Repeated-drive adaptive feedback identification of network topologies

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The identification of the topological structures of complex networks from dynamical information is a significant inverse problem. How to infer the information of network topology from short-time dynamical data is a challenging topic. The presence of synchronization among nodes makes the identification of network topology difficult. In this paper we present an efficient method called the repeated-drive adaptive feedback scheme to reveal the network connectivity from short-time dynamics. By applying the short asynchronous transient data as a repeated drive, the adjacency matrix can be successfully determined in terms of the modified adaptive feedback scheme. This improved scheme is valid for both synchronous and asynchronous cases of the network and is especially efficient in the presence of global or local synchronization, where the transient drive can be obtained by perturbing the system to get a very short asynchronous transient. The detection speed of our scheme exhibits the optimized effect by adjusting the time-series segment length and the coupling strength among nodes in the network.

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I. INTRODUCTION

Understanding the relationship between dynamics and the network structure is a central issue in studying network dynamics [1–3]. Network synchronization has been extensively studied for miscellaneous systems, and the role played by the topology can be marvelously separated and appreciated by analyzing the master stability function [4–6]. Such progress has greatly enhanced the significance of identification and detection of these important topological characteristics [7]. Arenas *et al.* studied the relationship between topological scales and dynamic time scales in complex networks [8]. It was found that modular structures corresponding to well-defined communities of nodes emerge in different time scales in a hierarchical way.

An important inverse problem, i.e., how to infer the topological properties of complex networks from dynamical information, still remains a great challenge. For a network with known dynamics but unknown network topology, how can we reveal the structure of this "black box," i.e., the topology of this network? This issue is significant because we may frequently encounter the difficulty of direct detections of a network. It is well known that the output data we get from any node possess the information of links of nodes in a network. However, how to excavate the topological information from this dynamical information seems to be a harder while more significant issue.

Progress has been made recently on the dynamical estimations of the network topology, and some methods in addressing this inverse problem have been proposed. Yu *et al.* applied the adaptive feedback approach for estimating the topology of a network [9], which had been extensively adopted in estimating parameters of nonlinear systems [10]. Timme showed that the information about network connectivity can be obtained by exploiting the response dynamics to the driving [11]. Bu and Jiang suggested a scheme in estimating the degree distribution in coupled chaotic oscillator networks in terms of the reconstruction of phase space based on the output time series and found the link between node degree and the deformation of a reconstructed attractor [12]. Nawrath *et al.* made a detailed study in distinguishing direct from indirect interactions in oscillatory networks [13]. Ren *et al.* studied the relationship between dynamical properties and interaction patterns in oscillatory networks in the presence of noise and revealed a general correspondence between the dynamical correlation and the node connections in various networks [14]. Very recently, Pikovsky's group proposed an ensemble method to reconstruct the network structure [15]. Related works include the detection of communities and modules of networks [16], identification of weighted networks [17–19], measuring interactions among neurons from spike trains [20], Granger analysis on topology detections of complex networks with stochastic perturbations [21], time-series-based prediction of complex oscillator networks via compressive sensing [22], and inferring network topology by analyzing the mean first passage time [23].

In practice one often faces the situation that only partial dynamical data can be obtained. It should be valuable if one can infer the network structure from very limited data. This often happens when there exists synchronization among nodes in a network, where the network dynamics becomes degenerated [24]. Most of the previous schemes cannot be successfully applied to topology detections in the presence of synchronous motion on a network. A direct idea of extracting topology information from dynamical data is to drive the system away from the synchronous state by applying external perturbations. When the external drive is applied for a long time, practically this drive may do harm to the network system. When one exerts a short-term perturbation, one can obtain a finite-time asynchronous data segment before the system relaxes back to the synchronous state. The connection information among nodes is embedded in this short asynchronous transient. This again naturally gives rise to the above question of detecting network structure in terms of finite dynamical data. It is our motivation to exploit this issue. We present, in this paper, an efficient scheme in revealing network connectivity from

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transient dynamics. We show that, by applying the transient dynamical signals as a periodic drive, the adjacency matrix can be determined in terms of the finite-time adaptive feedback scheme. This improved scheme is found to be very efficient and useful in the presence of global or local synchronization.

II. THE ADAPTIVE FEEDBACK SCHEME: SYNCHRONIZATION HINDERS NETWORK IDENTIFICATION

The adaptive feedback scheme (AFS) is a method to estimate system parameters by using chaos synchronization [10]. It has been successfully extended to the topology detection of networks [9], where the parameters under detection here are the couplings among oscillators. Consider a dynamical network of N coupled oscillators, where each node is described by an n-dimensional dynamical system. The dynamics of the network can be written as the following coupled ordinary differential equations:

$$\dot{\mathbf{x}}_i = \mathbf{F}(\mathbf{x}_i) + s\sum_{j=1}^{N} c_{ij}\mathbf{H}(\mathbf{x}_j), \tag{1}$$

where $\mathbf{x}_i(t)$ describes the state of the ith oscillator ($i = 1,2,\ldots,N$), $\mathbf{F}$ governs the dynamics of isolated oscillators, $\mathbf{H}$ is an output function describing the detail of the interaction, the adjacency matrix $\mathbf{C} = \{c_{ij}\}$ describes all the topological information of the weighted network, and s is the global coupling strength. We assume that the topology of the network, i.e., the adjacency matrix $\mathbf{C}$, is unknown, which could be determined based on the dynamics of $\mathbf{X}(t) = \{\mathbf{x}_i(t)\}$. To do this, we first resort to the adaptive feedback control method [9,17,24]. One can design a *drive-response* system, where the drive system is described by Eqs. (1) and the response system $\mathbf{Y}(t) = \{\mathbf{y}_i(t)\}$ is built as follows:

$$\dot{\mathbf{y}}_i = \mathbf{F}(\mathbf{y}_i) + s\sum_{j=1}^{N} d_{ij}(t)\mathbf{H}(\mathbf{y}_j) + \mathbf{u}_i, \tag{2}$$

where $\mathbf{u}_i = -k_i\mathbf{e}_i$ drives the response network in a feedback manner, and $\mathbf{e}_i = \mathbf{y}_i - \mathbf{x}_i$. The parameter k_i is governed by $\dot{k}_i = \parallel \mathbf{e}_i \parallel^2/2$. The adjacency matrix of the response network $d_{ij}(t)$ evolves with time and adapts itself to the drive network $\dot{d}_{ij} = -\mathbf{e}_i^T\mathbf{H}(\mathbf{y}_j)$ [9,17,24]. When the response network is synchronized by the drive system, i.e., $y_i(t) = x_i(t)$, the response adjacency matrix d_{ij} approaches a steady one. As long as $d_{ij}(t) \to c_{ij}$ when $t \to \infty$, this indicates that the topology of the network described by the adjacency matrix c_{ij} can be successfully detected by the response matrix $d_{ij}(t)$ in terms of the above adaptive feedback scheme.

The availability of the AFS can be justified by means of the Lyapunov-function analysis. If the nonlinear functions $\mathbf{F}_i$ and the output function $\mathbf{H}$ are Lipschitzian, i.e., if there exist positive constants $\mathcal{L}_i$ and $\mathcal{L}_H$ such that $\parallel \mathbf{F}_i(\mathbf{x}) - \mathbf{F}_i(\mathbf{y}) \parallel \leqslant \mathcal{L}_i \parallel \mathbf{x}-\mathbf{y} \parallel$ and $\parallel \mathbf{H}(\mathbf{x}) - \mathbf{H}(\mathbf{y}) \parallel \leqslant \mathcal{L}_H \parallel \mathbf{x}-\mathbf{y} \parallel$, we can define the following Lyapunov function:

$$V = \frac{1}{2}\sum_{i=1}^{N}\mathbf{e}_i^T\mathbf{e}_i + \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N}(d_{ij}-c_{ij})^2 + \sum_{i=1}^{N}(k_i-\rho)^2, \tag{3}$$

where ρ is a positive constant. It has been proved [9] that there exists a positive constant ρ, such that $\dot{V} \leqslant 0$. When $t \to \infty$, $d_{ij}(t) \to d_{ij}^*$. $\sum_{j=1}^{N}(d_{ij}(t) - c_{ij})\mathbf{H}(\mathbf{x}_j(t)) \to 0$, and this requires that the coefficients $d_{ij}^* - c_{ij} = 0$ for $j = 1,2,\ldots,N$. However, the values of d_{ij}^* do not necessarily coincide with c_{ij} when nodes in the network, for example, say, i and j, are synchronous to each other [$\mathbf{x}_i(t) = \mathbf{x}_j(t)$] [24]. When all nodes are not synchronized, the output functions $\mathbf{H}[\mathbf{x}_j(t)]$ are linearly independent of each other, and the coefficients $d_{ij}(t) - c_{ij} \to d_{ij}^* - c_{ij} = 0$. This condition implies that the topology of the drive network cannot be correctly detected when there exists synchronization (complete or partial) among nodes.

Let us demonstrate the above identification scheme by considering a network consisting of $N = 4$ Lorenz chaotic oscillators:

$$\begin{aligned} F_1(\mathbf{x}) &= 10(x_2 - x_1),\\ F_2(\mathbf{x}) &= 28x_1 - x_1x_3 - x_2,\\ F_3(\mathbf{x}) &= x_1x_2 - 8x_3/3. \end{aligned} \tag{4}$$

The topology is shown in Fig. 1(a), whose adjacency matrix $\mathbf{C}$ is weighted and asymmetric with elements $c_{2,4} = c_{4,2} = 0, c_{3,2} = 2$ and the other nondiagonal elements $c_{ij} = 1$. The network dynamics is described by Eqs. (1), and the coupling function $\mathbf{H}(\mathbf{x}) = \mathbf{x}$.

We first study the synchronization of nodes in this network and estimate the critical coupling strength for synchronization in terms of the master-stability-function (MSF) analysis [4,5].

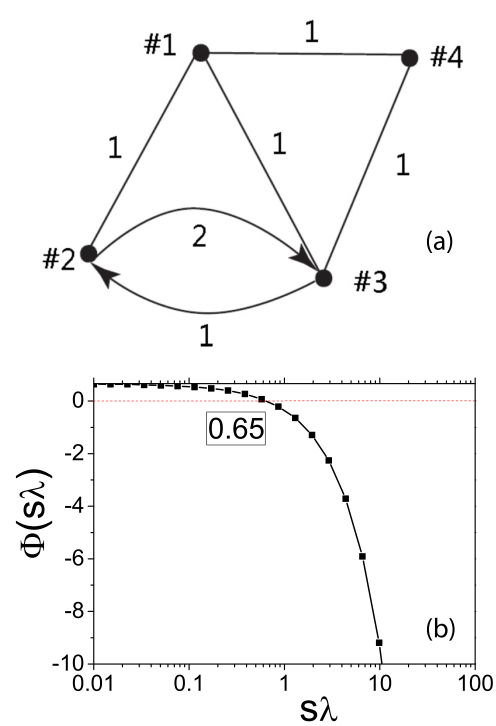

FIG. 1. (Color online) (a) The topology of a network with four nodes. (b) The normalized global coupling strength $s\lambda$ against the master-stability function $\Psi(s\lambda)$ for a network of Lorenz oscillators. The curve of $\Psi(s\lambda)$ is below the dotted line $\Psi = 0$ in the interval $s\lambda > 0.65$.

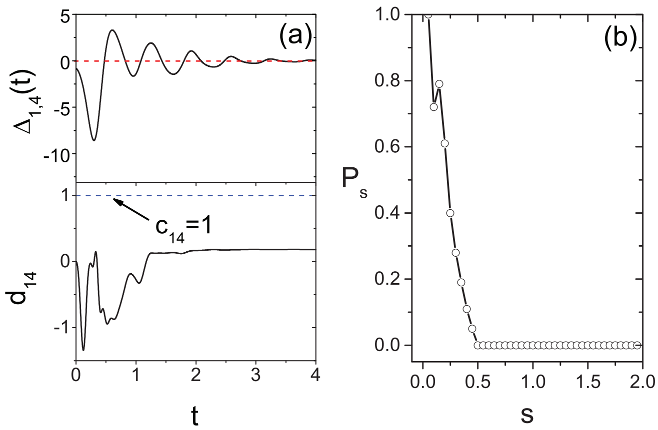

FIG. 2. (Color online) (a) Top: The evolution of the difference in the first component of nodes 1 and 4, $\Delta_{1,4}(t)$, for the global coupling strength $s = 1$. The network evolves from the asynchronous state to the synchronous state. Bottom: The evolution of one element in the estimator $d_{14}(t)$ driven by this transient time series, and the true value $c_{14} = 1$ is labeled by dotted line. (b) The probability of successful identification, P_s, varies with the global coupling strength s by using the traditional AFS.

In Fig. 1(b), the normalized global coupling strength $s\lambda$ versus the master-stability function $\Psi(s\lambda)$ is given, with λ being the second largest eigenvalue of the Laplacian matrix $-\mathbf{C}$. It can be found that when $s > 0.325$ the synchronous state is stable [for the network in Fig. 1(a), $\lambda = 2$].

In the upper part of Fig. 2(a), the evolutions of two nodes $i = 1,4$ are given for $s = 1$. $\Delta_{1,4}(t) = x_1^1(t) - x_1^4(t) \to 0$ as $t \to \infty$ implies the synchronous evolution of these two nodes. We further check the validity of the AFS to identify the matrix element c_{14} in representing the linking between nodes 1 and 4. We plot the evolution of the response element d_{14} in the lower part of Fig. 2(a). It can be found that $d_{14}(t)$ evolves to a value far from $c_{14} = 1$. This indicates that synchronization among nodes in a network degenerates the linking information among nodes and thus hinders the identification of network topology [24].

Let us introduce the success rate, i.e., the probability of successful identification, $P_s(s)$. Practically $P_s(s)$ is the portion of successful identifications in a finite time (usually long enough) by starting from uniformly random initial states $\{\mathbf{X}_i(0)\}$ for a given coupling strength s. In Fig. 2(b), the success rate P_s, which varies with the global coupling strength s, is plotted. It can be found that P_s decreases with increasing s. For small couplings, when there is no synchrony among nodes, $P_s = 100\%$; when $s > 0.5$, $P_s = 0\%$. This indicates that the traditional AFS fails in the presence of network synchronization.

III. THE REPEATED-DRIVE ADAPTIVE FEEDBACK SCHEME

A natural way of extracting the linking information among nodes in the presence of synchronization is to desynchronize the network and extract the linking information from these desynchronized data. Desynchronization can be achieved by changing the global coupling strength s or the output function $\mathbf{H}$ [24]. However, it is usually difficult or even impossible to significantly change the parameters and skeletons of the system. One may also drive the network dynamics away from the synchronous state by applying external perturbations. If the external perturbation is removed, the system will usually evolve back to the synchronous state, and one can only get transient asynchronous data. This asynchronous segment contains the linking information, but it is not long enough to apply the original AFS to correctly extract the network connectivity [25].

One option to tackle this difficulty is to produce many segments of short asynchronous data $\{\mathbf{x}(t), t_1^l \leqslant t \leqslant t_2^l\}, l = 1,2,\ldots\}$ and merge into a "longer" time series. To get these segments, a common way is to apply an external perturbation many times to get an ensemble of copies [15]. However, this is not the most efficient choice since one should apply many perturbations. If one uses only *one* asynchronous segment $\{\mathbf{x}(t), t_1 \leqslant t \leqslant t_2\}$, by copying it into many *copies* and combining them, one can get a longer segment. One can use this longer asynchronous segment as the driver. By replacing the driver $\mathbf{x}(t)$ used in the traditional AFS and adopting the traditional AFS, one expects the successful detection of network topology. We call this improved scheme the *repeated-drive adaptive feedback scheme* (RDAFS). However, is it possible to successfully identify the network topology by using the RDAFS?

To check the availability of RDAFS, we refer back to the Lyapunov analysis of AFS. It has been proved that the Lyapunov function defined for the AFS [9] decreases monotonically with time. This monotonicity remains valid even when synchronization among nodes occurs, as shown in our numerical observation in Fig. 2(b), where d_{ij} has a tendency of approaching c_{ij} when the response network is driven by an asynchronous transient. With this in mind, we check the validity and efficiency of our proposed RDAFS.

Suppose Δt to be the length of the short transient time series. During the lth period of the repeated drive, one uses the transient time series to drive the response starting from the new initial values $\mathbf{D}^l(0) = \mathbf{D}^{l-1}(\Delta t)$ and $k_i^l(0) = k_i^{l-1}(\Delta t)$. By repeating this procedure many times, we obtain a series $\{\mathbf{D}^l(\Delta t) \mid l = 1,2,3,\ldots\}$. We expect that $\mathbf{D}^l(\Delta t) \to \mathbf{C}$ when $l \to \infty$. By adopting the Lyapunov function proposed for the AFS in Sec. II, $\dot{V} \leqslant 0$ still remains valid within the lth driving period. Therefore,

$$V^l(\Delta t) \leqslant V^l(0). \tag{5}$$

Considering that ρ is a positive constant, because $\dot{k}_i \geqslant 0$, $\dot{k}_i(t)[k_i(t) - \rho] \geqslant 0$ can be satisfied for all $t > 0$ as long as initially $k_i(0) \geqslant \rho$. Therefore, one has the relation

$$\sum_{i=1}^{N} \left[k_i^l(\Delta t) - \rho\right]^2 \geqslant \sum_{i=1}^{N} \left[k_i^l(0) - \rho\right]^2. \tag{6}$$

Furthermore, relation (6) becomes

$$\sum_{i=1}^{N} \left[k_i^l(\Delta t) - \rho\right]^2 \gg \sum_{i=1}^{N} \left[k_i^l(0) - \rho\right]^2 \tag{7}$$

if $k_i(0) \geqslant \rho$ are sufficiently large, i.e., $k_i(0) \gg \rho$. Although $[\mathbf{e}_i^l(\Delta t)]^T \mathbf{e}_i^l(\Delta t) \leqslant [\mathbf{e}_i^l(0)]^T \mathbf{e}_i^l(0)$, the inequality

$$\sum_{i=1}^{N}\sum_{j=1}^{N}\left[d_{ij}^l(\Delta t) - c_{ij}\right]^2 \leqslant \sum_{i=1}^{N}\sum_{j=1}^{N}\left[d_{ij}^l(0) - c_{ij}\right]^2 \quad (8)$$

can still be ensured as long as one has relation (7).

If the time series $\{\mathbf{H}[\mathbf{x}_j(t)], t \in (0,\Delta t)\}$ are linearly independent, the two sides of relation (8) become equal only if $\mathbf{D}^l(0) = \mathbf{C}$. This conclusion can be proved in the same way proposed by Chen and Lu [24]. Relation (8) confirms that if the information obtained by the previous drive after each drive is utilized as the next drive, i.e., $\mathbf{D}^{l+1}(0) = \mathbf{D}^l(\Delta t)$, all the information of the drive network topology can be obtained: $\lim_{l\to\infty} \mathbf{D}^l(\Delta t) = \mathbf{C}$.

IV. NUMERICAL SIMULATIONS

We perform numerical simulations to study the topology identification of the network by using the above scheme. We still employ the four-node network shown in Fig. 1(a) with node dynamics given by the Lorenz oscillator Eq. (4). The global coupling strength $s = 1$. For this network, $\rho = 10$ is a sufficiently large constant to guarantee $\dot{V} \leqslant 0$. We set the initial value $k_i^1(0) = 10$.

In Figs. 3(a)–3(d), the evolutions of the three components of the Lyapunov function $V(t) = V_1(t) + V_2(t) + V_3(t)$, i.e., $V_1(t) = \frac{1}{2}\sum_{i=1}^{N} \mathbf{e}_i^T \mathbf{e}_i$, $V_2(t) = \sum_{i=1}^{N}(k_i - \rho)^2$, and $V_3(t) = \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N}(d_{ij} - c_{ij})^2$, respectively, are shown when repeated drives are applied. At the end of each drive, we reset $t = 0$ to be the starting point of the next drive. It can be clearly observed that all three components of the Lyapunov function evolve and approach the steady values. This conforms to the above analysis of the Lyapunov function.

In Fig. 4(a), we give an example of the RDAFS for identifying the element c_{14} of the adjacency matrix of the network presented in Fig. 1(a) for $s = 1.0$. The usual AFS fails to predict the network links [see Fig. 2(a)]. A segment of asynchronous transient with a time duration $\Delta t = 2.0$ is applied as the repeated drive, as shown in the top of Fig. 2(a). By applying this periodic asynchronous drive, it can be found that the matrix element d_{14} evolves in a steplike way and approaches c_{14} quickly (here we only adopt $L \leqslant 10$ drive periods). This indicates that in the presence of network synchronization the RDAFS exhibits an excellent and efficient ability of topology identification. In fact, all the elements of the adjacency matrix $\mathbf{C}$ can be correctly predicted in terms of this procedure. Moreover, the error of the estimation decreases exponentially, as can be clearly seen in Fig. 4(a).

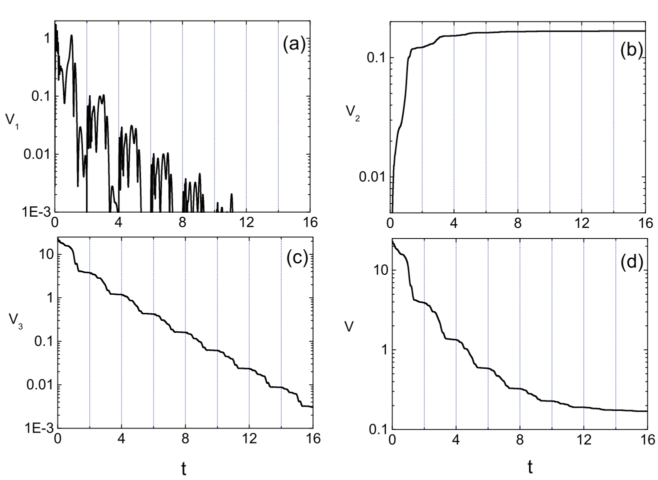

FIG. 3. (Color online) The evolutions of different terms in the Lyapunov function [Eq. (3)] and the Lyapunov function $V(t)$ under the repeated asynchronous drives ($\Delta t = 2.0$), where the global coupling strength $s = 1$. At the end of every drive, we reset $t = 0$. (a) $V_1(t)$, (b) $V_2(t)$, (c) $V_3(t)$, and (d) the Lyapunov function $V(t)$.

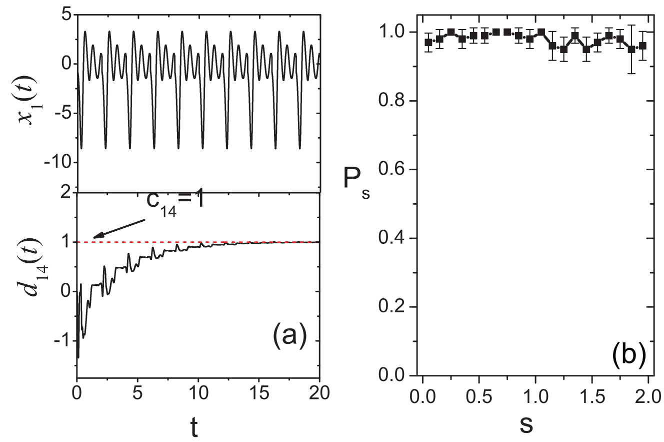

FIG. 4. (Color online) The numerical simulation results for the RDAFS, where the global coupling strength $s = 1$. (a) Top: The repeated transient time series $\{\mathbf{x}_i(t), t \in (0,2)\}$ with each segment the same as in Fig. 2(a). Bottom: The evolution of $d_{14}(t)$, and $c_{14} = 1$ is labeled by the dotted line. Coincidence can be clearly found. (b) The rate of successful identification, P_s, versus the global coupling strength s for 2000 drives.

We are concerned with the success rate of this method. In Fig. 4(b), we give the probability of successful identification P_s in $l = 2000$ drives for different coupling strengths. It can be found that when $s \in (0,2.0)$, $P_s > 90\%$. We find that as the number of driving segments l increases, the success rate increases. As $l \to \infty$, $P_s \to 1$. For example, when $l = 20\,000$, $P_s \approx 100\%$. This indicates the availability of our approach. Moreover, our approach works well at very strong couplings, where global synchronization is robust (e.g., $s = 10.0$).

As seen from the above analysis and Fig. 4(b), the RDAFS is naturally applicable to cases of different coupling strengths. For the asynchronous state, one can still successfully detect the adjacency matrix by using a short segment of dynamic data as the repeated drive in terms of the RDAFS. This can be well understood by using the above Lyapunov-function analysis.

V. OPTIMIZATION OF NETWORK IDENTIFICATION

The length of the asynchronous segment used for the repeating drive plays an important role in the topology identification process. If the segment length is too short, there will not be enough information for a successful identification of network structure. On the other hand, if the length of each segment is too long, it may include the nearly synchronous information that slows down the estimation process. Therefore,

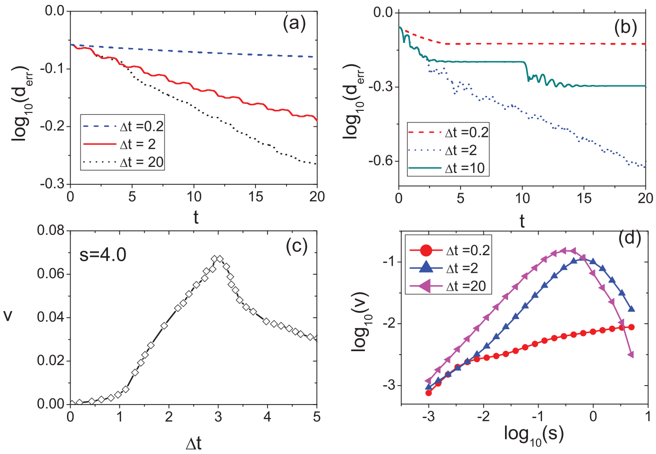

FIG. 5. (Color online) (a) The evolution of $d_{\rm err}(t)$ for $s = 0.01$ (asynchronous regime) and different segment lengths $\Delta t = 0.2,2,20$. (b) The evolution of $d_{\rm err}(t)$ for $s = 4.0$ (global synchronization) and different segment lengths $\Delta t = 0.2,2,10$. (c) The identification speed v varying against the segment length Δt for the global coupling $s = 4.0$ (global synchronization). (d) The identification speed v varying against the global coupling s for different segment lengths $\Delta t = 0.2,2,20$.

one expects an optimized transient length for networks with synchronization among some nodes.

To measure the speed of the identification, let us introduce an average relative identification error as

$$d_{\rm err}(t) = \frac{1}{N^2}\sum_{i=1}^{N}\sum_{j=1}^{N}\frac{|d_{ij}(t) - c_{ij}|}{c'_{ij}}, \tag{9}$$

where $c'_{ij} = \max\{|c_{ij}|,\tilde{c}\}$, and $\tilde{c}$ is the value of the minimal nonzero element in the matrix $\mathbf{C}$. This quantity measures the efficiency of the identification of the adjacency matrix (network topology). Obviously the error $d_{\rm err}(t) \to 0$ for a successful identification of the adjacency matrix, and in this case $\lim_{t\to\infty}\mathbf{D}(t) \to \mathbf{C}$. In numerical simulations, $d_{\rm err}(t)$ is found to decrease approximately in an exponential way, i.e., $d_{\rm err}(t) \propto \exp(-vt)$. Therefore, the exponent v can be used to measure the identification rate.

We perform numerical simulations of the Lorenz network shown in Fig. 1(a) for different coupling strengths. To make a comparison, we study the identification efficiency by using the repeated procedure with different lengths of the time-series segments. In Fig. 5(a), the evolution of the error $d_{\rm err}(t)$ for $s = 0.01$ is given, when the motion of oscillators in a network is asynchronous. It can be clearly found from Fig. 5(a) that the relative error $d_{\rm err}(t)$ decays exponentially for different segment length Δt. Moreover, for a longer segment, the error decays more rapidly, implying a faster identification. This can be well understood because a longer asynchronous segment contains more linking information of a network. For a very long segment, the identification rate v approaches the value of the AFS ($\Delta t \to \infty$). When synchronization exists among some nodes in a network, the length of the asynchronous segment becomes finite due to the relaxation to synchronous motion. Therefore, even if one extracts a longer segment, the nearly synchronous part does not provide valuable identification of the network topology. The competition between the asynchronous and synchronous parts in a segment leads to an optimization of the identification. In Fig. 5(b), the evolution of $d_{\rm err}(t)$ at a very strong coupling $s = 4.0$ is plotted, where all oscillators in the network are synchronized. In this case, it can be observed that $d_{\rm err}(t) \to 0$ for different segment lengths, indicating that the RDAFS can successfully estimate the network structure. However, the fastest estimation is found for an intermediate length $\Delta t = 2.0$, and a very long segment may slow the identification process. In Fig. 5(c), we compute the rate v varying with the segment length Δt for $s = 4.0$. It can be found that the adjacency matrix of the network cannot be successfully identified for very short asynchronous segments, and the identification efficiency becomes low if the segment is long. The speed v becomes the largest at $\Delta t \approx 3$, implying an optimization of the identification process.

In Fig. 5(d), the identification speed v varying against s is plotted for different Δt. For small s, it can be found that $v \propto s^{\nu}$, where $\nu \approx 1.0$. This indicates that when there is no synchronization in a network, the identification speed of the RDAFS increases with increasing coupling strength. As some nodes become synchronous, one can find that v reaches the maximum at certain coupling strength and then decreases with further increasing the coupling. This indicates an optimization of identification by varying the coupling strength. Our preliminary result indicates that the optimal Δt is closely related to the relaxation time scale of the synchronization among nodes in a network, and a deeper study in understanding the mechanism of this optimization is necessary.

VI. CONCLUSIONS

To summarize, in this paper we explored the identification of network topology in the presence of limited dynamical data. How to dig out the topological information from limited data is a challenging and significant issue. The presence of synchronization among nodes degenerates the linking information between them and may deteriorate the ability of network topology detection. In this work, we showed that the identification process can be successfully executed by adopting the adaptive feedback scheme that repeatedly uses the asynchronous data as the drive. This improved scheme is found to be very useful in the presence of global or local synchronization, where the transient drive can be obtained by perturbing the system to get an asynchronous transient. We also studied the detection speed of this scheme and found the optimized effect of network detection against the time-series segment length and the coupling strength among nodes in the network. Moreover, the validity of the RDAFS proposed here and the optimization in detections are numerically tested in other cases, for example, different number of nodes, different network topologies (e.g., weighted and directed networks), different linking ways, different node dynamics, and so on.

Since the AFS has been extensively applied to numerous cases of parameter detections of nonlinear systems, the RDAFS we proposed here can also be naturally applied to chaos-synchronization-based parameter identifications. There are some merits in our approach. First, only a very short segment of dynamical data is needed to perform the parameter identification by using the RDAFS, and this is very economic in practice. Second, different from the traditional AFS, one does not need to stay online to monitor the system under detection. The gathered data segment can be stored offline and used at any time for convenience. This provides a high possibility in applying the RDAFS proposed in this paper to practical situations.

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