

Transmission of linear regression patterns between time series: From relationship in time series to complex networks

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The linear regression parameters between two time series can be different under different lengths of observation period. If we study the whole period by the sliding window of a short period, the change of the linear regression parameters is a process of dynamic transmission over time. We tackle fundamental research that presents a simple and efficient computational scheme: a *linear regression patterns transmission algorithm*, which transforms linear regression patterns into directed and weighted networks. The linear regression patterns (nodes) are defined by the combination of intervals of the linear regression parameters and the results of the significance testing under different sizes of the sliding window. The transmissions between adjacent patterns are defined as edges, and the weights of the edges are the frequency of the transmissions. The major patterns, the distance, and the medium in the process of the transmission can be captured. The statistical results of weighted out-degree and betweenness centrality are mapped on timelines, which shows the features of the distribution of the results. Many measurements in different areas that involve two related time series variables could take advantage of this algorithm to characterize the dynamic relationships between the time series from a new perspective.

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I. INTRODUCTION

In the real world there exist linear or nonlinear relationships between variables in many fields. We can probe the changes in one variable in terms of the changes in another variable using a linear regression model, which is one of the most widely used methods. Although the relationships between some variables are nonlinear, the newly created variables that result from some appropriate mathematical transformations have linear or approximately linear relationships. The cointegration theory that was proposed by Engle and Granger has been the most important approach to analyze the long-term equilibrium relationships between time series [1]. Most of the linear regression models are good at indicating the correlations using functions, and they provide information about the linear relationship between two time series for a period of time. However, at this point, we should focus on the issue that variables are fluctuating over time; thus, the relationships between them are also changing at the same time. The Granger representation theorem has proved that there exists a process that the short-term fluctuation adjusts toward to the long-term equilibrium [1,2]. However, there are few of studies on the specific dynamic mechanism of the adjustment.

Although scholars can denote the relationship by means of piecewise functions or dynamic linear regression models [3,4], these models cannot contain the integrated information of the evolution of the linear regression in the whole period. The time series is continuous and so is the relationship between variables; these models were also limited in demonstrating the

transmission of the dynamic relationship between variables. If we build linear equations by sliding windows, the linear relationship between two time series can be characterized more accurately. However, this process will produce a very large equation set. It is difficult to find the solution of the equation set, and also difficult to capture the transmission features of the linear relationship and the dynamic processes of the transmission.

In order to study the dynamic characteristics in time series, with the development of complex network theory [5–7], some studies have transformed time series into networks. Zhang, Small, and Xu introduced a method to address the pseudoperiodic time series and found that the structure of the corresponding network depended on the dynamics of the series [8,9]. Li *et al.* presented a scheme to extract a multiscale state space network from a single-molecule time series [10]. Some researchers transformed a linear model containing terms with different time delays into complex networks [11]. The time series was divided into fragments that have fixed sizes [12–14]. Lacasa *et al.* proposed the visibility graph algorithm, which can map all types of time series into networks [15]. Then, the Hurst exponent of fractional Brownian motion is studied by means of the visibility algorithm [16]. Thus far, the visibility algorithm has been diversely used in many areas [17,18]. In this sense, complex network theory is effective in analyzing the nonlinear dynamic characteristics [19].

Most recent studies mainly focus on describing the linear relationships by establishing various linear regression models, but they ignore the fact that the evolution of linear regression of time series is a process of dynamic transmission. Moreover, there are different linear regression patterns in different fragments of the whole period. These patterns change and transform into each other over time. Thus, it is necessary

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to introduce an approach to capture the inner transmission mechanism so as to help us to understand the fluctuation of relationships between two variables over time.

II. ALGORITHM DESCRIPTION

In this article we present a simple and efficient algorithm based on econophysics to capture the inner transmission mechanism and dynamic characteristics of the evolution of linear regression. It can help us to understand the adjustment process (the short-term fluctuation adjusts toward the long-term equilibrium). The *linear regression patterns transmission algorithm* (LRPTA) we proposed maps the transmission of the linear regression patterns between two time series into a network. Although the LRPTA also encodes the time series into a network, it is fundamentally different from other methods [8,16]. The previous algorithms focus on characterizing a univariate time series, but there remains a challenging issue to define the relation patterns between two time series and the corresponding transmission networks of the relation patterns. This article focuses on the transmission of the evolution of linear regression instead of the fluctuation of the variables. We propose the scheme for the algorithm in Fig. 1.

Step 1. Defining the size of the sliding window ω and dividing the whole period into fragments. We divide the whole time series set (x and y) into fragments by the sliding windows with the sliding step length of 1 based on the idea of the phase space reconstruction theory [20]. Each fragment contains ω pairs of values of x and y . The advantage of utilizing the method is that the fragments have the feature of memory and transitivity [21]. The value of ω is the length of the fragments; thus, the number of fragments is $n - \omega + 1$. Moreover, we can set different ω depending on different needs of analysis. If you want to study the features of the transmission of the linear regression patterns based on short periods, you can set a smaller value of ω . If you want to understand the features of the transmission based on long periods, you can set a larger value of ω .

Step 2. Estimating the values of the parameters α and β in the linear regression equation of each fragment. In

fact, the linear regression model is simple and is based on the straightforward functional form $y = \beta + \alpha x$, but it is effective in revealing the linear relationships between variables. In a linear regression model, the values of the parameters α and β can reflect the patterns of the linear relationship between the independent variable x and the dependent variable y . It means that the dependent variable y will change the α unit when the independent variable x changes 1 unit and the parameter β is the intercept. The parameters estimation plays an important role in the process of constructing the linear regression models. The most basic and effective method of ordinary least square (OLS) is utilized for estimating the two parameter values. Hence, we can obtain a series of values for the parameters α and $\beta \{ \alpha_1, \alpha_2, \alpha_3, \dots, \alpha_{n-\omega+1} \}$. Each combination of the two parameters means a linear regression equation, which describes the linear relationship between the two time series in a fragment of the whole period under the sliding window size of ω .

Step 3. Significance test of the parameters α of the linear regression equation of each fragment. The parameter α indicates the regression level between two time series. Thus, this step is necessary because passing the test means that the independent variable x has a significant impact on the dependent variable y and the linear regression equation is effective. Whether the significance test is passed depends on the P value through Student's t test. Specifically, if the P value is less than 5%, then the result of the significance is acceptable. In Fig. 1 the symbol P denotes that the linear regression of the fragment passes the significance test; otherwise, if the linear regression of the corresponding fragment does not pass the significance test, we use N as a mark.

Step 4. Building the linear regression patterns. In this step we first allocate the parameters α and β to the different intervals. We define 0.1 as the interval extent of the parameters α and 5 as the interval extent of the parameters β . Thus, the combination of α and β can be allocated into different intervals. Then, the combination of intervals of the parameters α and β is defined as linear regression patterns. In addition, if the fragment does not pass the significance test, then we mark N at the end of the patterns. For example, in Fig. 1, in the first fragment (i.e., fragment₁), the parameters are $\alpha_1 = 0.95, \beta_1 = 4.2$ and the parameter α passes the significance test. Thus, the

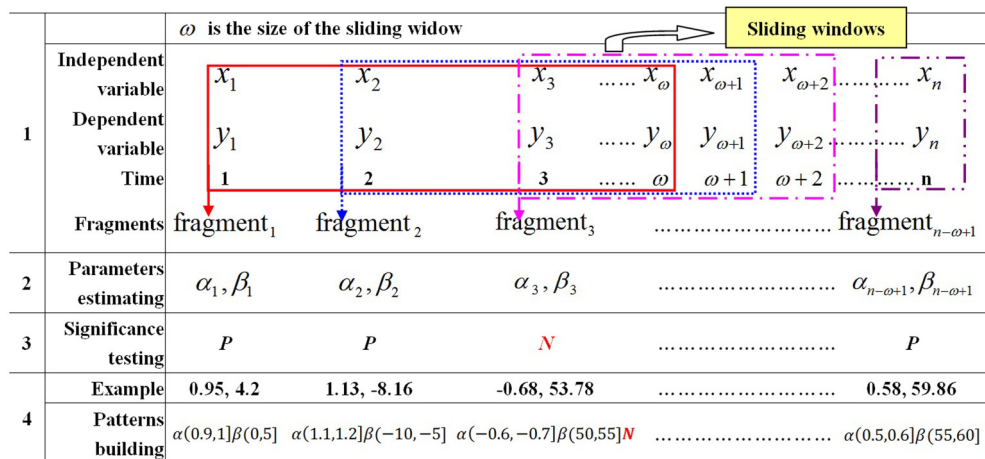


FIG. 1. (Color online) The process of building patterns.

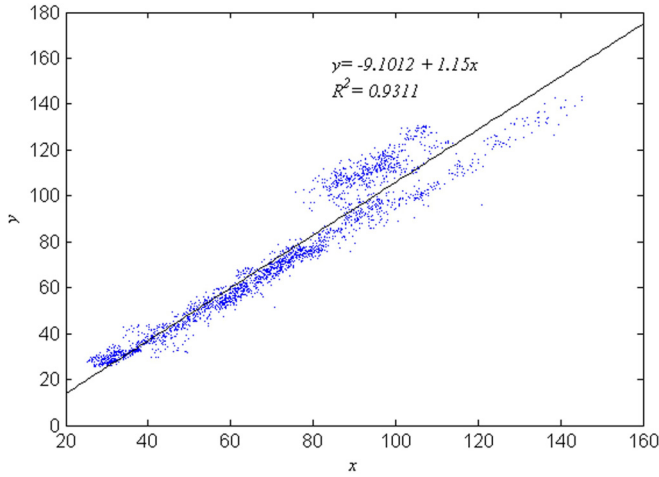


FIG. 2. (Color online) The linear regression between the West Texas Intermediate crude oil future price series and the Daqing China crude oil spot price series. The fitting linear equation is $y = -9.1012 + 1.15x$, the goodness of fit $R^2 = 0.9311$, and the P value $p \ll 1\%$, which passes the significance test.

linear regression pattern of this fragment is $\alpha(0.9, 1]\beta(0, 5]$. The purpose of this step is that we utilize the limited patterns to show the intrinsic features of the transmission between the continuous patterns. At the same time, the interzone patterns are necessary for constructing the transmission networks.

Step 5. Constructing the transmission network. We therefore obtain the sequence of the linear regression patterns $\{\alpha(0.9, 1]\beta(0, 5], \alpha(1.1, 1.2]\beta(-10, -5], \alpha(-0.6, -0.7]\beta(50, 55]N, \dots, \alpha(0.5, 0.6]\beta(55, 60)\}$ from step 4. The linear regression patterns evolve over time $\{\alpha(0.9, 1]\beta(0, 5] \rightarrow \alpha(1.1, 1.2]\beta(-10, -5] \rightarrow \alpha(-0.6, 0.7]\beta(50, 55]N \rightarrow \dots \rightarrow \alpha(0.5, 0.6]\beta(55, 60)\}$. There are $n - \omega + 1$ patterns in the sequence. However, after allocating the values of the parameters to the different intervals, there are not many types of patterns. For example, we can obtain 2721 patterns when $\omega = 20$. However, there are only 327 types of patterns in the 2721 patterns. Thus, 327 types of linear regression patterns transform into each other and then form a transmission

matrix $\begin{bmatrix} w_{1,1} & w_{1,2} & \dots & w_{1,327} \\ w_{2,1} & w_{2,2} & \dots & w_{2,327} \\ \vdots & \vdots & \ddots & \vdots \\ w_{327,1} & w_{327,2} & \dots & w_{327,327} \end{bmatrix}$ ($w_{i,j}$ is the frequency of the transmission from the i th pattern to j th pattern). Then, we define the 327 types of linear regression patterns as nodes and the transformations as edges. The weight of an edge is the frequency of the transmission between the two types of patterns. Thereby, we obtain a directed and weighted transmission network.

III. RESULTS

A. Statistical characteristics

In the economic area, there exist cross correlations between economic time series [22–24] and the long-term equilibrium relationship between future prices and spot prices have been proven in many studies [25–27]. The relationship between future prices and spot prices is an important issue that people arbitrage based on the fluctuation of the relationship. Thus, we need to understand the relationship structure by mathematic form. For example, there exists a linear relationship between the West Texas Intermediate crude oil future price series (x) and Daqing China crude oil spot price series (y) on a large scale. We selected 2740 sets of data from 2002 to 2013 (see Fig. 2).

From Fig. 2 we find that the relationship between future and spot prices exhibits good linear regression over the entire series. The fitting linear regression equation shows a high level of the goodness of fit and passes the significance test. There is a long-term equilibrium relationship between two variables. However, in fact, the linear regression relationship not always follows this fitting equation. As shown in Fig. 3 there are different linear regression patterns in different periods following different fitting equations (if we just divide the time series data into four periods based on the trend of fluctuations). When we divide the whole time series into different smaller-scale fragments by sliding windows, we can obtain the linear regression pattern of each fragment.

Then, we can estimate the corresponding values of α and β for each fragment. A series of the values of α and β can be gained for different sizes of sliding window. We took $\omega = 20$

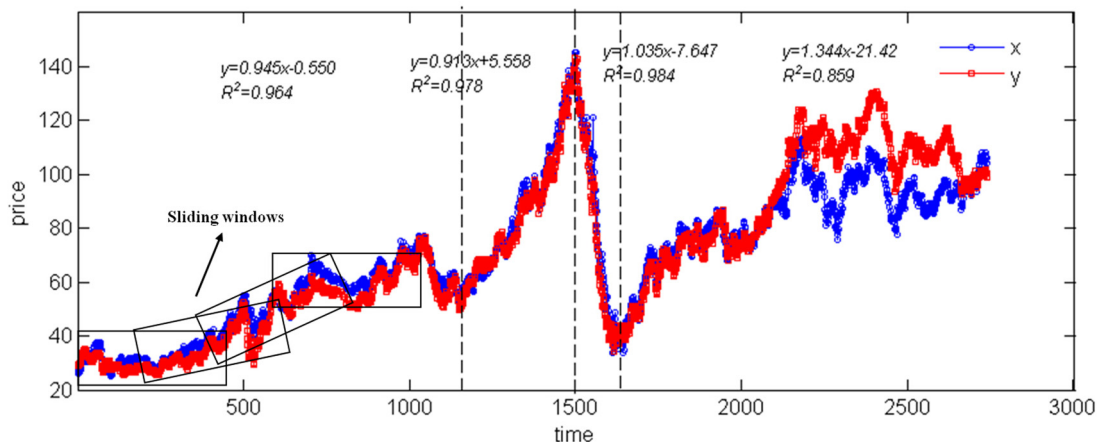


FIG. 3. (Color online) Different linear regression patterns in different periods.

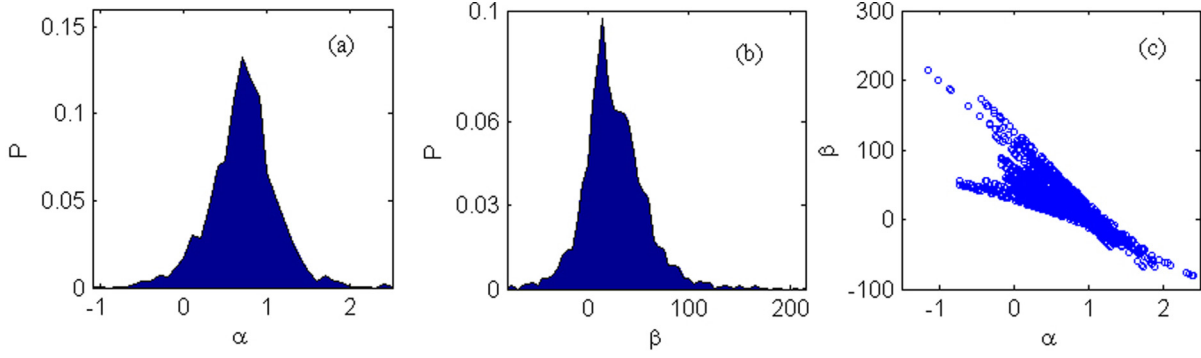


FIG. 4. (Color online) The frequencies and scatter diagram of the values of the parameters α and β ($\omega = 20$). (a) The frequencies of the values of the parameters α . (b) The frequencies of the values of the parameters β . (c) The scatter diagram of the values of the parameters α and β .

as an example. The distributions of the values of α and β are shown in Figs. 4(a) and 4(b), respectively. We find that the values of α mainly distribute between 0.6 and 0.9 instead of 1.15, and the values of β mainly distribute between 10 and 20 rather than -9.1012 in the fitting linear equation. In Fig. 4(c) the scatter diagram of α and β shows that there are many types of linear regression patterns.

B. Identifying the major patterns (weighted out-degree distribution)

According to the above steps, we can obtain $n - \omega + 1 = 2721$ fragments when $n = 2740$ and $\omega = 20$, in which 2234 of these fragments pass the significance test (account for 82.1%) and the leftover 487 patterns are marked with “N” (account for 17.9%). However, only 327 types of patterns appear in the transmission network.

In Fig. 5 we plot the behavior of the transmission of the linear regression patterns over time by weighted out-degree:

$$w_i^{\text{out}} = \sum_{j \in N_i} w_{i,j}, \quad (1)$$

where N_i is the set of neighbors of node i and w_{ij} is the weight of the edge from node i to node j . The higher weighted out-degree a node has, the more important it is in the transmission network.

First, there are major patterns in the trajectory of the pattern transmissions. From Fig. 5(a), the image is mainly filled with the colors white and yellow. This arrangement means that few types of patterns play a major role in the transmission. Additionally, the condition can also be proved in the first subgraph of Fig. 6 that the weighted out-degree distribution follows a power law $p(w) \sim w^{-\lambda}$ with $\lambda = 1.09 \pm 0.01$. It means that a few types of patterns play a major role in the time series. The most important two linear regression patterns

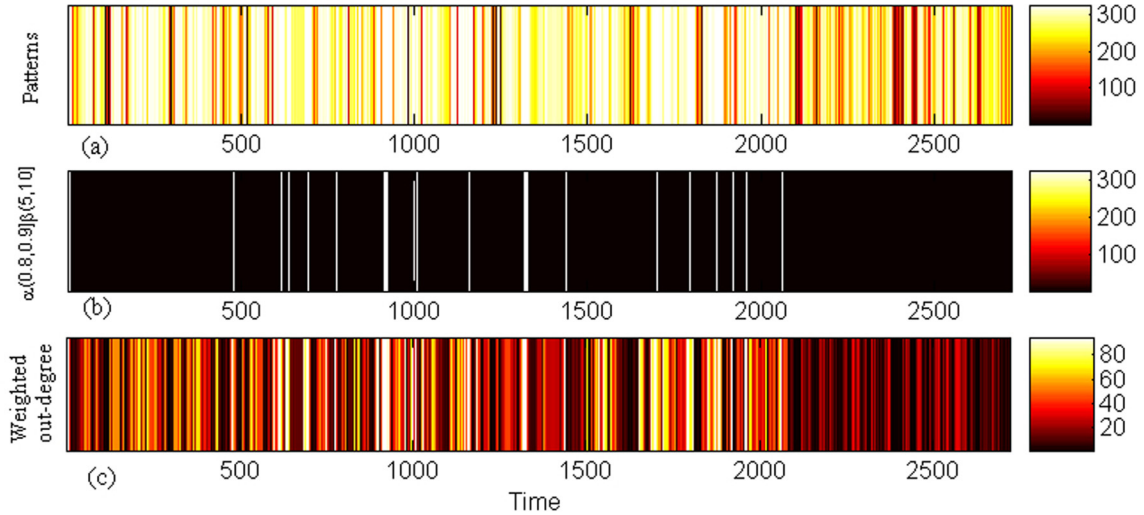


FIG. 5. (Color online) The transmission of the linear regression patterns and the weighted out-degree distribution over time ($\omega = 20$). (a) The distribution of the weighted out-degree of the 327 types of patterns over time. There are 327 types of colors, which indicate the 327 types of patterns, respectively. The higher the weighted out-degree of the patterns, the higher the value of the color bar. (b) An example of the distribution of a pattern with the highest weighted out-degree in the time series (the color bar value is 327). (c) The distribution of the weighted out-degree distribution over time. There are 95 types of colors, which indicate the 95 values of the weighted out-degree. The higher the weighted out-degree, the higher the value of the color bar.

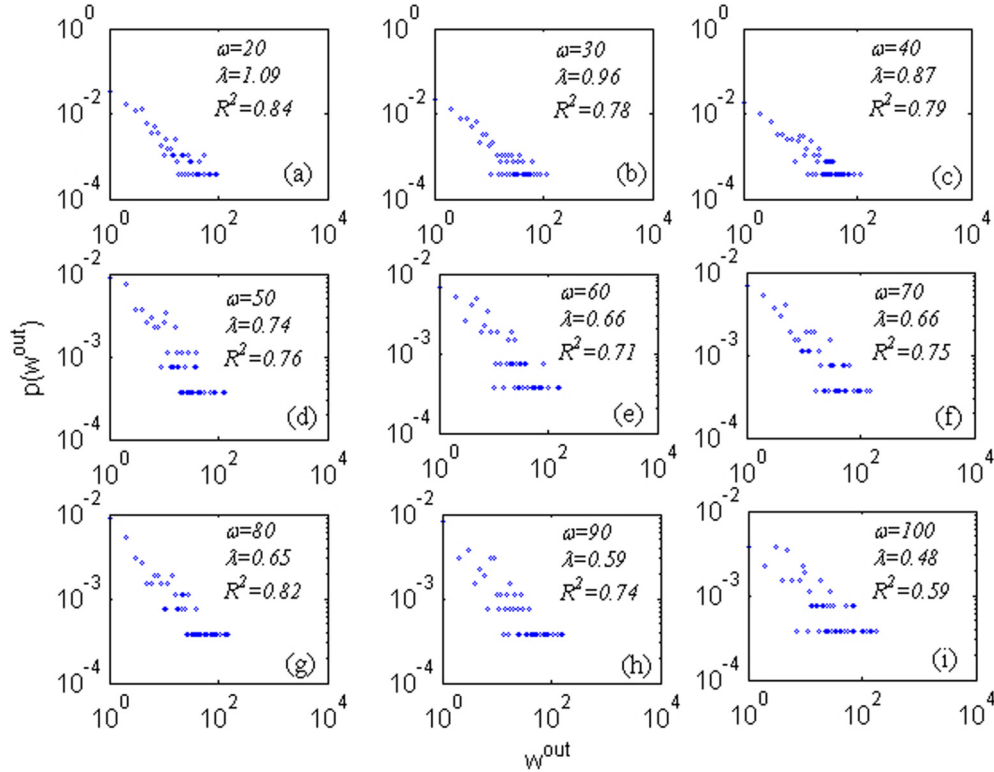


FIG. 6. (Color online) The weighted out-degree distributions under the different sizes of sliding window. (R^2 is goodness of fit of linear regression under the double logarithmic coordinates.)

are $\alpha(0.8,0.9]\beta(5,10]$ and $\alpha(0.7,0.8]\beta(10,15]$. Thus, when we identify the relationship between two variables in a short period (20 days), we should refer to these major patterns but not $y = -9.1012 + 1.15x$.

Second, the process of the transmission is a mutational form. The colors in Fig. 5(a) are neither gradient changing nor stable. They are obviously split, which indicates that a linear regression pattern stays for a period and then leaps and transfers into another pattern that could be very different from the previous linear regression pattern. Additionally, the appearance of each pattern can be isolated from the whole transmission process [see Fig. 5(b)]. This approach can help us to understand the time distribution when a certain pattern appears or to identify which pattern is a major pattern during a period.

Third, the transmission between patterns becomes more variegated over time. As shown in Fig. 5(c), we find that the patterns that have a larger weighted out-degree are mainly concentrated at the earlier part of the time span. The distribution of the weighted out-degree on the timeline is uneven; i.e., a few major patterns appear during a short period of time, from approximately time 500 to time 1000. Notably, after time 2000, the image fills with patterns that have a lower weighted out-degree, which means that there are a variety of patterns during this period. This finding also proves that the linear regression is a dynamic and fluctuating process.

Based on the results described above, we can identify the major linear regression patterns. This method not only can capture the character of the distributions in the transmission

of the linear regression patterns over time but also can help us to identify the significant period or the key area in a sequence.

We also plot the weighted out-degree distribution of the different transmission networks for the different sizes of sliding window, as shown in Fig. 6. All of the weighted out-degree distributions follow the power law $p(w^{\text{out}}) \sim w^{\text{out}-\lambda}$. This result implies that a natural feature of the transmission networks is that there exist only a few patterns that play a major role during the process of the transmission under the different sizes of sliding window. Different sizes of sliding window exhibit different major linear regression patterns. For example, the primary pattern is $\alpha(0.8,0.9]\beta(5,10]$ when the size is 30 days. But when the size is 100 days the primary pattern is $\alpha(0.9,1]\beta(0,5]$. Thus, when we observe the linear relationships under a certain length of time, we could refer to the major patterns under the size of sliding window of the same length.

C. Identifying inflection points and the distance of the transmission

With an increase in the sizes of sliding window, the number of nodes N in the transmission networks decreases [the types of linear regression patterns become fewer as shown in Fig. 7(a)]. Moreover, we find that the number of nodes decays quickly at small sizes of sliding windows. There are inflection points in the curve at the large scale shown in Fig. 7(a) which are barely discernible. We can gain only a macroscopic linear regression equation (similar to Fig. 2) when the sizes of

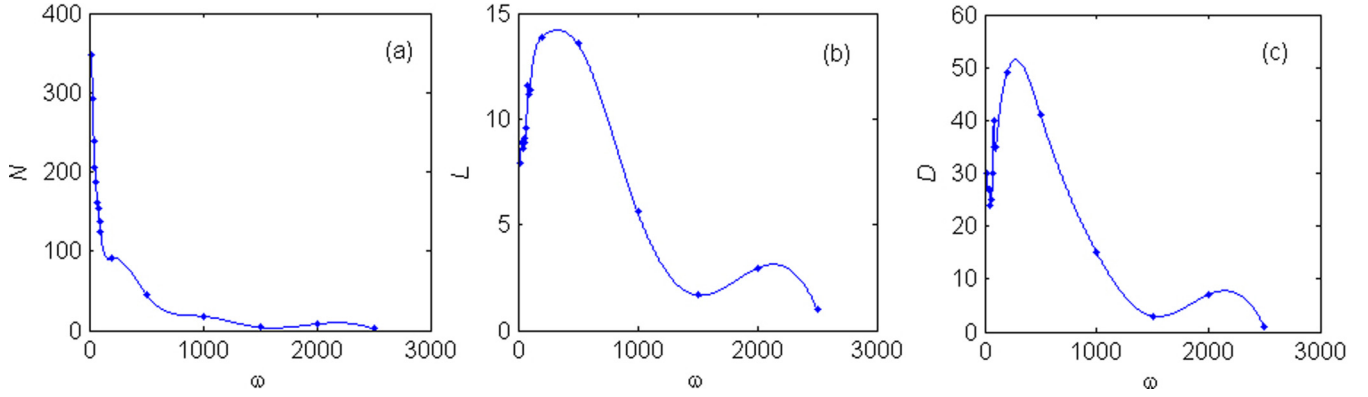


FIG. 7. (Color online) The numbers of nodes N , the average path length L , and the network diameter D , for different scales.

sliding windows are larger than the inflection points. It will hide the characteristics of fluctuation of the linear regression because the linear regression equations of the fragments are more similar to that of the whole period. The inflection points can tell us the certain threshold value when we study the long term equilibrium. If we want to understand more exactly the fluctuation of the linear regression, we would better analyze a shorter period according to the inflection points.

The distance of the transmission between the two patterns can help us understand the transmission path. The transmission network has different average path length L and network diameter D under different sizes of sliding windows. The changes of L and D describe the process of the fluctuant decay upon the increase in the size of sliding window. The average path length L is calculated as follows:

$$L = \frac{1}{N(N-1)} \sum_{i \neq j} l_{ij}, \quad (2)$$

where N is the number of nodes in the transmission complex network and l_{ij} is the distance between node i and j . The network diameter D is the maximum L .

Thus, the method can identify the average transmission distance for different sizes of sliding windows. As shown in

Fig. 7(a), if a type of pattern transforms into another, it will basically convert via few types of patterns. Because the major patterns in different sizes of sliding windows appear in the transmission process with high frequency, then if the current pattern is not a major pattern, it will not take a long time for the current pattern to transform into a major pattern.

On the other hand, according to the network diameter D , we find that although there are hundreds of linear regression patterns, the maximum transmission scope is not large. The change of the network diameter D has the same trend as the change of the average path length L upon an increase in the size of sliding window [see Figs. 7(b) and 7(c)].

D. Identifying the transmission medium

If a linear regression pattern stands in the short path between two patterns, it plays the role of transmission medium in the transmission process. The media capabilities of each pattern can be denoted by the normalized betweenness centrality BC_i :

$$BC_i = \frac{\sum_j \sum_k g_{jk}(i)/g_{jk}}{n^2 - 3n + 2}, \quad j \neq k \neq i, j < k, \quad (3)$$

where $g_{jk}(i)$ is the number of shortest paths between node j and k which pass the node i . g_{jk} is the total number of

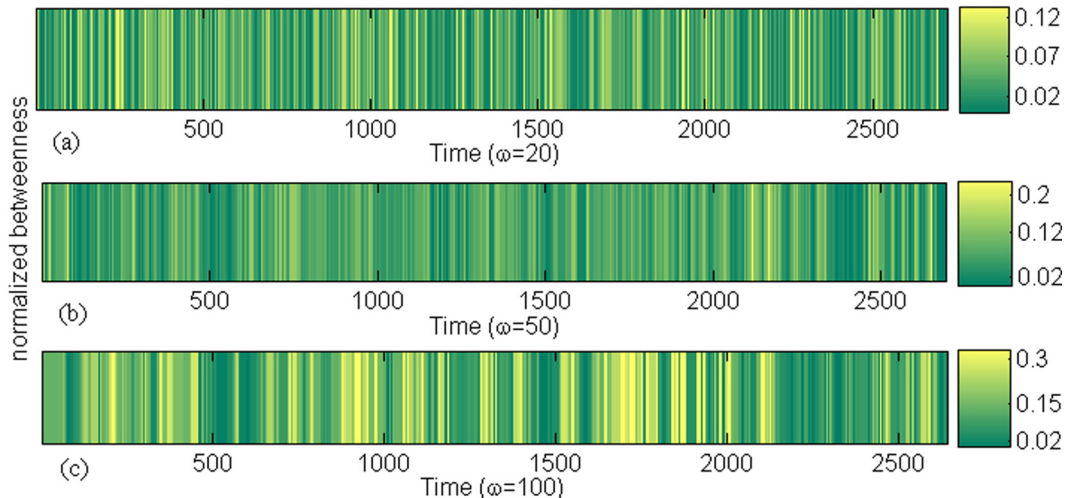


FIG. 8. (Color online) The distribution of the normalized betweenness centrality over time for different sizes of sliding windows.

shortest paths between node j and k . The higher normalized betweenness centrality, the stronger media capability the pattern has.

The transition period of the linear regression can be identified by the distribution of the media capabilities on the timeline (see Fig. 8). For example, for the sizes of sliding windows from 20 to 100, the value of betweenness centrality becomes larger, which means that the transmission-controlling capabilities of some patterns become stronger. We can identify the transitional period by finding the linear regression patterns with high normalized betweenness centrality. The linear regression patterns of the next period will change when the current period is a transitional period and we cannot evaluate the linear regression parameters by previous regression equations directly.

IV. CONCLUSIONS

In this article we have focused on the linear regression between two time series from the viewpoint of econophysics. Previously, a natural bridge between complex network theory and time series analysis had been built by Lacasa *et al.* [15]. Now, we have designed an algorithm to transform linear regression patterns between two time series into directed and weighted networks. With this method we need to ensure the two variables have the correlation. If the pass rate of the significances of the fragments is acceptable, the method is feasible. Different sizes of sliding windows can be set for multiscale research. The mapping of the statistical results on the timelines shows the distribution of the results over time. The major patterns, the distance, and the medium in the process of the transmission can be revealed. Thus, some direct applications of the algorithm can be proposed.

Different lengths of observation periods can provide different results of linear regression. People should refer to

different regressive parameters according to their terms of decision. If one wants to study the short-term relationship between two variables, one cannot refer to the long-term linear regression equation. For example, when we make short-term policies (30 days) on crude oil price, $\alpha(0.8,0.9]\beta(5,10]$ is the most important linear regression pattern. $\alpha(0.9,1]\beta(0,5]$ is the best reference for long-term policies (100 days). It means that variable y changes 0.8 to 0.9 unit instead of 0.9 to 1.0 units when variable x changes 1 unit under the scale of 30 days. We can identify the distances of the transmission and the transmission mediums to develop strategies for different lengths of time.

The algorithm can be applied to a large number of areas related to time series variables or series data (e.g., probably, seismic wave time series data, protein series, or gene series). The important or effective information can be identified by the timelines. Moreover, many areas involve more than two variables and a variable is often impacted by multiple variables. The diffusion issue of the multivariable linear regression patterns will be addressed in a future study, and the essential characteristics will be extracted to provide insight into more complex natural phenomena and human behaviors.

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