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Attention competition with advertisement

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In the new digital age, information is available in large quantities. Since information consumes primarily the attention of its recipients, the scarcity of attention is becoming the main limiting factor. In this study, we investigate the impact of advertisement pressure on a cultural market where consumers have a limited attention capacity. A model of competition for attention is developed and investigated analytically and by simulation. Advertisement is found to be much more effective when the attention capacity of agents is extremely scarce. We have observed that the market share of the advertised item improves if dummy items are introduced to the market while the strength of the advertisement is kept constant.

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I. MARKETS

Traditionally every product or service has a price tag. In order to get it, one has to pay the price. Nowadays, the price of items in some markets becomes so low, even to the point of being free of charge, that this concept of "pay to get" is challenged, especially in the era of the Internet. It is quite a common fact that one can get many products and services and pay absolutely nothing. Among these are Internet searches (Google, Yahoo), e-mail (gmail, Hotmail), storage (DropBox, Google, Yahoo), social networks (Facebook, Twitter, LinkedIn), movie storage (Youtube), communication (Skype, WhatsApp), document formats (PDF, RTF, HTML), various software platforms (Linux, LaTeX, eclipse, Java), and open educational resources (open course materials and massive open online courses).

Companies providing services for which users pay nothing at all are difficult to explain in economics. But even though these products are free to users, there is still a sound business plan behind them. To obtain a large market share is the key to their business plan as in the cases of Google, Facebook, LinkedIn, and Skype. Once it becomes widely used, a company starts to use its customer base to create money.

A. New market concepts

In order to understand such markets new concepts such as two-sided markets and attention economy have been developed. In a *two-sided market*, a company acts as a bridge between two different types of consumers [1]. It provides two products: one is free and the other has a price. Free products are used to capture the attention. Products with a price are used to monetize this attention. A set of very interesting examples of two-sided markets, including credit cards, operating systems, computer games, and stock exchanges, can be found in Ref. [1].

Suppose there are many competing products on the free side of a two-sided market. In theory, a customer can get all the products available. In practice, this is hardly the case. The abundance of immediately available products can easily exceed the customer's capacity to consume them. One way to look at this phenomenon is that products compete for the attention of users, which is referred to as *attention economy* in the literature [2–4].

Attention scarcity due to the vast amount of immediately available products is also the case for cultural markets. A *cultural market* is assumed to have an infinite supply of cultural products and it is assumed that an individual's consumption behavior is not independent of others' consumption decisions [5,6].

B. Compulsive markets

We focus on markets that are slightly different, where the customer compulsively purchases the item once he or she is aware of it. Clearly, this kind of compulsive buying behavior cannot happen for high-priced items such as cars and houses. On the other hand, it can be the case for relatively low-priced items such as movie DVDs and music CDs. This pattern of "compulsive purchasing" behavior becomes clearly acceptable if the items become free, as in the case of Web sites, video clips, music files, and free software, especially free mobile applications. There are a number of services that provide such items including Youtube, Sourceforge, and AppStore.

We call such markets *compulsive markets* and we consider the dynamics of the consumers rather then the economics of it. This new kind of market calls for new models. In this work, the simple recommendation model of Refs. [7,8] is extended to such a model. We use the extended model to answer the following questions: Under what conditions does the advertisement mechanism outperform the recommendation process? How much advertisement is enough to obtain a certain market share? We first present our analytic approach and then compare it with simulation results.

II. BACKGROUND

A compulsive buyer becomes aware of a product in two ways: (i) by local interactions within his or her social network, i.e., by means of word-of-mouth; and (ii) by global interactions, i.e., by means of advertisement. Word-of-mouth recommendations by friends make products socially contagious. Research on social contagion can provide answers to the question of how things become popular. Gladwell

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states, "Ideas, products, messages and behaviors spread like viruses do" [9]. He claims that the best way to understand the emergence of fashion trends is to think of them as epidemics. Infectious disease modeling is also useful for understanding opinion formation dynamics. Specifically, the transmission of ideas within a population is treated as if it were the transmission of an infectious disease. Various models have been proposed to examine this relationship [6,7,10–14]. There exist recent works whose essential assumption is the fact that an old idea is never repeated once abandoned [15,16]. In other words, agents become immune to older ideas as in the susceptible-infected-recovered (SIR) model. However, behaviors, trends, etc., can occur many times, over and over again. In this case they can be modeled as the susceptible-infected-susceptible (SIS) model. In a completely different context, limited attention and its relation to income distribution are investigated [17].

A. Epidemic spreading

The study of how ideas spread is often referred to as social contagion [18]. Opinions can spread from one person to another like diseases. An agent is called *infected* iff it has the virus. It is called *susceptible* iff it does not have the virus.

Using the SIS model of epidemics, the system can be modeled as a Markov chain. Consider a population of N agents. Let S_i be the state in which the number of infected agents is i. The state space is composed of $N + 1$ states, $\{S_0,S_1,\ldots,S_N\}$ with S_0 and S_N being the reflecting boundaries. The system starts with the state S_0 where nobody is infected.

Let $\mathbf{T} = [t_{ij}]$ be the $(N + 1)\times(N + 1)$ transition matrix of the Markov chain, where t_{ij} is the transition probability from state S_i to state S_j. As a result of a single recommendation, there are three possible state transitions: The number of infected agents can increase or decrease by 1 or stay unchanged. Such a system is called the birth-death process [19]. Hence, $\mathbf{T}$ is a tridiagonal matrix with entries given as

$$t_{ij} = \begin{cases} p_i, & j = i + 1, \\ l_i, & j = i, \\ q_i, & j = i - 1, \\ 0 & \text{otherwise,} \end{cases}$$

where p_i, l_i, and q_i are the transition probabilities. Then the stationary distribution $\boldsymbol{\pi} = [\pi_0 \ldots \pi_N]^\top$ of the Markov chain can be obtained from its transition matrix [19], which satisfies

$$\pi_i = \prod_{k=1}^{i} \frac{p_{k-1}}{q_k}\pi_0 \quad \text{and} \quad \sum_{i=0}^{N}\pi_i = 1. \tag{1}$$

B. Simple recommendation model

The *simple recommendation model* (SRM) reveals the relation between the fame and the memory size of the agents [7,8]. The SRM investigates how individuals become popular among agents with a limited memory size and analyzes the word-of-mouth effect in its simplest form. The SRM differs from many previous models in its emphasis on the scarcity of memory. In the SRM, agents, which have a strictly constant memory size M, learn each other solely via recommendations.

A *giver* agent selects an agent that he or she knows and *recommends* the agent to a *taker* agent. Since memory space is restricted to M, the taker *forgets* an agent to make space for the *recommended* one. This dynamics is called a *recommendation* and is reported more formally in Sec. III C. Note that (i) the selections have no sophisticated mechanisms—all selections are made uniformly at random; (ii) any agent can recommend to any other agent, therefore underlying network of interactions is a complete graph; and (iii) the taker has to accept the recommendation, that is, he or she does not have the option to reject.

In the SRM, no agent initially is different from the other. So the initial fames of agents are set to be the same, where the *fame* of an agent is defined as the ratio of the population that knows the agent. Recommendations break the symmetry of equal fames. As recommendations proceed, a few agents get very high fames, while the majority of agents get extremely low fames, even to the level of no fame at all. Once an agent's fame becomes 0, that is, *completely forgotten*, there is no way for it to come back. In the limit, the system reaches an *absorbing state* where exactly M agents are known by every one, i.e., a fame of 1, and the rest become completely forgotten, i.e., a fame of 0. The SRM offers many possibilities for extension. It is applied to minority communities living in a majority [20]. A recent work extends the forgetting mechanism by introducing familiarity [21].

III. PROPOSED MODEL

In the SRM, (i) the spread of information throughout the system is managed by recommendation only, and (ii) the results are obtained by simulations [7,8]. In this article, we propose the simple recommendation model with advertisement (SRMwA), which extends the SRM in the following ways: (i) In addition to recommendation, advertisement pressure as a new dynamic is introduced; and (ii) moreover, an analytical approach is developed, as well as simulations. Distinctively, via the SRMwA, we investigate the conditions under which social manipulation by advertisement overcomes pure recommendation.

A. New interpretation of the SRM

In the original model of the SRM, agents recommend other agents and the term *memory size* is used for the number of agents one can remember [7,8]. As one agent is known more and more by other agents, his or her fame increases. In the extended model of the SRMwA, agents recommend items rather than agents. Since items consume the limited attention of agents, there is competition among items for attention. For these reasons, we prefer to use the term "attention capacity" despite the term memory size for the amount of information an agent can handle. The focus of the work is no longer the fame of the agents but the attention competition among items.

Note that the proposed model allows us to consider items in a wider sense. Rather than a unique object such as the *Mona Lisa of Leonardo*, we consider items that are easily reproduced so that there are enough of them for everybody to have one, if they want it. Therefore items are not only products and services but also political ideas, fashion trends, and cultural products as in the case in Ref. [6].

B. Advertisement

We extend the SRM to answer the following question: What happens if some items are deliberately promoted? Suppose a new item, denoted a, is *advertised* to the overall population. At each recommendation, the taker has to select between the recommended item r and the advertised one a. The item that is selected by the taker is called the *purchased item*, denoted β.

C. Model

Adapting the terminology of the SRM [7] to the SRMwA, a *giver* agent g recommends an item, which he or she already owns, to an individual. The item and the individual are called the *recommended* r and the *taker* t, respectively. The taker pays attention to, that is, *purchases*, either the recommended or the advertised item. When the attention capacity becomes exhausted, in order to get space for the purchased item, an item f that is already owned by the taker is *discarded*. The *market share* of an item is defined to be the ratio of the population that owns the item.

The SRMwA is formally defined as follows. Let $\mathcal{N} = \{1,2,\ldots,N\}$ and $\mathcal{I} = \{1,2,\ldots,I\}$ be the sets of agents and items, respectively. Let $g,t \in \mathcal{N}$ and $r,f,\beta \in \mathcal{I} \cup \{a\}$ represent the *giver* and the *taker* agents and the *recommended*, the *discarded*, and the *purchased* items, respectively.

The attention "stock" of an agent i, denoted $m(i)$, is the set of distinct items that i owns. We say that agent $i \in \mathcal{N}$ *owns* item $j \in \mathcal{I}$ iff $j \in m(i)$. For the sake of simplicity, we assume that all agents have the same *attention capacity* M, that is, $|m(i)| = M$ for all $i \in \mathcal{N}$. The attention capacity of an agent is limited in the sense that no one can pay attention to the entire set of items, just to a small fraction of it, that is, $M \ll I$. Instead of directly using M, we relate M to I by means of the *attention capacity ratio*, defined as $\rho = M/I$. Since $0 \leqslant M \leqslant I$, we have $0 \leqslant \rho \leqslant 1$.

The recommendation and advertisement dynamics compete. The taker agent select either the recommended or the advertised item as the purchased one. Let the *advertisement pressure*, p, be the probability of selecting the advertised item as the purchased item.

The modified recommendation is composed of the following steps:

(i) g is selected.

(ii) t is selected.

(iii) $r \in m(g)$ is selected by g for recommendation.

(iv) t selects β, where β is set to a with probability p and to r with probability $1 - p$.

(v) The recommendation stops if β is already owned by t.

(vi) Otherwise, $f \in m(t)$ is selected by t for discarding and β is put in the space emptied by f.

Note that all selections are uniformly at random. With these changes, the SRMwA becomes a model for compulsive markets with advertisement.

D. Some special cases

In general, one expects that the market share of the advertised item will increase as advertisement get stronger. Depending on the strength of the advertisement, there are a number of special cases, the dynamics of which can be explained without any further investigation.

(i) *No advertisement.* Note that in this case, the original SRM is obtained since the purchased item is always the recommended item, i.e., $\beta = r$. In this case, the advertised item has no chance and its market share is 0.

(ii) *Pure advertisement.* When the taker has no choice but to get the advertised item, i.e., $\beta = a$, recommendation has no effect. In this case after every agent becomes a taker once, the market share of the advertised is 1. Note that in this case the system will stop evolving any further. Interestingly, this is a different state than the absorbing states of the SRM.

(iii) *Strong advertisement.* In the case of very strong advertisement, the taker almost always select the advertised item. Once all agents have the advertised item, the market share of the advertised item is 1 and the system becomes the SRM but with the attention capacity of $M - 1$.

IV. ANALYTICAL APPROACH

Note that the SRMwA resembles epidemic spreading. We explore epidemic spreading to explain the SRMwA as far as we can. Consider the advertised item as a virus. Agent j is called *infected* iff it has the advertised item in its attention stock, that is, $a \in m(j)$; otherwise it is called *susceptible*, that is, $a \notin m(j)$. Then the stationary distribution $\boldsymbol{\pi}$ provides the probability of the number of agents owning the advertised item when the system operates for infinitely long durations. Hence, the mean value of the stationary distribution $\boldsymbol{\pi}$ reveals our prediction for the number of infected agents. In other words, the expected number of agents that adopted the advertised item is the mean value of this distribution. That is, using Eq. (1), one obtains

$$\langle \boldsymbol{\pi} \rangle = \sum_{i=0}^{N} i\pi_i = \pi_0 \sum_{i=0}^{N} i \prod_{k=1}^{i} \frac{p_{k-1}}{q_k}.$$

Hence, the expected market share of the advertised item becomes

$$\langle F_a \rangle = \frac{\langle \boldsymbol{\pi} \rangle}{N},$$

where F_a is the market share of the advertised item.

A. Calculation of transition probabilities

In order to obtain the expected market share of the advertised item, we need to figure out the stationary distribution $\boldsymbol{\pi}$, which, in turn, calls for the transition probabilities p_i, l_i, and q_i. Suppose the system is in S_i and follow the steps of the recommendation given in Sec. III C. The possible selections can be represented by the tree given in Fig. 1. A path starting from the root S_i to a leaf in the tree corresponds to a recommendation. The paths that increase the number of infected agents are marked with a $\oplus$ sign at the leaf. Similarly, recommendations resulting in a transition of $S_i \to S_{i-1}$ are marked with a $\ominus$. The remaining paths, which correspond to no state change, are marked with a $\odot$.

Note that there three $\oplus$ and two $\ominus$ paths. Note also that the correspondence between the levels in the tree and the steps of the recommendation given in Sec. III C. At each level one

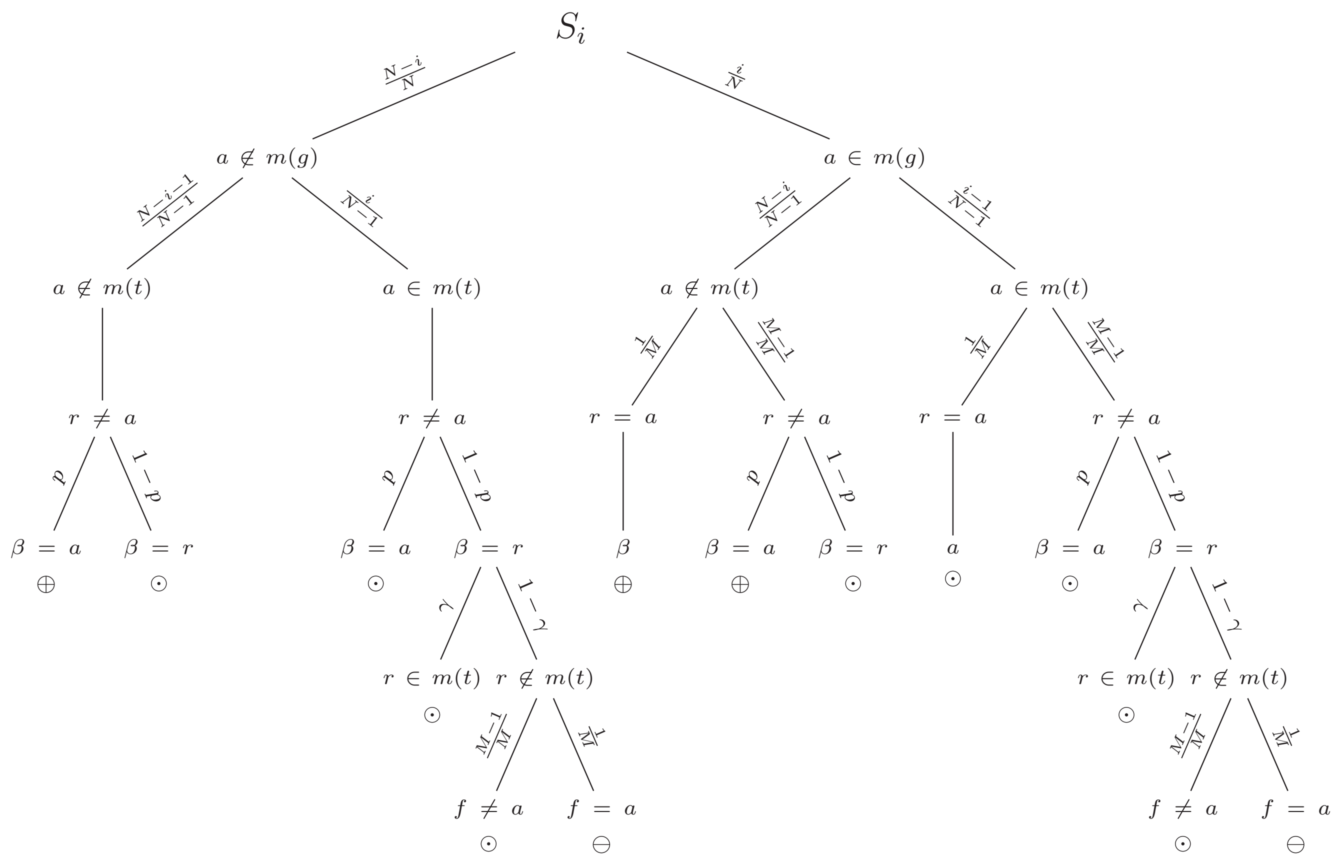

FIG. 1. Tree diagram for possible selections.

particular selection is made and the corresponding probability is assigned.

(i) $a \in m(g)$ level. The first-level branching in Fig. 1 corresponds to the selection of an infected or susceptible giver. There are N possible agents to be selected as g. If a system is in state S_i, then the probability of selecting an infected giver is $\frac{i}{N}$.

(ii) $a \in m(t)$ level. The second-level branching is due to the selection of an infected or susceptible taker. Once g is selected, there are $N - 1$ candidates left for t. The probability of selecting an infected taker depends on whether or not the selected giver is infected. For example, in the rightmost path, g is infected. So, the probability of selecting an infected taker in this case is $\frac{i-1}{N-1}$.

(iii) $r = a$ level. Now consider what the giver recommends. Depending on the path, the giver could be infected and could recommend the advertised item. Then the probability of an infected giver recommending a is $\frac{1}{M}$, since there are M items in its stock.

(iv) $\beta = a$ level. The fourth level illustrates the taker's purchase decision. The taker agent either follows the advertisement with probability p or accepts the recommended item with probability $1 - p$.

(v) $r \in m(t)$ level. Let γ be the probability of r already being owned by the taker agent. In this case, the taker agent does not make any changes in his or her stock.

(vi) $f = a$ level. It is possible that a can be chosen to be forgotten.

The transition probabilities can be obtained from Fig. 1 as

$$p_i = \frac{N-i}{N(N-1)}\left[\left(N-1-\frac{i}{M}\right)p + \frac{i}{M}\right], \quad (2)$$

$$q_i = \frac{i(1-p)(1-\gamma)}{N(N-1)M}\left[N-i+\frac{(i-1)(M-1)}{M}\right], \quad (3)$$

$$l_i = 1-(p_i+q_i). \quad (4)$$

Note that (i) these equations satisfy the expected boundary conditions $q_0 = 0$ and $p_N = 0$; (ii) $p_i > 0$ for all $i = 0,\ldots,N-1$; and (iii) $q_i = 0$ for all i when $p = 1$ or $\gamma = 1$. Therefore, for $p = 1$ or $\gamma = 1$, the system drifts to S_N and stays there forever.

B. Discussion of the value of γ

The stationary distribution can be obtained by means of Eqs. (1)–(3). The only unknown in these equations is γ, which is introduced in step v of the recommendation given in Sec. III C. γ is defined as the probability of the recommended item's already being owned by the taker agent. Unfortunately, γ cannot be obtained analytically except for the extreme case of $M = 1$. Therefore, we should find ways to approximate its value.

A first-order estimate for γ could be $\rho = M/I$, since the taker owns M items out of I in total. γ is close to 1 when M is in the range of I, since every agent owns almost all the items. The situation is quite different for $M \ll I$. Since every item

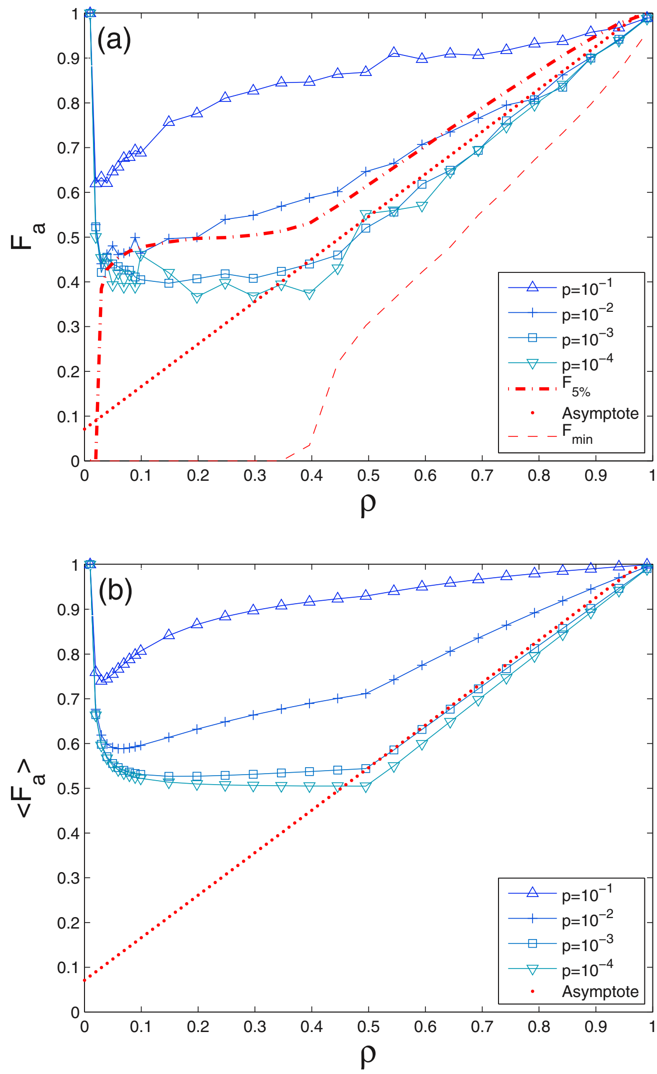

FIG. 2. (Color online) The market share of the advertised item as a function of the attention capacity ratio by (a) simulation and (b) analytic approaches. $F_{5\%}$, $F_{\min}$, and the asymptote line from Ref. [7] are given for comparison.

initially has the same market share, γ starts with a small value at the beginning. As the recommendations proceeds, we know that some items becomes completely forgotten [7]. Therefore γ increases as the number of recommendations increases and becomes 1 when the system reaches one of its absorbing states. In this respect, γ can be interpreted as the degree of closeness to an absorbing state. In order to investigate near-absorbing-state behavior, we set $\gamma = \max\{0.5, M/I\}$ in our analytic results shown in Fig. 2(b), where 0.5 is arbitrarily selected.

C. Extremely scarce attention capacity

For the extremely scarce attention capacity of $M = 1$, γ can be evaluated. Consider the paths in Fig. 1. For $M = 1$, the paths which contain an $(M-1)/M$ edge become paths with 0 probabilities. The only non0 probability path, involving γ, is the one terminating at the left $\ominus$ leaf. In this path the giver does not know the advertised item, $a \notin m(g)$, while the taker does, $a \in m(t)$. Since attention capacity is limited to 1, the giver and the taker do own different items. Therefore, the item recommended by the giver cannot be owned by the taker. Hence, $\gamma = 0$.

For $M = 1$ and $\gamma = 0$, Eqs. (2) and (3) lead to

$$\frac{p_i}{q_i} = 1 + \frac{N-1}{i}\frac{p}{1-p}$$

for $0 \leqslant i < N$. For $p \neq 0$, $p_i/q_i > 1$. This means that, for even a very small positive advertisement, the system inevitably drifts to state S_N, and once S_N is reached the system stays there forever since $q_N = 0$. Note that S_N, which corresponds to the state where all agents own the advertised item, is the unique absorbing state for this particular case.

V. SIMULATION APPROACH

In order to simulate the model, a number of decisions have to be made. The simulations start in configurations such that all I items have the same market share and no agent knows the advertised item. So that system is initially symmetric with respect to nonadvertised items. When to terminate the simulation is a critical issue. We set the average number of interactions $\nu = 10^3$. Since there are N^2 pairwise interactions among agents in both directions, the total number of recommendations is set to be νN^2.

(i) We run our simulation for a population size of $N = 100$ and an item size of $I = 100$.

(ii) The behavior of the system strongly depends on the attention capacity ratio ρ. We take ρ as a model parameter and run simulations for various values of ρ.

(iii) The advertisement pressure p is another model parameter. We use 10^{-1}, 10^{-2}, 10^{-3}, and 10^{-4} for p.

VI. OBSERVATIONS AND DISCUSSION

We investigate the effect of the advertisement pressure p and the capacity ratio ρ on the market share F_a of the advertised item. In order to make a quantitative comparison of the simulation results, being in the top 5% is arbitrarily set as our criterion. Let $F_{5\%}$ denote the lowest market share for an item to be in the top 5%. Then the advertised item is in the top 5% whenever $F_a > F_{5\%}$. Let $F_{\min}$ be the minimum market share among all the items.

In Fig. 2, the simulation results of F_a, averaged over 20 realizations and versus the analytical results for $\langle F_a \rangle$ are shown for each value of $p \in \{10^{-1}, 10^{-2}, 10^{-3}, 10^{-4}\}$ as a function of ρ. A number of observations can be made:

(i) The analytic results shown in Fig. 2(b) are in agreement with the simulation results in Fig. 2(a). Model predictions of $\langle F_a \rangle$ can quantitatively reproduce the simulation results for F_a, although we use an approximated value for γ. We observe that for larger ν, the similarity between analytical and simulation results gets even better.

(ii) The curves of $F_{5\%}$ in Fig. 2(a) resemble that in Ref. [7], although advertisement is not the case for the latter. Line $y = 0.95x + 0.071$, which is given as an asymptote for $F_{5\%}$ for large values of N in Ref. [7], is also plotted in Fig. 2(a) for comparison purposes.

(iii) Note that for $\rho < 0.05$, all F_a curves approach 1 and $F_{5\%}$ becomes 0. This is due to the finite-size effect. In an absorbing state, there would be exactly the same M items purchased by all the agents and the remaining items would be completely forgotten. For $I = 100$, $\rho < 0.05$ means that $M < 5$. That is, there is no space left for the fifth item. Hence, in a near-absorbing state, the market share of the fifth item, $F_{5\%}$, approaches 0. On the other hand, any promotion, i.e., $p > 0$, is enough to push the advertised item into the top M items.

(iv) The minimum market share $F_{\min}$ becomes 0 when at least one item is completely forgotten. This occurs for $\rho < 0.35$ in Fig. 2(a), which is consistent with Ref. [7]. We also observe that for larger ν, the advertised item leaves a smaller share of attention to others, which forces the 0 crossing of $F_{\min}$ to occur at a higher level of ρ.

(v) As expected, a strong advertisement, i.e., $p = 10^{-1}$, easily gets the advertised item into the top 5%, since the F_a curve for $p = 10^{-1}$ is always higher than the $F_{5\%}$ curve in Fig. 2(a), while a weak promotion such as $p = 10^{-3}$ or 10^{-4} does not. The case of $p = 10^{-2} \approx \frac{1}{I+1}$ for $I = 100$ is interesting. For small and moderate values of ρ, i.e., $\rho < 0.6$, the advertised item is in the top 5% with one exception: for large values of ρ, this is not the case.

(vi) How do agents allocate their attention when the attention capacity becomes a limiting factor? This is a critical question for markets of attention economy. Consider the extreme case of attention capacity $M = 1$, which corresponds to $\rho = 0.01$ in Fig. 2. In this case, surprisingly, even a very small positive value of p is enough for the entire population to get the advertised item, i.e., $F_a = 1$, when $M = 1$. This observation is analytically investigated in Sec. IV C.

A. Item size effect

We run new simulations with different item sizes of I when N is fixed to 100. Let $F_a(I = k)$ denote the market share of the advertised item when $I = k$. Then we accept $F_a(I = 100)$ as the reference market share and define the relative market share $R_{I=k}$ with respect to $I = 100$ as follows:

$$R_{I=k} = \frac{F_a(I = k)}{F_a(I = 100)}.$$

In Fig. 3, we observe that for all $k \in \{100,200,300,500\}$, $R_{I=k} \geqslant 1$ when p is fixed to 10^{-1} except for $\rho = 0.01$. The case of $\rho = 0.01$ corresponds to $M = 1$ for $I = 100$. As explained in Sec. IV C, F_a gets its maximum value, 1, for $M = 1$. That is why $R_{I=k} \leqslant 1$ for $\rho = 0.01$.

We have observed that the market share of the advertised item improves while the number of items is increased even if the advertisement pressure is kept constant. Increasing the advertisement pressure in order to push the market share up is not usually an option in practical life. This could be an interesting interpretation. If one cannot increase the intensity of advertisement, i.e., p, it is better to have a greater number of items, i.e., I. When this happens, the advertised item has a better chance of getting into the top 5% . In order to obtain this operating point, one may purposefully introduce some dummy items. This unexpected prediction of the model needs to be further investigated.

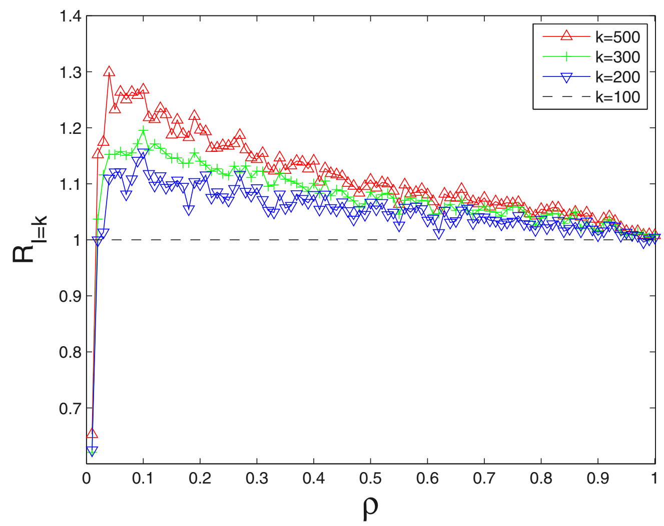

FIG. 3. (Color online) Effect of item size on the market share of the advertised item for $p = 10^{-1}$ is investigated as a function of the attention capacity ratio.

B. Closeness to the absorbing state

The system gets closer to one of its absorbing states as the number of recommendations increases; this is controlled by the simulation parameter ν. Let $F_a(\nu = k)$ be the market share of the advertised item after νN^2 recommendations. We define the relative market share $R_{\nu=k}$ at $\nu = k$ with respect to $\nu = 10^2$ as

$$R_{\nu=k} = \frac{F_a(\nu = k)}{F_a(\nu = 10^2)}.$$

The relative market share at $\nu = 10^3$ is given in Fig. 4 for different values of $p \in \{10^{-1},10^{-2},10^{-3},10^{-4}\}$ when $N = I = 100$.

We consider the system stationary if $R_{\nu=k}$ becomes 1, that is, the system stops changing with ν. We observe in Fig. 4 that as the attention capacity or the advertisement pressure gets

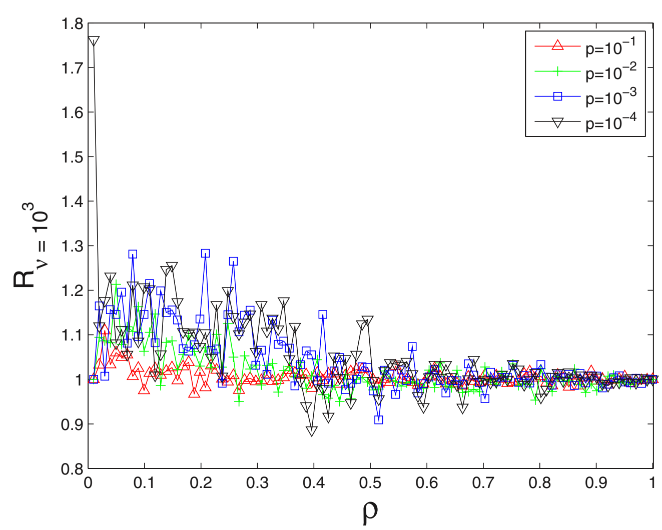

FIG. 4. (Color online) The relative market share $R_{\nu=k}$ at $\nu = 10^3$ is investigated as a function of the attention capacity ratio.

higher, the model gets closer to stationarity. Increasing the advertisement pressure is not very different from increasing the number of iterations. Both are favorable for the market share of the advertised item.

VII. CONCLUSIONS

The SRM as a model for pure word-of-mouth marketing is studied in Refs. [7,8]. We extend the SRM to attention markets with advertisement. This model constructs a theoretical framework not just for items but for study of the propagation of any phenomena such as ideas or trends under limited attention.

The model is investigated analytically and by simulation. The analytical results agree with the simulations. As expected, strong advertisement forces everyone to get the advertised item under all conditions.

Interestingly, when the attention capacity is small compared to the number of items, even a very weak advertisement can do the job. This behavior is shown analytically for the case of $M = 1$ and observed in the results of both simulations and analytic calculations as ρ approaches 0. This can be interpreted as meaning that when individuals have a limited attention capacity, they tend to adopt what is promoted globally rather than what is recommended locally. We have also found that introducing more standard items to the market is good for the market share of the advertised item. This observation may lead to interesting political consequences in terms of public attention and political administration. For example, public opinion can be kept under control by means of increasing the number of issues, possibly by means of artificial ones, so that the promoted idea is easily accepted by large audiences. This prediction calls for further investigation.

In the current work, there is a unique advertised item. The model can be extended to cover more than one promoted item. All selections are uniformly at random. One may investigate the effects of some other selection mechanism as in the case of Ref. [21]. We have a complete graph as the graph of interactions. One can investigate other graphs of interactions such as Scale-Free, Small-World, regular, or random graphs. The structure of interactions can also be improved by introducing a radius of influence. One may extend the model by introducing the concept of quality for items or letting agents prefer some items intrinsically as in Ref. [6].

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