Re-examining the self-contained quantum refrigerator in the strong-coupling regime

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We revisit the self-contained quantum refrigerator in the strong-internal-coupling regime by employing the quantum optical master equation. It is shown that strong internal coupling reduces the cooling ability of the refrigerator. In contrast to the weak-coupling case, strong internal coupling could lead to quite different and even converse thermodynamic behaviors.

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I. INTRODUCTION

Thermodynamics is one of the four pillars of theoretical physics and provides us with an essential way to study the thermodynamic process such as the heat engine, which can be dated back to Carnot [1]. When we consider the physical nature down to the quantum level, quantum thermodynamics, which is the intersection of thermodynamics and quantum mechanics, provides a new approach to investigating microscopic physics. Quantum thermodynamics has attracted more and more interest such as in Refs. [2–4] and references therein. In particular, the quantum heat engine has been extensively studied [5–12]. It was shown that the quantum heat engine is remarkably similar to the classical engines which obey macroscopic dynamics, and Carnot efficiency has been a well-established limit for some quantum heat engines [13–17]. A lot of work has been done especially related to quantum analogs of Carnot engines [18–22], while some other cycles such as Otto cycles [23–26] and Brownian motions [27] have also been covered with considerable progress. All the above provide microscopic alternatives for testing the fundamental laws of thermodynamics and deepening our understanding of quantum thermodynamics.

Recently, the concept of the self-contained quantum refrigerator has been raised for questions about the fundamental limitation on the size of thermal machines and their relevant topics [28–32]. It is shown that "self-contained" means that (i) all degrees of freedom of the refrigerator are taken into account; (ii) no external source of work is allowed; and (iii) in particular, time-dependent Hamiltonians or prescribed unitary transformations are not allowed. However, the key of their model is that they required the interaction (the coupling) among their three qubits to be weak enough, but the coupling and the decay rate are of the same order. In other words, the self-contained refrigerator works in the regime of weak internal coupling. Since the three-qubit interaction is the vital driving mechanism for cooling, could a strong internal interaction (coupling) provide more effective power?

In this paper, we revisit the same model proposed in Ref. [28] in the strong-internal-coupling regime. We employ the quantum optical master equation (QOME) to study the steady-state heat currents and the cooling efficiency. As the main result, we find that strong internal coupling has a negative effect on the cooling ability. The thermodynamic properties of such a model could also be different from, and even opposite to, those in the weak-internal-coupling regime. In addition, it is shown that our results will be consistent with those in Ref. [28] (weak internal coupling) if we reduce the internal coupling strength, although our master equation, in principle, is only suitable for strong internal coupling. This implies that the validity of the application of the quantum master equation deserves further consideration. This paper is organized as follows. In Sec. II, we briefly introduce the interacting mechanism of the refrigerator and derive the master equation. In Sec. III, we present our main results and do some necessary analysis. Finally, Sec. IV presents the conclusions.

II. THE MODEL AND THE MASTER EQUATION

The refrigerator we consider here is made up of three atoms, denoted, respectively, R, C, and H. The free Hamiltonian of the three-atom system is given by

$$H_0 = H_R + H_C + H_H, \tag{1}$$

where $H_\mu = \frac{\omega_\mu}{2}\sigma_\mu^z$, ω_μ, $\mu = R, C$, and H, is the transition frequency of atom μ, and $\sigma^z = |e\rangle\langle e| - |g\rangle\langle g|$, with $|e\rangle$ and $|g\rangle$ denoting the excited state and the ground state, respectively. In particular, in order to guarantee the resonant interaction, it is required that $\omega_R = \omega_H + \omega_C$. Suppose that the interaction of the three atoms is described by the Hamiltonian H_I,

$$H_I = g(\sigma_H^+\sigma_R^-\sigma_C^+ + \sigma_H^-\sigma_R^+\sigma_C^-), \tag{2}$$

with g the coupling constant, $\sigma^+ = |e\rangle\langle g|$, and $\sigma^- = |g\rangle\langle e|$; then the Hamiltonian of the closed system reads

$$H_S = H_0 + H_I. \tag{3}$$

Here we set the Planck constant and Boltzmann's constant to be unit, i.e., $\hbar = k_B = 1$. In the framework of self-contained refrigerator [28,30], all the atoms should interact with a reservoir, respectively, instead of a real working source. So we let atom H be connected with a hot reservoir, with the temperature denoted by T_H; atom R be in contact with a "room" reservoir of temperature T_R; and atom C interact with a cold reservoir of temperature T_C. Thus It is naturally implied that $T_H > T_R > T_C$. Here we assume that all the reservoirs consist of infinite harmonic oscillators with closely spaced frequencies $\nu_{\mu k}$ and annihilation operators $b_{\mu k}$. Note that the subscript μ labels the atom which the corresponding reservoir interacts with. Thus one can write the total Hamiltonian of the

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open system as

$$H = H_S + \sum_\mu (H_{\mu 0} + H_\mu), \tag{4}$$

where $H_{\mu 0} = \sum_k \nu_{\mu k} b_{\mu k}^\dagger b_{\mu k}$ is the free Hamiltonian of the μth reservoir, and

$$H_\mu = \sum_k f_{\mu k}(b_{\mu k}^\dagger \sigma_\mu^- + b_{\mu k}\sigma_\mu^+), \tag{5}$$

with $f_{\mu k}$ denoting the coupling constant, describes the interaction between the μth atom and its thermal reservoir. From Eq. (4), i.e., the total Hamiltonian, in principle, one can obtain all the dynamics of the refrigerator and the reservoirs. To do so, we have to derive a master equation that governs the evolution of the system of interests. Next, we follow the standard procedure [33,34] to find such a master equation.

Since the refrigerator (excluding the reservoirs) is a composite quantum system, the first step is to diagonalize the refrigerator Hamiltonian H_S. It is shown that the diagonalized H_S can be written as $H_S = \sum \epsilon_i |\lambda_i\rangle\langle\lambda_i|$, where the eigenvalues are given by

$$[\epsilon_1, \epsilon_2, \ldots, \epsilon_8] = [\omega_R, \omega_H, g, -\omega_C, \omega_C, -g, -\omega_H, -\omega_R], \tag{6}$$

and $|\lambda_i\rangle$ denote the corresponding eigenvectors, with the concrete form omitted here. In H_S representation, the Hamiltonian H can be rewritten as

$$H = \sum_{i=1}^{8} \epsilon_i |\lambda_i\rangle\langle\lambda_i| + \sum_{\mu,j}(H_{\mu 0} + H'_{\mu j}), \tag{7}$$

where

$$H'_{\mu j} = \sum_k f_{\mu k}(b_{\mu k}^\dagger V_{\mu j}(w_{\mu j}) + b_{\mu k} V_{\mu j}^\dagger(w_{\mu j})), \tag{8}$$

with $V_{\mu j}(w_{\mu j}))$ denoting the eigenoperators of the refrigerator Hamiltonian H_S such that $[H_S, V_{\mu j}(w_{\mu j})] = -w_{\mu j} V_{\mu j}(w_{\mu j})$ and $w_{\mu j}$ standing for the eigenfrequency. In particular, $V_{\mu j}(\nu_j)$ can be explicitly given as

$$V_{11} = |\lambda_5\rangle\langle\lambda_1| + |\lambda_8\rangle\langle\lambda_4|, \quad w_{11} = \omega_H, \tag{9}$$

$$V_{12} = \frac{1}{\sqrt{2}}(|\lambda_3\rangle\langle\lambda_2| + |\lambda_7\rangle\langle\lambda_6|), \quad w_{12} = \omega_H - g, \tag{10}$$

$$V_{13} = \frac{1}{\sqrt{2}}(|\lambda_7\rangle\langle\lambda_3| - |\lambda_6\rangle\langle\lambda_2|), \quad w_{13} = \omega_H + g, \tag{11}$$

$$V_{21} = \frac{1}{\sqrt{2}}(|\lambda_3\rangle\langle\lambda_1| - |\lambda_8\rangle\langle\lambda_6|), \quad w_{21} = \omega_R - g, \tag{12}$$

$$V_{22} = |\lambda_4\rangle\langle\lambda_2| + |\lambda_7\rangle\langle\lambda_5|, \quad w_{22} = \omega_R, \tag{13}$$

$$V_{23} = \frac{1}{\sqrt{2}}(|\lambda_8\rangle\langle\lambda_3| + |\lambda_6\rangle\langle\lambda_1|), \quad w_{23} = \omega_R + g, \tag{14}$$

$$V_{31} = \frac{1}{\sqrt{2}}(|\lambda_3\rangle\langle\lambda_5| + |\lambda_4\rangle\langle\lambda_6|), \quad w_{31} = \omega_C - g, \tag{15}$$

$$V_{32} = \frac{1}{\sqrt{2}}(|\lambda_4\rangle\langle\lambda_3| - |\lambda_6\rangle\langle\lambda_5|), \quad w_{32} = \omega_C + g, \tag{16}$$

$$V_{33} = \frac{1}{\sqrt{2}}(|\lambda_2\rangle\langle\lambda_1| + |\lambda_8\rangle\langle\lambda_7|), \quad w_{33} = \omega_C, \tag{17}$$

where $w_{\mu j} > 0$ is implied; otherwise, $V_{\mu j} = V_{\mu j}^\dagger$. Suppose that the system and their reservoirs are initially separable and the initial states of the reservoirs are the thermal equilibrium states. In particular, we assume that the coupling between the system and the reservoirs is weak enough. Based on the Born-Markovian approximations, one can derive the master equation as

$$\dot{\rho} = \mathcal{L}_C[\rho] + \mathcal{L}_R[\rho] + \mathcal{L}_H[\rho], \tag{18}$$

where the dissipators read

$$\begin{aligned}\mathcal{L}_\mu[\rho] = \sum_j & J_\mu(-w_{\mu j})[2V_{\mu j}(w_{\mu j})\rho V_{\mu j}^\dagger(w_{\mu j}) \\ & - V_{\mu j}^\dagger(w_{\mu j})V_{\mu j}\rho(w_{\mu j}) - \rho V_{\mu j}^\dagger(w_{\mu j})V_{\mu j}(w_{\mu j})] \\ & + J_\mu(w_{\mu j})[2V_{\mu j}^\dagger(w_{\mu j})\rho V_{\mu j}(w_{\mu j}) \\ & - V_{\mu j}(w_{\mu j})V_{\mu j}^\dagger(w_{\mu j})\rho - \rho V_{\mu j}(w_{\mu j})V_{\mu j}^\dagger(w_{\mu j})].\end{aligned} \tag{19}$$

The spectral density in Eq. (19) is given by

$$J_\mu(w_{\mu j}) = \gamma_\mu(w_{\mu j})\bar{n}(w_{\mu j}), \tag{20}$$

$$J_\mu(-w_{\mu j}) = \gamma_\mu(w_{\mu j})[\bar{n}(w_{\mu j}) + 1], \tag{21}$$

where $\bar{n}(w_{\mu j})$ is the average photon number, which depends on the temperature of the reservoir; i.e.,

$$\bar{n}(w_{\mu j}) = \frac{1}{e^{\frac{w_{\mu j}}{T_\mu}} - 1}. \tag{22}$$

Here we suppose that $\gamma_\mu(w_{\mu j}) = \gamma_\mu$ is frequency independent, for simplicity. In addition, we employed the rotating-wave approximation, which implies $\gamma_\mu \ll |\omega_\mu - \omega_\nu \pm 2g|, g$. This condition requires that the master equation is only suitable for large g. However, so far there has not been an explicit constraint on the degree to which g is larger than γ_μ [34,35].

III. RESULTS AND DISCUSSION

In order to study the thermodynamical behavior of the stationary state, we will find the stationary-state solution ρ^S of the master equation given by Eq. (18). To do so, we let ρ^S have the vanishing derivative on t; i.e.,

$$\dot{\rho}^S = 0. \tag{23}$$

Thus we arrive at the equations

$$M|\rho\rangle = 0, \tag{24}$$

$$\rho_{ij}^S = 0, \quad i \neq j, \tag{25}$$

where $|\rho\rangle = [\rho_{11}^S, \rho_{22}^S, \rho_{33}^S, \rho_{44}^S, \rho_{55}^S, \rho_{66}^S, \rho_{77}^S, \rho_{88}^S]^T$ is the vector made up of the diagonal entries of the stationary density matrix ρ^S, and

$$M = \sum_{\mu=1}^{3} M_\mu. \tag{26}$$

In order to give the explicit expression for M_μ, we first define some new quantities m_{ij}, $i,j = 1,2,3$, as

$$m_{11} = 2\mathbf{J}_{11} \otimes (\mathbf{1}_+ \otimes \mathbf{1}_+ + \mathbf{1}_- \otimes \mathbf{1}_-), \quad (27)$$

$$m_{12} = \mathbf{1} \otimes C_{23}(\mathbf{J}_{12} \otimes \mathbf{1}_-)C_{23}^\dagger, \quad (28)$$

$$m_{13} = \mathbf{J}_{13} \otimes (\mathbf{1}_+ \otimes \mathbf{1}_- + \mathbf{1}_- \otimes \mathbf{1}_+), \quad (29)$$

$$m_{21} = (\mathbf{1}_+ \otimes \mathbf{J}_{21} \otimes \mathbf{1}_+ + \mathbf{1}_- \otimes \mathbf{J}_{21} \otimes \mathbf{1}_-), \quad (30)$$

$$m_{22} = 2(\mathbf{1}_+ \otimes \mathbf{J}_{22} \otimes \mathbf{1}_- + \mathbf{1}_- \otimes \mathbf{J}_{22} \otimes \mathbf{1}_+), \quad (31)$$

$$m_{23} = C_{13}(\mathbf{J}_{23} \otimes \mathbf{1} \otimes \mathbf{1}_+)C_{13}^\dagger, \quad (32)$$

$$m_{31} = C_{21}(\mathbf{1}_- \otimes \mathbf{J}_{31} \otimes \mathbf{1})C_{21}^\dagger, \quad (33)$$

$$m_{32} = (\mathbf{1}_+ \otimes \mathbf{1}_- + \mathbf{1}_- \otimes \mathbf{1}_+) \otimes \mathbf{J}_{32}, \quad (34)$$

$$m_{33} = 2(\mathbf{1}_+ \otimes \mathbf{1}_+ + \mathbf{1}_- \otimes \mathbf{1}_-) \otimes \mathbf{J}_{33}, \quad (35)$$

where

$$\mathbf{1} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad \mathbf{1}_\pm = \frac{\mathbf{1} \pm \sigma_z}{2}, \quad (36)$$

and C_{jk}, $j,k = 1,2,3$, denotes the control-not gate, with j standing for the control qubit and k representing the target qubit. For example,

$$C_{12} = (\mathbf{1} \oplus \sigma_x) \otimes \mathbf{1}. \quad (37)$$

In addition, $\mathbf{J}_{\mu j}$ in Eqs. (27)–(35) is a matrix with its entries corresponding to the spectral density. It can be explicitly represented by

$$\mathbf{J}_{\mu j} = \begin{pmatrix} -J_\mu(-w_{\mu j}) & J_\mu(w_{\mu j}) \\ J_\mu(-w_{\mu j}) & -J_\mu(w_{\mu j}) \end{pmatrix}. \quad (38)$$

Based on Eqs. (27)–(35), M_μ can be explicitly written as

$$M_\mu = \sum_j m_{\mu j}. \quad (39)$$

M_μ apparently includes three terms which are related to three atoms, respectively. Using the definition of the heat current [34,36], we can find that the heat current subject to the μth reservoir reads

$$\dot{Q}_\mu = \mathrm{Tr}\{H_S \mathcal{L}_\mu[\rho^S]\} = \langle \epsilon | M_\mu | \rho \rangle. \quad (40)$$

It is obvious that $\dot{Q}_\mu$ corresponding to M_μ is uniquely determined by the steady state $|\rho\rangle$. It is fortunate that $\dot{Q}_\mu$ can be explicitly calculated, because Eq. (23) can be analytically solved. However, the concrete form of $|\rho\rangle$ is so tedious that we cannot write it here. Therefore, in the subsequent part, we have to give a numerical analysis based on the analytical $|\rho\rangle$ (even though it is not given here).

First, we would like to consider the weak-coupling case, i.e., $g \sim \gamma_\mu$. Based on Eq. (40), we plot the heat currents in Fig. 1. Here we suppose that $\omega_H = 3$ and $\omega_C = 1$, so $\omega_R = 4$. In addition, we let the room temperature be 21 K and the temperature of the cold reservoir be 18 K. (Of course, if the other parameters are chosen, one will get similar results.) When the temperature of the hot reservoir is low, the heat will flow into the cold reservoir. So the cold atom C is heated. However, with the temperature of the hot reservoir increasing, one can find that all the heat currents will become 0 simultaneously when $T_H = T_v = \frac{\omega_H}{\frac{\omega_R}{T_R} - \frac{\omega_C}{T_C}} \simeq 22.24$ K (as long as the coupling g and the decay rate γ are small enough.). This virtual temperature T_v is just consistent with Ref. [32], which is closely related to Ref. [28]. In addition, one can also see that the heat currents increase with an increase in T_H. That is, the thermodynamic machine works as a refrigerator. An obvious feature is that the heat currents subject to the hot reservoir and the cold reservoir have the same direction (sign) and the sign is determined by the virtual temperature T_v. However, if $\frac{\omega_R}{T_R} = \frac{\omega_C}{T_C}$, one will find that no matter how large T_H is, $\dot{Q}_C$ is always less than 0. In addition, we also consider the efficiency of the quantum refrigerator, which is illustrated in Fig. 2. Here the efficiency η is defined by $\eta = \frac{\dot{Q}_C}{\dot{Q}_H}$, which was studied in detail in Ref. [30]. In a simple way, it can be understood that, by extracting heat (current) $\dot{Q}_H$ from the hot reservoir, we are able to extract heat (current) $\dot{Q}_C$ from the cold reservoir while dumping heat (flow) $\dot{Q}_R$ into the reservoir R. It was also shown that η for the self-contained refrigerator in Ref. [28] was given by $\frac{\omega_C}{\omega_H}$. Taking the current parameters

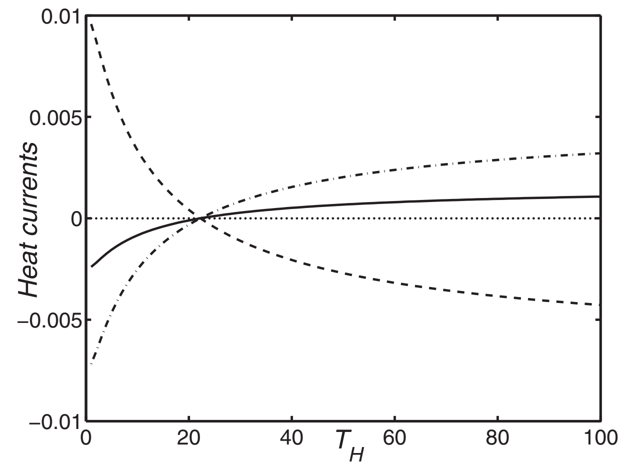

FIG. 1. Heat currents $\dot{Q}_\mu[J/s]$ versus $T_H[K]$ in the weak-coupling regime. Solid line, $\dot{Q}_C$; dashed line, $\dot{Q}_H$; and dash-dotted line, $\dot{Q}_R$. Here $g = 0.001\omega_H$. In particular, we set $\gamma = 0.001\omega_H$ throughout the paper.

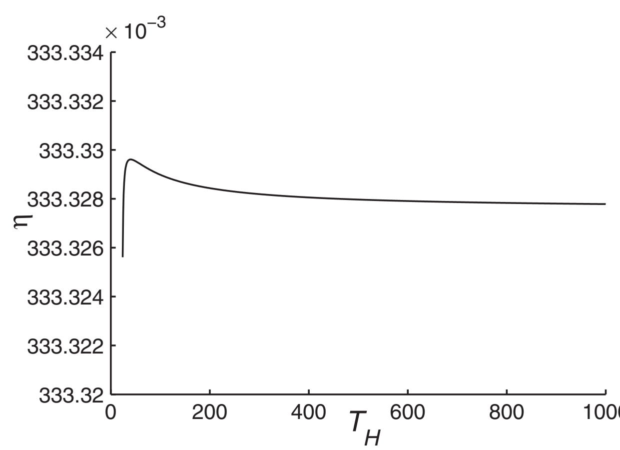

FIG. 2. Efficiency η of the refrigerator versus $T_H[K]$ in the weak-coupling regime. The efficiency changes slightly and it can be considered to be almost invariant within a good approximation, which is also supported by Fig. 5.

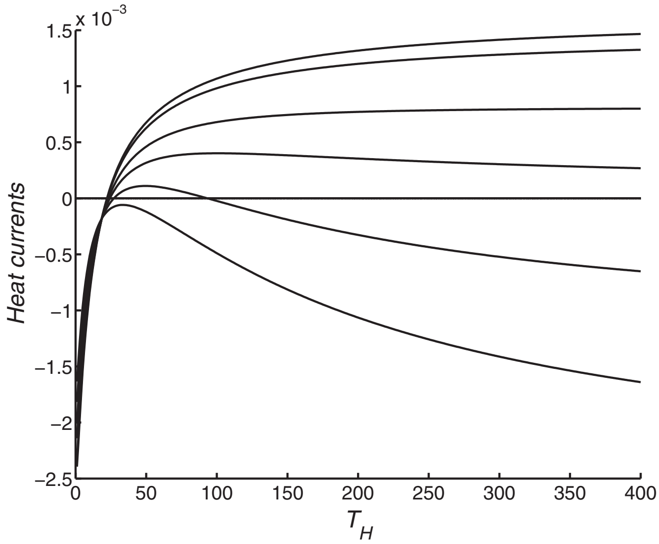

FIG. 3. Heat currents $\dot{Q}_C[J/s]$ versus $T_H[K]$ for different coupling constants. From top to bottom, $g = 0.001\omega_H$, $g = 0.1\omega_H$, $g = 0.2\omega_H$, $g = 0.25\omega_H$, $g = 0.3\omega_H$, and $g = 0.35\omega_H$. The straight line represents zero heat current.

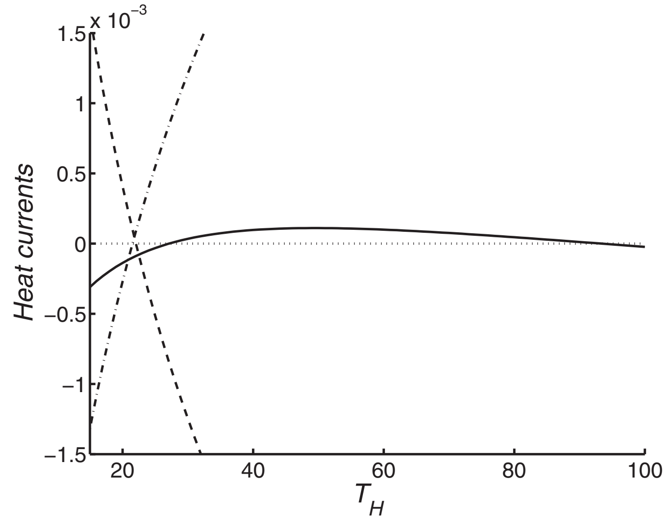

FIG. 4. Heat currents versus $T_H[J/s]$ in the strong-coupling regime. Here $g = 0.3\omega_H$ and $\gamma = 0.001\omega_H$. The dotted line represents the zero heat current and the other lines denote the same heat currents as in Fig. 1. $\dot{Q}_C$ is first pushed in the positive direction and then suppressed back to the negative direction.

into account, it should be $\eta = \frac{1}{3}$. From our Fig. 2, at first glance, the efficiency seems to have a peak somewhere. But one further finds that, to some acceptable approximation, η can be considered to be invariant with T_H and just equal to $\frac{1}{3}$. The peak is explained in the next part. All this evidence shows that in the weak-coupling regime, the treatment with respect to the QOME shows good consistency with the results given in Ref. [28]. This could imply that the QOME is not sensitive to the rotating-wave approximation corresponding to $g \gg \gamma_\mu$ in this case, which could mean that the QOME is valid here. Now let us turn to our main results, i.e., $g \gg \gamma_\mu$. To determine the influence of the coupling strength g, we keep ω_μ and γ_μ invariant and plot the heat currents in Fig. 3 at different values of g. One will immediately see that a large g directly leads to the suppression of the heat current $\dot{Q}_C$. Compared with the case of weak coupling, the high-temperature T_H could have a negative effect on the cooling of the cold atom. It is obvious that atom C cannot be cooled if the coupling strength g is too high, which is opposite to the case of the weak-internal-coupling regime. In particular, given T_C, T_R, and all the frequencies, one sees that cooling only happens within some range of T_H, which is also shown in Fig. 4. Thus the direct conclusion is that strong coupling is not beneficial to cooling from the refrigerator point of view. In addition, one also finds that the heat currents do not meet at a single point (temperature), which is quite different from the weak-coupling case. The heat currents do not change their direction simultaneously. In particular, the heat current $\dot{Q}_C$ seems not to be directly relevant to the virtual temperature T_v. The machine becomes a refrigerator only when $\dot{Q}_C > 0$, where one will find $T_H \simeq 27.25\ \mathrm{K} \neq 22.24\ \mathrm{K} = T_v$. For a refrigerator, the efficiency depends on the coupling constant g. The numerical results are given in Fig. 5. It is shown that the efficiency will become higher if g decreases. It will arrive at a constant efficiency ($\eta = \frac{\dot{Q}_C}{\dot{Q}_H} = \frac{\omega_C}{\omega_H} = \frac{1}{3}$) when it reaches the weak-coupling limit. However, the efficiency will change with T_H if it is still in the strong-coupling regime. The peak of the efficiency mainly results from the suppression of cooling induced by strong coupling. In particular, the suppression becomes strong for large T_H. When the reservoir H is hot enough, it can be heated instead of cooled, which can be obviously found for $g = 0.3\omega_H$. When the internal coupling becomes weak, the suppression will be weakened. If it is weak enough, the suppression will not be so apparent that the peak can be neglected to some good approximation, which is illustrated in Fig. 2. When $\frac{\omega_R}{T_R} = \frac{\omega_C}{T_C}$, one also finds that, no matter what the coupling constant is, it is impossible to make a refrigerator. However, from a different angle, we find that in

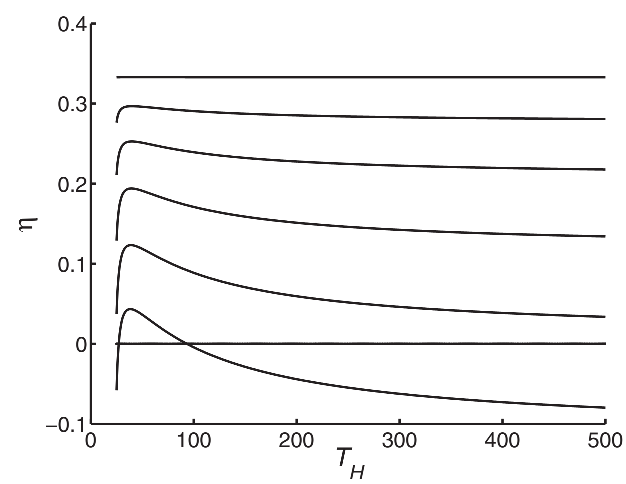

FIG. 5. Efficiency η versus $T_H[K]$ with different coupling constants. The lower straight line corresponds to zero efficiency. From top to bottom, the lines correspond to $g = 0.001\omega_H$, $g = 0.1\omega_H$, $g = 0.15\omega_H$, $g = 0.2\omega_H$, $g = 0.25\omega_H$, and $g = 0.3\omega_H$. In particular, $\eta =$ const. within acceptable approximations for $g = 0.001\omega_H$, which is consistent with Fig. 2.

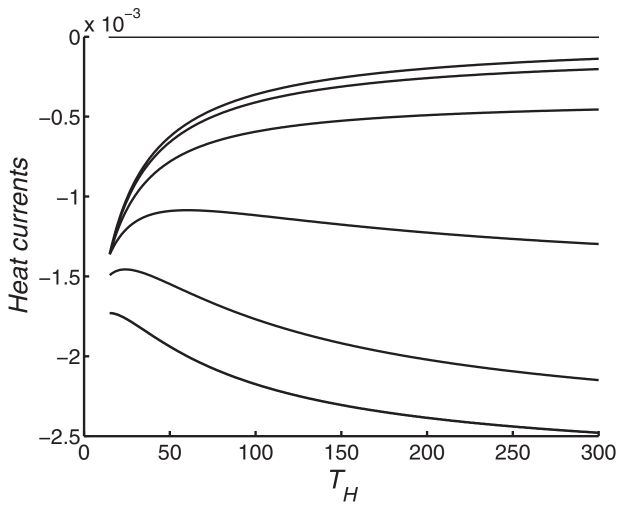

FIG. 6. Heat currents $\dot{Q}_C[J/s]$ versus $T_H[K]$ for different coupling constants. Here we let $T_C = 10$ K and $T_R = 40$ K in order to satisfy $\frac{\omega_C}{T_C} = \frac{\omega_R}{T_R} = \frac{1}{10}$. The upper straight line represents zero heat current. From top to bottom, the lines correspond to $g = 0.001\omega_H$, $g = 0.1\omega_H$, $g = 0.2\omega_H$, $g = 0.3\omega_H$, $g = 0.4\omega_H$, and $g = 0.5\omega_H$, respectively.

the weak-coupling limit, $\dot{Q}_C$ is reduced if we increase T_H. On the contrary, when the coupling is strong, $\dot{Q}_C$ becomes large with increasing T_H. This is shown in Fig. 6.

IV. CONCLUSIONS

In summary, we have revisited the self-contained refrigerator in the strong-internal-coupling regime by employing the QOME. We find that strong internal coupling reduces the cooling ability. In particular, in this regime, the considered machine demonstrates quite different (and even converse) thermodynamic behaviors compared with that in Ref. [28]. In addition, we find that the QOME provides results consistent with Ref. [28] in the weak-internal-coupling regime, even though the rotating-wave approximation, in principle, does not allow weak internal coupling. This could shed new light on the validity of the master equation.

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[1] S. Carnot, *Réflexions sur la Puissance Motrice du Feu* (Bachelier Libraire, Paris, 1824).
[2] G. Gemma, M. Michel, and G. Mahler, *Quantum Thermodynamics* (Springer, Berlin, 2004).
[3] E. P. Gyftopoulos and G. P. Beretta, *Thermodynamics: Foundations and Appications* (Dover, Mineola, NY, 2005).
[4] A. E. Allahverdyan, R. Ballian, and Th. M. Nieuwenhuizen, J. Mod. Opt. **51**, 2703 (2004).
[5] F. Tonner and G. Mahler, Phys. Rev. E **72**, 066118 (2005).
[6] M. Henrich, M. Michel, and G. Mahler, Europhys. Lett. **76**, 1057 (2006).
[7] M. Youssef, G. Mahler, and A.-S. F. Obada, Phys. Rev. E **80**, 061129 (2009).
[8] M. Henrich, G. Mahler, and M. Michel, Phys. Rev. E **75**, 051118 (2007).
[9] A. E. Allehverdyan and Th. M. Nieuwenhuizen, Phys. Rev. Lett. **85**, 1799 (2000).
[10] T. D. Kien, Phys. Rev. Lett. **93**, 140403 (2004).
[11] T. E. Humphrey and H. Linke, Physica E **29**, 390 (2005).
[12] D. Janzing *et al.*, Int. J. Th. Phys. **39**, 2717 (2000).
[13] J. Geusic, E. S. du Bois, R. D. Grasse, and H. Scovil, J. Appl. Phys. **30**, 1113 (1959).
[14] H. Scovil and E. S. du Bois, Phys. Rev. Lett. **2**, 262 (1959).
[15] J. Geusic, E. S. du Bois, and H. Scovil, Phys. Rev. **156**, 343 (1967).
[16] R. D. Levine and O. Kafri, Chem. Phys. Lett. **27**, 175 (1974).
[17] A. Ben-Shaul and R. D. Levine, J. Non-Equilib. Thermodyn. **4**, 363 (1979).
[18] D. Segal and A. Nitzan, Phys. Rev. E **73**, 026109 (2006).
[19] E. Geva and R. Kosloff, J. Chem. Phys. **96**, 3054 (1992).
[20] E. Geva and R. Kosloff, J. Chem. Phys. **104**, 7681 (1996).
[21] T. Feldmann and R. Kosloff, Phys. Rev. E **61**, 4774 (2000).
[22] J. P. Palao, R. Kosloff, and J. M. Gordon, Phys. Rev. E **64**, 056130 (2001).
[23] R. Kosloff and T. Feldmann, Phys. Rev. E **82**, 011134 (2010).
[24] T. Feldmann, E. Geva, and P. Salamon, Am. J. Phys. **64**, 485 (1996).
[25] T. Feldmann and R. Kosloff, Phys. Rev. E **68**, 016101 (2003).
[26] H. T. Quan *et al.*, Phys. Rev. E **76**, 031105 (2007).
[27] T. E. Humphrey, R. Newbury, R. P. Taylor, and H. Linke, Phys. Rev. Lett. **89**, 116801 (2002).
[28] N. Linden, S. Popescu, and P. Skrzypczyk, Phys. Rev. Lett. **105**, 130401 (2010).
[29] N. Linden, S. Popescu, and P. Skrzypczyk, arXiv:1010.6029v1 [quant-ph].
[30] P. Skrzypczyk, N. Brunner, N. Linden, and S. Popescu, J. Phys. A: Math. Theor. **44**, 492002 (2011).
[31] S. Popescu, arXiv:1009.2536v1 [quant-ph].
[32] N. Brunner, N. Linden, S. Popescu, and P. Skrzypczyk, Phys. Rev. E **85**, 051117 (2012).
[33] D. F. Walls and G. J. Milburn, *Quantum Optics* (Springer-Verlag, Berlin, 1994).
[34] H. P. Breuer and F. Petruccione, *The Theory of Open Quantum Systems* (Oxford University Press, Oxford, UK, 2002).
[35] C. Majenz, T. Albash, H.-P. Breuer, and D. A. Lidar, Phys. Rev. A **88**, 012103 (2013).
[36] T. M. Nieuwenhuizen and A. E. Allahverdyan, Phys. Rev. E **66**, 036102 (2002).