Correlated cluster mean-field theory for spin-glass systems

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The competition between cluster spin glass (CSG) and ferromagnetism or antiferromagnetism is studied in this work. The model considers clusters of spins with short-range ferromagnetic or antiferromagnetic (FE-AF) interactions (J_0) and long-range disordered couplings (J) between clusters. The problem is treated by adapting the correlated cluster mean-field theory of D. Yamamoto [Phys. Rev. B **79**, 144427 (2009)]. Phase diagrams $T/J \times J_0/J$ are obtained for different cluster sizes n_s. The results show that the CSG phase is found below the freezing temperature T_f for lower intensities of J_0/J. The increase of short-range FE interaction can favor the CSG phase, while the AF one reduces the CSG region by decreasing the T_f. However, there are always critical values of J_0 where AF or FE orders become stable. The results also indicate a strong influence of the cluster size in the competition of magnetic phases. For AF cluster, the increase of n_s diminishes T_f reducing the CSG phase region, which indicates that the cluster surface spins can play an important role in the CSG arising.

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I. INTRODUCTION

Disorder in spin systems is a permanent source of challenging problems. The spin-glass (SG) state is one of the most interesting examples showing that disorder can provide a new physics. The SG state appears when disorder is combined with competition between ferromagnetic (FE) and antiferromagnetic (AF) interactions, which leads the magnetic moments to a conflict situation. This avoids any conventional long-range order, but raises a richness of physical properties [1–6]. Among the current problems in disordered spin systems, an interesting one occurs when there are clusters of spins instead of canonical spins. (Here we use cluster to also denote nanoparticle.) Recently, the competition between AF or FE orders and cluster SG (CSG) behavior has motivated several experimental studies [7–10]. In some of these systems, the effects of cluster surface spins can play an important role to produce SG-like behavior. This is a clear indication that alternative theoretical approaches are still needed to account for the cluster effects in disordered magnetism.

An earlier example of CSG theory at mean-field level is the approach provided by Soukoulis and Levin [11–13]. In this approach, two distinct interactions are considered: (i) a uniform intracluster short-range FE-AF one; (ii) a disordered infinite-range intercluster one following a Gaussian distribution as the SK model for canonical SG [14]. This particular cluster mean-field theory has proved to be adequate to include short-range FE-AF correlations in SG problems. For instance, this approach was recently applied in different problems, such as the inverse freezing transition [15,16], or to account for the role of geometrical frustration in the SG state [17]. However, in order to obtain long-range FE-AF order competing with a SG state, the Gaussian distribution for the disordered infinite-range intercluster interactions has to be displaced from the origin. This procedure implies that the intraclusters FE-AF interactions are completely disconnected from the intercluster ones providing a rather artificial account for the cluster problem. Moreover, this approach treats the internal degrees of freedom of the cluster in such a way that there is no difference between surface and bulk of the cluster.

Recently, Yamamoto has proposed the so-called correlated cluster mean-field (CCMF) theory to improve the mean-field approximation for the canonical spin systems [18]. This theory divides the original spin lattice in clusters in such a way that the resulting system of clusters follows the original lattice symmetry. The presence of clusters in the CCMF approach is, in fact, an artefact to incorporate spin correlations. In this method, the internal field applied to a given cluster (called central cluster) is produced by the remaining clusters and it depends on the spin configuration of the central cluster itself. Thus, the CCMF method is able to improve considerably the mean-field description of thermodynamics, e.g., the Curie temperature T_c and the magnetic susceptibility when compared with the usual mean-field theory (see Ref. [18] and references therein). It should be remarked that there is not any disorder in the CCMF theory.

In this work, we propose an improvement for the CSG mean-field theory. The basic innovation is to adapt the CCMF theory to reformulate the earlier cluster mean-field theory introduced by Soukoulis and Levin [11–13]. Therefore, FE-AF long-range orderings competing with CSG state can be developed directly from short-range intracluster FE-AF interactions. The CSG phase is still obtained from the intercluster disordered interactions, which are given as the van Hemmen (vH) model for canonical spins [19]. It is important to remark that the van Hemmen model avoids the use of the replica method being, therefore, the main reason for using it in the present report. Albeit, the van Hemmen model represents a limit of weak frustration in the sense that not the entire spins of the system are frustrated, the model still retains the important signatures of the SG phase transition, such as the behavior of the susceptibility and the specific heat as a function of temperature. It should be noticed that in this new approach, the freezing temperature can be dependent on the cluster size. For the AF intracluster interaction, it would be an indication that the cluster surface can play a role in the appearance of CSG phase.

The paper is organized as follows: in Sec. II, the model and the method are presented and exploited in order to produce phase diagrams and susceptibility behavior. In Sec. III, a detailed discussion of the numerical results is presented. The last section is reserved for the conclusion.

II. MODEL

We start from the Ising model that is divided into clusters with n_s spins each:

$$H = -\sum_{\nu\lambda}^{N_{\rm cl}}\sum_{ij}^{n_s} J_{ij}^{\nu\lambda}\sigma_{\nu_i}\sigma_{\lambda_j} - \sum_{\nu}^{N_{\rm cl}}\sum_{i,j}^{n_s} J_{ij}^{\nu}\sigma_{\nu_i}\sigma_{\nu_j}, \tag{1}$$

where $\sigma_{\nu_i} = \pm 1$ is the spin of site i of cluster ν and $N_{\rm cl}$ is the number of clusters. The first and second terms of Eq. (1) correspond to intercluster and intracluster interactions, respectively. The intracluster interactions are assumed to be short-range couplings $J_{ij}^{\nu} = J_0$, while the intercluster interactions $J_{ij}^{\nu\lambda}$ consider two types of couplings: an infinite-range disordered among all pairs of clusters ($J^{\nu\lambda}$) and the short-range between nearest-neighbor spins of neighbor clusters (J_0).

The resulting model can be expressed as

$$H = -\sum_{\nu\lambda}^{N_{\rm cl}} J^{\nu\lambda} S_\nu S_\lambda - \sum_{(\nu_i,\lambda_j)} J_0\sigma_{\nu_i}\sigma_{\lambda_j} - \sum_{\nu}^{N_{\rm cl}} J_0 \sum_{(i,j)}^{n_s}\sigma_{\nu_i}\sigma_{\nu_j}, \tag{2}$$

where $(\cdots)$ represents nearest neighbors and $S_\nu = \sum_i \sigma_{\nu_i}$ is the total magnetic moment of cluster ν. In particular, the disorder acts between total magnetic moment of different clusters. It means that the disorder determines the cluster-like behavior in this approach.

For the intercluster disorder the SG van Hemmen interaction is assumed as follows [20]: $J^{\nu\lambda} = \frac{J}{N_{\rm cl}n_s}(\xi_\nu\eta_\lambda + \xi_\lambda\eta_\nu)$, where ξ_ν's and η_λ's are independent random variables that follow identical Gaussian distributions with variance one. J is associated with the strength of disorder. This kind of disorder allows us to write the first term of Eq. (2) in a separable form:

$$\sum_{\nu\lambda}^{N_{\rm cl}} J^{\nu\lambda} S_\nu S_\lambda = \frac{J}{N_{\rm cl}n_s}\left\{\left[\sum_\nu(\xi_\nu+\eta_\lambda)S_\nu\right]^2 - \left(\sum_\nu \xi_\nu S_\nu\right)^2 - \left(\sum_\nu \eta_\lambda S_\nu\right)^2 - 2\sum_\nu \xi_\nu\eta_\lambda\right\}. \tag{3}$$

The partition function for a particular set of fixed distribution of $\{\xi,\eta\}$ can be obtained as

$$Z(\xi,\eta) = {\rm Tr}e^{-\beta H(\xi,\eta)} = \int Du \exp\left\{-N\left[\frac{(q_3^2 - q_1^2 - q_2^2)}{2\beta J} - \frac{1}{N}\ln{\rm Tr}\exp(-\beta H_{\rm eff})\right]\right\}, \tag{4}$$

where $Du \propto dq_1dq_2dq_3$ with $\{q_n(\xi,\eta)\}$ ($n = 1, 2,$ and 3) is introduced to linearize the terms of Eq. (3) and $H_{\rm eff} = -J\sum_\nu^{N_{\rm cl}}[(\xi_\nu+\eta_\nu)q_3 - \xi_\nu q_1 - \eta_\nu q_2]S_\nu + A_{J_0}.A_{J_0}$ represents the last two terms of Eq. (2). The functional integrals over $\{q_n(\xi,\eta)\}$ in Eq. (4) can be solved by using the steepest descent method in the thermodynamic limit ($N_{cl} \to \infty$), which gives $q_3 = q_1 + q_2$, $q = q_1 = q_2$. The SG order parameter q is obtained from [19,20]

$$q = \frac{1}{N}\left\langle \frac{{\rm Tr}\sum_\nu \frac{(\xi_\nu+\eta_\nu)}{2} S_\nu e^{-\beta H_{\rm eff}}}{{\rm Tr}e^{-\beta H_{\rm eff}}}\right\rangle_{\xi,\eta}, \tag{5}$$

where

$$H_{\rm eff} = \sum_\nu^{N_{\rm cl}}\left[-J(\xi_\nu+\eta_\nu)qS_\nu - J_0\sum_{(i,j)\in\nu}\sigma_i\sigma_j\right] - \sum_{(\nu_i,\lambda_j)} J_0\sigma_{\nu_i}\sigma_{\nu_j}, \tag{6}$$

and $\langle\cdots\rangle_{\xi,\eta}$ represents the average over the random variables ξ and η.

The intercluster disorder has been evaluated within a usual mean-field treatment in Eqs. (4)–(6). However, there is still a short-range coupling (FE or AF) between neighbor clusters [see last term of Eq. (6)]. In the present work, this interaction between spins in different clusters is treated in the framework of the CCMF approach. The CCMF method allows us to decouple the clusters by treating the remaining interactions with simplicity and good accuracy in determining the critical temperature [18]. In the following, we analyze disordered cluster systems with ferromagnetic and antiferromagnetic short-range interactions.

A. Ferromagnetic short-range interactions

We consider FE short-range interactions in a square lattice that is divided into clusters with four sites ($n_s = 4$). The resulting one-cluster model can then be written as

$$H_{\rm eff} = -J(\xi+\eta)q\sum_{i\in\nu}\sigma_i - \sum_{(i,j)\in\nu} J_0\sigma_i\sigma_j - \sum_{i\in\nu} h_i^{\rm eff}\sigma_i, \tag{7}$$

where the effective field $h_i^{\rm eff}$ acting on spin σ_i is introduced as in the standard routine of CCMF [18]. In this approach, $h_i^{\rm eff} = J_0(m^{\sigma_i\sigma_j} + m^{\sigma_i\sigma_k})$ [see Fig. 1(a)], in which the mean fields $m^{\sigma_i\sigma_j}$ and $m^{\sigma_i\sigma_k}$ are strongly dependent on the spin states of site i (σ_i) and its nearest neighbors (σ_j and σ_k) belonging

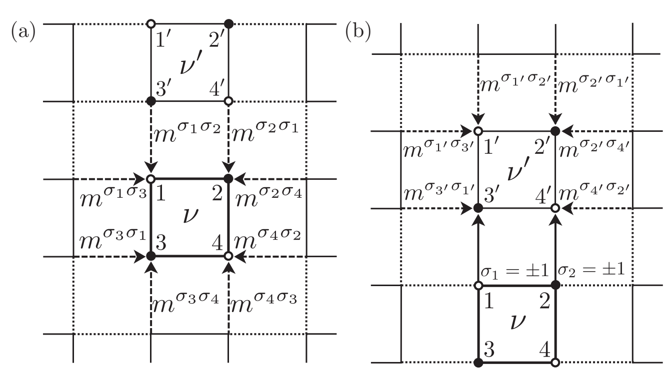

FIG. 1. Schematic representation for a square lattice divided into clusters with $n_s = 4$. The mean fields are pointed by arrows that represent (a) the interactions between cluster ν with its neighbors and (b) the interactions on ν' used to evaluate the $m^{ss'}$. For the AF case it is considered two sublattices that are represented by solid and open circles.

to the same cluster ν. For instance, $m^{\sigma_1\sigma_2}$ can assume four different values: one for each spin configuration of the pair of sites 1 and 2 ($\sigma_1\sigma_2 = ++, +-, -+, --$).

The mean fields are obtained by considering the nearby connected clusters [18]. Explicitly, $m^{ss'}$ is determined from the connected cluster ν' when the states of sites 1 and 2 of the cluster ν assume spins s and s', respectively [see Fig. 1(b)]. The values of $m^{ss'}$ are numerically computed by solving the self-consistently set of equations

$$m^{ss'} = \left\langle \frac{\mathrm{Tr}\sigma_{k'} \exp(-\beta H'_{\mathrm{eff}})}{\mathrm{Tr}\exp(-\beta H'_{\mathrm{eff}})} \right\rangle_{\xi\eta}, \tag{8}$$

where $k' = 3'$ is the site of cluster ν' that is neighbor of site $i = 1$ of cluster ν. The Hamiltonian of cluster ν' is given by

$$H'_{\mathrm{eff}} = J(\xi+\eta)q \sum_{i\in\nu'} \sigma_i - J_0 \sum_{(i,j)\in\nu'} \sigma_i\sigma_j - \sum_{\substack{i\in\nu' \\ \{i\}\neq\{3',4'\}}} h_i^{\mathrm{eff}}\sigma_i - J_0 s\sigma_{3'} - J_0 s'\sigma_{4'}, \tag{9}$$

in which it is considered the spin configurations assumed by sites 1 and 2 (s and s') of cluster ν. Furthermore, it is important to remark that the set of equations for the mean fields does not depend on a specific site due to the symmetry of $n_s = 4$ square-shape cluster.

In particular, this problem differs from the original CCMF method. Here, we have also to consider the configurational average and solve Eqs. (5) and (8) self-consistently, where H_{eff} in Eq. (5) is given by Eq. (7). One can now obtain the SG order parameter [see Eq. (A1)] and the magnetization:

$$m = \frac{1}{n_s}\left\langle \frac{\mathrm{Tr}\sum_{i\in\nu}\sigma_i \exp(-\beta H_{\mathrm{eff}})}{\mathrm{Tr}\exp(-\beta H_{\mathrm{eff}})} \right\rangle_{\xi\eta}. \tag{10}$$

B. Antiferromagnetic short-range interactions

Disordered clusters with AF short-range interactions are analyzed for clusters of small size ($n_s = 4$). To this purpose, the system is first divided into two sublattices: A and B (see Fig. 1). The disorder treatment follows the same procedure as before, resulting in Eqs. (5) and (6) with the sublattice structure. The CCMF can also help us obtain an effective one-cluster problem. However, the cluster model has to consider explicitly the distinction between the sublattices, which results in the effective Hamiltonian

$$H_{\mathrm{eff}}^{\mathrm{AF}} = -\sum_{(i,j)\in\nu} J_0\sigma_i^p\sigma_j^{p'} - \sum_p \sum_{i\in\nu,p} \left(J(\xi+\eta)q + h_{p,i}^{\mathrm{eff}}\right)\sigma_i^p, \tag{11}$$

where $p = A$ or B ($p' = B$ or A) indicates the sublattice.

The effective field $h_{p,i}^{\mathrm{eff}} = J_0(m_p^{\sigma_i^p\sigma_j^{p'}} + m_p^{\sigma_i^p\sigma_k^{p'}})$ acting on the site i of sublattice p depends on the states of site i and its neighbors j and k of the same cluster. However, the sites j and k belong to the sublattice p'. Therefore, there are eight possible mean fields (four for each sublattice): $m_A^{\sigma_i^A\sigma_j^B}$ and $m_B^{\sigma_j^B\sigma_i^A}$. Nevertheless, we can explore the symmetry between the sublattices to reduce the number of independent mean fields: $m_p^{ss'} = -m_{p'}^{\bar{s}\bar{s}'}$, where $s = -\bar{s}$ and $s' = -\bar{s}'$ (e.g., $m_A^{+-} = -m_B^{-+}$). It means that only four mean-fields have to be evaluated as in the FE case. Following the procedure introduced in Ref. [18], the mean fields are obtained from

$$m_A^{ss'} = \left\langle \frac{\mathrm{Tr}\sigma_{3'}^B \exp\left(-\beta H'^{\mathrm{AF}}_{\mathrm{eff}}\right)}{\mathrm{Tr}\exp\left(-\beta H'^{\mathrm{AF}}_{\mathrm{eff}}\right)} \right\rangle_{\xi\eta} = -m_B^{\bar{s}\bar{s}'}, \tag{12}$$

where the Hamiltonian of cluster ν' is given by

$$H'^{\mathrm{AF}}_{\mathrm{eff}} = -\sum_{(i,j)\in\nu'} J_0\sigma_i^p\sigma_j^{p'} + \sum_p \left[J(\xi+\eta)q \sum_{i\in\nu',p} \sigma_i^p - \sum_{\substack{i\in\nu',p \\ i\neq 3',4'}} h_{p,i}^{\mathrm{eff}}\sigma_i^p \right] - J_0 s\sigma_{4'}^A - J_0 s'\sigma_{3'}^B. \tag{13}$$

The effective fields are obtained from the self-consistent computation of Eqs. (5) and (12), where Hamiltonian Eq. (11) is used in Eq. (5). The magnetization is then substituted by the staggered magnetization: $m_s = |m_p - m_{p'}|/n_s$, where $m_p = <\mathrm{Tr}\sum_i \sigma_i^p \exp(-\beta H_{\mathrm{eff}}^{\mathrm{AF}})/\mathrm{Tr}\exp(-\beta H_{\mathrm{eff}}^{\mathrm{AF}}) >_{\xi\eta}$ is computed as in Eq. (10) by considering only sites belonging to the sublattice p.

III. RESULTS

Numerical results are obtained by solving the equations for the magnetizations and the SG order parameter. For the FE case ($J_0 > 0$), Eqs. (5) and (8) are solved self-consistently. After the magnetization Eq. (10) is obtained. To calculate the traces, the Ising base of states is used. The CSG phase is characterized by $q > 0$ with $m = 0$ and the FE order occurs when $q = 0$ with $m > 0$. The case with AF interactions $J_0 < 0$ follows an analogous procedure. The AF phase appears when $m_s > 0$ and $q = 0$. In particular, the limits $J_0 = 0$ and $J = 0$ recover the results for the van Hemmen model without cluster and the Ising model treated with CCMF, respectively.

The phase diagram in Fig. 2 exhibits the competition between disorder and J_0 interaction, where the range of coupling J_0 is from the AF (left side) to the FE (right side) for a square-lattice cluster with $n_s = 4$. This phase diagram shows clearly that the freezing temperature T_f increases with the FE short-range coupling. However, the FE phase becomes the stable one for larger values of J_0/J. It means that the CSG phase is favored by the presence of clusters with short-range FE interactions. On the other hand, FE interactions can also introduce the FE long-range order. It depends on the relation between intercluster disorder J and J_0. To put it another way, the magnetic moment of clusters increase with J_0, which can favor the intercluster disordered interactions and, consequently, the CSG phase. But, for larger J_0 the disorder cannot be enough to bring the CSG phase and the FE order is found. This phase diagram differs from that obtained by the vH model without clusters, in which the T_f is independent of J_0 [20].

Disordered clusters with AF short-range interactions present a phase diagram with a different behavior for the PM-CSG transition [see left side of Fig. (2)]. The freezing temperature is gradually reduced by increasing the strength of AF interaction until the AF order is found. Therefore, the AF cluster formation is contrary to the CSG phase. In fact, the

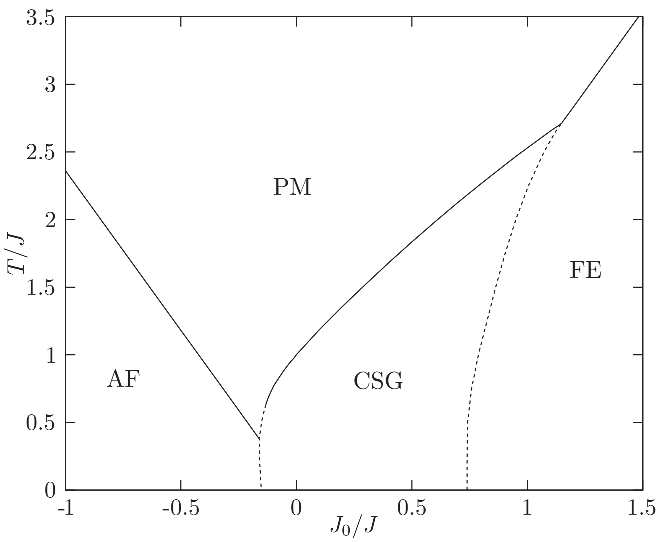

FIG. 2. Phase diagram of T/J vs. J_0/J for square-lattice clusters with four spins. The FE order is found for large, short-range interactions $J_0/J > 0$, while the AF can appear at negative values of J_0/J. Solid and dashed lines indicate continuous transition and spinodal of CSG phase, respectively. The cluster SG phase occurs for small interactions J_0/J.

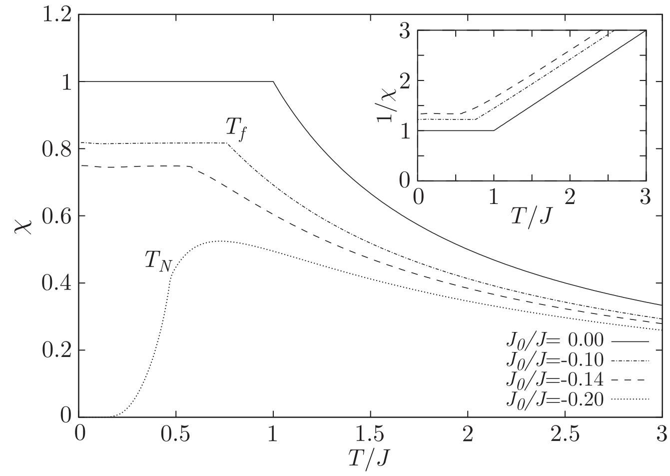

FIG. 4. χ versus T/J for AF J_0/J values. The inset exhibits the behavior of $1/\chi$.

total cluster magnetic moment is reduced by the AF couplings, which can cause decreasing in the disordered intercluster couplings. When the FE and AF cases are compared, we can see that a small strength of AF coupling is already able to destroy the CSG phase.

In Fig. 3 the magnetic susceptibility χ indicates a typical canonical SG behavior for $J_0/J = 0$ (without cluster), in which χ presents a cusp at T_f and it is independent of the temperature inside the SG phase [3,19]. However, this behavior changes when $J_0/J > 0$. χ becomes temperature dependent for $T < T_f$ increasing as T diminishes. In specific, the χ dependence on J_0/J shows a divergence inside the SG phase as J_0/J increases [see the inset of Fig. (3)]. This indicates that the ferromagnetic interactions progressively overcome the CSG phase at low temperatures, where the FE long-range order is found for larger J_0/J. This divergence is used to locate the stability limit of the CSG phase, which is indicated by the dashed line in Fig. 2.

For AF interactions ($J_0/J < 0$), the magnetic susceptibility also presents a discontinuity at T_f (see Fig. 4), but the χ shows a different behavior inside the CSG phase when compared with the FE case. χ follows practically temperature independent in the CSG phase and it decreases as the strength of AF interactions increases. It also exhibits an AF Curie-Weiss coefficient as can be observed in the inset of Fig. 4. The AF characteristic increases with the intensity of J_0 and χ shows a typical Ising AF behavior for high enough values of J_0 (see Fig. 4 for $J_0/J = -0.20$), where only a PM-AF transition is observed.

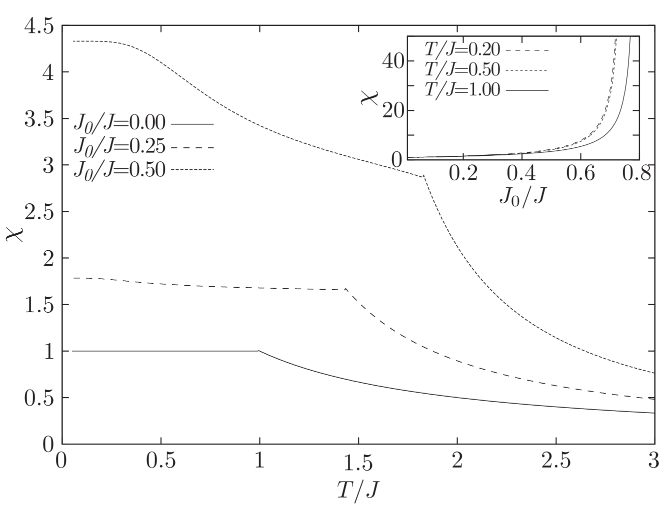

FIG. 3. Magnetic susceptibility χ as a function of the temperature for several values of J_0/J. The inset exhibits χ vs. J_0/J for low temperatures. The increase of ferromagnetic short-range interactions destroys the CSG phase introducing the FE order.

A. Clusters with large size

The effects caused by increasing the cluster size are now analyzed. We first discuss the original CCMF theory for square lattice clusters with more than four sites. In this case, the number of mean fields increases considerable. For instance, a cluster with 16 sites (see Fig. 5) has four border sites with 2^4 spin possible configurations. It means that we should evaluated 2^5 mean fields (2^4 for the corner spins and 2^4 for other spins in the cluster border). However, we assume a new approach for the CCMF in order to allow a solution with a smaller number of mean fields, which we call adapted CCMF. We suggest that only the state of the site i and its nearest neighbors in the same frontier of cluster ν are taken into account to obtain the mean fields that act on the site i. In this case, there are four possible mean fields acting on the corner site clusters and eight possible mean fields if the site i is not on the corner.

For instance, let us consider a square lattice cluster with 16 sites (Fig. 5). The mean-field that acts on site 2 ($m^{\sigma_1\sigma_2\sigma_3}$) depends on the spin states of sites 1, 2, and 3, while the mean fields on site 1 ($m^{\sigma_1\sigma_2}$ and $m^{\sigma_1\sigma_5}$) depend on the spin states of

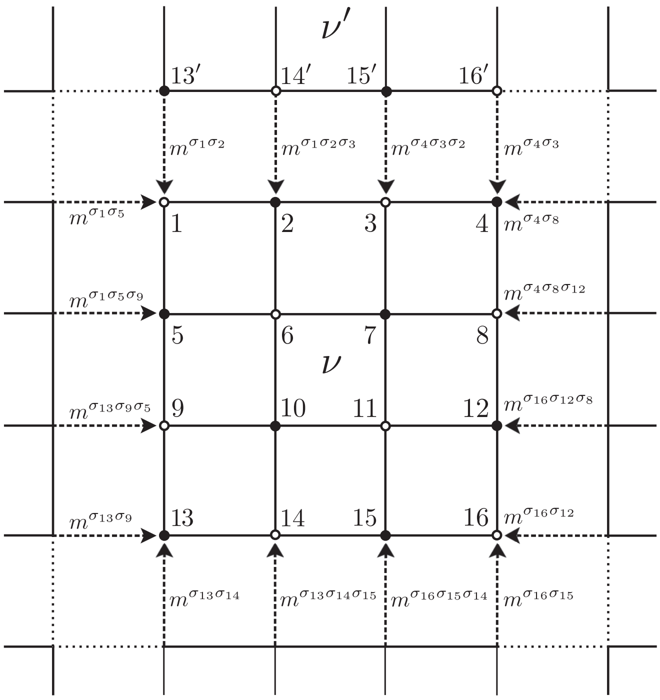

FIG. 5. Schematic representation for a cluster with $n_s = 16$. It is used in the same convention as described in the legend of Fig. 1.

sites 1 and 2, and 1 and 5. Therefore, instead of computing 2^5 mean fields in the original CCMF approach, we can decouple the clusters evaluating at maximum 12 mean fields with the present adapted CCMF. This condition for the number of mean fields is also valid to large clusters. It means that in the adapted CCMF the number of mean fields is limited to 12 no matter the cluster size.

This procedure is also used to deal with the cluster SG model when $n_s > 4$. The long-range disordered interactions are treated introducing the SG order parameter as before. However, the interactions J_0 between neighbor clusters are approached as described above by the adapted CCMF. The effective one-cluster model can then be written as

$$H_{\text{eff}} = -J(\xi+\eta)q\sum_{i\in\nu}\sigma_i - \sum_{(i,j)\in\nu} J_0\sigma_i\sigma_j - \sum_{i\in\bar{\nu}} h_i^{\text{eff}}\sigma_i, \tag{14}$$

where $\bar{\nu}$ represents border sites of cluster ν. The effective field $h_i^{\text{eff}} = J_0(m^{\sigma_i\sigma_j} + m^{\sigma_i\sigma_k})$ if the site i belongs to the corner of cluster ν or $h_i^{\text{eff}} = J_0 m^{\sigma_k\sigma_i\sigma_j}$ for others sites of $\bar{\nu}$ (j and k are the nearest neighbors of site i belonging to the border). For example, we consider below clusters with $n_s = 16$ for both cases: FE and AF short-range interactions.

1. Ferromagnetic interactions

The fields $m^{ss'}$ and $m^{ss's''}$ are evaluated by following an analogous procedure as in Sec. II A. $m^{ss's''}$ is obtained from

$$m^{ss's''} = \left\langle \frac{\text{Tr}\sigma_{k'}\exp(-\beta H'_{\text{eff}})}{\text{Tr}\exp(-\beta H'_{\text{eff}})} \right\rangle_{\xi\eta}, \tag{15}$$

TABLE I. Comparison of the T_c/J_0 obtained for the case without disorder and several n_s. The exact result is also exhibited.

n_s	4	9	16	20	Exact
T_c/J_0	2.362	2.362	2.361	2.358	2.269

where $k' = 14'$ (see Fig. 5) and the Hamiltonian of cluster ν' considers that spins $\sigma_{13'}$, $\sigma_{14'}$, and $\sigma_{15'}$ couple with spins s, s', and s'' of cluster ν (sites 1, 2, and 3), respectively. Explicitly,

$$H'_{\text{eff}} = -J(\xi+\eta)q\sum_{i\in\nu'}\sigma_i - \sum_{(i,j)\in\nu'} J_0\sigma_i\sigma_j - \sum_{\substack{i\in\bar{\nu}' \\ i\neq\{13',14',15'\}}} h_i^{\text{eff}}\sigma_i - J_0 s\sigma_{13'} - J_0 s'\sigma_{14'} - J_0 s''\sigma_{15'}. \tag{16}$$

For computing $m^{ss'}$, we consider Eq. (8) with $k' = 13'$ and H'_{eff} defined in Eq. (16), in which $J_0 s''$ is replaced by $h_{15'}^{\text{eff}}$. In this case, spins $\sigma_{13'}$ and $\sigma_{14'}$ couple with spins s (σ_1) and s' (σ_2), respectively.

It is important to observe that $m^{ss'}$, $m^{ss's''}$, and q [see Eq. (5) with H_{eff} defined in Eq. (14)] have to be solved self-consistently. After the magnetization can be obtained. The results for clusters with $n_s = 4$ are recovered (fields $m^{ss's''}$ are not present).

This adapted CCMF treatment (without disorder) can improve the results for T_c when n_s increases. For example, Table I shows the temperature T_c/J_0 for PM-FE phase transition when $J = 0$ with several n_s. The T_c becomes closer to the exact result for square lattice Ising model ($T_c/J_0 = 2.269$) as n_s increases. However, the computational cost also increases with n_s.

In this work, the main advantage of increasing the cluster size is to study the competition between the ordered phases and the CSG phase. For this purpose, we construct phase diagrams of the critical temperatures as a function of J_0/J when n_s is increased (see Fig. 6). We can clearly see that the CSG phase is favored by increasing the cluster size. In particular, the FE short-range interactions can increase the total magnetic moment of clusters S_ν, which is also affected by n_s. For the present approach, the increase of S_ν can amplify the effect of disordered interactions.

The T_f can also be located by using the expansion of the Appendix, which helps to understand the relation between T_f and the cluster magnetic moment. From Eq. (A3), $T_f/J = \langle S_\nu S_\nu\rangle_0/n_s$, where $\langle S_\nu S_\nu\rangle_0$ is associated to the intensity of cluster magnetic moment at the PM-CSG transition. The correlation $\langle S_\nu S_\nu\rangle_0$ increases with n_s for FE short-range interactions. As a result, the T_f can be displaced to higher temperatures (see the inset of Fig. 6). However, there is always a critical value of J_0/J where the FE order becomes dominant. On the other hand, the T_c/J is little affected by the increase of n_s. At the scale of Fig. 6 the T_c changed for different n_s becomes almost imperceptible.

2. Antiferromagnetic interactions

To study the AF-CSG competition for large n_s we consider a two-sublattice structure. Equations (14)–(16) are then written with an explicit distinction between the sublattices, say A and B. The symmetry of the mean fields can also be explored: $m_A^{ss'} = -m_B^{\bar{s}\bar{s}'}$ and $m_A^{ss's''} = -m_B^{\bar{s}\bar{s}'\bar{s}''}$. The numerical evaluations

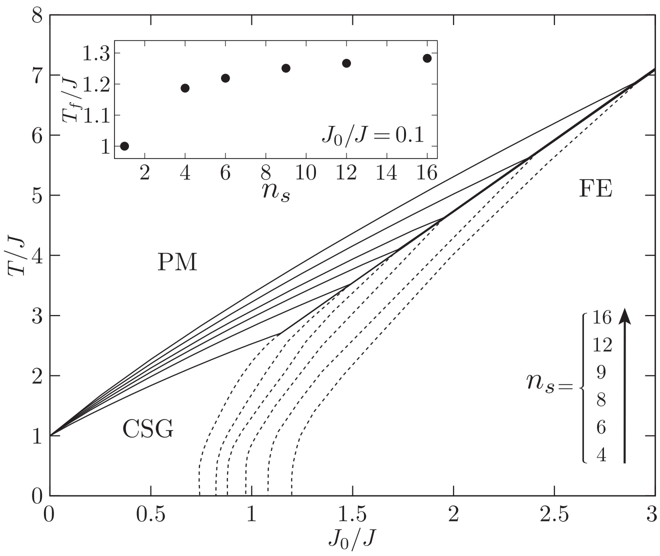

FIG. 6. Phase diagrams T/J vs. J_0/J for several cluster sizes: 4, 6, 8, 9, 12, and 16. The inset exhibits T_f/J as a function of n_s for $J_0/J=0.1$. The T_f increases with n_s and J_0 at the same time that the enhancement of J_0 can stabilize the FE phase.

for the mean fields, SG order parameter, and staggered magnetization follow analogous to the case treated in Sec. II B, however, with the adapted CCMF for large clusters.

For AF clusters without disorder, the T_N gradually converges to the exact result when n_s increases as in the FE case (T_N presents the same values of T_c found in Table I). For AF clusters with disorder, the effects of increasing n_s and J_0 on the freezing temperature are presented in the phase diagrams of Fig. 7. The CSG phase is always decreased by increasing the AF coupling strength. As pointed before, this reduction is attributed to the decreasing of the total magnetic moment of clusters due to the presence of short-range AF interactions

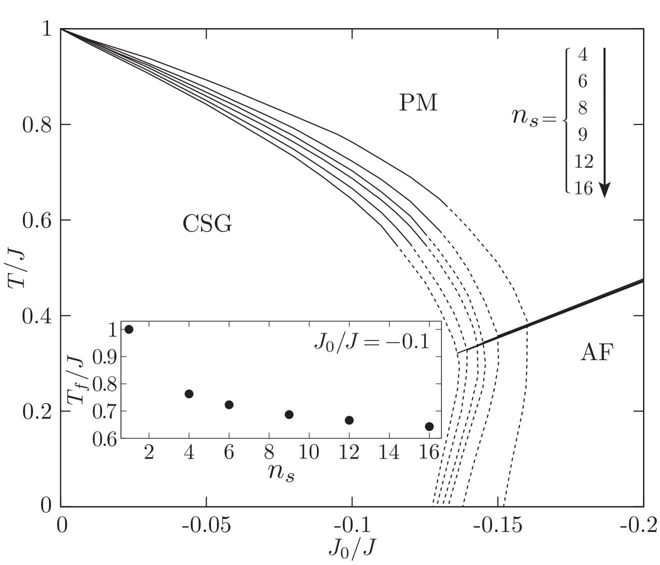

FIG. 7. Phase diagrams T/J vs. J_0/J for antiferromagnetic interactions and several cluster size. The inset exhibits the behavior of T_f when n_s increases for a constant $J_0/J=-0.1$.

(cluster with compensated spins). These interactions affect the disordered intercluster coupling (see Appendix) at the same time that can favor the AF order.

The increase in cluster size also reduces the CSG region. However, different from J_0, n_s can increase the number of spins that couple antiferromagnetically inside the clusters. This situation can energetically favor cluster configurations with a high number of AF spin couplings and low S_ν. As a consequence, the disordered intercluster interactions are weakened and the CSG phase occurs only at lower temperatures (see the inset of Fig. 7). It means that the increase of n_s intensifies the effects of J_0 on the CSG phase. In addition, for strong AF coupling, the AF order is found where the Néel temperature is practically independent of the cluster size.

IV. CONCLUSION

We have presented a study of the competition between cluster SG, FE, and AF. Ising spin clusters with vH type of disorder and short-range interactions have been considered. The disorder has been computed within the mean-field theory without the replica trick, in which the CCMF treatment has been used to decouple the FE-AF intercluster interactions. The original CCMF theory has been adapted to consider the AF case and several cluster sizes for the square-lattice. The resulting effective one-cluster model was then evaluated exactly.

The results show that the presence of FE interaction inside the clusters favor the SG behavior. The short-range FE interactions potentialize the disordered couplings between magnetic moments of clusters, which brings the CSG phase to higher temperatures. The increase of cluster size also plays an analogous role. However, there is always a critical value of J_0/J in which the long-range FE order is found. In particular, the increase of J_0 introduces FE correlations that gradually overcome the CSG phase.

The presence of short-range AF interactions reduces the CSG phase. These interactions cause a spin-cluster compensation that decreases the cluster magnetic moment weakening the intercluster disordered coupling. The CSG phase is replaced by the AF order that becomes stable for small intensities of J_0/J. When the cluster size increases, the CSG region diminishes. However, the critical temperature T_N (and also T_c) is weakly dependent of cluster size. The cluster interpretation has meaning when the disordered interactions are able to produce a SG behavior in the present approach. Furthermore, the behavior of the freezing temperatures with n_s suggests that the cluster surface spins can be important to the disordered intercluster couplings in order to introduce the CSG phase.

Although these results are obtained for a specific type of cluster geometry and disorder, the present approach can be extended for others types of cluster geometry and disorder. Therefore, it can be useful to study problems where the effects of surface of the cluster are relevant to the CSG behavior as proposed in Refs. [7–10].

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APPENDIX: SG-PM PHASE TRANSITION

The SG order parameter, Eq. (5), can be expressed as

$$q = \frac{1}{n_s}\int_{-\infty}^{\infty}\frac{dx e^{-x^2/2}}{\sqrt{2\pi}}\frac{\mathrm{Tr}\frac{\sqrt{2}x}{2}S_\nu e^{-\beta H_{\mathrm{eff}}}}{\mathrm{Tr}e^{-\beta H_{\mathrm{eff}}}}, \tag{A1}$$

where the average over the Gaussian probability distributions ξ and η were explicitly used and evaluated as an integral over x.

The continuous PM-CSG phase transition can also be located by expanding the SG order parameter in powers of q:

$$q = \frac{1}{n_s}\langle S_\nu\rangle_0 + \frac{\beta J}{n_s}[\langle S_\nu S_\nu\rangle_0 - (\langle S_\nu\rangle_0)^2]q + O(q^2), \tag{A2}$$

where $\langle\cdots\rangle_0 = \mathrm{Tr}\cdots e^{-\beta H^0_{\mathrm{eff}}}/\mathrm{Tr}e^{-\beta H^0_{\mathrm{eff}}}$ with $H^0_{\mathrm{eff}} = H_{\mathrm{eff}}(q = 0)$. At the PM-CSG phase transition, $\langle S_\nu\rangle_0$ represents the magnetization that is zero. Therefore, the freezing temperature T_f $(\beta_f = 1/T_f)$ for continuous PM-CSG phase transition can be located by solving

$$1 - \frac{\beta_f J}{n_s}\langle S_\nu S_\nu\rangle_0 = 0, \tag{A3}$$

where $\langle S_\nu S_\nu\rangle_0$ can be interpreted as the intensity of the total magnetic moment of cluster.

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