Symbolic dynamics-based error analysis on chaos synchronization via noisy channels

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In this study, symbolic dynamics is used to research the error of chaos synchronization via noisy channels. The theory of symbolic dynamics reduces chaos to a shift map that acts on a discrete set of symbols, each of which contains information about the system state. Using this transformation, a coder-decoder scheme is proposed. A model for the relationship among word length, region number of a partition, and synchronization error is provided. According to the model, the fundamental trade-off between word length and region number can be optimized to minimize the synchronization error. Numerical simulations provide support for our results.

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I. INTRODUCTION

Synchronization of nonlinear systems, particularly chaotic systems, has attracted the attention of many researchers [1,2]. Many control techniques have been devised for chaos synchronization [3]. Recently, the limitations of control under constraints imposed by a finite-capacity information channel have been investigated in detail in the literature of control theory. (See [4–8] and the references therein.) It has been shown that stabilization under information constraints is possible if and only if the capacity of the information channel exceeds the entropy production of the system at equilibrium. However, the results of previous work on control system analysis under information constraints do not apply to synchronization of systems, since in a synchronization problem the trajectories in phase space converge to a set rather than to a point; i.e., the problem cannot be reduced to simple stabilization. To address this problem, chaos synchronization via a communication channel of limited information capacity has been investigated [9,10].

Symbolic dynamics provides a rigorous way to investigate chaotic behaviors with finite precision [11], and it can be used in combination with information theory [12]. When using symbolic dynamics to analyze a chaotic system, one carefully divides its state space into small cells (regions), labels each cell with a symbol, and records the visit of evolving states in cells with a list of symbols (infinite word). In this way, the drive and response states, respectively, correspond to the drive and response infinite words. When infinite words and dynamical states are in one-to-one correspondence, the symbolic dynamics is equivalent to its underlying dynamics. This result is achieved for the so-called generating partition. However, the generating partitions of high-dimensional chaotic systems are difficult to build [11,13,14] if they actually exist. Therefore, it is not easy to guide the practice of synchronizing high-dimensional chaotic systems with these results [15–17]. To circumvent this difficulty, chaos synchronization based on symbolic dynamics using nongenerating partitions has been investigated [18]. To simplify the analysis, previous work on chaos synchronization via limited-capacity communication channels assumes that the observations are not corrupted by observation noise and that the transmission delay and transmission channel distortions may be neglected. However, in complex real-world networked control systems, data arrival times are often delayed, irregular, time varying, and not precisely known. Data might arrive out of order. Moreover, data transferred via a communication network might be corrupted or even lost due to noise in the communication medium, congestion of the communication network, or protocol malfunction [19].

In practice, synchronization error is the primary factor that affects the performance of a synchronized system. Until recently, most research work has focused on designing the controller to meet synchronization. The bound of synchronization error in a noisy environment is seldom considered. It is important to evaluate synchronization error under a given encoder-controller scheme in a noisy environment, which provides a theoretical basis to choose suitable parameters for optimal synchronization performance. For this purpose, we focus this study on the error analysis on chaos synchronization via noisy channels. Using symbolic dynamics, an efficient coder-decoder scheme is proposed, and a model on the relationship among synchronization error, word length, and region number of a partition is developed. Based on the model, we show that the trade-off between word length and region number can be optimized to minimize the synchronization error.

II. SYMBOLIC DYNAMICS AND CODER-DECODER

To describe our coder-decoder scheme expediently, we consider N-dimensional discrete-time chaotic systems. Let one of the two synchronized systems be controlled by a scalar control function $u(Y_{n+1}^k, w_{n+1})$ implementing the coupling between two systems. The model of the coupled systems is as follows:

$$\mathbf{x}_{n+1} = f(\mathbf{x}_n),\ \mathbf{x}_n \in \mathbf{X} \subset \mathbb{R}^N,\qquad y_{n+1} = \mathbf{C}\mathbf{x}_{n+1}, \tag{1}$$

$$\mathbf{z}_{n+1} = f(\mathbf{z}_n) + u\big(Y_{n+1}^k, w_{n+1}\big),\ \mathbf{z}_n \in \mathbf{X},\qquad w_{n+1} = \mathbf{C}\mathbf{z}_{n+1}, \tag{2}$$

*Corresponding author: linda_740609@aliyun.com where $\mathbf{x}_n$ and $\mathbf{z}_n$ are N-dimensional vectors of state variables, y_{n+1} and w_{n+1} are scalar output variables, $\mathbf{C}$ is a $1 \times n$ matrix, f is a continuous nonlinear function, and Y_{n+1}^k is the word of length k describing the output y_{n+1} with certain precision. Let $y = \{y_0, y_1, y_2 \cdots\}$. We name chaotic systems (1) and (2) the drive and response systems, respectively, and assume the trajectories of both chaotic systems to be in state space $\mathbf{X} \subset \mathbb{R}^N$. In this study, it is sufficient to assume that the controller can meet the needs of sustaining synchronization. The coding procedure is described in the following.

Here we introduce the basic idea of symbolic dynamics, which can efficiently describe dynamical behaviors with finite precision [15,17,18]. First, we divide y into a set of m disjoint regions $\{B_i | 0 \leqslant i \leqslant m-1\}$ and assign a unique symbol $i \in M = \{0,1,2,\ldots,m-1\}$ to region B_i. The set $\beta = \{B_i | 0 \leqslant i \leqslant m-1\}$ is a partition. Recording the jump of the state among regions with symbols, we can get an infinite word $Y_0^\infty = Y_0 Y_1 Y_2 \cdots$, where $Y_j \in M, j = 0,1,2,\cdots$. The jth symbol Y_j of the infinite word implies that the output is y_j, which lies in region B_{Y_j}.

Denote the mapping between a state and an infinite word with μ_β, then $\mu_\beta(y_0) = Y_0^\infty$. Symbolic dynamics turns chaotic system iterations into infinite word shifts; i.e., if $\mu_\beta(y_0) = Y_0 Y_1 Y_2 Y_3 \cdots$ then $\mu_\beta[Cf(x_0)] = \mu_\beta(y_1) = Y_1 Y_2 Y_3 \cdots$. This simple fact plays an important role in symbolic dynamics. Note that the μ_β mapping is not necessarily one to one. In most cases, one infinite word Y_0^∞ corresponds to more than one state although one state can only be mapped to one infinite word. If different states are mapped to different infinite words, except possibly for a subset of y of measure zero, then β is a generating partition. Otherwise it is a nongenerating partition.

A finite symbol sequence $Y_0^k = Y_0 Y_1 Y_2 \cdots Y_{k-1}$ is a word of length k. A fixed-length word shifts one symbol to its left in each iteration: $Y_0^k = Y_0 Y_1 Y_2 \cdots Y_{k-1}$ shifts to $Y_1^k = Y_1 Y_2 Y_3 \cdots Y_k$, and so on. In this study, a word of length k is the output of the coder. A word Y_0^k corresponds to a subset of y, which is $B_{Y_0^k}$. The word $Y_0^k = Y_0 Y_1 Y_2 \cdots Y_{k-1}$ will be obtained if and only if the chaotic system iterates from $B_{Y_0^k}$. Note that $B_{Y_0^k}$ is a subset of B_{Y_0} because the first symbol of Y_0^k is Y_0. If we denote $\{B_{Y_0^k} | Y_0^k \in M^k, Y_0^k \text{ is admissible}\}$ as β^k, then β^k is also a partition of y and is finer than β; that is, $\text{diam}(\beta^k) \leqslant \text{diam}(\beta)$, where diam() is the maximum diameter of all regions of the partition. In symbolic dynamics, β^k is called the partition refinement of partition β at stage k. In fact, with the increase in k, $\text{diam}(\beta^k)$ is monotonically nonincreasing. Furthermore $\lim_{k\to\infty} \text{diam}(\beta^k) = 0$ if β is a generating partition, otherwise $\lim_{k\to\infty} \text{diam}(\beta^k) > 0$.

Symbolic dynamics is used to represent the continuous trajectory of a chaotic system using discrete samples with finite precision. To generate a symbolic representation of a trajectory, the state space containing the chaotic attractor is partitioned into regions, each of which is labeled with a symbol such as "0", "1", etc. Whenever the state enters a new region, the corresponding symbol is generated. For flows represented by a return map, a new symbol is generated for each return. With regard to synchronization, the symbolic dynamics representation immediately shows that the channel only has to transmit one symbol per cycle to maintain synchronization under the controller $u(Y_{n+1}^k, w_{n+1})$ between drive and response systems to any level of precision. The block diagram for drive-response controlled synchronization is given in Fig. 1. To illustrate the algorithm of the decoder, the flow chart of the decoder algorithm is shown in Fig. 2.

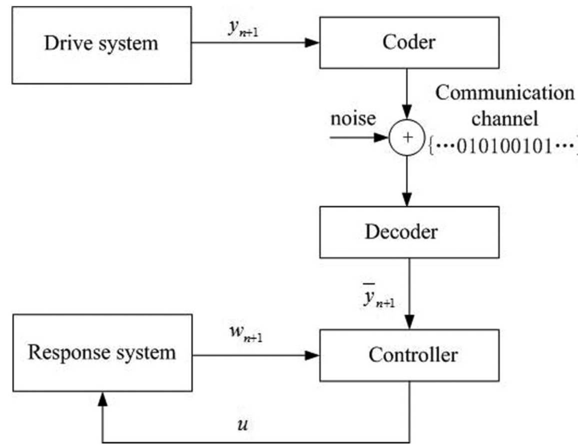

FIG. 1. Block diagram for drive-response controlled synchronization.

III. SYNCHRONIZATION ERROR ANALYSIS

We now find a relationship among synchronization error e, word length k, and region number m of a partition.

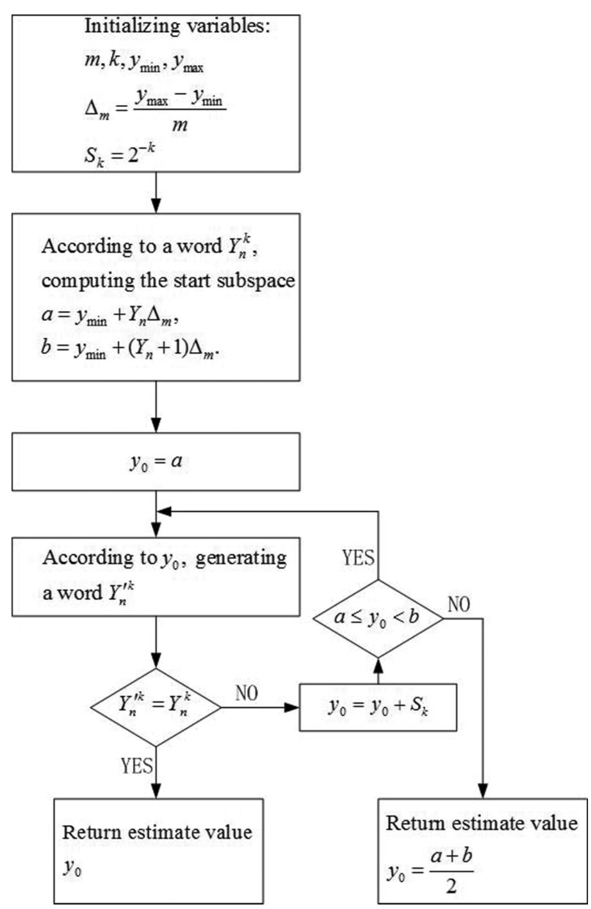

FIG. 2. Flow chart of the decoder algorithm.

As discussed above, a chaotic system provides a sequence of information bits, which are to be transmitted through a communication channel. The information bits occur at a uniform rate of R bits per cycle. The reciprocal of R is the bit interval T_b. The modulator takes q bits at a time and maps them into one of $m = 2^q$ signal wave forms. Each block of q bits is a symbol. The time interval available to transmit each symbol is $T = qT_b$. Hence, T is the symbol interval.

In most of this study, we use $m = 2^q$ multidimensional orthogonal signal wave forms to transmit information through an additive white Gaussian noise (AWGN) channel. Then, the probability of symbol error is given by [20]

$$P_m = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \{1 - [1 - Q(w)]^{m-1}\} e^{-(w-\sqrt{2qE_b/N_0})^2/2} dw, \tag{3}$$

where $Q(w) = \frac{1}{2\pi}\int_w^\infty e^{-v^2/2}dv$, E_b is the energy per bit, and the white Gaussian noise has a spectral power density of $N_0/2$ W/Hz. Note that for $m = 2^q$, q bits are required for the representation of each level. This means that if the bandwidth of the analog signal is W and if sampling is done at the Nyquist rate, the required bandwidth for the transmission of the pulse-code modulation (PCM) signal is qW. However, in practice a bandwidth of $1.5qW$ is closer to reality. In an example described in Sec. IV B, we also use $m = 2^q$ carrier-phase modulation signal wave forms to transmit information through an AWGN channel. Then, the probability of symbol error is given by [20]

$$P_m \approx 2Q\left(\sqrt{\frac{2qE_b}{N_0}}\sin\frac{\pi}{m}\right), \tag{4}$$

where $Q(w) = \frac{1}{2\pi}\int_w^\infty e^{-v^2/2}dv$, E_b is the energy per bit, and the white Gaussian noise has a spectral power density of $N_0/2$ W/Hz.

According to the coder-decoder pair, the error that limits the accuracy of synchronization in the absence of noise is

$$e_{\text{ideal}} = \Delta_m S_k, \tag{5}$$

where $\Delta_m = (y_{\max} - y_{\min})/m$ and $S_k = 1/2^k$; Δ_m denotes the interval length and S_k denotes the step length, which is applied in the decoder algorithm; $y_{\max}$ and $y_{\min}$ are the maximum and minimum of y, respectively. The error e_{ideal} is intrinsic; for a generating partition, $\lim_{k\to\infty} e_{\text{ideal}} = 0$. In practice, when transmitting q bits per cycle via a noisy channel, the probability of symbol error is given by Eq. (3). (A symbol consists of q bits.) In this study, we assume that the distance between two incorrect symbols is larger than k; i.e., there are at least $k+1$ symbols between two consecutive incorrect symbols. For a word of length k, which is composed of k symbols, an incorrect symbol leads to k words being incorrect. For example, when a word $Y_{n-1}^k = Y_{n-1}Y_n \cdots Y_{n+k-2}$ is correct and a word $Y_n^k = Y_n Y_{n+1} \cdots Y_{n+k-1}$ is incorrect, the symbol Y_{n+k-1} is incorrect; then the word $Y_{n+1}^k = Y_{n+1}Y_{n+2}\cdots Y_{n+k-1}Y_{n+k}$ is also incorrect, and so on. Here we assume that each symbol has the same probability of error. According to the above discussion and the decoder algorithm, the synchronization error via a noisy channel is modeled as

$$e_{\text{noise}} = P_m \frac{\Delta_m}{2}(k-1) + P_m \frac{(m-1)\Delta_m}{2}. \tag{6}$$

The first term on the right-hand side of Eq. (6) denotes the synchronization error when the first symbol in a word of length k is correct; the second term denotes the synchronization error when the first symbol in a word of length k is incorrect. Thus, the total synchronization error is modeled as

$$e = g(k,m) = P_m \frac{\Delta_m}{2}(k-1) + P_m \frac{(m-1)\Delta_m}{2} + \Delta_m S_k. \tag{7}$$

From Eq. (7), we can obtain the optimal word length k^* such that the synchronization error is minimized:

$$k^* = -\log_2 [P_m/(2\ln 2)]. \tag{8}$$

By communication theory, the symbol error probability P_m is determined by the signal-to-noise ratio (SNR) and the region number m of a partition. Therefore, the optimal word length k^* is determined by the SNR and the region number m of a partition. From Eq. (8), we find that $\lim_{P_m\to 0} k^* = \infty$, which implies that $\lim_{k\to\infty} e_{\text{ideal}} = 0$ for a generating partition. Note that the synchronization error e is the mean absolute error (MAE). For a given chaotic system, the range of output y is certain. Proper region number m of a partition and word length k are chosen to meet synchronization. As seen in Eqs. (3) and (4), the symbol error probability P_m depends on the wave forms used to transmit information. For any digital transmission approach, it is only necessary to change the symbol error probability P_m. Therefore, this error model is general for symbolic dynamics-based error analysis on chaos synchronization.

IV. SIMULATION RESULTS

To verify the above model of synchronization error, we choose some chaotic systems as examples. Here we use the controller [10,17]

$$u(Y_{n+1}^k, w_{n+1}) = K[w_{n+1} - \eta(Y_{n+1}^k)], \tag{9}$$

where $K \in \mathbb{R}$ is the coupling coefficient and the function η denotes the decoder output, which is used to estimate y_{n+1}. The decoder algorithm chart is shown in Fig. 2.

A. Logistic map

The drive system is

$$\begin{aligned} x_{n+1} &= f(x_n) = 4x_n(1-x_n),\ x_n \in [0,1], \\ y_{n+1} &= x_{n+1}. \end{aligned} \tag{10}$$

The controlled response system is

$$\begin{aligned} z_{n+1} &= f(z_n) = 4z_n(1-z_n) + u(Y_{n+1}^k, w_{n+1}), \\ z_n &\in [0,1], \quad w_{n+1} = z_{n+1}. \end{aligned} \tag{11}$$

In the following, two different partitions are used for comparison.

First, we choose the partition as $\beta = \{B_0, B_1, B_2, B_3\}$, where $B_0 = [0,0.25)$, $B_1 = [0.25,0.5)$, $B_2 = [0.5,0.75)$, and $B_3 =$

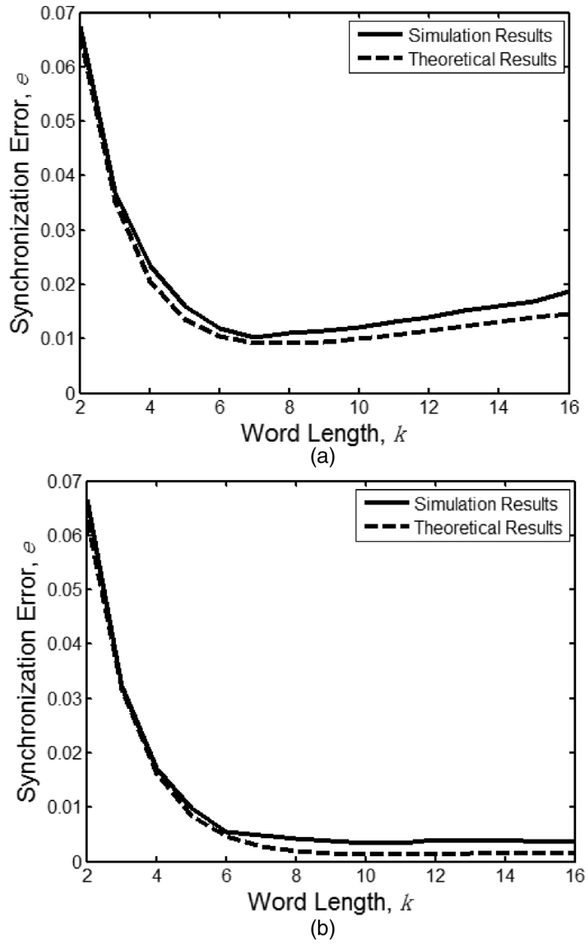

FIG. 3. Synchronization error of a logistic map with $m = 4$ as a function of word length for a SNR of (a) $\gamma_{\rm SNR} = 4$ dB and (b) $\gamma_{\rm SNR} = 6$ dB. The solid curves represent simulation results, and the dashed curves represent the exact theoretical solutions of Eq. (7).

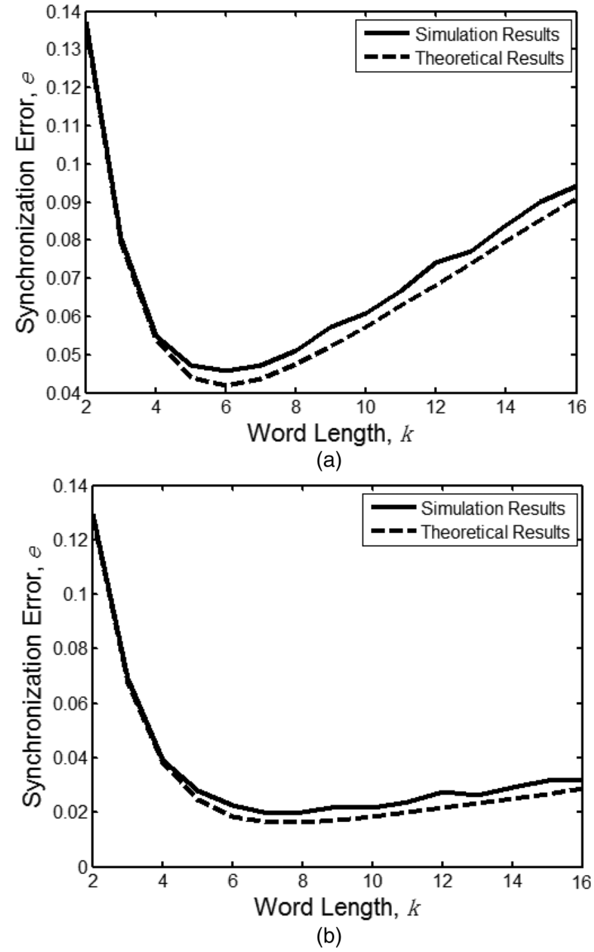

FIG. 4. Synchronization error of a logistic map with $m = 2$ as a function of word length for a SNR of (a) $\gamma_{\rm SNR} = 4$ dB and (b) $\gamma_{\rm SNR} = 6$ dB. The solid curves represent simulation results, and the dashed curves represent the exact theoretical solutions of Eq. (7).

[0.75,1]; thus, the region number of the partition is $m = 4$, $y_{\max} = 1$, $y_{\min} = 0$, and $\Delta_m = (y_{\max} - y_{\min})/m = 1/4$. The coupling coefficient is chosen to be $K = 0.8$. We use $m = 4$ multidimensional orthogonal signal wave forms to transmit information through an AWGN channel so that P_m is given by Eq. (3). The relationship between the synchronization error e and the word length k is plotted in Fig. 3. Here, we choose two different SNR values of $\gamma_{\rm SNR} = E_{\rm b}/N_0 = 4$ dB and $\gamma_{\rm SNR} = E_{\rm b}/N_0 = 6$ dB as simulation parameters for the data shown in Figs. 3(a) and 3(b), respectively. To obtain the simulation results, we run 10 000 times at each word length k, and compute the MAE. The simulation results (solid curves) are compared to the exact theoretical solutions (dashed curves) of Eq. (7) using P_m given by Eq. (3).

Next, we choose the partitions as $\beta = \{B_0, B_1\}$, where $B_0 = [0,0.5)$ and $B_1 = [0.5,1]$; thus, the region number of the partition is $m = 2$, $y_{\max} = 1$, $y_{\min} = 0$, and $\Delta_m = (y_{\max} - y_{\min})/m = 1/2$. The coupling coefficient is chosen to be $K = 0.8$. We use $m = 2$ multidimensional orthogonal signal wave forms to transmit information through an AWGN channel so that P_m is given by Eq. (3). The relationship between the synchronization error e and the word length k is plotted in Fig. 4. Here, we choose two different SNR values of $\gamma_{\rm SNR} = E_{\rm b}/N_0 = 4$ dB and $\gamma_{\rm SNR} = E_{\rm b}/N_0 = 6$ dB as simulation parameters for the data shown in Figs. 4(a) and 4(b), respectively. To obtain the simulation results, we run 10 000 times at each word length k, and compute the MAE. The simulation results (solid curves) are compared to the exact theoretical solutions (dashed curves) of Eq. (7) using P_m given by Eq. (3).

B. Chua system

The drive system is given by

$$\begin{aligned} \dot{x}_1 &= a_1[-x_1 + \varphi(x_1) + x_2], \\ \dot{x}_2 &= x_1 - x_2 + x_3, \\ \dot{x}_3 &= -a_2 x_2, \\ y &= x_1, \end{aligned} \tag{12}$$

where $-5 \leqslant y \leqslant 5$ is the drive system output; a_1 and a_2 are known parameters; $\varphi(x_1)$ is a piecewise-linear function, having the form

$$\varphi(x_1) = b_0 x_1 + b_1(|x_1 + 1| - |x_1 - 1|), \tag{13}$$

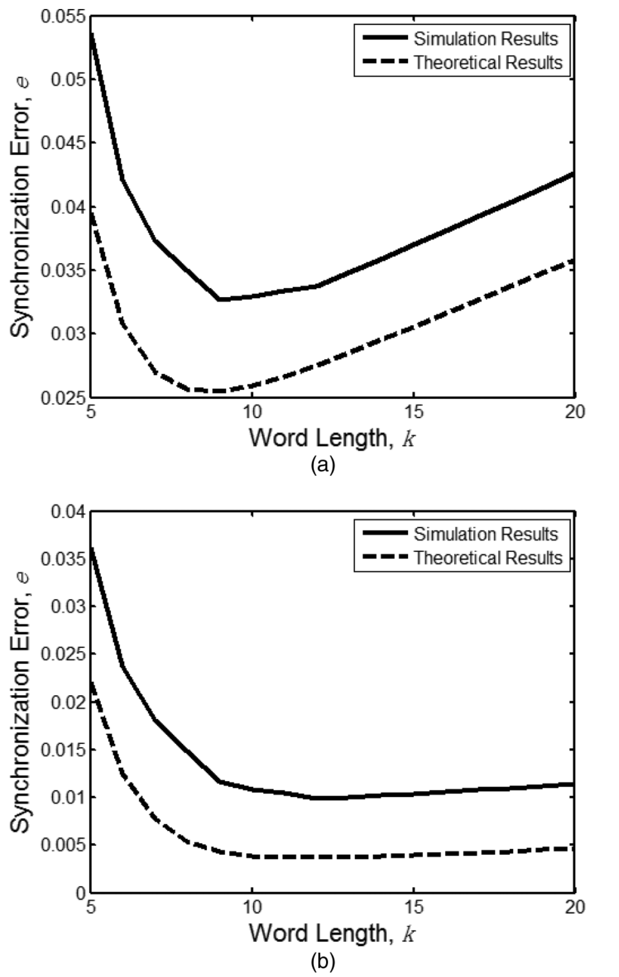

FIG. 5. Synchronization error of a Chua system with $m = 16$ as a function of word length for a SNR of (a) $\gamma_{\mathrm{SNR}} = 3$ dB and (b) $\gamma_{\mathrm{SNR}} = 4$ dB. The solid curves represent the simulation results, and the dashed curves represent the exact theoretical solutions of Eq. (7).

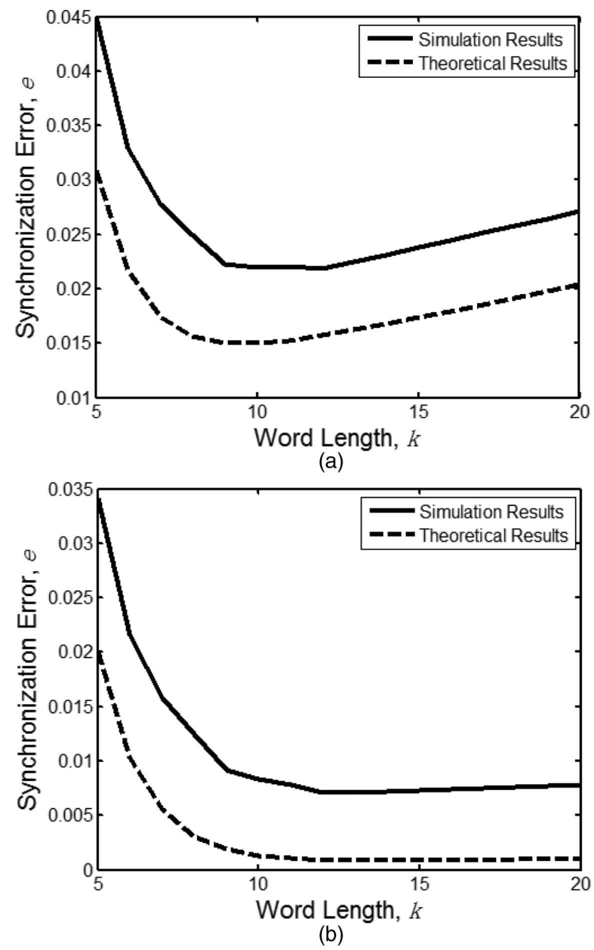

FIG. 6. Synchronization error of a Chua system with $m = 16$ as a function of word length for a SNR of (a) $\gamma_{\mathrm{SNR}} = 15$ dB and (b) $\gamma_{\mathrm{SNR}} = 17$ dB. The solid curves represent the simulation results, and the dashed curves represent the exact theoretical solutions of Eq. (7).

where b_0 and b_1 are given parameters. The controlled response system is given by

$$\begin{aligned} \dot{z}_1 &= a_1\left[-z_1 + \varphi(z_1) + z_2 + u\left(Y_{n+1}^k, w_{n+1}\right)\right], \\ \dot{z}_2 &= z_1 - z_2 + z_3, \\ \dot{z}_3 &= -a_2 z_2, \\ w &= z_1, \end{aligned} \tag{14}$$

where $\varphi(z_1)$ is defined by the function in Eq. (13).

Here, the output y is equally divided into 16 disjoint regions. The following parameter values are taken for the simulation: The Chua system parameters are $a_1 = 10$, $a_2 = 15.6$, $b_0 = 0.33$, and $b_1 = 0.945$; region number $m = 16$ of the partition, $y_{\max} = 5$, $y_{\min} = -5$, $\Delta_m = (y_{\max} - y_{\min})/m = 10/16$. The coupling coefficient is chosen to be $K = 1$. We use $m = 16$ multidimensional orthogonal signal wave forms to transmit information through an AWGN channel so that P_m is given by Eq. (3). The relationships between synchronization error e and word length k are plotted in Fig. 5. Here, we choose two different SNR values of $\gamma_{\mathrm{SNR}} = E_{\mathrm{b}}/N_0 = 3$ dB and $\gamma_{\mathrm{SNR}} = E_{\mathrm{b}}/N_0 = 4$ dB as simulation parameters for the data shown in Figs. 5(a) and 5(b), respectively. To obtain the simulation results, we run 10 000 times at each word length k, and compute the MAE. The simulation results (solid curves) are compared to the exact theoretical solutions (dashed curves) of Eq. (7) using P_m given by Eq. (3).

To further verify the above model of synchronization error, we use $m = 16$ carrier-phase modulation signal wave forms to transmit information through an AWGN channel so that P_m is given by Eq. (4). The following parameter values are taken for the simulation: The Chua system parameters are $a_1 = 10$, $a_2 = 15.6$, $b_0 = 0.33$, and $b_1 = 0.945$; region number $m = 16$ of the partition, $y_{\max} = 5$, $y_{\min} = -5$, $\Delta_m = (y_{\max} - y_{\min})/m = 10/16$. The coupling coefficient is chosen to be $K = 1$. The relationships between synchronization error e and word length k are plotted in Fig. 6. Here, we choose two different SNR values of $\gamma_{\mathrm{SNR}} = E_{\mathrm{b}}/N_0 = 15$ dB and $\gamma_{\mathrm{SNR}} = E_{\mathrm{b}}/N_0 = 17$ dB as simulation parameters for the data shown in Figs. 6(a) and 6(b), respectively. To obtain the simulation results, we run 10 000 times at each word length k, and compute the MAE. The simulation results (solid curves) are compared to the exact theoretical solutions (dashed curves) of Eq. (7) using P_m given by Eq. (4).

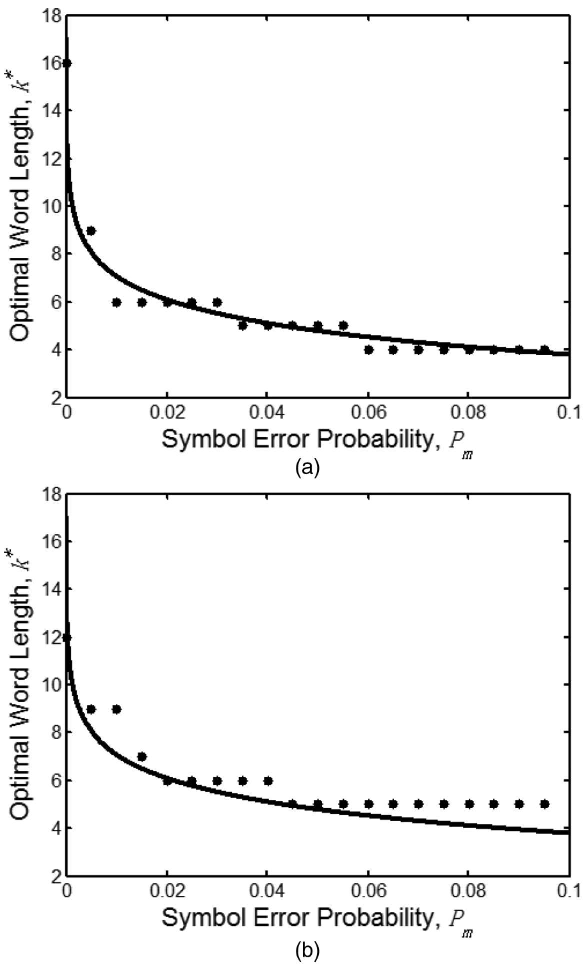

FIG. 7. Optimal word length with $10^{-5} \leqslant P_m \leqslant 0.1$ as a function of symbol error probability for (a) logistic map and (b) Chua system. The dots represent the simulation results, and the solid curves represent the exact theoretical solutions of Eq. (8).

C. Optimal word length

Consider a given chaotic system; there exists a proper region number m of a partition for a given communication system. Thus, the symbol error probability P_m can be obtained from the SNR for a given digital transmission approach. For this reason, the optimal word length k^* can be evaluated using Eq. (8), which is shown in Fig. 7 as a function of symbol error probability P_m. The simulation results make it possible to choose the optimal word length k^* for a given chaotic system. Here, we use a logistic map and a Chua system as examples for the data shown in Figs. 7(a) and 7(b), respectively.

From the simulation results, we can see that for symbolic dynamics-based synchronization there exists an optimal word length k^* for a given combination of SNR and region number m such that the synchronization error is minimized. Furthermore, for a given SNR, the synchronization error monotonically decreases with increasing region number m of the partition; for a given partition region number m, the synchronization error monotonically decreases with increasing SNR. By communication theory, with the increase in the region number m of a partition, the required bandwidth also increases. In general, the supplied bandwidth is constant for a given communication system, which implies that the maximum of the required partition region number m is constant for a given communication system. Therefore, for symbolic dynamics-based synchronization there exists a proper partition region number m for a given communication system. When a proper region number m of a partition is chosen, the symbol error probability P_m is determined by the SNR for a given digital transmission approach.

Based on the above discussion, the synchronization error e is determined by the symbol error probability P_m and the word length k. In practice, one can use Eq. (8) to select the optimal word length k^* to minimize the synchronization error for a given symbol error probability P_m once P_m is found from the given SNR and partition region number m.

V. CONCLUSIONS

In this study, we propose a coder-decoder scheme based on symbolic dynamics and provide a model for the relationship among word length, region number of a partition, and synchronization error. From the statistics viewpoint, the model is general and can be used to optimize the trade-off among the region number of a partition, the word length, and the synchronization error via noisy channels. This general model is independent of the particular system. In practice, Eq. (8) can be used to effectively select the optimal word length k^* to meet synchronization performance. The lower bound of error is also given by the model, which is important for estimating the synchronization performance of a system.

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[1] I. I. Blekhman, *Synchronization in Science and Technology* (ASME Press, New York, 1988).
[2] A. Pikovsky, M. Rosenblum, and J. Kurths, *Synchronization: A Universal Concept in Nonlinear Sciences* (Cambridge University Press, Cambridge, UK, 2001).
[3] J. M. G. Miranda, *Synchronization and Control of Chaos: An Introduction for Scientists and Engineers* (Imperial College Press, London, 2004).
[4] G. N. Nair and R. J. Evans, Automatica **39**, 585 (2003).

[5] G. N. Nair and R. J. Evans, SIAM J. Control Optim. **43**, 413 (2004).

[6] G. N. Nair, R. J. Evans, I. Mareels, and W. Moran, IEEE Trans. Autom. Control **49**, 1585 (2004).

[7] L. M. J. Bazzi and S. K. Mitter, IEEE Trans. Inf. Theory **51**, 2103 (2005).

[8] G. N. Nair, F. Fagnani, S. Zampieri, and R. Evans, Proc. IEEE **95**, 108 (2007).

[9] A. L. Fradkov, B. Andrievsky, and R. J. Evans, Phys. Rev. E **73**, 066209 (2006).

[10] A. L. Fradkov, B. Andrievsky, and R. J. Evans, Phys. Rev. E **78**, 036210 (2008).

[11] B. L. Hao and W. M. Zheng, *Applied Symbolic Dynamics and Chaos* (World Scientific, Singapore, 1998).

[12] C. E. Shannon, Bell Syst. Tech. J. **27**, 379 (1948).

[13] Y. Hirata, K. Judd, and D. Kilminster, Phys. Rev. E **70**, 016215 (2004).

[14] M. B. Kennel and M. Buhl, Phys. Rev. Lett. **91**, 084102 (2003).

[15] S. D. Pethel, N. J. Corron, Q. R. Underwood, and K. Myneni, Phys. Rev. Lett. **90**, 254101 (2003).

[16] T. Stojanovski, L. Kocarev, and R. Harris, IEEE Trans. Circuits Syst., I: Fundam. Theory Appl. **44**, 1014 (1997).

[17] M. Wang, X. Wang, Z. Liu, and H. Zhang, Chaos **21**, 013107 (2011).

[18] X. Wang, M. Wang, and Z. Liu, Chaos **19**, 023108 (2009).

[19] A. S. Matveev and A. V. Savkin, *Estimation and Control over Communication Networks* (Birkhäuser, Boston, 2009).

[20] J. G. Proakis and M. Salehi, *Contemporary Communication Systems using MATLAB* (PWS Publishing Company, Boston, 1998).