Transport through quasi-one-dimensional wires with correlated disorder

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We study transport properties of bulk-disordered quasi-one-dimensional (Q1D) wires paying main attention to the role of long-range correlations embedded into the disorder. First, we show that for stratified disorder for which the disorder is the same for all individual chains forming the Q1D wire, the transport properties can be analytically described provided the disorder is weak. When the disorder in every chain is not the same, however it has the same binary correlator, the general theory is absent. Thus, we consider the case when only one channel is open and all others are closed. For this situation we suggest a semianalytical approach which is quite effective for the description of the total transmission coefficient. Our numerical data confirm the validity of this approach.

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I. INTRODUCTION

According to the theory of Anderson localization, all eigenstates in one-dimensional (1D) disordered models are exponentially localized independently on the degree of randomness [1–3]. The same result has been rigorously proved for quasi-one-dimensional (Q1D) systems [4]. To date, many details of global transport properties are known for Q1D systems describing multichannel wires with continuous random potentials (see for example [5,6]). On the other hand, some problems remain open for Q1D tight-binding models. Among these problems are the definition of the localization length and its relevance to mean free paths and the analytical description of transmission characteristics in terms of the localization length (see discussion and references in [7–14]). These problems are mainly caused by the nonhomogeneous character of transport in the channel space, a fact that cannot be neglected, especially when the number of channels M is not very large. Thus, in order to correctly describe all details of the transmission, one needs to know M characteristic lengths, which is in contrast to the single-parameter scaling emerging for 1D disordered models [15].

The situation turns out to be even more complicated for tight-binding Q1D models that can be considered as M-leg chains with interchain coupling. For example, for weakly disordered coupling between different channels (off-diagonal disorder) in the M-leg tight-binding hopping model the eigenstates may be delocalized at the center of the energy band [16,17]. Clearly, this effect is attributed to the off-diagonal disorder, however, even for a purely diagonal disorder the role of the interchannel interaction is not fully understood. The interest on tight-binding models with $M > 1$ has recently increased due to their relevance to specific physical systems, such as DNA models [18–22] and synthetic photonic lattices [23–25]. In such models a quite natural question emerges about the role of statistical correlations that are described by colored-noise potentials.

To be compared, in 1D disordered models the role of short- and long-range correlations has been already studied in detail (see for example review [26] and references therein). The interest on the problem of localization for 1D colored-noise potentials has been triggered by numerical studies of discrete dimer models for which the onset of delocalization has been observed numerically [27,28]. Later it was understood that such delocalization, appearing at discrete energy values, does not contradict the general statement of the theory of localization according to which the delocalization is not possible for 1D disordered potentials in *finite-width* energy windows. On the other hand, since in reality the size of disordered samples is always finite, one can speak about an *effective* delocalization when the localization length is much larger than the system size.

One of the results of the studies of tight-binding models with colored-noise disorder is the discovery that long-range correlations can lead to a vanishing Lyapunov exponent in a finite range of energy inside allowed energy bands [29,30]. Although this result is obtained for weak disorder in the first-order perturbation theory, one can speak about an emergence of effective mobility edges dividing the regions with localized and extended states. The theoretical prediction of arranging controlled energy windows with perfect transmission has been experimentally confirmed in microwave experiments with pointlike scatterers inserted into one-channel waveguides [31,32]. Alternatively, it was shown that with long-range correlations one can also strongly enhance the localization even when the disorder is very weak [33]. The important point is that such localization of eigenstates can be arranged in quite narrow energy regions, thus resulting in a strong selective reflection of scattering waves. Both numerical and experimental results have demonstrated robust anomalous properties of the scattering even if the sample size is quite small. It should be stressed that apart from specific colored-noise potentials that can be experimentally arranged, there are many physical situations where long-range correlations are not avoidable and have to be taken into account. One such situation occurs in experiments with interacting bosons in 1D optical lattices (see for example [34–36]).

In contrast to the problem of correlated disorder in 1D systems for which the theory is practically developed, the transport properties of Q1D systems with colored noise are studied very poorly. Among few models that were under close attention one can mention the tight-binding Anderson model with two-coupled chains [21], the models with a *layered* bulk

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disorder [37,38], and multimode waveguides with long-range correlations in surface profiles [39].

The aim of this presentation is to contribute to the theory of correlated disorder in the Q1D geometry. Specifically, we consider the Q1D model of the Anderson type in connection with the results obtained for 1D disordered models. As the first step we analyze the situation for which the scattering potential has specific long-range correlations in the longitudinal direction, however, does not depend on the transverse coordinate. As is shown in Ref. [37], for such *stratified* disorder the problem can be solved by the reduction of the Q1D scattering problem to the analysis of the scattering along M independent 1D channels. Indeed, the independence of the disorder on the transverse coordinate allows one to reduce the model to a coset of noninteracting channels characterized by different localization lengths [39,40]. However, still there is a problem to express the total transmission coefficient in terms of these lengths. A similar problem emerges even for 1D tight-binding disordered models for which the presence of resonant values of energy does not allow us to derive general expressions for the transmission and reflection coefficients valid for any value of energy inside the energy band (see discussion in Ref. [26]). There is no such a problem for continuous 1D random potentials for which the resonances are absent. Thus, the analytical treatment of the scattering problem for Q1D tight-binding models with stratified disorder presented in Ref. [37] is not complete. In this paper we suggest a semianalytical approach that allows us to solve this problem by borrowing the analytical expression for the transmission coefficient from the theory of localization fully developed for 1D models with continuous potentials. A similar procedure has been recently used in Ref. [41] for a potential consisting of a random set of barriers and/or wells of fixed thickness and random heights. Our numerical results demonstrate a very good agreement between numerics and the analytical expression, even if the disorder is not very weak and sample sizes are finite. It should be stressed that for Q1D models with stratified correlated disorder one gets a quite unusual (nonmonotonic) dependence for the total conductance as a function of energy [37,38].

As a second step, we study a much more complicated model where the stratification of the disorder is broken. Specifically, we consider the case when along every chain forming the Q1D wire the long-range correlations are of the same kind, however, specific realizations of the disorder are different for each channel. Thus, there are no correlations in the transverse direction, however, in the longitudinal direction the correlations persist. For the case of two coupled chains such a model has a clear relevance to the DNA molecules for which long-range correlations are found from experimental data. The analytical treatment for more than two channels, $M > 2$, is absent due to an extremely complicated character of the transmission along the chains, and to an unavoidable mixing between various channels of propagation. In this situation, we have found a way to suggest a phenomenological expression for the Lyapunov exponent that appears to work quite well for a specific case when only one channel is open in the energy space and all other $M-1$ channels are closed.

This paper is organized as follows. In the next section we define the model of Q1D wire and give basic relations for the characterization of scattering properties. Specifically, we show how the total transmission coefficient can be expressed in terms of the independent partial transmission coefficients corresponding to the 1D wires composing the Q1D structure. In Sec. III we explain the approach according to which we numerically compute the scattering properties of the Q1D correlated wires, based on an effective non-Hermitian Hamiltonian approach to scattering. In Sec. IV for the stratified disorder we verify our analytical predictions by comparing them with numerical data, and show that for long-range correlations the conductance reveals a highly unexpected nonhomogeneous energy dependence. In Sec. V we study the specific case when along each chain the long-range correlations are of the same kind, however, individual realizations in the channels are different. Thus, in the transverse direction the correlations are absent. For this situation we suggest a procedure according to which one can get an analytical description of the transmission coefficient in dependence of the control parameters of the model. Finally, in Sec. VI we draw our conclusions.

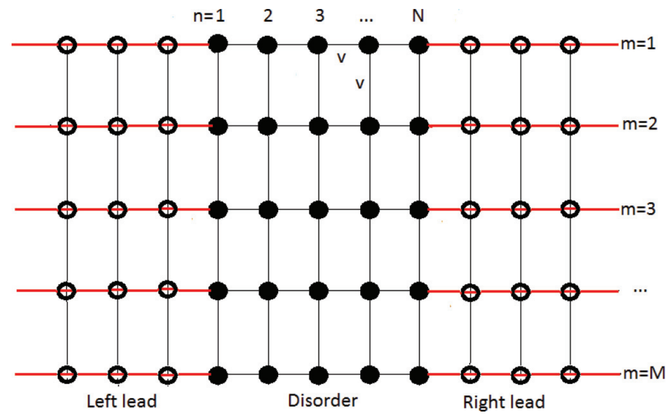

FIG. 1. (Color online) Disordered Q1D wire of length N and width M (full circles) connected at both ends to semi-infinite ideal leads of width M (open circles).

II. MODEL AND SCATTERING SETUP

The model consists of a rectangular array of sites of length N and width $M \ll N$, with nearest-neighbor couplings; see Fig. 1. The Hamiltonian corresponding to this setup has the following form:

$$\langle n,m|H|n',m'\rangle = \epsilon_{nm}\delta_{nn'}\delta_{mm'} - v(\delta_{nn'}\delta_{m,m'+1} + \delta_{nn'}\delta_{m,m'-1} + \delta_{n,n'+1}\delta_{mm'} + \delta_{n,n'-1}\delta_{mm'}). \tag{1}$$

The on-site entries ϵ_{nm} are assumed to be random numbers whose statistical properties will be specified below, while the coupling amplitudes v between sites are considered to be constant. The disordered wire is connected to semi-infinite tight-binding leads of width M marked in Fig. 1 by open circles. In the leads, for $n \leqslant 0$ and $n > N$, the disorder is absent and the coupling amplitudes between the sites in the leads are also fixed to v.

The corresponding stationary Schrödinger equation for the eigenstates ψ_{nm} of energy E reads

$$v(\psi_{n,m+1} + \psi_{n,m-1} + \psi_{n+1,m} + \psi_{n-1,m}) = (E - \epsilon_{nm})\psi_{nm}. \tag{2}$$

Without disorder, $\epsilon_{nm} = 0$, the solutions ψ_{nm} are plane waves with wave numbers μ_q in the transverse direction. In the following, the index $q = 1,\ldots,M$ is treated as the channel (or mode) number. In our model we assume zero boundary conditions in the transverse direction, $\psi_{n,0} = \psi_{n,M+1} = 0$, therefore, the dispersion relation takes the form [12]

$$2v\cos\mu_q = E - 2v\cos\left(\frac{\pi q}{M+1}\right), \quad q = 1,\ldots,M. \tag{3}$$

From this relation one can see that the qth channel is open as long as the energy fulfills the condition inside the interval,

$$-2v \leqslant E - 2v\cos\left(\frac{\pi q}{M+1}\right) \leqslant 2v, \tag{4}$$

and outside it μ_q becomes imaginary. In fact, the latter equation determines the number $M_1(E)$ of open modes in dependence of the energy. For example, at the band center, $E = 0$, all modes are open and $M_1 = M$. While for $|E|/2v > 1 + \cos(\pi/M + 1)$ all modes are closed. Taking into account that

$$\cos\left(\frac{\pi q_1}{M+1}\right) > \cos\left(\frac{\pi q_2}{M+1}\right)$$

for $q_2 > q_1$, one can obtain the expression for the number of open modes in terms of the energy and channel number q,

$$M_1 = \begin{cases} q & \text{if} \quad \cos\left(\frac{(q+1)\pi}{M+1}\right) < \frac{|E|}{2v} - 1 < \cos\left(\frac{q\pi}{M+1}\right), \\ 0 & \text{if} \quad \frac{|E|}{2v} > 1 + \cos\left(\frac{\pi}{M+1}\right). \end{cases} \tag{5}$$

Then, once the model for Q1D wires has been introduced, in the next section we describe the non-Hermitian Hamiltonian approach, i.e., the approach we shall use to describe the transmission through Q1D disordered wires.

III. NON-HERMITIAN HAMILTONIAN APPROACH

In order to analyze the transport properties of our model, in what follows we use the *non-Hermitian Hamiltonian* approach (see for example [42–45]). The key point of this approach is based on the projection of the total Hermitian Hamiltonian $\mathcal{H}$ [disordered part plus leads; see Eq. (1)] onto the basis defined by the Hamiltonian $H^{(\text{int})}$ describing the properties of the *closed* model (only disordered part in Fig. 1). In this way the leads are considered as a continuum to which the disordered part is coupled according to given boundary conditions. The knowledge of the effective Hamiltonian $H^{(\text{int})}$ allows one to construct the scattering matrix and, as a result, all transport properties can be obtained.

For our model the non-Hermitian Hamiltonian expressed in the site basis $|n,m\rangle$ has the following form:

$$\langle n,m|\mathcal{H}(E)|n',m'\rangle = \langle n,m|H^{(\text{int})}|n',m'\rangle - \sum_{q=1}^{M_1} e^{i\mu_q}(\gamma_L\delta_{n1} + \gamma_R\delta_{nN})\delta_{nn'}P_{qm'}P_{qm}. \tag{6}$$

Here $H^{(\text{int})}$ is the Hermitian Hamiltonian of the internal system, i.e., $H^{(\text{int})}$ represents the Q1D wire (of length N and width M) and has zero boundary conditions at $n = 0$ and $n = N + 1$. The second term in the right hand side of Eq. (6) corresponds to the coupling of the internal system to the leads. In the general case the coupling is characterized by two parameters, γ_L and γ_R, with L and R denoting the left and right leads, respectively. In our study, for simplicity, we assume symmetric couplings, $\gamma_L = \gamma_R = \gamma$. As is defined above, μ_q stands for the wave number at the qth channel and is related to the energy E through the dispersion relation (3). The elements P_{ij} in Eq. (6) are the eigenstates of a 1D tight-binding chain of size M in the absence of disorder,

$$P_{ij} = \sqrt{\frac{2}{M+1}}\sin\left(\frac{\pi ij}{M+1}\right). \tag{7}$$

Equation (6) can be written in the matrix form as follows:

$$\mathcal{H}(E) = \mathbf{H}^{(\mathbf{int})} + 2\pi\mathbf{A}\mathbf{Q}(E)\mathbf{A}^T - i\pi\mathbf{A}\mathbf{A}^T. \tag{8}$$

Here $\mathbf{H}^{(\mathbf{int})}$ is the $NM \times NM$ Hermitian matrix with ordered matrix elements $\langle n,m|H^{\text{int}}(E)|n',m'\rangle$. The second Hermitian and third non-Hermitian terms in the right hand side of Eq. (8) represent the real and imaginary parts of the coupling to the leads, respectively. As for the coupling matrix $\mathbf{A}$ of size $MN \times 2M_1$, it is composed by the ordered coupling amplitudes $\mathbf{A} = \{A_{mn}^{(Lq)}, A_{mn}^{(Rq)}\}$ between the internal states $|n,m\rangle$ and the open left and right channels Lq and Rq, respectively. Thus, the *coupling amplitudes* are given by

$$\begin{aligned} A_{mn}^{Lq} &= (\gamma_L/\pi)^{1/2}\sqrt{\sin\mu_q}\,P_{qm}\delta_{n1}, \\ A_{mn}^{Rq} &= (\gamma_R/\pi)^{1/2}\sqrt{\sin\mu_q}\,P_{qm}\delta_{nN}. \end{aligned} \tag{9}$$

$\mathbf{Q}(E)$ is the $2M_1 \times 2M_1$ diagonal matrix with real elements ordered as $\{Q_1,Q_2,\ldots,Q_{M_1},Q_1,Q_2,\ldots,Q_{M_1}\}$, where

$$Q_q(E) = -(\cot\mu_q)/2, \quad q = 1,\ldots,M_1(E). \tag{10}$$

Now, from the effective non-Hermitian Hamiltonian we can write down the scattering $\mathbf{S}$ matrix in the channel space as

$$\mathbf{S} = \begin{pmatrix} \mathbf{t} & \mathbf{r}' \\ \mathbf{r} & \mathbf{t}' \end{pmatrix} = \frac{1 + \mathcal{C}^\dagger\mathcal{K}}{1 + \mathcal{C}\mathcal{K}}, \tag{11}$$

where $\mathbf{t}$, $\mathbf{t}'$, $\mathbf{r}$, and $\mathbf{r}'$ are $M_1 \times M_1$ transmission and reflection matrices. Below, we chose the coupling parameter γ in such a way that the transmission in each channel is maximal. This means that we consider the so-called *perfect coupling* for which the average scattering matrix is zero, $\langle S\rangle = 0$. In our case, both for the 1D model with $M = 1$ and for the Q1D model with $M > 1$, the perfect coupling corresponds to $\gamma \approx v$ [45].

It can be shown that the matrix $\mathcal{C}$ in Eq. (11) of size $2M_1 \times 2M_1$ has the following structure:

$$\mathcal{C} = i\pi - 2\pi\mathbf{Q}(E).$$

As for the *reaction matrix* $\mathcal{K}$ (of the same size, $2M_1 \times 2M_1$), its matrix elements are defined by

$$\mathcal{K}_{ab}(E) = \sum_{nm}\frac{\widehat{A}_{nm}^{(a)}\widehat{A}_{nm}^{(b)}}{E - E_{nm}}, \quad \widehat{A}_{nm}^{(a,b)} = \sum_{rs}A_{rs}^{(a,b)}\psi_{rs}(E_{nm}). \tag{12}$$

Here, ψ_{rs} are the components of the eigenvector of the matrix $\mathbf{H}^{(\mathrm{int})}$ having the eigenvalue E_{nm}, and we have introduced the channel index $a,b \equiv cq$ that indicates which lead $c = L,R$ and mode q we refer to. Once the S matrix is known one can calculate the dimensionless conductance $g = (2e^2/h)T$, where $T = \mathrm{Tr}(tt^\dagger)$ is the transmission coefficient [46], with e and h being the charge of the electron and the Planck constant, respectively.

It should be noted that at the band center the relation (8) reduces to the simple form [47]

$$\mathcal{H} = \mathbf{H}^{(\mathbf{int})} - i\pi \mathbf{A}\mathbf{A}^T,$$

in which the coupling to continuum is described by the imaginary term only. In addition, in the 1D case the scattering matrix (11) takes the well known form

$$\mathbf{S} = \frac{1 - i\pi\mathcal{K}}{1 + i\pi\mathcal{K}}.$$

In the next section, before presenting numerical results for the transmission through Q1D disordered wires, we first elaborate on the stratified disordered case to be able to propose an analytical expression that will allow us to predict the transmission as a function of the energy of the incident wave.

IV. CORRELATED STRATIFIED DISORDER

A. Analytical results

In this section we consider the so-called stratified disorder for which the potential is independent of the transverse coordinate quantized by the index m, i.e.,

$$\epsilon_{n1} = \epsilon_{n2} = \cdots = \epsilon_{nM} \equiv \epsilon_n. \tag{13}$$

In this case our model can be reduced to a set of M 1D independent chains which are nothing but 1D tight-binding Anderson models [37]. To show this, first, we rewrite the Schrödinger equation (2) in the matrix form,

$$v(\mathbf{a}^{(n+1)} + \mathbf{a}^{(n-1)}) = (E - \mathbf{B}^{(n)} - \mathbf{C})\mathbf{a}^{(n)}, \tag{14}$$

where $\mathbf{a}^{(n)}$ is the vector with components ψ_{nm} ($m = 1,\ldots,M$). Here $\mathbf{B}^{(n)}$ and $\mathbf{C}$ are $M \times M$ matrices with elements given by

$$B_{ij}^{(n)} = \epsilon_{nj}\delta_{ij}, \quad C_{ij} = v(\delta_{i,j+1} + \delta_{i,j-1}). \tag{15}$$

Then, we pass to a new unperturbed basis through the transformation,

$$\mathbf{b}^{(n)} = \mathbf{P}\mathbf{a}^{(n)}, \tag{16}$$

where the columns of the matrix $\mathbf{P}$ are the eigenvectors of the Hamiltonian matrix $\mathbf{C}$ that correspond to the 1D Anderson model of size M with vanishing disorder, $\epsilon_i = 0$, and zero boundary conditions. Note that the corresponding eigenvectors and eigenvalues are analytically known. In this representation the Schrödinger equation (14) takes the form

$$v(\mathbf{b}^{(n+1)} + \mathbf{b}^{(n-1)}) = (E - \mathbf{P}^T\mathbf{B}^{(n)}\mathbf{P} - \mathbf{D})\mathbf{b}^{(n)}, \tag{17}$$

where the elements of the $M \times M$ matrix $\mathbf{D}$ are given by

$$D_{ij} = 2v\cos\left(\frac{\pi i}{M+1}\right)\delta_{ij}, \tag{18}$$

while the elements of the $M \times M$ matrix $\mathbf{P}$ are given by Eq. (7).

For the stratified disorder Eqs. (17) become uncoupled since $\mathbf{P}^T\mathbf{B}^{(n)}\mathbf{P} = \mathbf{B}^{(n)}$ is a diagonal matrix. Hence, the Q1D Anderson model is reduced to a set of M 1D chains, where the energy for each chain is given by

$$E_q = E - 2v\cos\left(\frac{\pi q}{M+1}\right), \quad q = 1,\ldots,M. \tag{19}$$

Let us now specify the properties of the stratified disorder. First, we assume the zero mean and small variance σ^2 of the site energies,

$$\langle\epsilon_n\rangle = 0, \quad \sigma^2 = \langle\epsilon_n^2\rangle \ll 1, \tag{20}$$

where $\langle\cdots\rangle$ denotes the average over different realizations of disorder. Apart from that, we assume that the statistical properties of disorder are defined by the two-point correlator describing long-range correlations. Therefore, an additional ingredient of the disorder is the specific form of the normalized binary correlator,

$$\chi(k) = \frac{\langle\epsilon_n\epsilon_{n+k}\rangle}{\sigma^2}, \tag{21}$$

to be defined below.

Since our model with the stratified disorder can be rigorously reduced to a set of M 1D chains, one can try to apply the theory of 1D localization developed for continuous potentials. According to this theory for weak disorder and in the limit $N \to \infty$, the eigenstates $b_q^{(n)}$ [in our case the components of the vectors $\mathbf{b}^{(n)}$; see Eq. (17)] are exponentially localized with the characteristic length $l_\infty^{(q)}$ related to the qth channel. As is known, the inverse localization length can be defined in terms of the Lyapunov exponent λ_q, where the analytical expression for the latter has the following form:

$$\lambda_q \equiv \frac{1}{l_\infty^{(q)}} = \frac{\sigma^2}{8\sin^2\mu_q}W(2\mu_q),$$
$$W(2\mu_q) = 1 + 2\sum_{k=1}^{\infty}\chi(k)\cos 2\mu_q k. \tag{22}$$

These equations were obtained by the use of the *Hamiltonian map approach* [48] following the procedure described in Refs. [29,49] (see also Ref. [26]). As one can see from (22), the correlation properties of the random sequence $\{\epsilon_n\}$ are entirely defined by the power spectrum $W(\mu)$ of the binary correlator. Note that the expression for λ_q is correct for weak disorder, $\sigma^2 \ll 1$; as for the higher moments of the correlations they may contribute to the localization length only in the next order of perturbation theory in the disorder parameter σ^2. We also would like to stress that Eq. (22) is generically valid for any random potential, provided it is statistically homogeneous; that is, the binary correlator $\chi(k)$ can be introduced according to Eq. (21). Indeed, Eq. (22) has been successfully put to test in the case of short- and long-range correlated potentials (see Ref. [26] and references therein). Also note that for a white-noise disorder we have $W(2\mu_q) = 1$.

Now we focus on the problem of scattering through the disordered region represented by full circles in Fig. 1. Since for stratified disorder the transmission along every 1D channel is independent from those along the other channels, the total transmission coefficient T can be expressed as the sum of partial coefficients T_q corresponding to the propagation of incident plane waves along the qth independent channels,

$$T(E) = \sum_{q=1}^{M_1} T_q(E). \tag{23}$$

This expression agrees with the Landauer concept of conductance [46]. Here $M_1(E)$ is the number of open modes in the leads given by Eq. (5).

Above we announced that we plan to use the analytical results developed for 1D *continuous* models. However, the model under consideration is a discrete model for which the theoretical analysis is restricted due to the presence of resonances in the energy space (see for example [26,50] and references therein). Specifically, there are no rigorous results for the probability distribution of T, or equivalently, for the corresponding moments for any energy inside the allowed energy band. To the contrary, for continuous random potentials with weak disorder the problem of scattering through finite 1D wires was rigorously solved by various analytical approaches [15,26]. In particular, there is an exact expression for the average transmission coefficient in terms of the ratio of the localization length l_∞ to the length N of the sample [15,26],

$$\langle T_q \rangle = \sqrt{\frac{2x_q^3}{\pi}} \exp\left(-\frac{1}{2x_q}\right) \int_0^\infty \frac{z^2}{\cosh z} \exp\left(-\frac{z^2 x_q}{2}\right) dz,$$
$$x_q = l_\infty^{(q)}/N. \tag{24}$$

In Eq. (24), we have added the index q in order to indicate to which channel we are referring. Here, the brackets stand for the average over a number of different realizations of the correlated disorder.

As was found in Ref. [51], expression (24) turns out to be very good even for the 1D tight-binding Anderson model, provided the energy values are not very close to the resonances. Moreover, recently a different approach has been developed in Ref. [41] allowing one to modify the standard perturbation theory in such a way that it gives good results also at the resonant energies. In particular, it was shown that Eq. (24) still gives a good description of numerical results at the resonances, provided the expression for the localization length takes into account the influence of those resonances. Thus, the relation (24), together with Eqs. (22) and (23), give us the possibility to obtain the expression for the transmission coefficient $\langle T(E)\rangle$ in dependence on the model parameters. Below we verify the validity of Eq. (24) by comparing it with numerical data.

B. Numerical data

Since the localization length $l_\infty^{(q)}$ of any qth conducting channel is fully determined by Eq. (22), if the power spectrum $W(2\mu_q)$ vanishes within some energy window, the corresponding channel will be fully transparent in that energy interval [29,30,52]. This prediction has been confirmed both numerically and experimentally for the 1D Anderson model (see details and references in Ref. [26]). Here our question is how can such an effect be seen in the Q1D model with the stratified disorder? To answer this question, we choose the following form of the binary correlator [26]:

$$\chi(k) = \frac{1}{2k(\mu_R - \mu_L)}(\sin 2\mu_R k - \sin 2\mu_L k). \tag{25}$$

As one can see, this correlator exhibits a power law decay which is typical for long-range correlated disorder. With this choice of $\chi(k)$, the correlator (25) results in the stepwise power spectrum,

$$W(E_q) = \begin{cases} W_0 & \text{if} \quad E_L \leqslant |E_q| \leqslant E_R, \\ 0 & \text{if} \quad |E_q| < E_L \text{ or } E_R < |E_q| \leqslant 2v, \end{cases} \tag{26}$$

having three well defined energy windows of total transparency in the 1D-chain case; see below and Ref. [53] where similar correlators $\chi(k)$ have been used. Here, μ_L and μ_R are related to E_L and E_R through the dispersion law for the 1D system, $E = 2\cos\mu$, and W_0 is determined by the normalization condition $\int_0^{\pi/2} W(\mu)d\mu = \pi/2$ (see details in Ref. [26]). Note that we have omitted the transverse index m since each chain has *the same* disorder sequence ϵ_n. In our numerical simulations we consider the following fixed values: $E_L = 0.4$, $E_R = 1.3$, $\sigma^2 = 0.02$, and $v = 1$.

Correlations with the power spectrum (26) have been already used in the literature to control the transport properties of 1D systems [31,52,54]. In Fig. 2 we demonstrate these

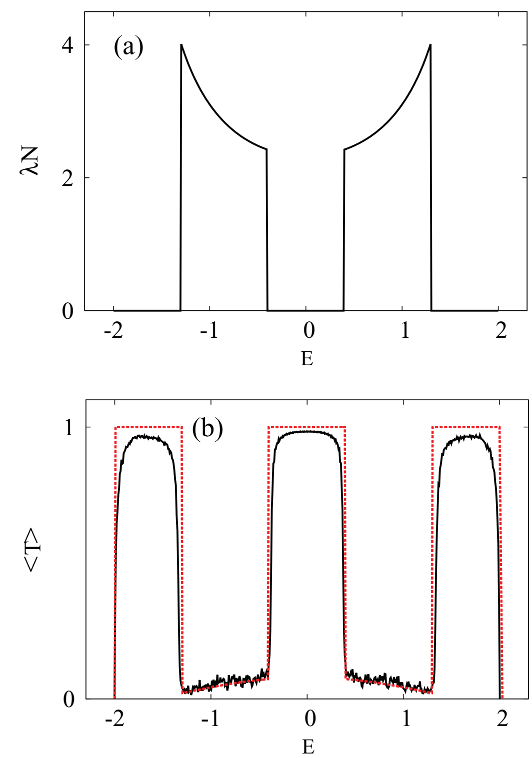

FIG. 2. (Color online) (a) Rescaled Lyapunov exponent λN as a function of the energy E for a one-chain wire, $M = 1$. The correlated disorder has been fixed by the stepwise power spectrum of Eq. (26) with $E_L = 0.4$, $E_R = 1.3$, $\sigma^2 = 0.02$, and $v = 1$. (b) Average transmission coefficient $\langle T\rangle$ as a function of E. The continuous curve corresponds to numerical data while the dashed one is the theoretical prediction from Eqs. (22)–(24). The average is taken over 100 realizations of disorder for a disordered region of length $N = 300$.

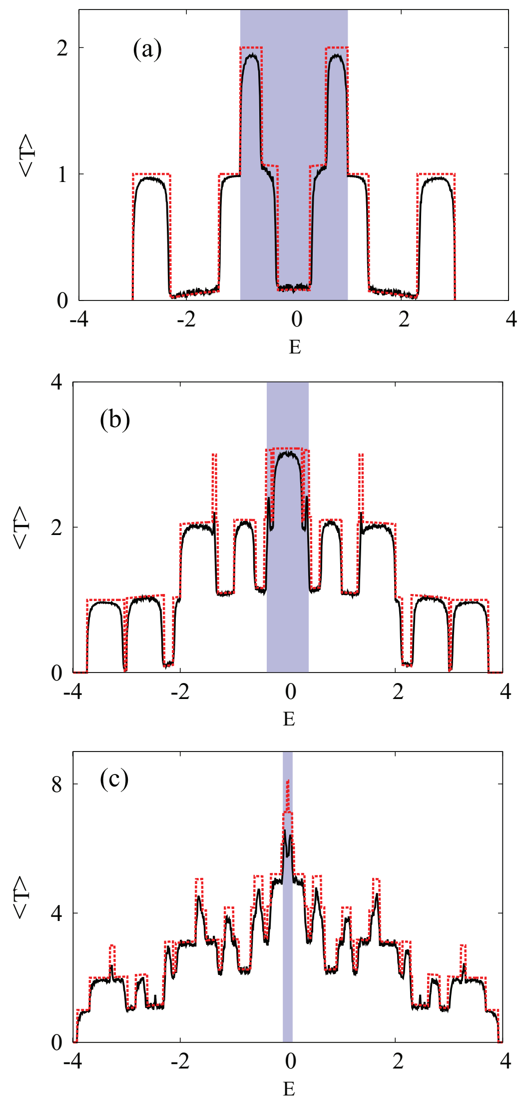

FIG. 3. (Color online) Same as in Fig. 2(b) for the Q1D model with (a) $M = 2$, (b) $M = 5$, and (c) $M = 10$. The energy window where all modes in the leads are open is shown by the shaded region.

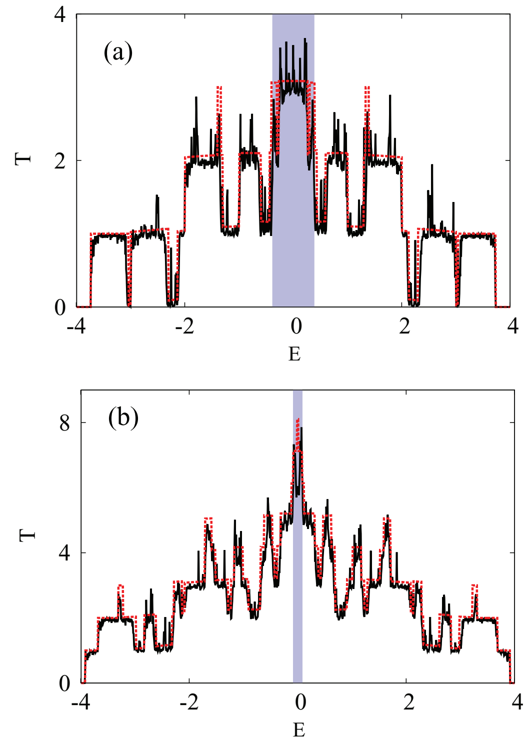

FIG. 4. (Color online) Transmission coefficient T as a function of E for the Q1D model with correlated stratified disorder of length $N = 300$ having (a) $M = 5$ and (b) $M = 10$. Continuous curves correspond to the numerical data while dashed curves are the theoretical predictions from Eqs. (22)–(24). A single realization of disorder was used. The energy window where all modes in the leads are open is indicated by the shaded region.

properties by considering our model with one chain only, $M = 1$. Specifically, we present the analytical expression for the Lyapunov exponent (22), together with the predicted and actual dependence of the transmission coefficient $\langle T \rangle$; see Eq. (24). One can see a good correspondence with the data showing the expected windows of transparency in dependence of the energy E.

For the Q1D model with stratified disorder the scenario is much more complicated. Indeed, for any of the qth channels the windows of transparency are defined by the energy shifted in accordance with Eq. (19). Specifically, for each channel there are three transparent energy windows given by Eq. (26). Outside of these windows the transmission vanishes due to the chosen type of correlations. Thus, the total transmission is obtained by the overlap of the energy dependencies corresponding to each channel. One can show that the average transmission coefficient is approximately defined by the integer number due to the following expression:

$$N_c(E) = \sum_{q=1}^{M}(\Theta[E_q + E_L] - \Theta[E_q - E_L] + \Theta[E_q - E_R] - \Theta[E_q + E_R] + \Theta[E_q + 2v] - \Theta[E_q - 2v]), \quad (27)$$

with $\Theta[x]$ as the Heaviside step function. The resulting stepwise behavior is shown in Fig. 3. Notice that the maximum value of the average conductance is reached, in most of the cases, in the energy region where without disorder all modes are open, therefore, the average transmission takes its maximum value M. The general properties of such behavior were predicted in Ref. [37]. As one can see, the main feature of the correlated disorder is the nonmonotonic dependence of the transmission in dependence on the energy.

The data presented in Fig. 3 are obtained by averaging over a large number of disorder realizations with the same kind of long-range correlations. However, from the experimental point of view it is important to know whether the nonmonotonic dependence of $T(E)$ can be practically seen for individual samples of *finite* size and disorder. To answer this question, in Fig. 4 we report $T(E)$ for a *single* disorder realization of the Q1D model of length $N = 300$ with $M = 5$ and $M = 10$, i.e., we use the same parameters as in Figs. 3(b) and 3(c), respectively. The numerical data reported in Fig. 4 demonstrate that the nonmonotonic stepwise dependence of $T(E)$ can be still detected. However, the fluctuations can wash out very narrow peaks; this can be clearly seen when the average is performed [compare Figs. 4(a) and 4(b) with Figs. 3(b) and 3(c), respectively]. In fact, according to the theory of 1D disordered systems in the ballistic regime, $N/l_\infty^{(q)} \ll 1$, the variance of T_q can be written as [26]

$$\mathrm{Var}(T_q) = 4\left(\frac{N}{l_\infty^{(q)}}\right)^2 + O\left[\left(\frac{N}{l_\infty^{(q)}}\right)^3\right]. \qquad (28)$$

Therefore, on the one hand, one can expect that if the qth channel is open (ballistic regime, $T_q \approx 1$), the fluctuations of T_q should not be very strong, i.e., proportional to $(N/l_\infty^{(q)})^2$. On the other hand, if due to specific long-range correlations the qth channel is closed (localized regime, $T_q \ll 1$) the fluctuations are negligible. Thus, we expect the fluctuations of the total transmission to be of the order of $(N/l_\infty^{(j)})^2$, with $l_\infty^{(j)}$ being the smallest localization length.

V. CORRELATED NONSTRATIFIED DISORDER

Another important problem refers to the Q1D model (1) with the disorder having the same correlation properties in the longitudinal direction for each chain, however, with no correlations in the vertical direction. Mathematically, this means that the disorder potential depends on both transverse and longitudinal coordinates, being correlated along the sample, however, completely uncorrelated transverse to it. In other words, the statistical properties for the site energies are defined as follows:

$$\langle \epsilon_{nm}\rangle = 0, \quad \sigma^2 = \langle \epsilon_{nm}^2\rangle, \quad \chi(k) = \frac{\langle \epsilon_{nm}\epsilon_{n+k,m'}\rangle}{\sigma^2}\delta_{mm'}, \qquad (29)$$

where $\chi(k)$ is the normalized binary correlator of the site energies. As before, we assume that the disorder is weak, $\sigma^2 \ll 1$. In the numerical calculations the specific form of the correlator is chosen according to Eq. (25) with the corresponding power spectrum (26). However, the presented results are valid for the systems with any form of the binary correlator $\chi(k)$.

When the correlated disorder is not stratified, the model cannot be represented by a set of independent 1D systems, and the analytical solution for the localization length is known for specific cases only. For example, for the two-chain model, $M = 2$, the expression for the Lyapunov exponent has been analytically derived in Ref. [21] for any kind of correlations, both in the longitudinal and transverse directions.

So far, an analytical solution for the Lyapunov exponent in the general case with $M > 2$ channels and correlated disorder is absent. Instead, in this section we focus on a particular case when only one channel is open to wave propagation and all the others are closed. Specifically, we are interested in the transmission coefficient T for the energy corresponding to the first open channel. Our approach is based on the expression for the localization length which is used in Eq. (24) obtained for 1D models with continuous random potentials. To do this, we employ the result obtained for a white-noise disorder [12],

$$\lambda_R = \frac{1}{N}\sum_{n,m=1}^{M}\langle |R_{nm}^{(N)}|^2\rangle = \frac{(3+\delta_{2,M+1})\sigma^2}{16(M+1)\sin^2\mu_1}, \qquad (30)$$

valid for any number of channels M, however, in the situation when one channel is open only. Here the Lyapunov exponent λ_R is defined through the reflection amplitudes $R_{nm}^{(N)}$ for an electron which is incident along the nth transverse mode and scattered back along the mth transverse mode; see details in Ref. [12]. The relation of the localization length $l_R = 1/\lambda_R$ to the localization length ξ_M defined via the conventional definition,

$$\xi_M^{-1} = -\lim_{N\to\infty}\frac{1}{2N}\left\langle \ln \sum_{n,m}^{M}|T_{nm}^{(N)}|^2\right\rangle, \qquad (31)$$

is discussed in Refs. [9,11,12,14]. Here $T_{nm}^{(N)}$ is a transmission amplitude for the transmission of the wave incoming into the nth channel and outgoing from the mth channel. Note that in both Eqs. (30) and (31) the summation is performed over *all* channels since the mixing between the channels should not be neglected.

With the use of this expression, we can suggest the phenomenological generalization valid for the correlated disorder as well,

$$\lambda(E) = \frac{(3+\delta_{2,M+1})\sigma^2}{16(M+1)\sin^2\mu_1}W(2\mu_1). \qquad (32)$$

Note that in the case of uncorrelated disorder, $W(2\mu_1) = 1$, the latter expression reduces to Eq. (30) and for $M = 2$ the result obtained in Ref. [21] is recovered. Quite remarkable is the prediction that the inverse localization length decreases as the total number M of chains increases.

It should be pointed out that here we study the case when the longitudinal and transverse hopping amplitudes are equal; see Fig. 1. Therefore, all the subbands (4) are overlapped. However, Eq. (32) can be generalized even for the case when the transverse hopping amplitudes are not equal to the longitudinal ones; this could lead to the nonoverlapping of some subbands. In this case one has also to take into account the influence of the evanescent modes as stated in Ref. [55]. Another related result can be found in Ref. [56] where the localization-delocalization transition was studied when considering the strength of the transverse hopping as an independent parameter.

Now, since only the open mode mainly contributes to transmission, one can suggest that the expression for the average total transmission (in the energy region where only one mode is open) can be obtained by inserting the localization length defined by Eq. (32) into Eq. (24). Indeed, our numerical data presented in Figs. 5 manifest that Eq. (32) give a very good description for the total transmission coefficient in the energy regions corresponding to only one open channel. One can also see that when more than one channel is open the energy dependence of the transmission coefficient acquires a quite complicated form, thus indicating an extreme difficulty in the analysis of the behavior of the transmission coefficient in other energy regions.

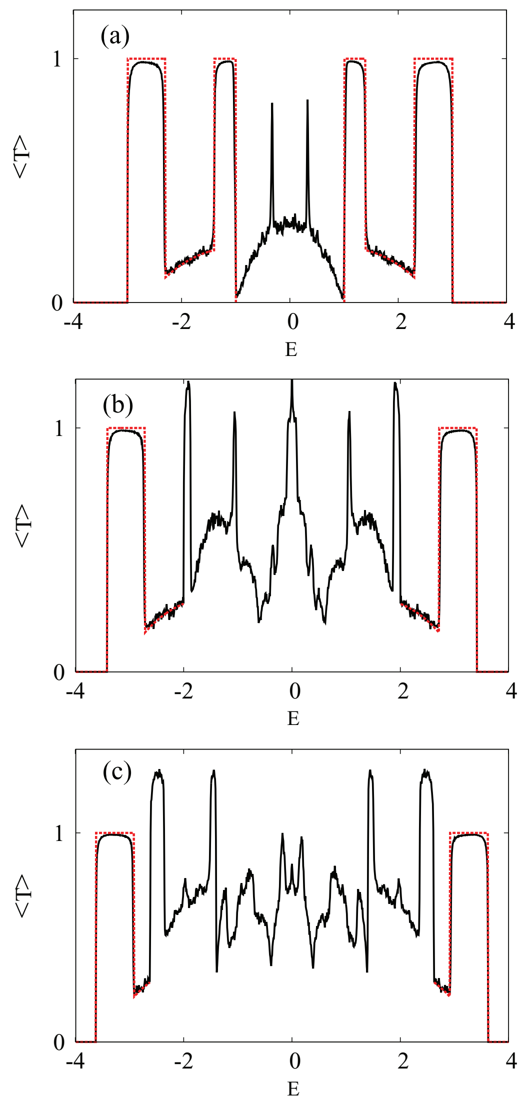

FIG. 5. (Color online) Average transmission coefficient as a function of the energy for nonstratified disorder. Continuous curves represent the numerical data and dashed curves are the theoretical prediction given by Eqs. (24) and (32). The length of the system is $N = 300$ with the disorder strength $\sigma^2 = 0.02$ for (a) $M = 2$, (b) $M = 3$, and (c) $M = 4$.

VI. CONCLUSIONS

We have studied the transport properties of bulk-disordered Q1D wires paying main attention to the role of long-range correlations along disordered structures of finite size. First, we have manifested that in the case of stratified disorder global transport properties can be fully explained by the semianalytical theory that uses results obtained for 1D continuous random potentials. As predicted in Ref. [37], in this case the expression for the total transmission coefficient T can be presented as a sum of partial coefficients T_q that correspond to independent 1D chains characterized by the index q. Since the theory of correlated disorder for 1D tight-binding models is fully developed (see discussion in [26]), this allows one to incorporate the obtained results into the problem of the correlated transport for Q1D disordered systems. This incorporation is not rigorous since it does not take into account the resonances emerging in discrete (tight-binding) models, however, it can be approximately used. Indeed, our numerical data demonstrate a perfect agreement with the analytical predictions. For the numerical study we have used the approach which is based on the non-Hermitian Hamiltonians from which one can construct the scattering matrix, therefore, to obtain all transport characteristics.

Second, we have analyzed the model in which the long-range correlations are taken in the same way as for the stratified disorder, however, the individual realizations of the disorder in the chains are independent from each other. In this case the disordered potential depends on both coordinates, the longitudinal and transverse ones; this leads to a mixing between different channels when the waves propagate through the Q1D structure. Since the general theory is absent, we have studied a specific, however, realistic case when one channel is open only while $M - 1$ other channels are closed. For this case the rigorous analysis is also absent, and we suggested an approach which results in a phenomenological expression for the transmission coefficient. This approach is based on the results obtained earlier for a white-noise disorder. The suggested expression turns out to be very good, as the comparison with the data shows. It should be noted that such a situation when the transport in Q1D systems is practically defined by one open channel can be arranged experimentally [57].

Finally, we have numerically demonstrated that specific long-range correlations can result in a strong enhancement of the localization, even when the disorder is weak; in the 1D disordered models this was confirmed experimentally (see results, discussion, and references in [26]). Such an effect of an enhancement of localization is clearly seen from our numerical data obtained for relatively short disordered samples. Specifically, the emergence of the energy windows where the transmission coefficient vanishes or becomes very small is a direct consequence of the enhancement of the localization. Interestingly enough, such an enhancement of localization in the selected energy windows is accompanied by the suppression of localization in the complementary energy windows within the energy band [26]. Therefore, the long-range correlations can be considered as the mechanism for the creation of eigenstates with both enhanced and suppressed localization, selectively embedded in the energy space. This effect can be used to manufacture devices with controlled transport properties in photonic heterostructures, semiconductor superlattices, electron nanoconductors, and microwave waveguides. As an example of an experimental realization of the Q1D correlated disorder we mention the recent study of transport properties of a Q1D waveguide with long-range correlations inside the scattering potential (for details see Ref. [58]).

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