

**Stochastic resonance in bistable spin-crossover compounds with light-induced transitions**

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This article presents a theoretical prediction of stochastic resonance in spin-crossover materials. The analysis of stochastic resonance phenomenon in a spin-crossover system is performed in the framework of the phenomenological kinetic model with light-induced transition described by dynamical potential in terms of the Lyapunov functions. By using numerical simulation of stochastic trajectories with white- and colored-noise action, the evaluation of stochastic resonance is carried out by signal-to-noise ratio of the system output. The corresponding signal-to-noise ratio features a two-peak behavior which is related to the asymmetric shape of the dynamic potential. For the case of the Ornstein-Uhlenbeck process, the variations of resonance condition with respect to different autocorrelation times are additionally studied.

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**I. INTRODUCTION**

Stochastic resonance (SR) is a cooperative phenomenon arising from the interplay between deterministic and random dynamics in a nonlinear system wherein the coherent response to a deterministic monochromatic signal can be enhanced by the presence of an optimal amount of noise. Since its inception in 1981 [1], SR [2–6] has been found in various systems including sensory neurons, mammalian neuronal tissue, chemical solutions, lasers, Schmitt triggers, SQUIDS, tunnel diodes, communications devices, etc. One of the main characteristics used to identify SR phenomena is the signal-to-noise ratio (SNR). The study of the SR phenomenon by SNR, in the framework of linear-response theory based on the rate equation approach, was developed by McNamara and Wiesenfeld and presented in [7]. SNR is a commonly used measure of the information content from the response of a system. The phenomenon of stochastic resonance has attracted considerable interest in the past decade due, among other aspects, to its potential technological applications for optimizing the output SNR in nonlinear dynamical systems. The function of SNR has a broad maximum for noise strengths chosen such that the escape time from one potential well into the other of the unmodulated system is comparable to the period of the harmonic force. This nonmonotonic behavior of the SNR is commonly referred to as SR. The common features to most of the systems showing SR phenomenon are bistability and two input signals—deterministic monochromatic periodic signal and random forcing. Although SR was studied analytically in some limit cases, the basic tool for its investigation is the numerical simulation of stochastic differential equations.

SR has also been identified in various magnetic systems [8–14]. In this paper we show that SR phenomenon also happens in spin-crossover compounds, an example of bistable magnetic molecular materials. Fe(II)-based spin-crossover (SC) compounds are prototypic bistable magnetic molecular systems. They possess two stable states with different total

spin number  $S$ : the so-called high spin (HS) paramagnetic state with  $S = 2$  and the diamagnetic low spin (LS) one with  $S = 0$ . The system can be switched from one state to another by means of temperature, pressure, magnetic field, or light. The occurrence of photoinduced spin transition is related to the light-induced excited spin state trapping (LIESST) effect [15]. The LIESST effect is an intra-atomic phenomenon widely used for molecular magnets based on Fe(II) HS sites [16]. The photoexcitation dynamics [17,18] and characteristic sigmoidal relaxation have been reported for highly cooperative systems and can be well described by a macroscopic evolution equation. The major practical interest of this area of research is related to the potential use of optical switching in data storage and display devices. Physical properties of these compounds undergo abrupt structural, optical, and magnetic changes. Generally, the behavior of SC materials is caused by magnetic transitions with spin multiplicity change in certain compounds having the central transition metal ion ( $d^4$ – $d^7$  electron configuration) located in octahedral ligand.

The paper is organized as follows. In the next section we focus on the bistable properties of the spin-crossover complexes induced by light irradiation. We first discuss the macroscopic phenomenological model for light-induced transitions and its application for SR phenomena. In Sec. III we analyze the SR in spin-crossover compounds under white noise. The study of SR in spin-crossover compounds driven by colored noise and deterministic monochromatic forcing is considered in Sec. IV. Finally, the conclusions are drawn in Sec. V.

**II. MODEL**

For this study, we chose the macroscopic phenomenological model of photoinduced transitions described in terms of competing photoexcitation and relaxation processes. The dynamics of the spin-crossover system may be described by the changes of fraction of HS molecules  $n_H$  according to the following evolution equation written in terms of transition rates [19–21]:

$$\frac{dn_H}{dt} = \beta(1 - n_H) - n_H \exp(-\alpha n_H) \equiv f(n_H). \quad (1)$$

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Here, the first term  $\beta(1 - n_H)$  describes the photoinduced transition from LS to HS system states with transition probability per time unit  $\beta = I_0\sigma/k_{HL}^\infty$ , where  $I_0$  is intensity of light irradiation,  $\sigma$  is absorption cross section of optically active element, and  $k_{HL}^\infty$  is the high temperature asymptotic of the relaxation rates at the end of the process ( $T \rightarrow \infty$ ). The second term gives the relaxation dynamics of a system without light and is characterized by self-acceleration factor  $\alpha$ , which is determined by intermolecular interactions and is dependent on temperature.

For a more comprehensive study of spin-crossover system dynamics with additive noise action it is convenient to use the dynamic potential, in terms of Lyapunov functions [22–24]:

$$\frac{dn_H}{dt} = -\frac{dU(n_H)}{dn_H} + \xi(t), \quad (2)$$

where  $U(n_H) = -\int f(n_H)dn_H$  is the unperturbed potential of the spin-crossover system. The nonequilibrium potential  $U(n_H)$  is similar to the free energy in equilibrium phenomena and characterizes the behavior of the processes in the system [22]. Here  $\xi(t)$  is the additive stochastic term that describes the Gaussian white-noise action with the intensity  $\varepsilon$  and obeys the following statistical characteristics:

$$\langle \xi(t) \rangle = 0, \quad \langle \xi(t)\xi(t') \rangle = 2\varepsilon^2\delta(t - t'). \quad (3)$$

In the system with additive noise action, the transition between its states is possible, but it is a very rare event unless the values of noise intensity are comparable or larger than the height of the potential barrier. When an external weak periodic modulation is introduced in the system, the transition between the system states may occur with some regularity for the values of noise intensity lower than the potential barrier height, i.e., the stochastic resonance takes place [2,5]. We assume that the external periodic modulation can be introduced in the experiment in various ways, such as the irradiation of the sample by additional modulated source of light or the modulation of the applied pressure. In this case the system dynamics may be described in the following way:

$$\frac{dn_H}{dt} = -\frac{dU(n_H)}{dn_H} + A \cos\left(\frac{2\pi}{T}t + \phi_0\right) + \xi(t), \quad (4)$$

where  $A$  is the amplitude of periodic signal,  $T$  is its period, and  $\phi_0$  is the initial phase. The arbitrary phase  $\phi_0$  can be chosen to be zero. The model (4) with conditions (3) describes the overdamped Brownian motion in the bistable asymmetric potential subject to a small periodic forcing.

The evolution of the system with periodic forcing potential driven by additive noise may be described by the linear Fokker-Planck equation for the probability density, which, in the Stratonovich interpretation, is the following [25,26]:

$$\frac{\partial P(n_H, t)}{\partial t} = \hat{L}P(n_H, t) - \frac{\partial}{\partial n_H} A \cos\left(\frac{2\pi}{T}t\right) P(n_H, t), \quad (5)$$

with the Fokker-Planck operator

$$\hat{L} = -\frac{\partial}{\partial n_H} \left( -U'_{n_H}(n_H) \cdot \right) + \varepsilon^2 \frac{\partial^2}{\partial n_H^2}. \quad (6)$$

Here, dots indicate where to put the objects upon which the operator acts, while  $U'_{n_H}$  represents the first derivative

of dynamic potential on  $n_H$ . The value of  $P(n_H, t)dt$  is the probability to find the spin-crossover system described by the kinetic equation (4) in the position of phase space  $n_H$  at time  $t$ . Since the period  $T$  is relatively large, there is enough time for the system to reach the local equilibrium during this period, so the adiabatic limit [7] is valid. Consequently, a quasistate regime is established and the probability distribution of spin-crossover system states is given by the quasisteady solution of Fokker-Planck equation (5). The corresponding quasisteady solution is found to be

$$P_{st}(n_H, t) = N \exp\left[-\frac{-U(n_H) + A \cos\left(\frac{2\pi}{T}t\right)n_H}{\varepsilon^2}\right], \quad (7)$$

where  $N$  is the normalization constant. If the control parameters of the system are chosen in such a way that leads to a bistable behavior, the probability distribution gives two peaks that coincide with the minima of the dynamic potential corresponding to the LS and HS state.

The analysis of stochastic resonance phenomenon was carried out by analyzing the signal-to-noise ratio (SNR), that was found as the ratio between the first Fourier component of the output power spectral density at the angular frequency  $\omega = 2\pi/T$  and the level of the background noise at the same frequency. The standard SNR definition reads as follows:

$$\text{SNR} = \frac{\int_{\omega-\Delta\omega}^{\omega+\Delta\omega} S(z)dz}{\int_{\omega-\Delta\omega}^{\omega+\Delta\omega} S_N(z)dz}. \quad (8)$$

Here,  $S$  is the spectral power of the resulting signal in the neighborhood of the frequency  $\omega$ , whereas  $S_N$  is the spectral power of the noise level at the same frequency. Qualitatively, power spectral density  $S(\omega)$  may be described as the superposition of a background power spectral density  $S_N(\omega)$  and a structure of  $\delta$  spikes centered at  $\omega = (2n + 1)\frac{2\pi}{T}$  with  $n = 0, \pm 1, \pm 2, \dots$ . The studying of SR phenomenon by SNR lies on SNR increase for a periodically modulated system with random noise, and relative to that, observed with no externally injected noise. Therefore, the SNR measures how much the system output contains the input signal frequency  $\omega$ . The ways to define SNR for the systems with different dynamic potential are considered in [27].

For our system, the SNR was computed from the stochastic experiments by finding the average Fourier transform over the ensemble of simulated stochastic trajectories. This scheme of calculating SNR is closer to the experimental ones [28–31].

### III. SR IN SPIN-CROSSOVER COMPOUNDS DRIVEN BY WHITE NOISE

As previously shown, the evolution of the spin-crossover system may be described by the dynamic potential in terms of Lyapunov function [22–24,32]. The various kinetic properties of LS and HS species of spin-crossover compounds lead to the asymmetrization of system potential from which diverse responses on external perturbation of physical fields are generated for stable and metastable states [32]. In this context, the problem of stochastic resonance can reveal very interesting and important properties for nonlinear spin-crossover systems. The LIESST long-lived metastable HS states have been

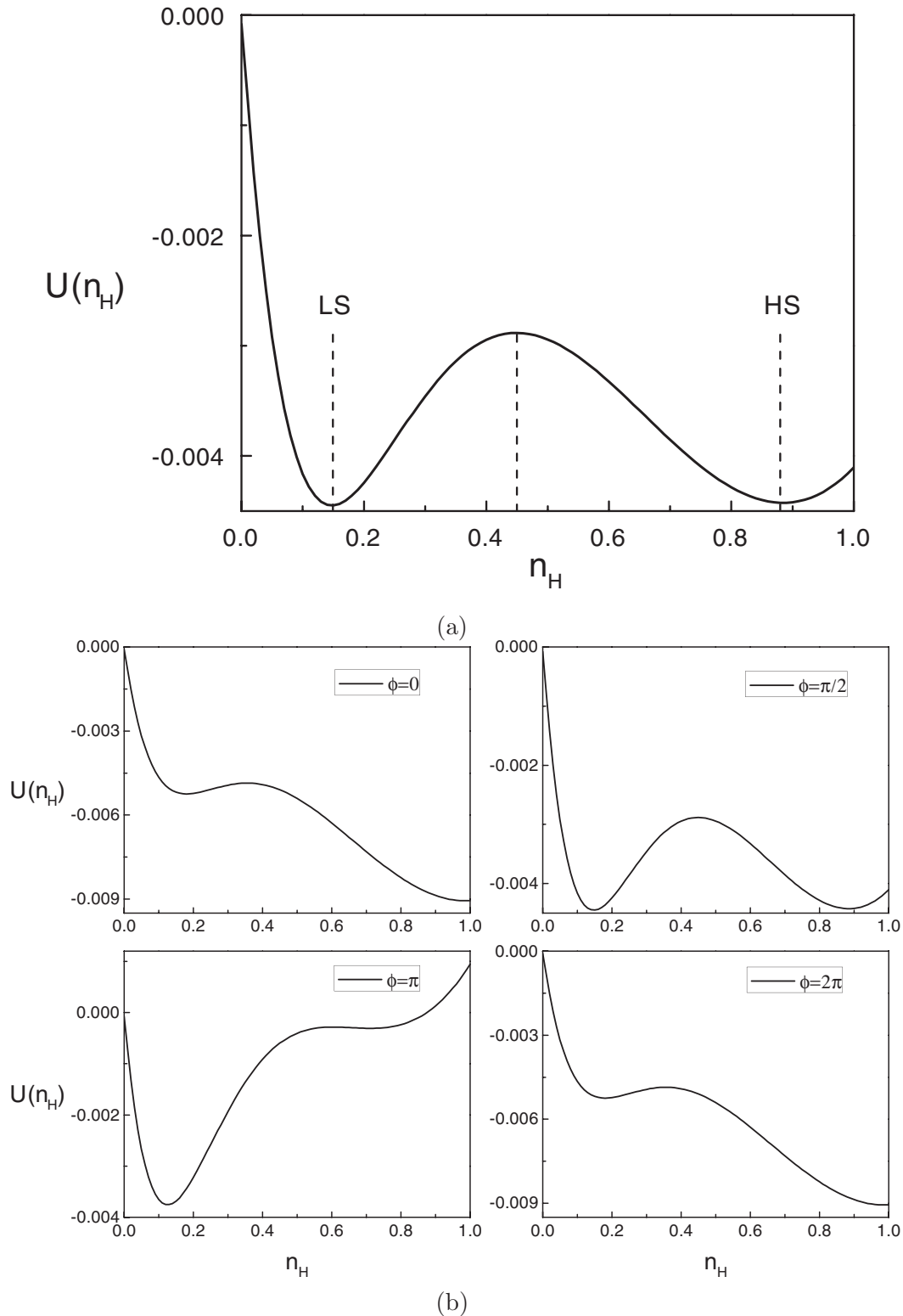


FIG. 1. Deterministic dynamic potential (a) and its changes due to the periodic forcing signal with the amplitude  $A = 0.005$  (b). Hereinafter the other parameters of periodic signal are  $T = 10000$  and  $\phi_0 = 0$ .

observed in various Fe(II) spin-crossover systems [33]. The performed simulations use the parameters that characterize  $[\text{Fe}(\text{ptz})_6](\text{BF}_4)_2$  but they can reflect the behavior of a wide range of Fe(II) spin-crossover systems. Thus a dynamic potential with the same values of the potential wells is used in

this study, corresponding to the light intensity  $\beta = 0.0811$  and the self-acceleration factor  $\alpha = 5.14$ . The positions of LS state ( $n_H = 0.15$ ) and HS state ( $n_H = 0.88$ ) are asymmetric relative to the unstable point of the dynamic potential ( $n_H = 0.45$ ), as reflected by Fig. 1(a). Although the depths of potential wells

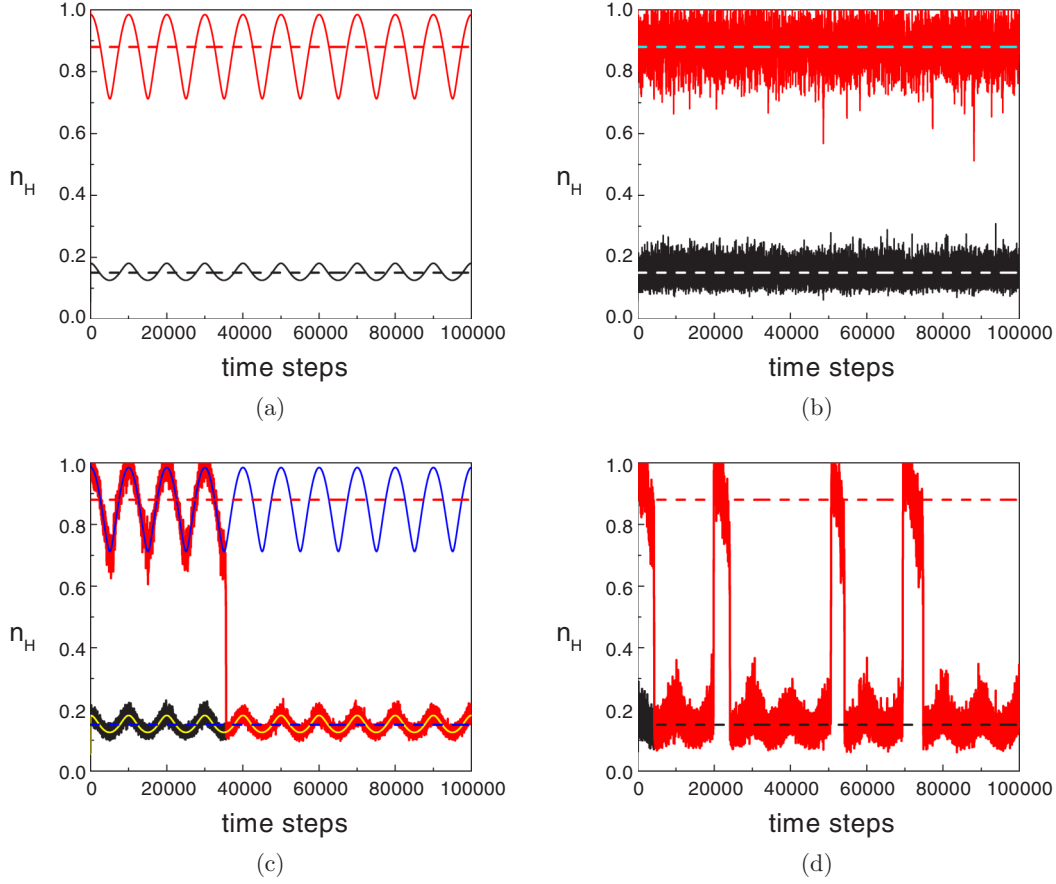


FIG. 2. (Color online) Evolution of the system perturbed only by the periodic signal with amplitude  $A = 0.005$  (a), only by noise with intensity  $\varepsilon = 0.015$  (b), and for simultaneous action of periodic signal with amplitude  $A = 0.005$  and noise with intensities  $\varepsilon = 0.006$  (c) and  $\varepsilon = 0.015$  (d). The system steady states are at  $n_H = 0.15$  for LS configuration and  $n_H = 0.88$  for HS configuration (dash lines).

are the same, the dynamic potential cannot be regarded as completely symmetric, and the asymmetric behavior of the system is still preserved.

Besides the action of the control field, we add the small external periodic signal that modulates the system potential. Depending on the phase of the periodic signal, a system state may change from stable to metastable, and vice versa, as shown in the Fig. 1(b). The chosen amplitude of periodic force is insufficient to overcome the potential barrier defined by the value of control parameter  $\beta = 0.0811$  and does not provide the transitions between the states, i.e., the deterministic system oscillates around its steady LS state or HS state, depending on the initial position of the system, with the period  $T$  of harmonic force.

The situation cardinally differs for the system with additive noise. The nonstationary dynamics of the system with periodic forcing and noise action is described by Eq. (4). This stochastic differential equation may be solved numerically by using Heuns methods [24,34]. The Heuns algorithm is based on the second-order Runge-Kutta-type method, and integrates the stochastic equation (4) by the following recursive formula:

$$n_H(t+h) = n_H(t) + \frac{h}{2}[F(n_H(t)) + F(y(t))] + \varepsilon\sqrt{\frac{h}{2}}u(t),$$

$$y(t) = n_H(t) + F(n_H(t))h + \varepsilon\sqrt{2hu}(t). \quad (9)$$

Here,  $F(n_H) = f(n_H) + A \cos(\frac{2\pi}{T}t)$  is the force derived from the bistable potential;  $h$  is the integration time step;  $u(t)$  represent the Gaussian distributed random number with variance one obtained by the Box-Muller algorithm and a pseudo-random-number generator [35].

The set of numerical solutions of Eq. (9) for each time step forms the sample stochastic trajectory, which describes the instantaneous position of the system in phase space. Figure 2 displays particular trajectories of the system for initial conditions corresponding to the LS or HS state: for periodic forcing with amplitude  $A = 0.005$  and zero noise intensity  $\varepsilon = 0$  (a); zero periodic forcing amplitude  $A = 0$  and noise intensity  $\varepsilon = 0.015$  (b); for simultaneous actions of periodic signals with amplitude  $A = 0.005$  and noise with intensity  $\varepsilon = 0.006$  (c) and for simultaneous actions of periodic signals with amplitude  $A = 0.005$  and noise with intensity  $\varepsilon = 0.015$  (d). In the deterministic (a) and noisy (b) dynamics with indicated parameters, the oscillations take place around the minima of the potential  $U(n_H)$  representing the steady states of the system. The simultaneous action of the periodic signal and the noise leads to transitions between the states, as shown in Fig. 2(c), and some kind of regular transition behavior becomes possible even for a subthreshold periodic signal, as shown in Fig. 2(d). For small noise intensity the system oscillates around the steady HS state by the period of deterministic signal up until the favorable conditions for

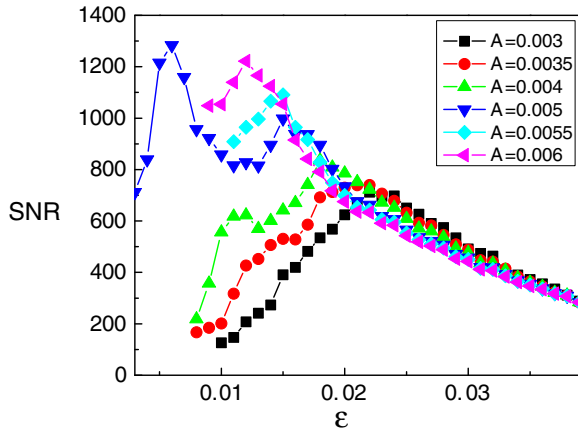


FIG. 3. (Color online) SNR versus additive noise intensity for fixed values of periodic signal amplitude.

transition appear. For unchanged conditions after transition, the system finally oscillates separately around the LS state if the noise intensity is sufficiently low, and transition may occur only as a very rare event. With further increasing of noise intensity the system becomes able to leave the LS state and transitions between the states are observed [see Fig. 2(d)]. In this case the system dynamics is partially synchronized with the periodic signal showing specific behavior for HS and LS state. As we can see further, the occurrence of such regular transitions assisted by noise represents the stochastic resonance phenomenon.

The trajectories starting from the LS and HS state are different due to the potential asymmetry, previously mentioned. For the stochastic case, the probabilities of population LS and HS states are unequal, although the depths of potential wells are the same, because the shape of the potential wells are different (see Fig. 1). That also generates different passage times between the two potential wells.

We characterize the SR phenomenon by SNR (8) obtained from numerical simulations carried out over an ensemble of 300 sample trajectories with 100 000 time steps each. The resulting SNR of the system was found as an average of SNR obtained for each trajectory calculated by Eq. (9) with initial condition  $n_H(0) = 1$ . Due to the unequal states response to the external perturbation, the system is sensitive to the choice of the initial value for numerical simulations [see Figs. 2(a)–2(c)]. However, the system behavior presented in Fig. 2(d) does not depend on initial conditions. For the sake of presentation, we show the simulations of system kinetics initialized from the more sensitive HS potential well. The resulting dependence of SNR on the noise intensity is presented in Fig. 3 for several values of the periodic signal amplitude. As we can see from these plots, the behavior of SNR is quite different from the one obtained for the classic symmetric double-well potential, where a single peak in the SNR curve is observed. Due to the asymmetry of the potential well, this spin-crossover system features two resonant noise intensities for a specific range of periodic amplitude. The apparition of a second peak in the SNR curve has been previously shown in [36] in the case of a periodically forced system subject to the simultaneous influence of additive and multiplicative noise. In the mentioned

work, the asymmetrization of the system dynamic potential is the result of multiplicative noise action. More sophisticated theory of phenomenon based on noise-induced transition and SR is known as double stochastic resonance [37]. This term emphasizes that additive noise causes a resonancelike behavior in the structure, which in its own turn is induced by multiplicative noise.

In our case, the two peaks in the SNR curve arise from different potential barrier height for LS and HS metastable phases of the dynamic potential and are a consequence of the system behavior described by Fig. 3 from [32]. The additive noise influence is similar to the action of temperature in thermodynamic equilibrium systems and leads to the effective reducing of the depths of potential wells together with its barrier height. The slope of the potential well for the HS state undergoes a more drastic change in comparison to the one for the LS state. The resonance intensity for the lower noise value corresponds to the escape from the HS metastable state as is indicated in Fig. 2(c). Due to a higher potential barrier of the LS state, the value of this intensity is not enough for a reverse transition. With further increasing of noise intensity, the transitions between LS and HS states take place [see Fig. 2(d)]. Thereby another peak for higher noise intensity is observed where more favorable conditions for transitions are realized. Thus there is a window of amplitude values for the periodic signal where two peaks in the SNR characteristic are observed. For the periodic amplitude lower than  $A = 0.0035$  and higher than  $A = 0.0055$ , only one resonance intensity appears, but the system dynamics for each case is different. For low periodic amplitudes, the peak on the SNR curve corresponding to small noise intensity cannot be observed due to the fact that its value is merged with the background ones. For high periodic amplitude, the metastable state is not realized and, consequently, the behavior of the spin-crossover system is similar to the one obtained for a monostable overdamped system [26].

The position of the SNR peaks is also dependent on the signal amplitude  $A$ . This dependence is presented in Fig. 4 for several noise strengths. The SNR shows a maximum as a function of modulation amplitude of the input periodic signal.

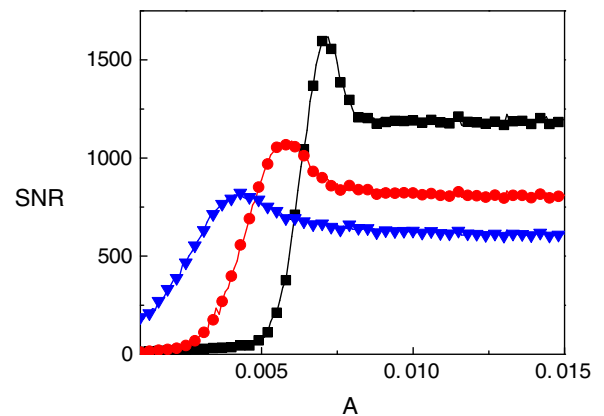


FIG. 4. (Color online) SNR versus modulation amplitude of periodic signal. Here, the white-noise strength is kept constant at the values  $\varepsilon = 0.01$  (black squares),  $\varepsilon = 0.015$  (red circles), and  $\varepsilon = 0.02$  (blue triangles).



With the increasing of the modulation amplitude  $A$  to a higher level, the SNR reaches the saturation. As one can see, the decrease in noise intensity  $\varepsilon$  leads to the increase of the SNR maximum, which is also shifted toward higher modulation amplitude. In addition, the SNR peak becomes sharper.

#### IV. SR IN SPIN-CROSSOVER COMPOUNDS DRIVEN BY COLORED NOISE

For the real spin-crossover compounds, the noise spectrum may have a large but finite bandwidth, i.e., the noise is colored. When an additive stochastic term is colored, the evolution equation of the system becomes intractable unless some assumptions are made regarding the noise nature. We consider here the Ornstein-Uhlenbeck (OU) process [38–40], which can be intuitively interpreted as a Brownian particle diffusing in a parabolic potential. Now the system dynamics may be described by the following equation:

$$\frac{dn_H}{dt} = -\frac{dU(n_H)}{dn_H} + A \cos\left(\frac{2\pi t + \phi_0}{T}\right) + \eta(t), \quad (10)$$

where the noisy relaxation process  $\eta(t)$  satisfies the stochastic differential equation:

$$\frac{d\eta(t)}{dt} = -\frac{1}{\tau}\eta(t) + \frac{\varepsilon}{\tau}\xi(t), \quad (11)$$

with the autocorrelation time of noise  $\tau$  representing the degree of noise coloration. The Ornstein-Uhlenbeck (OU) process  $\eta(t)$  with constant intensity  $\varepsilon^2$  has an exponential correlation function in the following form:

$$\langle \eta(t)\eta(t') \rangle = \frac{\varepsilon^2}{\tau} \exp\left(-\frac{|t-t'|}{\tau}\right). \quad (12)$$

The autocorrelation time is also related to the cutoff frequency characteristic to the Lorentzian power spectrum of OU noise:

$$S_\eta(\omega) = \frac{2\varepsilon^2}{\tau^2\omega^2 + 1}. \quad (13)$$

The correlation of white stochastic term  $\xi$ , with zero mean, between times  $t$  and  $t' > t$  is  $\langle \xi(t)\xi(t') \rangle = 2\delta(t' - t)$ . If we replace the term  $\eta(t)$  in Eq. (11) by its expression (10) one obtains the following general form of the kinetic equation for the system dynamics:

$$\tau \frac{dn_H^2}{dt^2} + \gamma(n_H, \tau) \frac{dn_H}{dt} - F(n_H) = \varepsilon \xi(t). \quad (14)$$

This is the equation of the stochastic nonlinear oscillator. For the system with simultaneous actions of periodic force and noise, the nonlinear damping function reads

$$\gamma(n_H, \tau) = 1 - \tau \left[ \frac{df(n_H)}{dn_H} - \frac{2\pi}{T} A \sin\left(\frac{2\pi}{T}t\right) \right]. \quad (15)$$

By using similar techniques for calculating SNR as in Sec. III, we found the SNR dependence on the intensity of the colored noise for the system dynamics described by Eq. (14). Sample results are shown in Fig. 5 for  $A = 0.003$  at fixed values of autocorrelation time  $\tau$ .

The system behavior under the influence on the colored OU noise features a relatively different SNR dependence on noise intensity than the one previously obtained for systems

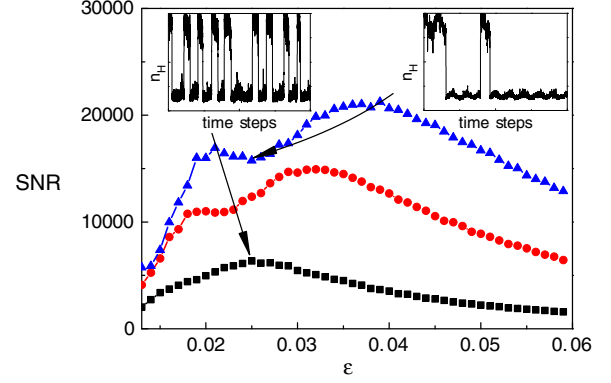
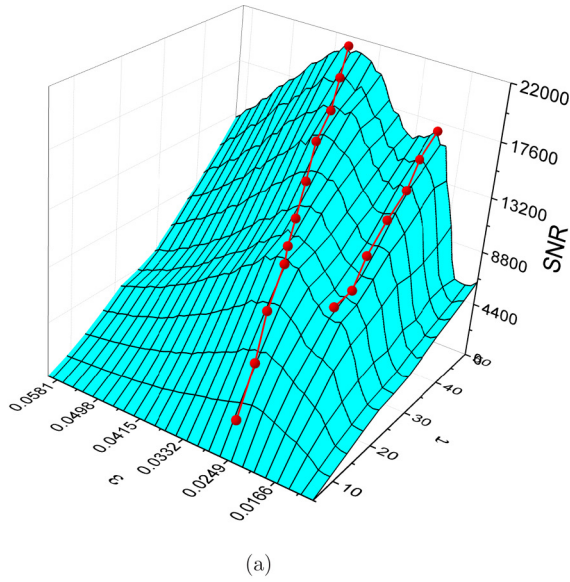


FIG. 5. (Color online) Changes of SNR with the increasing of the colored-noise intensity for fixed values of the autocorrelation time  $\tau$  and modulation amplitude  $A = 0.003$ . Here the curve marked by squares (black online) is for  $\tau = 10$ , circles (red online) are for  $\tau = 30$ , and triangles (blue online) are for  $\tau = 50$ . The insets show the sample trajectories for the noise intensity  $\varepsilon = 0.025$ .

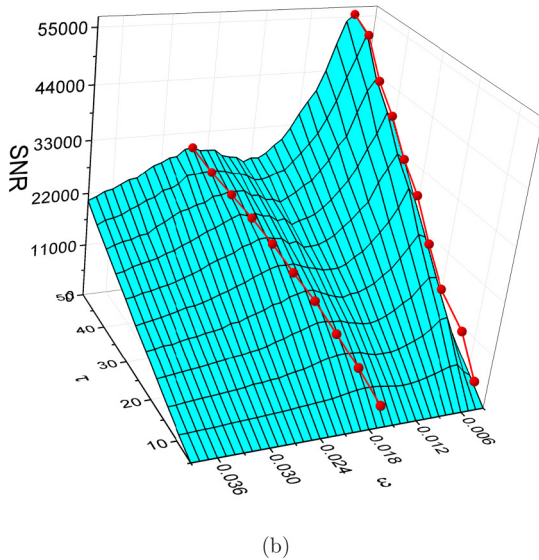
with white noise. For small autocorrelation time, only one peak in SNR dependence is observed; but with the increase of  $\tau$ , a second peak arises. The appearance of a second peak for small intensities of noise is related to the intrawells transition of the system that is illustrated in Fig. 2(c). Moreover, the increase of autocorrelation time  $\tau$  shifts the peaks of the SNR curve towards larger values of noise intensity and the SNR rises for all noise values. These increased values are related to the cutoff of the power spectrum, which leads to a lower contribution of noise in SNR. According to Eq. (13), for high autocorrelation time, the larger value of noise intensity is necessary to observe the SR phenomenon. In the insets of Fig. 5, we present two sample trajectories calculated for  $\tau = 10$  and  $\tau = 50$  corresponding to the noise intensity  $\varepsilon = 0.025$ , which is the resonant noise intensity for  $\tau = 10$ .

For a more comprehensive characterization of SNR behavior, we present the dependence of SNR on both  $\varepsilon$  and  $\tau$  in Fig. 6. As it is illustrated in this figure, the increase of  $\tau$  generates the shifts of SNR peaks, marked by spheres, towards higher  $\varepsilon$ , while a higher modulation amplitude leads to an increase of the SNR value. If the amplitude of the periodic signal is sufficiently high, the peak corresponding to intrawells transition becomes higher than the other one describing the resonance interwells transitions. This may be explained by the unequal sensitivity of LS and HS states on the same modulation signal and noise, which was illustrated in Fig. 2. As one can see, for the fixed noise intensity and the modulation amplitude, the system oscillations around the HS steady state are larger than those around the LS one.

In Fig. 7, we depict the dependence of noise intensity values  $\varepsilon$  of SNR peaks on the autocorrelation time of noise  $\tau$  for several values of periodic amplitude  $A$ . Here, the curves marked by squares and circles (black online) are obtained for periodic signal with amplitude  $A = 0.003$ , left and right triangles (red online) are for  $A = 0.0035$ , and up and down triangles (blue online) are for  $A = 0.004$ . These curves show the typical maxima that represent the fingerprint of the stochastic resonance phenomena. For a large amplitude of the periodic signal one can see two peaks of SNR. The situation



(a)



(b)

FIG. 6. (Color online) Dependence of SNR on  $\varepsilon$  and  $\tau$ . Here, the amplitude of periodic force is  $A = 0.003$  for (a) and  $A = 0.005$  for (b).

is different for a small periodic amplitude, where only one peak is observed for small  $\tau$ , but the second peak becomes apparent for higher autocorrelation times. The increasing of autocorrelation time  $\tau$  reduces the contribution of the noise, according to Eq. (13), and a second peak corresponding to the intrawells transition appears. As one can see from Fig. 7, with the increase of  $\tau$ , the shift of resonance noise intensity for the LS state is more evident than the one for the HS state. This is

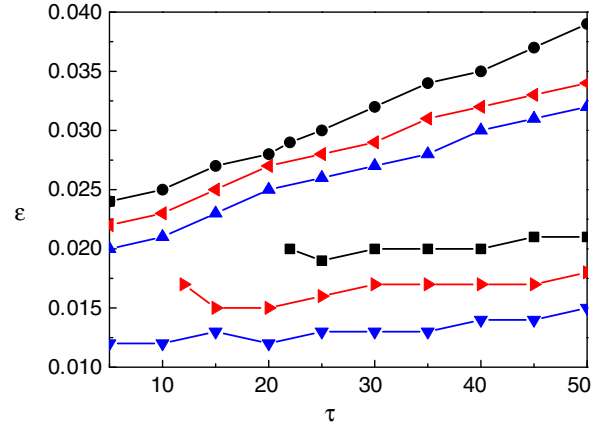


FIG. 7. (Color online) Behavior of SNR on  $\varepsilon$  and  $\tau$ . Here, the amplitudes of periodic signal are  $A = 0.003$  (black squares and circles),  $A = 0.0035$  (red left triangles and right triangles), and  $A = 0.004$  (blue up triangles and down triangles).

related to a greater sensitivity of the HS state to the external influence, in contrast to the LS one, as mentioned above.

## V. SUMMARY AND DISCUSSION

We have developed a theory for SR in spin-crossover compounds with photoinduced transitions. Based on the numerical simulation of the Langevin kinetic equation with a periodic modulating signal, we have analyzed the SNR in the case of white- and colored-noise actions.

The specific asymmetric dynamic potential of the spin-crossover system leads to unusual behavior during the SR phenomenon. The different responses of the LS and HS states to the external actions generate two peaks in SNR curves which correspond to various resonant noise intensity. The double-peaks behavior of the system is clearly observed only for a specific range of modulation amplitude. Similar behavior is observed in systems with colored-noise action, but it is influenced by the noise autocorrelation time. The increasing of noise autocorrelation time generates the shifting of SNR peaks, which correspond to the intrawells and interwells transitions, towards a higher noise intensity but with different shifts.

It is important to note that the concepts proposed in this paper may be also applied for other real bistable systems with an asymmetric dynamic potential.

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