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Spontaneous jamming and unjamming in a hopper with multiple exit orifices

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We show that the flow of granular material inside a two-dimensional flat bottomed hopper is altered significantly by having more than one exit orifice. For hoppers with small orifice widths, intermittent flow through one orifice enables the resumption of flow through the adjacent jammed orifice, thus displaying a sequence of jamming and unjamming events. Using discrete element simulations, we show that the total amount of granular material (i.e., avalanche size) emanating from all the orifices combined can be enhanced by about an order of magnitude difference by simply adjusting the interorifice distance. The unjamming is driven primarily by fluctuations alone when the interorifice distance is large, but when the orifices are brought close enough, the fluctuations along with the mean flow cause the flow to unjam.

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An assembly of discrete, noncohesive particles, also known as dry granular media, while trying to flow through a narrow opening, can clog or jam, an occurrence widely observed in particle drainage through silos and hoppers that are used ubiquitously in several industrial applications. The jamming at the orifice is caused by a few particles forming a stable arch at the exit [1,2], eventually causing the entire system to halt abruptly. The particles constituting the arch can be from anywhere in the system [2], and while the occurrence of an arch is quite unpredictable, some information can be obtained through the spatial distribution of the velocity fluctuations in the system [3]. The shape of the arch, though, can be predicted quite well using a random walk model [1].

For a three-dimensional hopper there exists a critical orifice width above which the system never jams [4] while no such limit exists for a two-dimensional system, which can jam for large enough orifice widths [5]. It has been shown that the output flow rate from a hopper can be increased by as much as 10% by placing suitable inserts at appropriate locations [6–8]. This also decreases the probability of jamming by about two orders of magnitude [9]. These studies have shown that the reduction in the tendency to jam is due to the reduction of the pressure in the region of arch formation. The motivation for placing inserts is derived from parallel interesting studies carried out to alter the flow behavior of pedestrians from a crowded room [10,11].

Here we report an interesting observation about the jamming behavior in a two-dimensional hopper having two exit orifices of the same width placed far away from each other. Whenever one of the orifices jams, which is expected given the orifice width incorporated, it unjams spontaneously if the flow is occurring through the other orifice. Effectively, flow continues to occur through both orifices, alternately or together, for a much longer duration than expected for a hopper with a single orifice. The overall duration of flow, consequently, the tendency to jam, can be altered simply by changing the interorifice distance. Such a nonlocal interaction between regions exhibiting differing flow behavior has been observed previously, but in different configurations [12,13] and under different flow conditions. In these studies, it was shown that the origin of such a nonlocal interaction can be attributed to a self-activated process within which the stress fluctuations induced by a localized shear cause the material to yield and flow elsewhere [13].

In the present system, we conjecture that the observed behavior of spontaneous unjamming occurs due to the rearrangement of the particles in the jammed region above one orifice. This unjamming behavior can be correlated with the fluctuations induced in the system due to flow from the nearby orifice. We believe that such a nonlocal interaction between two widely separate regions can be used to systematically and nonintrusively alter the jamming behavior in a hopper, which could be quite significant for several industrial operations. We have explored this behavior in great detail through discrete element method (DEM) simulations of soft particles. We measure the mean avalanche sizes for varying conditions and show their correlation with the fluctuations measured as root-mean-squared (rms) values.

The DEM simulations were carried out using the Large Atomic/Molecular Massively Parallel Simulator (LAMMPS) developed at Sandia National Laboratories [14,15]. The simulation employs Hookean force between two contacting particles, described in detail elsewhere [16]. All the simulation parameters were the same as those used in a previous systematic study of hopper flows [16] except for a higher normal elastic constant ($k_n = 2 \times 10^6 mgd$), which corresponds to

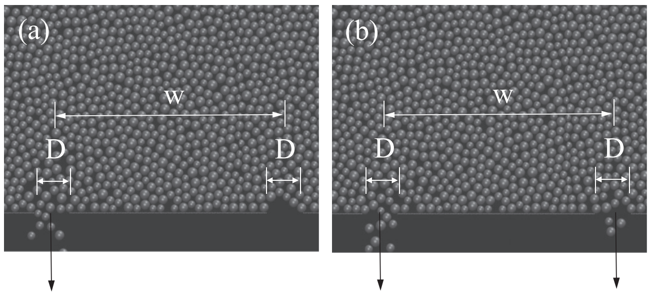

FIG. 1. Sample snapshots of the flow occurrence in a two-orifice hopper for a specified w and D. (a) Flow occurring through the left orifice while the right orifice is jammed. (b) Spontaneous flow reinitialization through the right orifice at a later time.

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a stiffer particle. The interparticle friction coefficient (μ) is varied from 0.2 to 0.8 with no qualitative difference between the results. Here, we report the results obtained for $\mu = 0.5$.

The simulation geometry consists of a two-dimensional hopper of height about 1.2 times its width and thickness $1d$, where d is the mean particle diameter with a polydispersity of 15%. The sidewalls are created out of the largest particles (rough wall) while the bottom surface is created from the smallest particles (relatively smooth wall) by freezing the particles so that their translational and angular velocities are kept zero throughout the simulation. The hopper has two orifice, each of width D, placed at an interorifice distance w, both defined in terms of mean particle diameter d. The hopper width is large enough to prevent any confinement effects due to the proximity of side walls to either orifice. The fill height is maintained constant by repositioning the particles exiting from the orifice just above the free surface. The hopper is filled using the sedimentation method as suggested previously [17], in which a dilute packing of nonoverlapping particles is created in a simulation box and allowed to settle under the influence of gravity. The simulation is run for a significant time so that the kinetic energy per particle is less than $10^{-8}mgd$, resulting in a quiescent packing of desired fill height H in the hopper.

The flow through the hopper is initiated by opening both orifices simultaneously. The orifice width is chosen to be small enough to cause jamming after a certain period of flow. After the flow is initialized, either of the orifices gets jammed but unjams again spontaneously. Note that this unjamming would not have been possible in the absence of the second orifice through which the flow occurs for a very short duration before jamming itself. The jamming-unjamming sequence can thus flip from one orifice to the other. Effectively, the flow occurs through either one or both of the orifices at any given time. After a few of these jamming-unjamming sequences, the flowing orifice jams before it can unjam the other orifice and the overall flow stops. It is quite evident that the particles above the flowing orifice transmit some information to the jammed orifice, causing it to unjam again. However, this information is available only for a short duration before the flowing orifice jams on its own. After both orifices jam, the flow is reinitiated by removing two to three particles from each of the arches. This procedure is continued to get a significant number of jamming events. The total number of particles flowing out from the hopper from the instant both orifices are opened until both are jammed is defined as the avalanche size (s). The value of s is found to depend on the values of D and w, which we discuss next. A sample snapshot of particles flowing through one orifice while other is jammed is shown in Fig. 1(a). A short time later, the jammed orifice starts to flow spontaneously, as shown in Fig. 1(b).

The distribution of avalanche sizes (s) for a given D and w, normalized by the mean avalanche size $\langle s \rangle$, exhibits an exponential tail for all cases, which is typical of the random nature of discrete avalanche events [9]. Figure 2(a) shows the mean avalanche size per orifice ($s_m = \langle s \rangle/2$) obtained for four different values of D and several interorifice distances (w). The behavior is qualitatively similar for all orifice sizes. The avalanche size (s_m) decreases monotonically with increasing w, and at infinitely large w it asymptotically approaches a

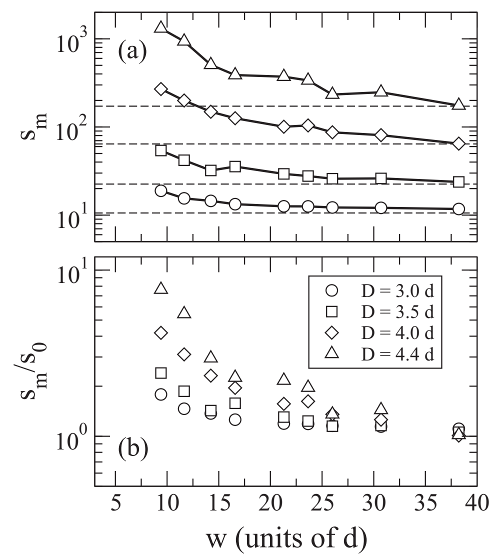

FIG. 2. Jamming characteristics for varying distances (w) between orifices of different widths (D). (a) Mean avalanche size per unit orifice (s_m) plotted against the interorifice distance (w). The dashed lines correspond to the avalanche size (s_0) for a hopper having a single orifice of width D. (b) Scaled mean avalanche size (s_m/s_0) plotted against the interorifice distance (w).

constant value, which corresponds to that for a single-orifice hopper [see Fig. 3(b)]. In this scenario, the two orifices will function, unaware of each other's existence, and no information is exchanged between the respective flowing regions. For w larger than those shown in Fig. 2(a) (but not studied), an unjamming event can happen, but with much less probability. Now, with decreasing w, the value of s_m increases and it grows quite rapidly for small enough w. This is a consequence of the flow from one orifice aiding that through the other in some way, thus effectively increasing the total time period over which flow occurs, hence larger s_m. In the limit when the two orifices are very close to each other (w of the order of D or lower), the flow now occurs as if through a wider orifice ($\sim 2D$). Eventually it approaches the asymptotic limit for a single orifice of width $2D$. Within the entire range of w, the value of s_m is observed to vary by almost an order of magnitude difference for the largest orifice width considered. The mean avalanche size (s_m) normalized by the single-orifice avalanche size (s_0) is shown in Fig. 2(b). The data show a reasonable collapse for $w > 20d$, while increasing scatter is observed with decreasing w, which, perhaps, indicates a nonlinear dependence on the orifice width (D) and interacting flow fields which cannot be scaled out by simple normalizing.

To elucidate the origin of the enhanced avalanche sizes, we perform simulations in a hopper fitted with a single orifice for different D [see Fig. 3(a)]. The width of the hopper is more than twice the maximum distance (w) used in a two-orifice system. For every orifice size, several avalanches are obtained by reinitiating the flow postjamming. The mean avalanche size shows an exponential-squared dependence on the orifice size ($e^{[0.26D^2]}$), as shown in Fig. 3(b), and is in accordance with experimental results obtained previously for a two-dimensional system [5], suggesting the absence of a

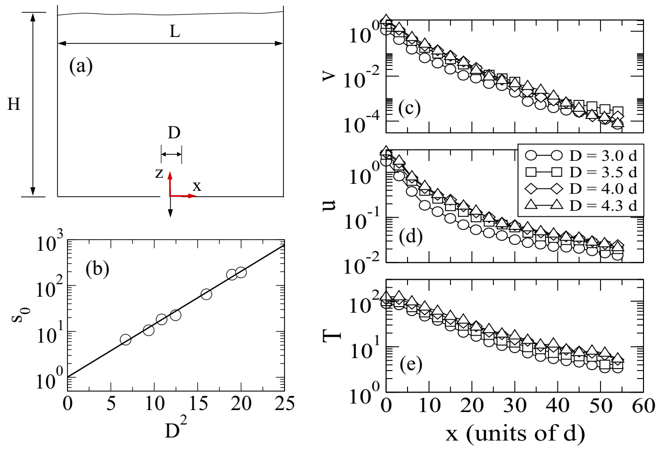

FIG. 3. (Color online) (a) Schematic of a single-orifice hopper of width L and height H. (b) Variation of mean avalanche size (s_0) for a hopper with a single orifice of width D. Variation of (c) mean velocity v, (d) rms velocity u, and (e) rms normal stress T along the horizontal direction (x) obtained over a region $3d$ in the z direction for four different orifice widths. Please refer to the text for the method to obtain the mean and rms quantities. The mean velocity and rms velocity are measured in units of d/τ, while the rms stress is measured in units of mg/d^2.

critical orifice width (D) separating the flowing and jamming conditions.

We next obtain the profiles of the mean velocity, velocity fluctuations, and normal stress fluctuations of the particles along the x direction. To calculate these quantities, we save snapshots of the particle positions at intervals of 0.0025τ, where $\tau = \sqrt{d/g}$, g is the gravitational acceleration, and the integration time step used in the simulations is $\delta t = 2.5 \times 10^{-5}\tau$. The mean velocity is defined as $v(x) = \sqrt{\langle c_x \rangle^2 + \langle c_z \rangle^2}$, while the average velocity fluctuations are measured in terms of the rms velocity defined as $u(x) = \sqrt{[\langle c_x^2 \rangle - \langle c_x \rangle^2] + [\langle c_z^2 \rangle - \langle c_z \rangle^2]}$. Here, c_x and c_z are the instantaneous horizontal and vertical velocity components of every particle obtained from the displacements between two successive snapshots. The simulation algorithm also outputs the horizontal (σ_x) and vertical (σ_z) components of the computed normal stress on each particle within each snapshot. The normal stress fluctuations are captured in terms of rms stress defined as $T(x) = \sqrt{[\langle \sigma_x^2 \rangle - \langle \sigma_x \rangle^2] + [\langle \sigma_z^2 \rangle - \langle \sigma_z \rangle^2]}$. In all of the above calculations, $\langle\ \rangle$ represents a temporal average over several time instants of flow and a spatial average over a $3d$ region in the z direction located about $4d$ above the orifice. This spatial region is chosen so as to capture the essential dynamics in a simple manner. All the three quantities are shown in Figs. 3(c)–3(e) for the four orifice widths. The profiles are symmetric about the orifice and hence only the right halves of the profiles are shown in each case.

The magnitudes of all the quantities increase with an increase in the orifice width, which is expected given the faster flow and, consequently, more collisions between particles. The profiles show similar behavior for different orifice sizes and collapse quite nicely (not shown) when normalized by the respective quantity at $x = 0$. The mean velocity decays more rapidly with distance from the orifice while the rms velocity and stress show a much gradual decay. The ratio of rms to the mean velocity increases progressively away from the orifice and reaches a value of about 100 by $x = 45d$. Almost similar numbers are obtained for all orifice sizes studied.

Now consider the scenario of a two-orifice hopper with one of the orifices located at $x = 0$, which is in the unjammed (flowing) state. Provided the other orifice located at some distance (w or x) is in a jammed state, the flow profile due to this orifice will be the same, as shown in Figs. 3(c)–3(e). The occurrence of flow through the unjammed orifice over a time period (which corresponds to the time interval between jamming and unjamming instances of the second orifice) causes particle rearrangements in the system and breaks the arch above the second orifice, leading to flow. For an interorifice distance of less than $10d$, with the mean and rms velocity magnitudes being of the same order, it is difficult to clearly isolate the effect of each on the spontaneous unjamming behavior. However, for larger interorifice distances ($>20d$), the mean velocity is over an order of magnitude lower than the rms velocity and the fluctuations are expected to dominate the spontaneous unjamming behavior. It is expected that the fluctuations too will decay at infinitely large distances and will not be able to reinitiate flow in the jammed orifice. The flow and jamming behavior of the two orifices in that case resembles that seen from two isolated orifices that are unaware of the other's existence. The mean avalanche size then approaches that for a single orifice [dashed lines in Fig. 2(b)].

Given the observed dependence of the mean avalanche size [see Fig. 2(a)] and that of the flow variables [see Figs. 3(c)–3(e)] on the interorifice distance, we now proceed to determine the relation between these quantities. We consider data only for interorifice distances greater than $10d$, beyond which the mean flow decreases rapidly compared to fluctuations. Figures 4(a) and 4(b) show the avalanche sizes (s_m) for different interorifice distances (w) plotted against the rms velocity $u(w)$ and rms normal stress $T(w)$, respectively. All quantities are evaluated at the same distance (w or x) in all the cases. An exponential relation (shown as dashed lines) is observed for all the cases with the y intercept approximately equal to the avalanche size (s_0) for a single-orifice hopper (i.e., two orifices infinitely far away from each other). The normalized mean avalanche size (s_m/s_0) plotted against the normalized rms velocity and normalized rms normal stress values is shown in Figs. 4(c) and 4(d), respectively. The rms velocity and normal stress are normalized with their corresponding values at $x = 0$. The collapse is much better than that observed in Fig. 2(b), which indicates that the fluctuations do play a role and w is not the only parameter influencing the flow behavior. The solid line shows an exponential fit. For smaller $u(w)/u(0)$ and $T(w)/T(0)$ ($w > 20d$), where the primary quantity responsible for unjamming is the fluctuations, the data collapse is quite good. However, the scaling shows increasing scatter at increasing values of $u(w)/u(0)$ and $T(w)/T(0)$ (or smaller w), which perhaps is indicative of the more complex dependence on the mean flow in addition to fluctuations and the interorifice width. A similar scaling behavior for both rms velocity and rms normal stress data is not quite surprising as both represent average fluctuations in the system and are quite interrelated to each other. A fluctuation in velocity is expected

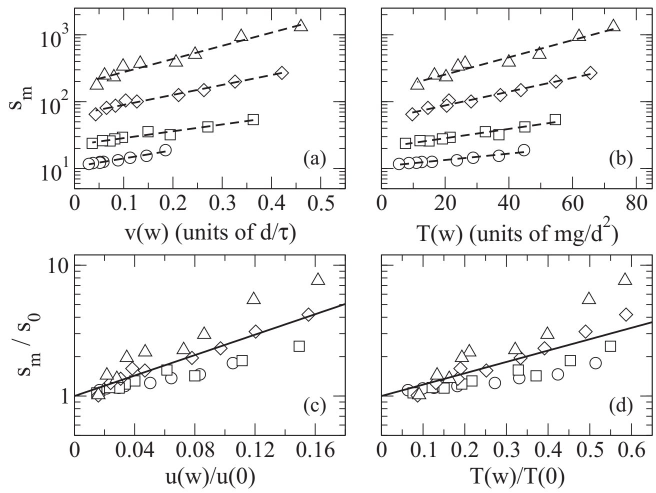

FIG. 4. Correlation between the mean avalanche size per unit orifice width (s_m) and (a) the rms velocity and (b) the rms normal stress. All quantities are obtained at distance (w). The dashed lines show an exponential fit. (c), (d) Data in (a) and (b), respectively, after normalizing s_m with the mean avalanche size (s_0) for a hopper with a single orifice. The solid line in (c) and (d) shows an exponential fit.

to generate collisions between particles, leading to fluctuations in stresses and vice versa.

In conclusion, our study suggest that the jamming occurrences within a hopper can be altered nonintrusively using multiple orifices by varying the interorifice distance. The hopper can be made to flow for a prolonged duration for a small enough orifice size by having another orifice of the same size at different distances. We observe that the fluctuations arising locally cause rearrangements of particles in the region located far away, which leads to spontaneous unjamming. We would also like to note that while the results have been reported for a strictly two-dimensional system, similar qualitative behavior is observed in simulations of three-dimensional or quasi-3D systems. This mechanism of spontaneous jamming and unjamming can be of immense importance for (a) more detailed explorations of the jamming characteristics for granular systems studied previously [18–21] and (b) investigating other disordered systems which exhibit jamming, viz., bubbles escaping from an orifice [22] or colloidal hard spheres flowing through a constriction [23]. As a system of practical utility, the multiorifice hopper can be operated as an efficient mixing device [24] wherein the mixing can be initialized through an interaction between different zones of a hopper through either random or controlled closing and opening of the orifice.

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