

Treelike networks accelerating capillary flowDahua Shou,^{1,*} Lin Ye,^{1,†} and Jintu Fan^{2,‡}¹*Centre for Advanced Materials Technology (CAMT), School of Aerospace, Mechanical and Mechatronic Engineering, The University of Sydney, NSW 2006, Australia*²*Department of Fiber Science & Apparel Design, College of Human Ecology, Cornell University, Ithaca, New York 14853, USA*

(Received 13 November 2013; published 12 May 2014)

Transport in treelike networks has received wide attention in natural systems, oil recovery, microelectronic cooling systems, and textiles. Existing studies are focused on transport behaviors under a constant potential difference (including pressure, temperature, and voltage) in a steady state [B. Yu and B. Li, *Phys. Rev. E* **73**, 066302 (2006); J. Chen, B. Yu, P. Xu, and Y. Li, *ibid.* **75**, 056301 (2007)]. However, dynamic (time-dependent) transport in such systems has rarely been concerned. In this work, we theoretically investigate the dynamics of capillary flow in treelike networks and design the distribution of radius and length of local branches for the fastest capillary flow. It is demonstrated that capillary flow in the optimized tree networks is faster than in traditional parallel tube nets under fixed constraints. As well, the flow time of the liquid is found to increase approximately linearly with penetration distance, which differs from Washburn's classic description that flow time increases as the square of penetration distance in a uniform tube.

DOI: [10.1103/PhysRevE.89.053007](https://doi.org/10.1103/PhysRevE.89.053007)

PACS number(s): 47.56.+r, 47.55.nb

I. INTRODUCTION

Treelike networks can be widely found in nature, including branches [1], vasculatures [2], and rivers [3], and have received increasing attention in industrial applications such as oil recovery [4], microelectronic cooling systems [5], desalination [6], and textiles [7]. The investigation of treelike networks can be dated back to 1926, when Murray [2] found the optimal radius ratio between parent and daughter branches in the cardiovascular system under the least expenditure of energy. It has also been shown that branch radii of tree networks are optimized for driving water in plants with the minimum flow resistance [8]. As well, the treelike geometry of mammalian airways indicates that the ratios for radius and length of bronchi lead to a minimization of the airflow power loss [9,10]. Following the study on the regular branching system, branching irregularities have been explored by functional modeling of the mammalian respiratory system [11,12]. When the branching is asymmetric rather than symmetric, the network appears optimal to supply fresh air at limited delivery time. Recently, it was found that the energetic efficiency, the geometric efficiency and the transit time efficiency are simultaneously optimized in the bronchial tree [13]. It was noted that asymmetric flow occurs in symmetric treelike structures at a high Reynolds number when inertial effects are considered [14–16]. The flow asymmetry depends on the Reynolds number and is critical to the morphogenesis and functions of the bronchial tree [14–16]. As well, the seepage for non-Newtonian fluids in treelike networks, such as blood in the vascular system, is of considerable current interest [17], and nonlinear flow for power-law fluids was found in disordered porous media [18] and was properly quantified by a modified permeabilitylike index and Reynolds number [19].

To explain the transport mechanisms of these natural transports, Bejan and Lorente [20] developed a constructal theory that optimized the flow distribution between one point and many points on an equidistant line or circle for the minimum overall resistance. The optimal design of the flow path leading to a treelike network was found [20]. The constructal theory was also extended to designate the natural tendency of flow systems (e.g., rivers, lungs, and tectonic plates) to morph in an evolutionary process towards greater flow access over time based on optimization at every scale and level [21]. Inspired by the branching vessel tree of arteries or veins, Cheng and Chen [22] designed a rectangular tree-shaped fractal network for the cooling of a microelectronic chip. They showed that the network increases the convective heat transfer rate but reduces the pressure drop in comparison to parallel tube nets [22]. Later, Yu and Li [23] found that the thermal conductivity of fractal-like tree networks can be much smaller than that of an equivalent single cylinder. In 2007, Chen *et al.* [24] derived the permeability model of porous constructs embedded with fractal-like tree networks and observed that the composite structures were much more permeable than parallel channel nets under a fixed constraint. Treelike networks or Bethe lattices were also used to describe power-law fluids in porous media [25]. Although the idealized treelike networks lack reconstructions as in the real porous media, they are able to derive closed-form expressions [25]. Recently, imitation of the branching structure in textile fabric demonstrated a positive impact on the sweat absorption property of the fabric [7]. In addition, the multilayer treelike fabrics had two to three times the evaporation rate from the free water surface [7].

Transport behaviors in treelike networks under a constant potential difference (including pressure, temperature, and voltage) have been explored extensively in numerical studies [23,24]. To our best of knowledge, however, there is little research work on the dynamics of capillary flow in treelike networks. In such structures, the only driving force for the self-motion of liquid is capillary pressure, which is dependent on the microstructure of the local branch where the fluid front

*dhshou@gmail.com

†Corresponding author: lin.ye@sydney.edu.au

‡jf456@cornell.edu

or meniscus arrives. As such, the capillary pressure varies in different levels of branches and so does the flow behavior.

Washburn's equation has been the basis for describing the time-dependent capillary phenomena since a century ago [26]. Washburn [26] modeled the dynamics of capillary flow in a straight tube driven by the negative capillary pressure and suggested a correlation between the distance of liquid movement z and the time t ,

$$z = \left(\frac{\gamma r \cos \theta}{2\eta} t \right)^{0.5}, \quad (1)$$

where γ and η stand for the (liquid-vapor) surface tension and the viscosity of the liquid, respectively, r is the tube radius, and θ is the contact angle between the liquid meniscus and the wall of the tube. This law holds in the limit $z \ll z_e$ [27] or when the tube is placed horizontally, where z_e is the final height of the liquid rise with the capillary force simultaneously being equal to the gravity of the liquid column. Washburn's equation was also found valid down to nanoscale pore sizes based on the molecular dynamics simulation [28]. Numerical simulations showed that the liquid meniscus suffered complex deformation during moving through tubes with variations in radii [29,30]. In particular, the liquid passing through a sudden expansion was pinned when the actual contact angle exceeded 90° [30]. To avoid the effect of liquid meniscus on the capillary flow, it is commonly assumed that the tubes are long with extremely mild contractions and expansions [31].

Capillary flow in tubes is solely driven by capillary pressure and there is no fixed overall pressure difference across the system. The displacing phase is a wetting viscous liquid and the displaced phase is a nonviscous phase (such as air). Here, the capillary flow is accelerated by the increase in the capillary pressure, which is proportional to the tube radius [32]. Conversely, the capillary flow slows down due to the decreased permeability of the tube, which scales with the square of the radius [33]. As a whole, the time of a liquid required to move for a constant distance is monotonously dependent on the pore radius in a uniform tube as presented in Eq. (1), based on the balance of the two mechanisms. The branching tubes in treelike networks have different radii and lengths, and the capillary flow has different sensitivity to microstructures of local branches at different levels. Thus, the fastest capillary flow may be found in the treelike networks on the basis of the dynamic competition between the above two mechanisms superimposed, in comparison to parallel uniform tubes under fixed constraints. In this article, we aim to theoretically describe the dynamics of capillary flow in treelike networks and expect to find a general criterion for the design of network structures for the fastest capillary flow.

II. THEORETICAL MODEL

Figure 1 shows a V-shaped treelike network and a parallel tube net. V-shaped treelike networks are widely found as arteries or veins in human beings and animals. For instance, the human arterial vessels contain around 30 branches from the aorta to the arterioles [22]. It is visible that the higher levels of branches have similar structures to those at the lower levels. Every vessel in the network system is divided into m branches at the next level with branching angle ϕ , as seen in Fig. 1. Thus,

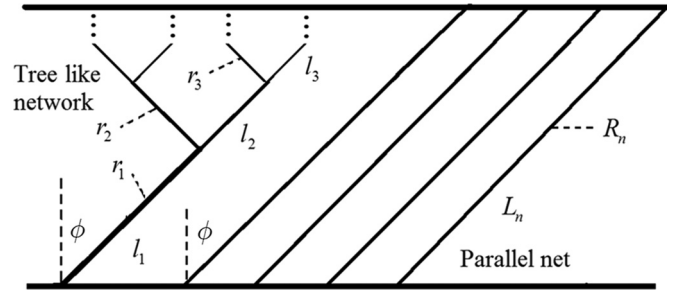


FIG. 1. A V-shaped treelike network (left) and a parallel tube net (right).

this network can be generated by repeating this algorithm from the first-level (single-tube) branch to the n th-level branches, where n is the total number of branching levels. The branches are composed of straight hollow tubes. We define the ratio of radius between the tubes at the $(j+1)$ th branching level and that at the j th branching level, viz.,

$$\alpha = r_{j+1}/r_j, \quad (2)$$

and therefore, it is easy to obtain

$$r_j = \alpha^{j-1} r_1, \quad (3)$$

where r_j and r_1 are the radii of the tubes at the j th and first branching levels, respectively. α is generally assumed to be smaller than 1 as in natural network systems. Similarly, the ratio of length of the tubes at the $(j+1)$ th branching level to that at the j th branching level is defined as

$$\beta = l_{j+1}/l_j, \quad (4)$$

and thus,

$$l_j = \beta^{j-1} l_1, \quad (5)$$

where l_j and l_1 are the lengths of the tubes at the j th and first branching levels, respectively.

In order to investigate the unique dynamics of capillary flow in treelike networks, we compare our model with the traditional parallel tube nets under the same cross area of the system, total length and total void volume (Fig. 1). To this end, we assume the number of parallel tubes is equal to the number of outlet tubes (at the n th level) of the treelike network. As well, the interdistance of the outlet tubes of the treelike network is identical to that of the parallel tubes. Thus, the parallel nets are defined similarly as those in Ref. [24]. In this work, we focus on investigating capillary flow in treelike networks whose tube radius is much smaller than the length. These networks have great applications such as plant-structured fabrics. In the networks, the effect of the junction between different branching levels on the flow is assumed negligible (see Supplemental Material [34]). As well, the effect of the branching angle is eliminated, since all tubes have the same branching angle in both parallel tube nets and treelike networks (Fig. 1).

Based on Fig. 1 and Eq. (5), the total length of the treelike network is given by

$$L_n = \sum_{j=1}^n l_j = \frac{1 - \beta^n}{1 - \beta} l_1. \quad (6)$$

The total volume of the treelike network is calculated as

$$V_n = \sum_{j=1}^n \pi k_j r_j^2 l_j = \pi r_1^2 l_1 \frac{1 - (m\alpha^2\beta)^n}{1 - m\alpha^2\beta}, \quad (7)$$

where k_j is the number of tubes at the j th level, which is expressed as $k_j = m^{j-1}$.

The parallel tube net has the same length and volume as the treelike network. The volume of the net composed of k_n tubes is given by

$$V_n = \pi k_n R_n^2 L_n, \quad (8)$$

where R_n is the tube radius of the net. R_n is calculated on the basis of Eqs. (7) and (8):

$$R_n = \left(\frac{\sum_{j=1}^n k_j r_j^2 l_j}{L_n k_n} \right)^{0.5} = r_1 \left[m^{1-n} \frac{1 - (m\alpha^2\beta)^n}{1 - m\alpha^2\beta} \frac{1 - \beta}{1 - \beta^n} \right]^{0.5}. \quad (9)$$

The capillary flow in all tubes of the treelike network is driven by capillary pressure. When the meniscus is in the k th-level tube, the capillary pressure is given by

$$\Delta p_{c,k} = -\frac{2\gamma \cos \theta}{r_k}. \quad (10)$$

The flow rate Q_k is obtained based on the Hagen-Poiseuille equation, viz.,

$$Q_k = \pi r_k^2 u_k = -\frac{\pi r_k^4}{8\eta} \frac{\partial p}{\partial x}, \quad \text{with} \quad u_k = \frac{\partial z}{\partial t}, \quad (11)$$

where u_k is the spontaneous velocity of the liquid at the k th level. Note we have $k_k Q_k = Q_1$ based on conservation of mass.

In the first level of the treelike network, the time of capillary flow is obtained in terms of the liquid penetration distance l based on Eq. (1), viz.,

$$t_{l,1} = \frac{C l^2}{2 r_1}, \quad (12)$$

where C is a constant defined as $\frac{4\eta}{\gamma \cos \theta}$.

It is easy to obtain the time T_1 required for the liquid to fill the tube of the first level with the length l_1 :

$$T_1 = \frac{C l_1^2}{2 r_1}. \quad (13)$$

In the second level of the treelike network, the flow rate in a tube is given by

$$Q_2 = \pi r_2^2 u_2 = -\frac{\pi r_2^4}{8\eta} \frac{\partial p}{\partial x}, \quad \text{with} \quad u_2 = \frac{\partial z}{\partial t}, \quad (14)$$

and the total hydrostatic pressure drop of the first and the second levels of the treelike network is obtained as follows:

$$\begin{aligned} \Delta p &= -8\eta Q_1 \int_0^{h_1} \frac{dx}{\pi r_1^4} - 8\eta Q_2 \int_{h_1}^z \frac{dx}{\pi r_2^4} \quad \text{with} \\ k_2 Q_2 &= Q_1 \quad \text{and} \quad k_2 = m, \end{aligned} \quad (15)$$

which is equal to the capillary pressure $\Delta p_{c,2}$ at the second level on the basis of Eq. (10):

$$\Delta p_{c,2} = -\frac{2\gamma \cos \theta}{r_2}. \quad (16)$$

Thus, we have

$$\frac{\gamma \cos \theta}{r_2} = 4m\eta Q_2 \int_0^{l_1} \frac{dx}{\pi r_1^4} + 4\eta Q_2 \int_{l_1}^z \frac{dx}{\pi r_2^4}. \quad (17)$$

Integrating Eq. (17) we have the time of capillary flow in the second level of the treelike network:

$$t = C \int_{l_1}^l \left(m \frac{r_2^3 l_1}{r_1^4} + \frac{z - l_1}{r_2} \right) dz. \quad (18)$$

The time corresponding to the liquid at the second level of the treelike network $t_{l,1}$ is obtained in terms of distance of the liquid movement l based on Eqs. (13) and (18), viz.,

$$t_{l,2} = C \left[m \frac{r_2^3 l_1 (l - l_1)}{r_1^4} + \frac{1}{2} \frac{(l - l_1)^2}{r_2} \right] + T_1. \quad (19)$$

The time t_2 required for the liquid to fill the tubes of the second level with the length l_2 is obtained by Eq. (18), viz.,

$$t_2 = C \left[m \frac{r_2^3 l_1 l_2}{r_1^4} + \frac{1}{2} \frac{l_2^2}{r_2} \right]. \quad (20)$$

Therefore, the total time T_2 for the liquid moving for the distance $(l_1 + l_2)$ is the sum of t_1 and t_2 , viz.,

$$T_2 = t_1 + t_2 = \frac{C l_1^2}{2 r_1} + C \left[m \frac{r_2^3 l_1 l_2}{r_1^4} + \frac{1}{2} \frac{l_2^2}{r_2} \right]. \quad (21)$$

In analogy, we can obtain the time t_j required for the liquid to fill the tubes of the j th level with the length l_j :

$$t_j = \frac{C l_j^2}{2 r_j} + C \sum_{k=2}^j \left(m^{j+1-k} \frac{r_j^3 l_{k-1} l_j}{r_{k-1}^4} \right). \quad (22)$$

The total time T_n for the liquid to move for the distance L_n is obtained as the sum of t_j , viz.,

$$T_n = \sum_{j=1}^n t_j = \frac{C}{2} \sum_{j=1}^n \frac{l_j^2}{r_j} + C \sum_{j=2}^n \sum_{k=2}^j \left(m^{j+1-k} \frac{r_j^3 l_{k-1} l_j}{r_{k-1}^4} \right). \quad (23)$$

The flow time corresponding to the liquid movement at the j th level of the treelike network $t_{l,j}$ is obtained in terms of the distance of the liquid movement l :

$$\begin{aligned} t_{l,j} &= \frac{C (l - L_{j-1})^2}{2 r_j} \\ &+ C \sum_{k=2}^j \left[m^{j+1-k} \frac{r_j^3 l_{k-1} (l - L_{j-1})}{r_{k-1}^4} \right] + T_{j-1}. \end{aligned} \quad (24)$$

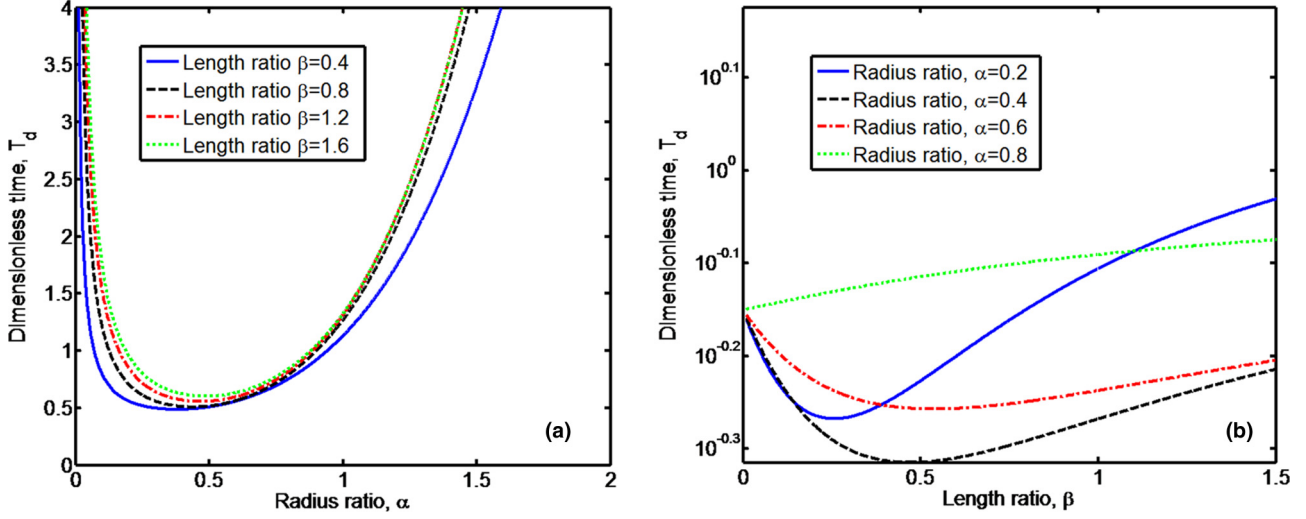


FIG. 2. (Color online) Dimensionless time of capillary flow in treelike networks versus (a) width radius α at different length ratios β , and (b) length ratio β at different radius ratios α when the level number $n = 2$.

Substituting Eqs. (4) and (5) into Eq. (23) results in T_n in terms of α and β :

$$T_n = \frac{C l_1^2}{2 r_1} \frac{1 - (\alpha^{-1} \beta^2)^n}{1 - \alpha^{-1} \beta^2} + C \frac{1}{1 - \alpha^{-4} m^{-1} \beta} \left[\frac{\alpha^3 m \beta - (\alpha^3 m \beta)^n}{1 - \alpha^3 m \beta} - \frac{\alpha^{-1} \beta^2 - (\alpha^{-1} \beta^2)^n}{1 - \alpha^{-1} \beta^2} \right]. \quad (25)$$

The total time for the liquid to fill the net of parallel tubes is obtained with the help of Eq. (13) for the single tube, viz.,

$$T_0 = \frac{C L_n^2}{2 R_n} = \frac{C l_1^2}{2 r_1} \frac{\left(\frac{1 - \beta^n}{1 - \beta} \right)^{1.5}}{\left[m^{1-n} \frac{1 - (m \alpha^2 \beta)^n}{1 - m \alpha^2 \beta} \right]^{0.5}}. \quad (26)$$

Thus, the dimensionless time of capillary flow in the treelike network is expressed as the ratio of Eqs. (25) to (26), viz.,

$$T_d = \frac{T_n}{T_0} = \frac{\left(m^{1-n} \frac{1 - (m \alpha^2 \beta)^n}{1 - m \alpha^2 \beta} \right)^{0.5} \left\{ \frac{1 - (\alpha^{-1} \beta^2)^n}{1 - \alpha^{-1} \beta^2} + \frac{2}{1 - \alpha^{-4} m^{-1} \beta} \left[\frac{\alpha^3 m \beta - (\alpha^3 m \beta)^n}{1 - \alpha^3 m \beta} - \frac{\alpha^{-1} \beta^2 - (\alpha^{-1} \beta^2)^n}{1 - \alpha^{-1} \beta^2} \right] \right\}}{\left(\frac{1 - \beta^n}{1 - \beta} \right)^{1.5}}. \quad (27)$$

III. RESULTS AND DISCUSSION

In this section, we investigate the V-shaped network with $m = 2$, a possible smallest dichotomous structure, to represent our work. With Eq. (27), it is easy to plot the dimensionless time of capillary flow in treelike networks in terms of the radius ratio α and the length ratio β in Fig. 2 ($n = 2$), Fig. 3 ($n = 3$), and Fig. 4 ($n = 4$).

The result is that T_d in Fig. 2(a) first decreases and then increases dramatically with the increase in α at four different values of β (i.e., $\beta = 0.4, 0.8, 1.2, 1.6$). The concave curves in Fig. 2(a) indicate the optimal α close to 0.5 that results in the minimum time required for the liquid arriving at the end of the network system or the tubes at the second level. It is expected that the first-level tube with larger radius has greater permeability and requires less time (i.e., T_1) for liquid penetration, whereas the two tubes at the second level with smaller radius create higher capillary forces to drive the liquid to move on. When α reaches the higher limit ($\alpha \gg 1$), the tube radius is extremely small in the first level and the

flow resistance rises markedly, significantly slowing down the process of liquid movement. On the other hand, when α is close to 0, the permeability of the two tubes at the second level, proportional to the square of the tube radius, is significantly reduced, and the extremely low permeability suppresses the effect of the high capillary pressure. Therefore, the minimum flow time versus α exists because of the two mechanisms superimposed: capillary pressure and permeability. Figure 2(a) also shows that the corresponding optimal values of α vary slightly with the variation in β .

Figure 2(b) presents the dimensionless time T_d versus β at different values of α ($\alpha = 0.2, 0.4, 0.6, 0.8$) in the two-level network. When $\alpha = 0.2, 0.4, 0.6, 0.8$ and β approaches 0, the length of the two tubes at the second level is close to 0, and the network becomes a single uniform tube. Thus, T_d converge to the same value for the four structures. When β increases higher than 1, however, different values of T_d respond to the varied α due to the competition between permeability and capillary pressure in the two-level network. The minimum T_d is found when $\alpha = 0.2, 0.4, 0.6$ by adjusting

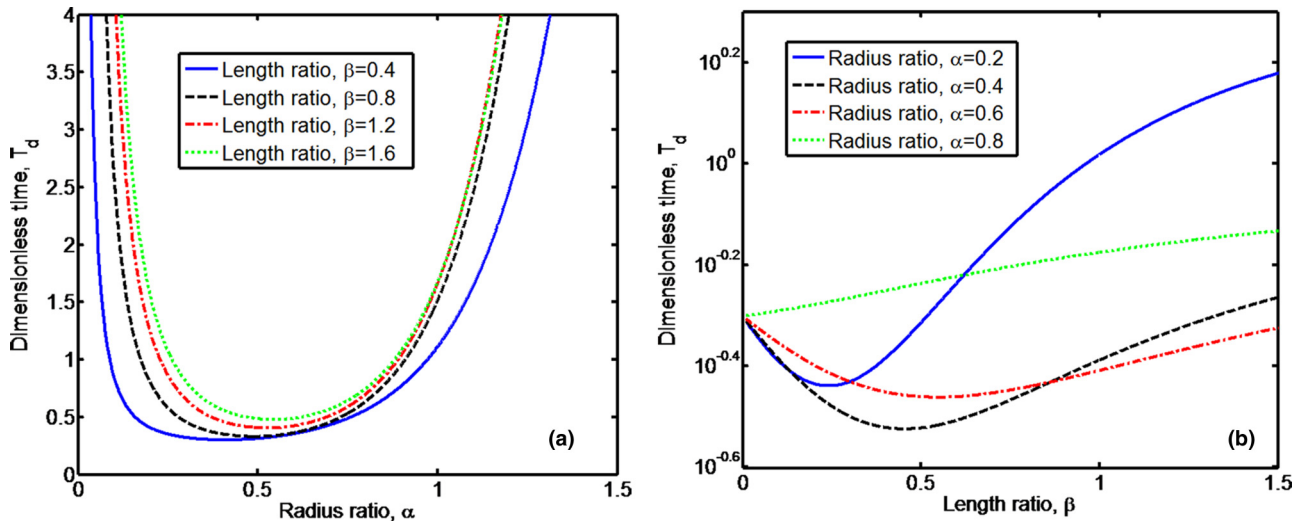


FIG. 3. (Color online) Dimensionless time of capillary flow in treelike networks versus (a) width radius α at different length ratios β , and (b) length ratio β at different radius ratios α when the level number $n = 3$.

the contribution of capillary flows in the two levels of local tubes, based on the phenomenon that the wider tube of the first level promotes higher permeability and simultaneously the two narrower tubes of the second level cause higher driving drags for the liquid. It is interesting to observe that T_d increases with the increase in β when $\alpha = 0.8$, because the radii of tubes in the second level are too large to generate an adequate capillary force for accelerating the capillary flow.

It is expected that the minimum T_d in terms of α and β can be simultaneously found by analyzing Fig. 2. With the help of MATLAB, the minimum T_d is derived, viz., 0.4841, with $\alpha = 0.3940$ and $\beta = 0.4672$, respectively. This result indicates it is possible to accelerate the speed of capillary flow for a two-level treelike network by around

52% at fixed total length and volume based on the structural optimization.

Analogous to the treelike network with $n = 2$, the minimum T_d 's for $n = 3$ versus the radius ratio α and the length ratio β are illustrated in Figs. 3(a) and 3(b), respectively. The three-level network has the same trend of capillary flow against α and β , though the optimal values of α and β accounting for the minimum T_d are different from those in the two-level case. As well, the minimum T_d in terms of α and β is found simultaneously for the optimal structure of the treelike network, viz., 0.2973, with $\alpha = 0.4306$ and $\beta = 0.4799$, respectively. This result indicates that T_d can be decreased by 70% in comparison to the parallel tube nets at fixed length and volume. For $n = 4$, the minimum T_d 's versus α and ratio β are illustrated in Figs. 4(a) and 4(b), respectively. The minimum T_d is also derived, viz.,

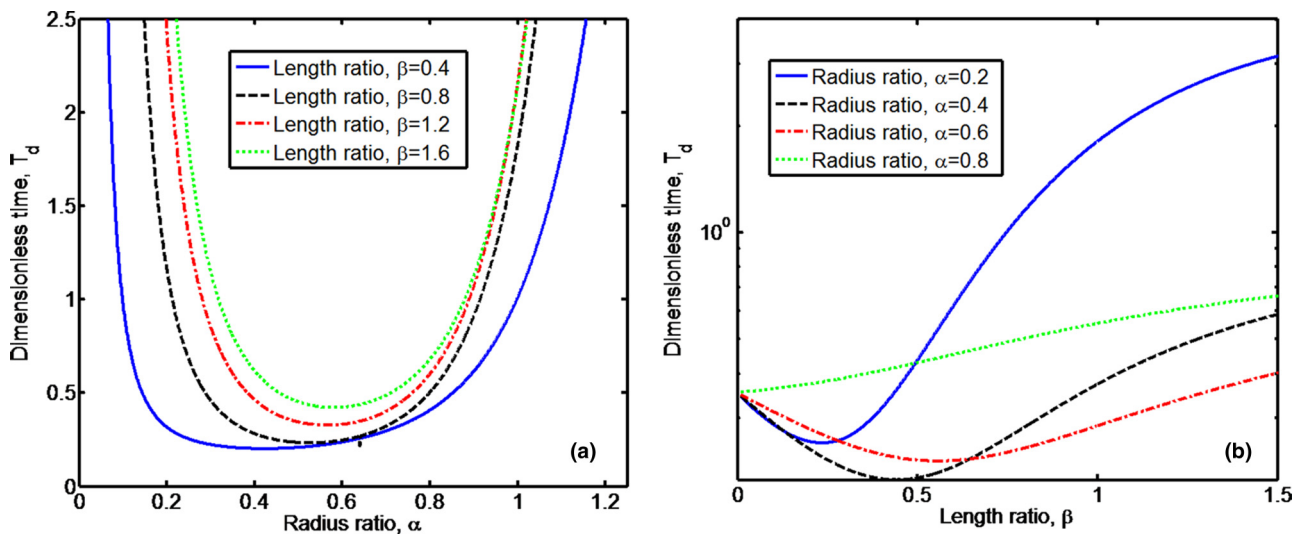


FIG. 4. (Color online) Dimensionless time of capillary flow in treelike networks versus (a) width radius α at different length ratios β , and (b) length ratio β at different radius ratios α when the level number $n = 4$.

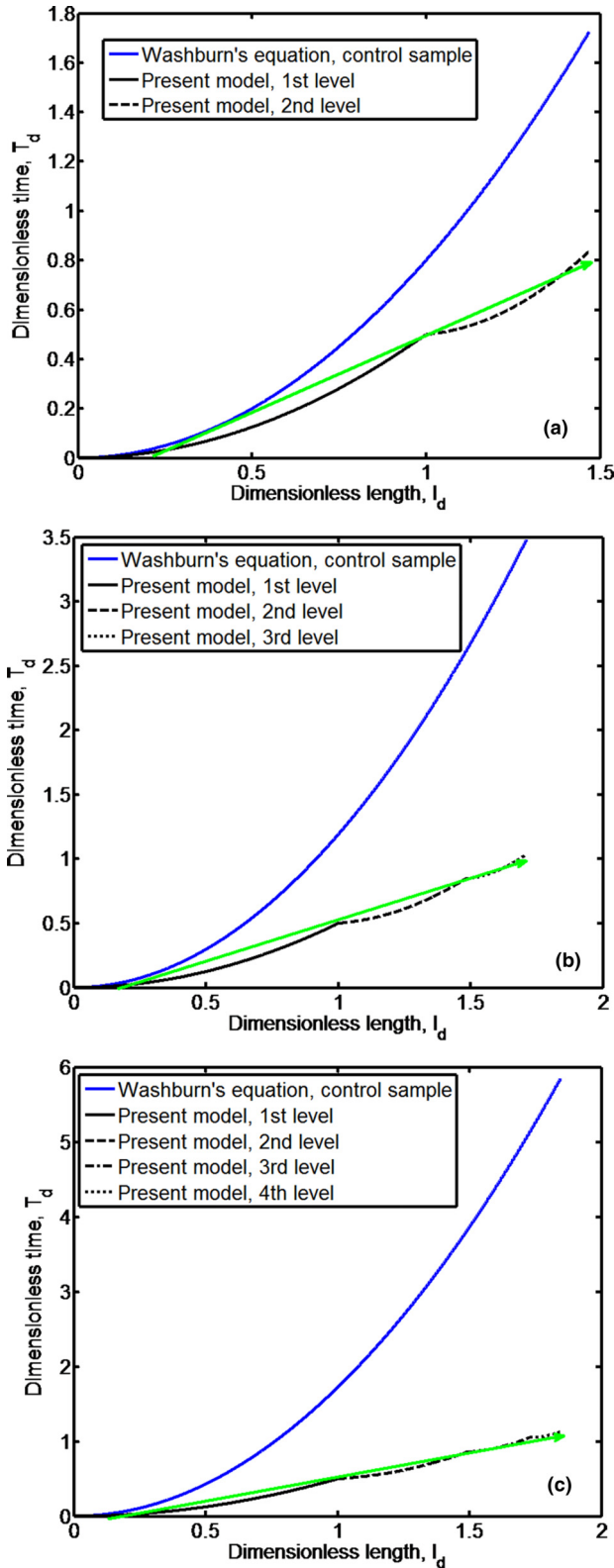


FIG. 5. (Color online) Comparison of dimensionless time of capillary flow versus dimensionless penetration distance between optimized treelike networks and parallel nets for (a) $n=2$, (b) $n=3$, and (c) $n=4$, where the solid (upper) blue line is the Washburn's equation and the solid (lower) black line is the present model of the first level.

0.1973, with $\alpha = 0.4540$ and $\beta = 0.4888$, respectively. The decreased magnitude of T_d in comparison to the control sample of parallel tube nets is around 80%. It can be expected that more levels of branches lead to a greater decrease in value of T_d .

Based on Eq. (27), flow time against penetration distance in the optimized treelike networks with the fastest flow is shown for $n=2$, $n=3$, and $n=4$, in Figs. 5(a)–5(c), separately. The parallel, traditional tube nets are also added for comparison. It is evident that time is diffusive to the distance of the liquid movement in the parallel tube nets and the flow behavior obeys Washburn's equation [Eq. (1)]. Furthermore, the results shown in Fig. 5 indicate that flow time in the optimized networks is much less than in the parallel tube nets at the given penetration distance, and the flow time increases approximately linearly as the distance. As such, the capillary flow in treelike networks differs from Washburn's classic description that the flow time increases as the square of penetration distance in a single uniform tube.

IV. CONCLUSION

A closed-form model has been developed to describe the dynamics of capillary flow in treelike networks. A treelike arrangement can accelerate the capillary flow comparing with the traditional parallel tube nets under a given total length and total volume. Moreover the design of different levels in the treelike network leads to the minimum time for the liquid to fill the whole network system, based on the optimized radius ratio and length ratio between different levels. After optimization of the network structure, the flow behaviors are found to differ from Washburn's equation, and the time of capillary flow is approximately proportional to the distance of liquid movement. In particular the time of the capillary flow of four-level, treelike networks can be reduced by around 80% in comparison to parallel tube nets, and more levels of branches lead to greater acceleration of capillary flow.

This study provides an initial theoretical exploration of the fastest capillary flow in treelike structures, but the proposed robust framework can be extended to more complex systems under different constraints. The present analysis can encourage us to design unique plant-structured fabrics with excellent liquid management properties, considering treelike networks as equivalent structures of these fabrics. As well, this work may help to explain some of the underlying mechanisms behind the natural phenomena, such as the effect of radius and length variations in different branching hierarchies of trees on efficient water absorption.

ACKNOWLEDGMENT

L.Y. is grateful for an ARC Discovery Project grant that supports this work.

- [1] T. D. Wheeler and A. D. Stroock, *Nature* **455**, 208 (2008).
- [2] C. D. Murray, *J. Gen. Physiol.* **9**, 835 (1926).
- [3] D. G. Tarboton, R. L. Bras, and I. Rodriguez-Iturbe, *Water Resour. Res.* **24**, 1317 (1988).
- [4] J. C. Cai, B. M. Yu, M. Q. Zou, and L. Luo, *Energy Fuels* **24**, 1860 (2010).
- [5] A. Bejan and M. R. Errera, *Fractals* **5**, 685 (1997).
- [6] M. S. Singleton, G. Heiss, and A. Hübler, *Phys. Rev. E* **83**, 016308 (2011).
- [7] J. T. Fan, M. K. Sarkar, Y. C. Szeto, and X. M. Tao, *Mater. Lett.* **61**, 561 (2007).
- [8] K. A. McCulloh, J. S. Sperry, and F. R. Adler, *Nature* **421**, 939 (2003).
- [9] E. R. Weibel and D. M. Gomez, *Science* **137**, 577 (1962).
- [10] B. Mauroy, M. Filoche, E. R. Weibel, and B. Sapoval, *Nature* **427**, 633 (2004).
- [11] M. Florens, B. Sapoval, and M. Filoche, *Phys. Rev. Lett.* **106**, 178104 (2011).
- [12] M. Florens, B. Sapoval, and M. Filoche, *J. Appl. Physiol.* **110**, 756 (2011).
- [13] B. Sapoval and M. Filoche, *Eur. Phys. J. E* **36**, 105 (2013).
- [14] J. S. Andrade, Jr., A. M. Alencar, M. P. Almeida, J. M. Filho, S. V. Buldyrev, S. Zapperi, H. E. Stanley, and B. Suki, *Phys. Rev. Lett.* **81**, 926 (1998).
- [15] B. Mauroy, M. Filoche, J. S. Andrade, and B. Sapoval, *Phys. Rev. Lett.* **90**, 148101 (2003).
- [16] M. P. Almeida, J. S. Andrade, S. V. Buldyrev, F. S. A. Cavalcante, H. E. Stanley, and B. Suki, *Phys. Rev. E* **60**, 5486 (1999).
- [17] S. F. Wang and B. M. Yu, *Transp. Porous Media* **87**, 191 (2011).
- [18] S. Rojas and J. Koplik, *Phys. Rev. E* **58**, 4776 (1998).
- [19] A. F. Morais, H. Seybold, H. J. Herrmann, and J. S. Andrade, *Phys. Rev. Lett.* **103**, 194502 (2009).
- [20] A. Bejan and S. Lorente, *J. Appl. Phys.* **100**, 041301 (2006).
- [21] A. H. Reis, *Appl. Mech. Rev.* **59**, 269 (2006).
- [22] Y. P. Chen and P. Cheng, *Int. J. Heat Mass Transfer* **45**, 2643 (2002).
- [23] B. Yu and B. Li, *Phys. Rev. E* **73**, 066302 (2006).
- [24] J. Chen, B. Yu, P. Xu, and Y. Li, *Phys. Rev. E* **75**, 056301 (2007).
- [25] C. B. Shah and Y. C. Yortsos, *AIChE J.* **41**, 1099 (1995).
- [26] E. W. Washburn, *Phys. Rev.* **17**, 273 (1921).
- [27] A. Ponomarenko, D. Quere, and C. Clanet, *J. Fluid Mech.* **666**, 146 (2011).
- [28] D. I. Dimitrov, A. Milchev, and K. Binder, *Phys. Rev. Lett.* **99**, 054501 (2007).
- [29] M. Chen, Y. C. Yortsos, and W. R. Rossen, *Phys. Rev. E* **73**, 036304 (2006).
- [30] H. Mehrabian, P. Gao, and J. J. Feng, *Phys. Fluids* **23**, 122108 (2011).
- [31] D. Erickson, D. Li, and C. B. Park, *J. Colloid Interface Sci.* **250**, 422 (2002).
- [32] S. Mendez, E. M. Fenton, G. R. Gallegos, D. N. Petsev, S. S. Sibbett, H. A. Stone, Y. Zhang, and G. P. Lopez, *Langmuir* **26**, 1380 (2010).
- [33] M. Akbari, D. Sinton, and M. Bahrami, *Int. J. Heat Mass Transfer* **54**, 3970 (2011).
- [34] See Supplemental Material at <http://link.aps.org/supplemental/10.1103/PhysRevE.89.053007> for model details.