size of balls).

positive correlation emerges. Note that a much weaker positive correlation emerges with *local* sector centrality (as measured by sector in and out strength).

In the high-resilience scenario, the sectors that triggered the largest avalanche sizes in most of the countries were wholesale (19 countries), other business services (19 countries), construction (18 countries), food and beverages (16 countries), and chemicals (14 countries). The "financial-intermediation" and "insurance" sectors triggered the largest avalanche sizes only in Luxembourg, although their impact was relevant throughout. Conversely, sectors that were more likely to trigger unit avalanche sizes were activities in the primary sector,

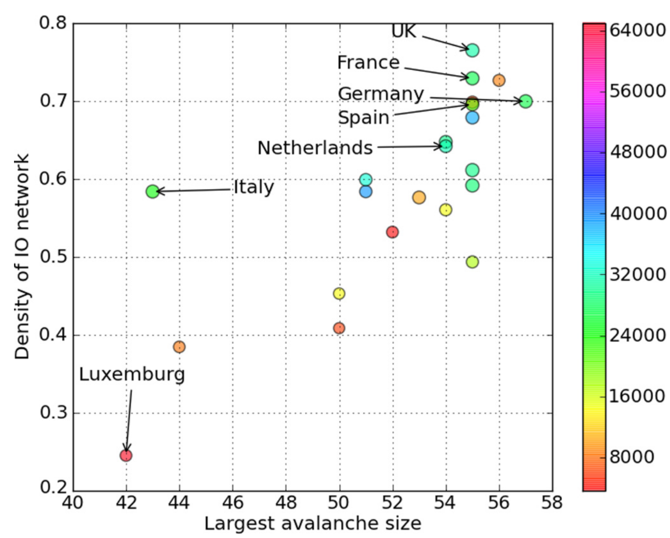

FIG. 6. (Color online) Model 2, high-resilience scenario: Density of the input-output network (y axis), largest avalanche size (x axis), logs of country GDP (size of balls), country GDP per capita (ball color, see color map).

except agriculture, such as forestry, fishing, coal and lignite peat, and metal ores.

Comparably, in the low-resilience case the list of sectors capable of inducing the largest avalanche sizes considerably expands. The most common triggers of the largest avalanches are chemicals (21 countries), wholesale (19 countries), other business services (19 countries), construction (17 countries), electrical energy and gas (15 countries), hotels and restaurants (13 countries), and food and beverages (12 countries). Compared to the previous simulations chemicals is now a trigger of the largest avalanches in seven more countries, electrical energy becomes a common trigger of the largest avalanches, and food and beverages becomes less common than before. In this setup, the countries that experienced the largest avalanches, covering 57 sectors, were France triggered by chemicals, construction, and other business services; Germany, triggered by chemicals, other business services, and public administration; Greece triggered by wholesale, and retail; Hungary, triggered by land transportation and food and beverages; and Spain, triggered by wholesale and chemicals. Other countries that experienced large avalanches of almost the totality of the economy (avalanches of size 56) were Belgium, Denmark, the Netherlands, and Slovenia.

Results indicate that despite the fact that the propagation of a shock used local rules involving the sectors' in and out strength [i.e., their local centrality; see Eq. (5)], the extent of the avalanches mostly depends on the overall embeddedness of a sector in the IO network, which depends also on the centrality of all other sectors involved in an avalanche.

Furthermore, we obtained the smallest variations in the size of the largest avalanches for small and big avalanche sizes, whereas the variability is higher for intermediate values of the avalanche size. Figure 7 shows these results where we color each observation proportionally to the cross-country standard errors associated with the average of largest avalanche sizes (on a blue to red range). This gives evidence of an inverse U-shaped relation between cross-country averages and standard errors of largest avalanche sizes. This is confirmed by the inset of Fig. 7. Similar results hold also for the low-resilience scenario and suggest that whenever a sector is able to induce either a large or a small average largest avalanche size, then it does so rather homogeneously across countries.

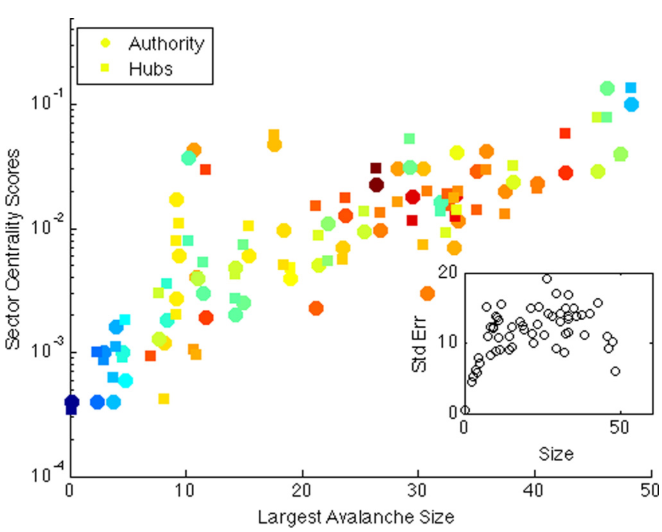

FIG. 7. (Color online) Model 2, high-resilience scenario: Cross-country averages of largest avalanche sizes vs cross-country averages of sector centrality score. Markers colored proportionally to the cross-country standard error of largest avalanche size. Inset: cross-country standard error of largest avalanche size vs average of largest avalanche size.

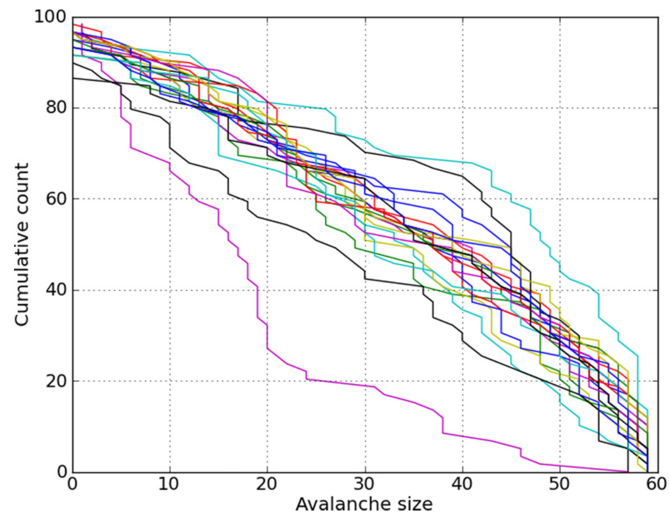

FIG. 8. (Color online) Model 3: Counter cumulative avalanche-size distributions for EU countries. High resilience ($f = 0.6$ and $c = 0.4$).

Model 3

We now assume that, when hit by a shock, a sector adjusts the level of its production according to Eq. (6). Simulations show that this additional adaptation mechanism typically reinforces the strength and scope of the ensuing avalanches, making countries more vulnerable. At the same time, avalanche size distributions become more concentrated around large values, cf. Figs. 8 and 9. Therefore, model 3 induces a cascading process which resembles that of model 1, but with considerably more heterogeneity. The tendency toward more homogeneous and large cascades is due to the fact that after adjusting, production sectors experience lower capacity thresholds. Therefore, the propagation of the shock along production chains becomes easier. In other words, negative shocks together with the capability of production adjustments trigger a reinforcing mechanism wherein economies become weaker and more vulnerable, even if the shock is small. This is due to the coupled effect of linkage and production updating.

Results for the high-resilience case imply that all countries, except Italy, experienced avalanches covering the entire economy, while Italy experienced avalanches of 57 sectors. All countries also experienced an increased number of avalanches of 58, 57, and 56 sectors.

The shift to the right of avalanche-size distributions and their increased homogeneity is more marked in the low-resilience case (Fig. 9). A higher shock and a lower capacity translated into a higher frequency of avalanches covering the entire economy. From the 22 countries in the sample, 11 of them experienced more than 30 sectors

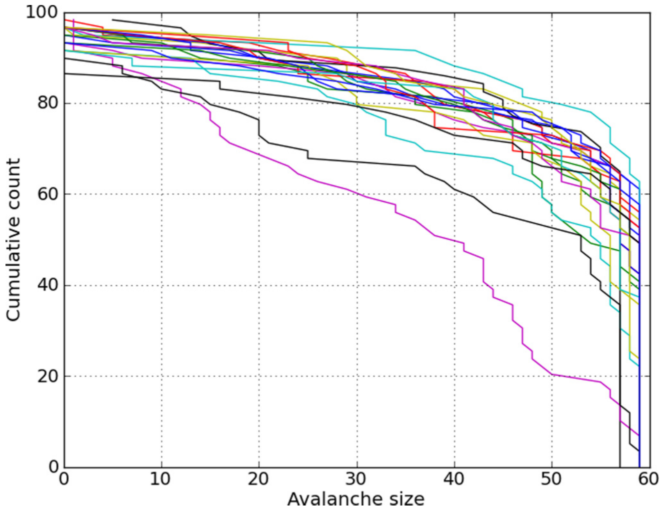

FIG. 9. (Color online) Model 3: Counter cumulative avalanche-size distributions for EU countries. Low resilience ($f = 0.7$ and $c = 0.1$).

triggering avalanches of the entire economy. Among the common triggers of the largest avalanches we now also find the agriculture and the financial sectors. Also the number of avalanches of size larger than 1 increased, thus reducing the frequency of avalanches of size 1.

Due to the fact that avalanche-size distributions are now very concentrated on their largest attainable values, the model with production updating does not feature robust correlation patterns between the largest avalanche size and country characteristics; cf. Figs. 10 and 11. Note how the increase in the number of medium and large avalanches in all countries entails lower coefficients of variation. Only a weak and negative relation is maintained between country size and the coefficient of variation of avalanche size distribution.

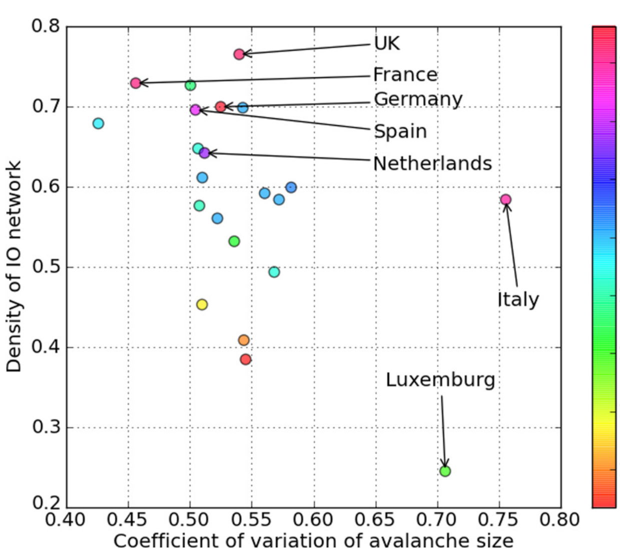

FIG. 10. (Color online) Model 3, high-resilience scenario: Density of the input-output network (y axis), coefficient of variation (ratio of sample standard deviation to sample mean) of avalanche size distribution (x axis), logs of country GDP (ball color, see color map), largest avalanche size (size of balls).

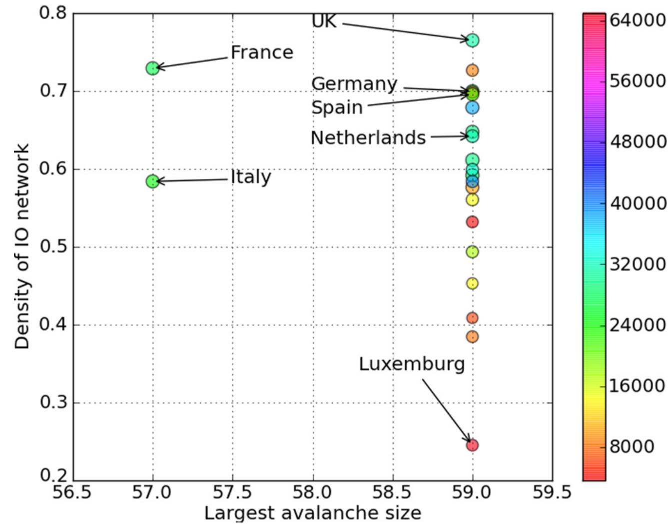

FIG. 11. (Color online) Model 3, high-resilience scenario: Density of the input-output network (y axis), largest avalanche size (x axis), logs of country GDP (size of balls), country GDP per capita (ball color, see color map).

However, the marked shift to the right of avalanche-size distributions induced by production updating in all countries did not affect the way in which different sectors trigger cascades in the economies under study. As discussed in detail in the Supplemental Material, a dominant role in generating the largest avalanches is still played by service and now the financial sectors. Furthermore, a stronger positive relation between sector hub and authority centralities and the largest avalanche size emerges; see Fig. 12. This implies that, even when sectors update their production during the propagation process, their global centrality mostly explains their importance in channeling and amplifying the initial shock.

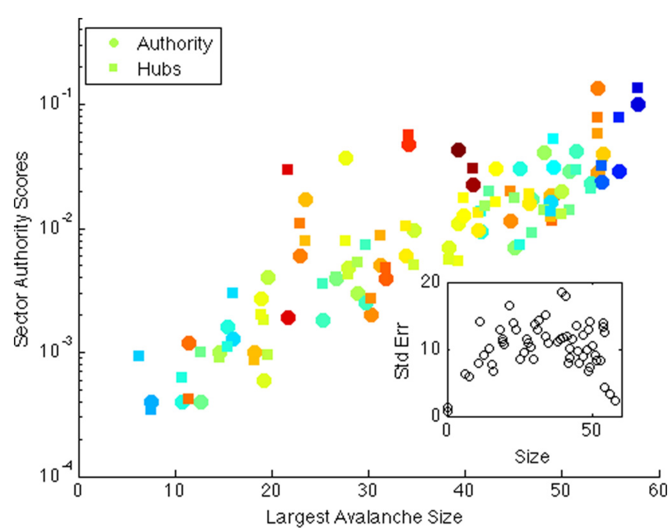

FIG. 12. (Color online) Model 3, high-resilience scenario: Cross-country averages of largest avalanche sizes vs cross-country averages of sector centrality score. Markers colored proportionally to the cross-country standard error of largest avalanche size. Inset: cross-country standard error of largest avalanche size vs average of largest avalanche size.

Finally, as in model 2, the relation between cross-country average and standard errors of largest avalanche sizes still follows an inverse U (see inset of Fig. 12).

IV. DISCUSSION AND CONCLUSIONS

In this paper, we have investigated the propagation mechanisms through which economic shocks are diffused and amplified throughout the input-output structure of national economies.

We studied three economically meaningful diffusion models on networks, properly calibrated using data for several European countries before the Great Depression. The models have been chosen so as to assess the role played by the nature of the shock and its impact on the economy. In particular, our models allow us to evaluate the relative importance of final demand- or production-driven shocks, as well as the relevance of diffusion mechanisms involving, during propagation, the update of input-output technological interlinkages, and/or sectoral production levels.

The framework of the first diffusion model is based on the IO model, and is used as a benchmark due to its foremost importance in economics. This framework models the diffusion of a shock on final demand of the exogenous sector composed by household consumption, government consumption, and capital. The production of the goods and services of the sectors in the economy instantaneously changes according to the input-output linkages. Production changes to perfectly match the decrease in final demand at the end of the propagation process.

The framework of the network diffusion models (1 and 2) is more adaptive. When a shock hits a sector (say i), this sector decreases the supply and demand of inputs to and from other sectors. If this type of shock is large, it is as if the sector had collapsed. An example of such a shock would be a natural disaster that hits a particular sector, like nuclear power plants in Japan after the 2011 Tōhoku earthquake and tsunami. The tsunami shut down the Fukushima nuclear power plant, which decreased the supply of electricity and the demand of other related goods and services. This, in turn, affected the automobile industry, which needed the supply of electricity to run and produce cars. This could in turn decrease the demand of inputs of the automobile industry such as the demand for water, rubber and plastic, steel, and research and development. Therefore, when a sector decreases its supply and demand of inputs after receiving a shock, it implies that the sector i is buying fewer inputs from its suppliers. The sectors that supply to sector i have less sales, thus, excess supply. This excess supply is implicitly assumed to be dissipate or thought of being useless in the period of analysis (i.e., during the propagation). Similarly, sector i is supplying fewer inputs to its clients, so it has excess supply which is assumed to dissipate as well. When the supply of inputs decreases, the demand is not satisfied. This excess demand, when large enough, is one of the reasons why a sector is hit by the propagating shock and collapses as well. With the unsatisfied demand of inputs a sector is not capable of producing and turns vulnerable to be hit by the propagating shock. Then, when the shock is propagated to the input suppliers and buyers of sector i, say sector j in the neighborhood of i that fulfilled the condition to propagate the shock, they too decrease the supply and demand of inputs. The same condition for sector i is true for all sectors that transact with i, and the sectors that transact with these sectors, and so on.

Whereas in model 2 only link weights are adapted, in model 3 also production is adjusted during propagation, so that at each stage of the propagation process the decrease in the supply and demand of inputs changes sectoral production. This adjustment process implicitly implies that sectors connected to the collapsed sector know they will be receiving fewer inputs so adjust their demand accordingly, and produce less with the fewer inputs they were able to buy. Similarly, a reduced production allows a lower supply of inputs to other sectors.

Simulation results show that the asymmetrical structure of the input-output networks of European countries makes their economies vulnerable to large avalanches. On the one hand, the propagation of shock according to model 1 is very large but homogeneous in each country. On the other hand, the avalanches of a propagating shock according to models 2 and 3 are heterogeneous. Nevertheless, European countries experienced larger avalanches when applying model 3. This is because model 3 triggered a reinforcing mechanism where economies become more vulnerable at each stage of the process. Heterogeneity allowed us to identify the triggers of the largest avalanches and relate their high impact to the structural properties of the network. We found that the most globally central sectors in each economy triggered the largest avalanches.

The strong relation between the largest avalanche size and the density of the input-output network of each country and the relation between country size and the size of the largest avalanche have policy implications. A country that is highly connected and that is big with respect to its production also experienced larger avalanches. This implies that countries that are "too big to fail" are also more vulnerable to large economic shocks. Furthermore, our results suggest that the systemic importance of industrial sectors should not be evaluated only by looking at their economic size (e.g., in terms of value added or employees), but also at their position and embeddedness in the complex fabric of input-output relations.

The foregoing analysis can be extended in several directions. First, one might investigate the impact of shocks not only in terms of avalanche size but also in terms of avalanche intensity. Indeed, in the exercises above we have focused our attention on avalanche-size distributions in general, and on the largest avalanches in particular. This has been done because we were interested in assessing the very possibility that a small shock can propagate or not in the entire economy. More generally, one might also want to target the total change in sectoral production induced by the shocks. Second, one can play with alternative models of diffusion on networks, possibly involving some (more sophisticated) microfoundation in terms of firm production behavior, in line with previous work [10,11,38].

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