

Simple geometric model to describe self-folding of polymer sheetsYing Liu,¹ Russell Mailen,² Yong Zhu,² Michael D. Dickey,^{1,*} and Jan Genzer^{1,†}¹*Department of Chemical & Biomolecular Engineering, North Carolina State University, 911 Partners Way, Raleigh, North Carolina 27695, USA*²*Department of Mechanical & Aerospace Engineering, North Carolina State University, 911 Oval Drive, Raleigh, North Carolina 27695, USA*

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Self-folding is the autonomous folding of two-dimensional shapes into three-dimensional forms in response to an external stimulus. This paper focuses on light-induced self-folding of prestrained polymer sheets patterned with black ink. The ink absorbs the light and the resulting heat induces the polymer beneath the ink to relax faster than the rest of the sheet. A simple geometric model captures both the folding angle and folding kinetics associated with this localized shrinkage. The model assumes that (1) the polymer in contact with the ink shrinks at a rate determined by the temporal temperature profile of the hinge surface; (2) the bottom of the sheet, which is cooler, does not shrink considerably; and (3) a linear gradient of strain relaxation exists across the film between these two extremes. Although there are more complex approaches for modeling folding, the appeal of this model is its simplicity and ease of use. Measurements of the macroscopic, thermally driven shrinkage behavior of the sheets help predict the kinetics of folding by determining how fast the top of the hinge shrinks as a function of temperature and time. These measurements also provide information about the temperature required to induce folding and offer indirect measurement of the glass transition temperature of the polymer that comprises the sheet.

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I. INTRODUCTION

This paper develops a predictive model of the dynamic folding angle of self-folding sheets driven by the absorption of light. Self-folding sheets are substrates that fold without human intervention, which is appealing for packaging, assembly, and actuation [1–6]. Self-folding is typically accomplished by predefining hinges in a two-dimensional (2D) substrate that facilitate the formation of three-dimensional (3D) structures [1,2]. Previously, we demonstrated a simple approach to self-folding of prestrained polymer sheets, i.e., shrink films. These films are shape memory polymers (SMPs) that shrink in plane when heated above their glass transition temperature (T_g). Black ink (e.g., toner from a desktop printer) printed on these sheets absorbs light preferentially relative to the rest of the optically transparent sheet [7]. Folding occurs when the inked regions get sufficiently hot to allow the prestrained polymer sheet to relax directly under the “hinge” in a gradient through the depth of the sheet (cf. Fig. 1). The appeal of this approach is its simplicity in converting 2D patterning of inexpensive plastic materials into 3D objects using only light.

We sought to understand and model the folding kinetics of the process illustrated in Fig. 1 (i.e., folding angle as a function of time) and the final folding angle, which is important for controlling the geometry of the folds. The basis for the self-folding is strain relaxation of the polymer in the hinged region. Elevated temperatures (i.e., above the T_g of the polymer) and large strains cause rapid strain relaxation, and thus, rapid folding. Temperature and strain both change dynamically during folding in a complex manner, which makes modeling a great challenge. Constitutive models exist for

amorphous SMPs that incorporate their thermomechanical behavior to capture strain relaxation [8–10]. However, these methods use complex finite element analysis that requires a number of intrinsic material parameters for the prediction of the shape memory effect [8,9,11–13]. Although we are pursuing similar methods to describe self-folding, we report here a simple geometric model that predicts the angle of the hinge. The model incorporates simple measurements of the macroscopic shrinkage behavior of prestrained polymer sheets in response to heat. These experiments require only a heat source (e.g., an oven or a lamp) and a ruler to measure strain relaxation. The appeal of our model is its simplicity and ease of use: The model predicts readily and accurately the folding angle knowing only the width of the hinge and the temperature of the surface of the hinge as a function of time.

II. EXPERIMENT

A desktop laser printer (HP-P3005dn) produced 2D black ink patterns (designed in CorelDRAW[®]) onto clear inkjet Shrink Film (Grafix[®]). After cutting with scissors, the polystyrene (PS) sheets had dimensions 25 mm \times 10 mm. Calipers confirmed the film thickness to be \approx 0.3 mm. To induce folding, the sheets were placed on a hot-plate at 90 °C and exposed to an infrared heat lamp (250 W, Phillips) with an intensity of 1 W/cm². The bending angles of the samples were recorded by video camera (Cannon Vixia HF S20) and the temperature measured by an infrared (IR) camera (FLIR A325).

Shrinkage of the sheets versus time was measured in both isothermal and constant heat flux modes. In both modes, the samples started at an initial temperature of 100 °C. All polymer sheets had initial dimensions of 20 mm \times 20 mm. A caliper measured the dimensions of the polymer sheet in the X and Y directions before and after shrinkage. Isothermal measurements were carried out within a hot stage (Mettler

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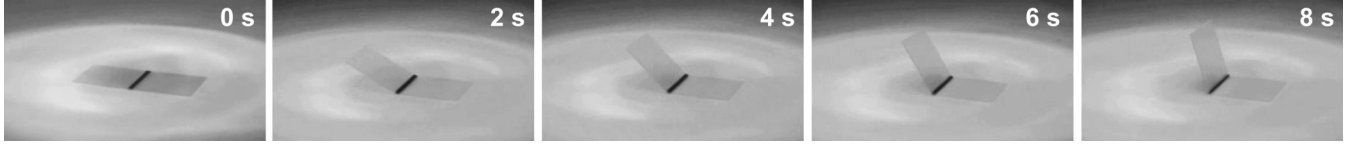


FIG. 1. Folding of a prestrained polymer sheet by printing a black line on the sheet and exposing the sample to high intensity light. The energy from the light gets absorbed preferentially by the black line, which acts as a hinge because it converts the light effectively into heat. The heat relaxes the polymer directly under the hinge region in a gradient across the sheet thickness. The dimensions of the black line are 10 mm \times 1 mm.

Toledo FP82HT). Non-isothermal shrinkage measurements at low heating rates ($<20^\circ\text{C}/\text{min}$) were carried out on a hot plate (EchoThermTM HS30, Torrey Pines Scientific). At higher heating rates ($>20^\circ\text{C}/\text{min}$), a halogen lamp (90 W, Phillips) and an IR heat lamp heated plain Grafix sheets coated with black ink.

III. RESULTS AND DISCUSSION

Our approach to self-folding relies on the local delivery of heat to a hinge relative to the rest of the sheet. Figure 2 compares the response of both a plain prestrained PS sheet and a sheet printed with black ink to light delivered via an IR heat lamp. The samples rest on a hot plate set to 90°C , which brings the sheet closer to the T_g of PS. An IR camera measures the average temperature of the top surface of the PS sheets as a function of exposure time to the lamp. Absorption of light by the black ink causes the coated PS sheet to heat up and shrink due to strain relaxation. Equation (1) defines the shrinkage (S) of the prestrained polymer sheet:

$$S = \frac{l_0 - l}{l_0}. \quad (1)$$

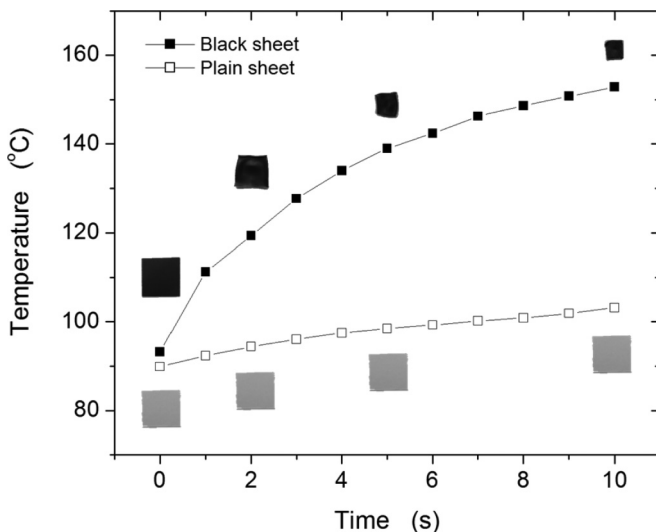


FIG. 2. Surface temperature vs time for a plain prestrained PS sheet (open symbols) and a prestrained PS sheet printed with black ink (solid symbols) exposed to an IR heat lamp. Photographs of representative black and plain PS sheets (photographed on a gray background) depict the geometry for selected data points. Curled samples are flattened for imaging. The PS sheets (initial dimension 20 mm \times 20 mm) rest on a hot plate at 90°C during the exposure to IR light.

In Eq. (1), l_0 is the original length of the polymer sheet and l is the length of the PS sheet during heating. For the conditions pertinent to our setup, shrinkage reaches its maximum value ($\approx 55\%$) over the course of ≈ 10 s of exposure to an IR lamp ($\approx 1 \text{ W}/\text{cm}^2$). This maximum shrinkage value is defined by the manufacturing process. In contrast, the plain (i.e., uninked) PS sheet maintains its original size over the course of ≈ 10 s of exposure to the IR lamp. The temperature of the polymer increases slightly due to absorption of the longer wavelengths (in the IR region) from the IR lamp, but does not warm up enough to induce measurable shrinkage within the time scale of the experiment. The sheet coated uniformly with black ink initially curls upwards because the top of the PS sheet is hotter than the bottom and therefore relaxes more rapidly on the inked surface. This curling behavior explains why black hinges induce self-folding, as shown in Fig. 1.

The goal of this paper is to understand and predict the folding angle as a function of the hinge width and folding time. We employ a simple geometric model based on the differential shrinkages between the top surface of the hinge (i.e., immediately below the black ink) and the bottom side of the PS sheet to predict the maximum folding angle and dynamic folding angle as a function of hinge width.

The model assumes four phenomena: (1) the polymer directly underneath the printed hinge shrinks as a function of time and temperature, (2) the bottom of the polymer sheet does not shrink considerably, (3) the strain relaxes linearly across the PS film thickness (H), and (4) the thickness of the film does not increase considerably with shrinkage. The latter assumption eventually fails as the sample shrinks (due to material conservation); we discuss the extent of the increase in film thickness during shrinking in a separate paper [14]. Assuming that these assumptions hold, the bending angle α_B , i.e., the angular displacement of the fold, can be determined geometrically from Eq. (2) as

$$\alpha_B = 180^\circ - 2 \times \tan^{-1} \left(\frac{2 \times H}{W \times S} \right). \quad (2)$$

In Eq. (2), W is the hinge width, H is the initial film thickness, and S is the shrinkage of the PS sheet as indicated in Fig. 3. Thus, determining S —which is a function of temperature and time—for a given H and W , predicts α_B . Here, we neglect the heat transport through the ink (containing a large fraction of carbon black) given the higher thermal conductivity ($\approx 0.3 \text{ W}/\text{m K}$) [15] and much smaller thickness of the ink layer ($\approx 10 \mu\text{m}$) relative to the shrink film [thermal conductivity $\approx 0.14 \text{ W}/\text{m K}$ measured by a thermal conductivity meter (TA instrument, DTC-300) and thickness $\approx 300 \mu\text{m}$].

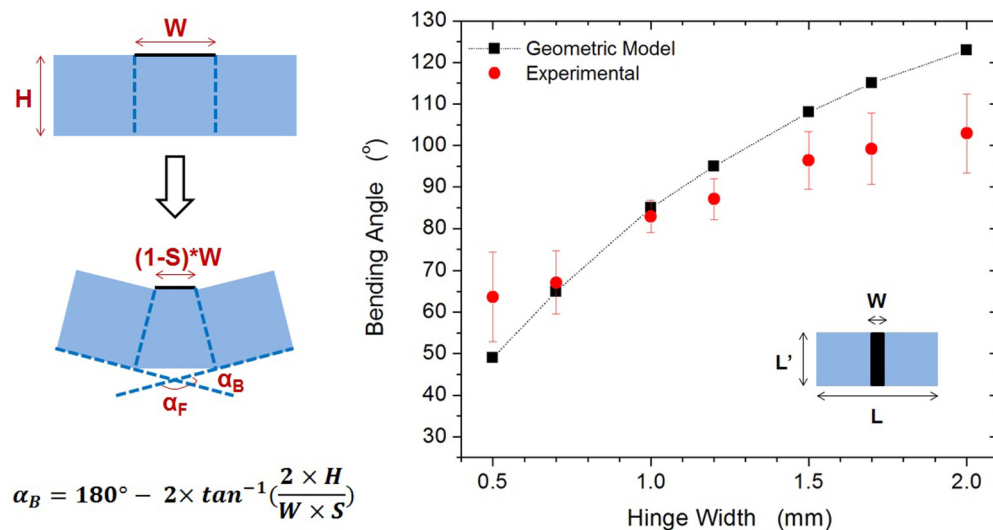


FIG. 3. (Color online) Left: Side-view schematic of the polymer sheet (blue) and ink (black) depicting the simple geometrical model used to calculate the bending angle. Right: Bending angles (α_B) as a function of hinge width (W) obtained from experiment (circles) and calculated using the geometric model (squares). Bending angle was measured by folding prestrained PS sheets (as depicted in the top-view schematic with dimensions $L' = 10$ mm and $L = 25$ mm) with a single line of ink patterned across the center of the sample. The sample rested on a hot plate at 90°C and was exposed to an IR light (≈ 1 W/cm 2).

First, we use this model to successfully predict the maximum folding angle as a function of hinge width induced by the exposure to an IR heat lamp (≈ 1 W/cm 2). The folding reaches the maximum α_B when the hinge surface achieves the maximum values of S , which is defined by the amount of prestrain incurred during the manufacturing process ($\approx 55\%$). The calculated α_B derived from the geometric model matches well with the experimental α_B , especially in the median range of hinge widths. Narrow hinges (≈ 0.5 mm) require longer exposure times to induce folding because heat dissipates at the edges of the hinge. As a result of this dissipation, the sheets take longer to fold and the polymer outside the inked region warms during that time, which causes the narrow hinges to fold with a similar angle as a sample with $W = 0.7$ mm. The discrepancy at wide hinge widths (1.5–2.0 mm) is likely due to the folded PS sheet blocking incoming light from reaching the printed hinge when $\alpha_B > 90^\circ$. Consequently, the experimental α_B value for wider hinges plateaus close to 90° and is thus lower than that predicted by the model. This model fits the experiment well when exposing the polymer sheet to light sources at high heating rates (e.g., heating with an IR light at ≈ 1 W/cm 2). At low heating rates (e.g., heating with an IR light at < 0.3 W/cm 2) or for samples that start from a lower initial temperature, it takes more time to reach the critical folding temperature; consequently, the heat dissipates outside the hinged region (for all hinge widths) and the shrinkage on the bottom side cannot be neglected (cf. Fig. S1 in the Supplemental Material [16]). This reality results in smaller experimental α_B than that predicted from the model. Long exposure times, which are associated with low heating rates, also cause the PS sheet to distort outside of the hinge (i.e., the “panels”). Thus, the model works best under conditions that give “ideal” folding (a sample resting on a hot plate at 90°C and exposure to light of ≈ 1 W/cm 2).

Next, we sought to predict the folding angle as a function of time and hinge width by developing expressions for S as a

function of time. The rate of shrinkage (i.e., strain relaxation) depends on the degree of strain and temperature, both of which change with time during folding. We measured the temperature of the hinges during folding as a function of time by an IR camera and found that the surface temperature rises quickly in the initial 2–3 s of exposure to the IR light for all hinge widths and then plateaus (120–130 $^\circ\text{C}$) (Fig. 4) despite the constant flux [7] (≈ 1 W/cm 2) of energy from the lamp. The plateau in Fig. 4 may be due to steady state heat dissipation, decreased area for light absorption due to the shrinkage of the

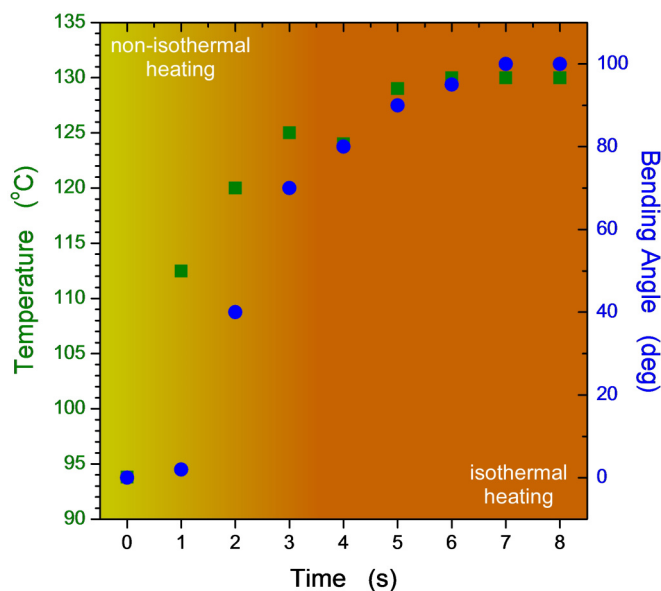


FIG. 4. (Color online) Hinge temperature (green squares, left ordinate) and bending angle (blue circles, right ordinate) versus exposure time to an IR heat lamp for a hinge having a width of 1.5 mm. The background gradient in the plot denotes the transition from nonisothermal to isothermal heating.

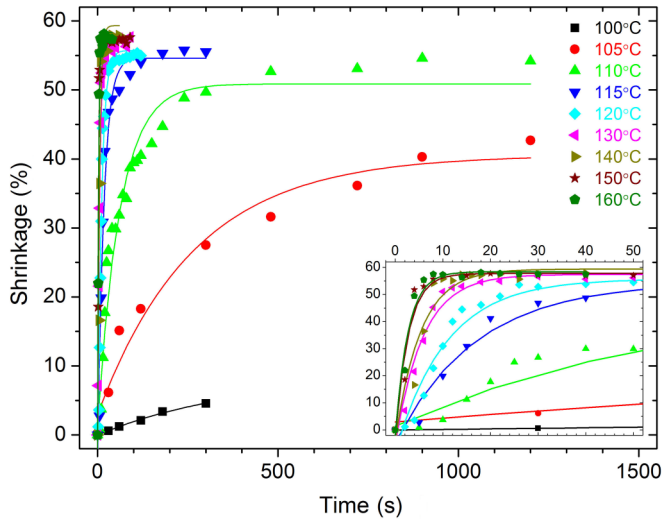


FIG. 5. (Color online) Thermal shrinkage of prestrained PS sheets (initial dimension 20 mm \times 20 mm) as a function of time at a constant temperature. The symbols represent the experimental data and the lines are best fit curves using Eq. (3). The inset magnifies the data from 0 to 50 s.

inked region on the PS sheet, or the simultaneous increase of mass near the hinge due to accumulation of shrunken material. Based on this temperature profile, the shrinkage performance of the hinge area for folding can be divided into two stages: (1) nonisothermal shrinkage under the constant heating rate of the light before reaching the plateau (first 2–3 s of exposure to light), and (2) isothermal shrinkage for the remaining light exposure. We therefore sought to find a mathematical relationship to predict S [i.e., Eq. (2)] as a function of constant heating rate (i.e., at short times) and constant temperature (i.e., at longer times) by measuring S

of the prestrained polymer sheets for various heating rates and various temperatures, respectively. These simple macroscopic measurements require only a heat source, a ruler, and a stop watch.

First, we studied the shrinkage behavior of PS sheets under isothermal conditions (cf. Fig. 5) to investigate the effect of temperature on S . We measured S of the polymer sheets versus time while holding the temperature constant within a hot stage. Figure S2 in the Supplemental Material [16] shows S of the polymer sheets as a function of time in both the horizontal (X) and the vertical (Y) directions of the sheet. The shrinkage takes place simultaneously in both X and Y directions, especially at high temperatures or high heating rates. Our analysis focuses on the shrinkage performance in one of the directions (say, X).

Figure 5 plots S of PS sheets under isothermal conditions as a function of time for various temperatures. As expected, the shrinkage rate, i.e., the change of shrinkage with time (dS/dt), increases with increasing temperature. Although there are fundamental macromolecular models to predict strain relaxation, we sought a simple mathematical fit with physical intuition that would easily insert into Eq. (2) to predict the bending angle. The isothermal S can be modeled analogously to first-order chemical reaction kinetics [17]:

$$S = S_{\infty}[1 - e^{-k_t(t-t_0)}]. \quad (3)$$

In Eq. (3), S is the shrinkage at time t ; S_{∞} is defined as the ultimate shrinkage obtained at the plateau of shrinkage on the time scale of our experiments; k_t is the shrinkage rate constant for a given temperature; and t_0 is the induction time (i.e., the time when the nonzero stress appears [18]) under isothermal conditions [17]. The model can be understood intuitively as having a maximum shrinkage rate when the PS strain is the largest and subsequently slowing down as strain (and stress) in the PS sheet relaxes due to shrinkage. The lines in Fig. 5 have

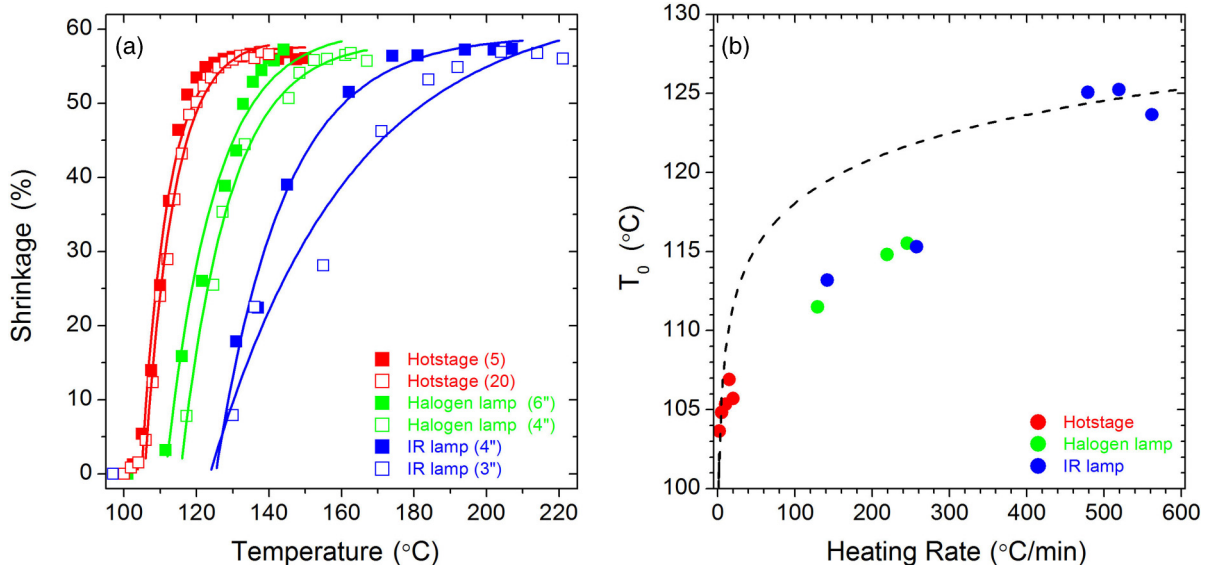


FIG. 6. (Color online) (a) Thermal shrinkage of prestrained PS sheets (initial dimension 20 mm \times 20 mm) as a function of temperature during constant heating rate. A wide range of heating rates is realized by using a hot stage (squares), halogen lamp (circles), and IR lamp (triangles), respectively. The numbers in parentheses denote different heating rates ($^{\circ}\text{C}/\text{min}$). The lines are best fits to Eq. (4). (b) Dependence of T_0 on the heating rates derived from Eq. (4). The dashed line represents the experimental variation of T_g (in $^{\circ}\text{C}$) with the heating rate reported for PS [$T_g = 99.5 + 4.03 \ln(dT/dt)$] [20].

been obtained by fitting the experimental data (symbols) to Eq. (3); the values of the fitting parameters are listed in Table SI in the Supplemental Material [16].

We also studied the shrinkage behavior of PS sheets under nonisothermal conditions (cf. Fig. 6) to investigate the effect of heating rate on sheet S in Eq. (2). Specifically, we measured S of the sheet as a function of time while keeping heating rates constant using a hot stage over a range of heating rates (ranging from 2 to 20 °C/min, the maximum of our hot stage). We carried out similar measurements with higher heating rates by using IR and halogen lamps and varied the heating rates by adjusting the distance of the lamp from the PS sheet. An IR camera measured the temperature of the PS sheets during these experiments. The first-order reaction kinetics, given by Eq. (4), can model the shrinkage kinetics at constant heating rate. This relationship works well except when the shrinkage is small (e.g., the first 1 to 2 data points when it first starts to shrink at temperatures close to 100 °C) [17].

$$S = S_{\infty}[1 - e^{-K(T-T_0)}]. \quad (4)$$

In Eq. (4), S is the shrinkage at temperature T , S_{∞} is the ultimate shrinkage, K is the shrinkage rate constant for a constant heating rate, and T_0 is the onset temperature for shrinkage. We calculated the corresponding heating rates for the various intensities by averaging the heating rate value over the interval $[0, t_{MS}]$, where t_{MS} is the time needed to achieve maximum shrinkage (i.e., $\approx 55\%$) (cf. Fig. S3 in the Supplemental Material [16]).

Figure 6(a) shows several representative shrinkage profiles as a function of temperature for different heating rates fit with the first-order reaction relationship described in Eq. (4). Figure S4 in the Supplemental Material [16] contains a more comprehensive data set of shrinkage as a function of heating rates. Table SII in the Supplemental Material [16] lists the parameters representing the best fits using Eq. (4). T_0 depends on the heating rate as shown in Fig. 6(b). High heating rates, achieved by exposing the polymer sheets to IR light, result in an onset shrink temperature ($T_0 = 115\text{--}120$ °C) much higher than the T_g of PS. This result is consistent with our previous observation that self-folding occurs when the surface of the hinge reaches ≈ 120 °C under the exposure of IR light [7].

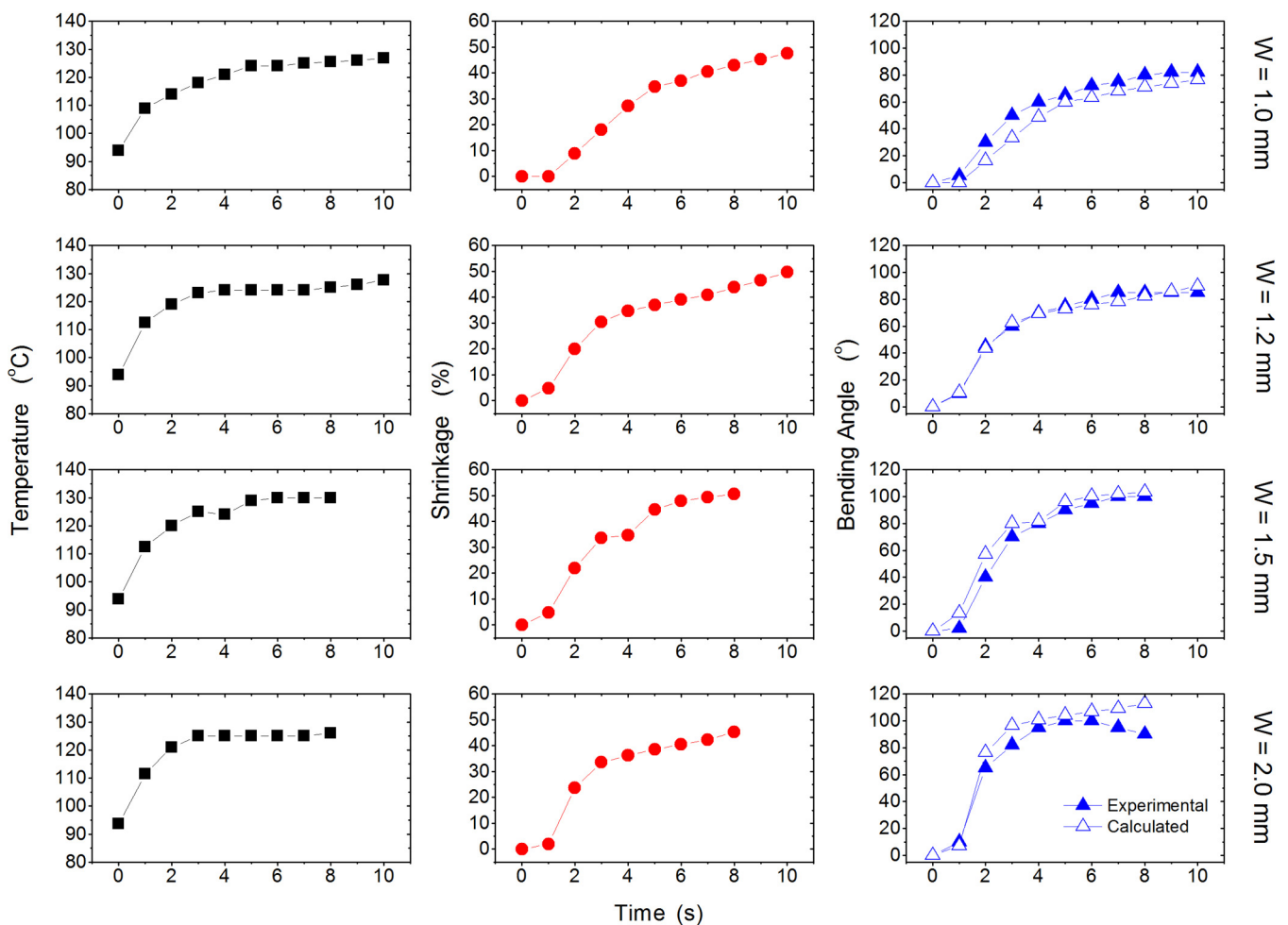


FIG. 7. (Color online) Left column: Temperature of the inked hinge (solid squares) measured by an IR camera during IR exposure. Middle column: Thermal shrinkages of the PS sheets on the top surface (solid circles) derived from the temperature temporal profile. Right column: Experimental bending angle change with time (solid triangles) compared with bending angle calculated based on the thermal shrinkage derived from the temperature temporal profile (open triangles). The different rows correspond to data with different hinge widths, W , equal to (from top to bottom) 1.0, 1.2, 1.5, and 2.0 mm.

Moreover, the dependency of T_0 in Eq. (4) on heating rate is analogous to the relationship between T_g and the heating rate in a traditional differential scanning calorimetry (DSC) measurement [19]. The data in Fig. 6(b) are in rough agreement with the reported variation of T_g on the heating and cooling rate reported for PS (dashed line). This result is remarkable considering the simplicity of the measurements and the nature of the experiment; i.e., performing macroscopic measurements and extracting molecular information.

To determine α_B as a function of time, we first measured the temperature of the hinge as a function of both time and hinge width (left column of Fig. 7). Based on the temperature profile, we calculated α_B by combining the nonisothermal and isothermal shrinkage models [i.e., Eqs. (3) and (4), respectively] to predict S as a function of time in Eq. (2) (the simple geometric model of folding, middle column of Fig. 7). The results agree very well with the measured α_B values for different hinge widths (cf. the right column of Fig. 7). The results in Fig. 7 suggest that measuring the surface temperature of the prestrained PS sheet at the hinge as a function of time provides sufficient information to calculate the folding angle as a function of time if the macroscopic strain relaxation properties of the polymer sheet are known. The model also confirms that wider hinges fold faster and lead to larger degrees of folding since there is greater absolute strain relaxation at the hinge.

In addition to modeling dynamics of sheet folding, we found that the macroscopic strain relaxation measurements (cf. Fig. 5) provide additional information about folding. Our previous work showed that folding occurs when the hinge reaches a temperature of $\approx 120^\circ\text{C}$, which is well above the T_g of the sheet. We sought to understand this observation by analyzing the k_t values from the data in Fig. 5.

In analogy to activation energy in chemical reactions, the natural logarithm of the shrinkage constant (k_t) in Eq. (3), derived from isothermal shrinkage experiments (cf. Fig. 5), is inversely proportional to the temperature, as governed by the well-known Arrhenius equation. The activation energy of thermal shrinkage can then be obtained from the slope of $\ln(k_t)$ vs $1/T$. From the plot in Fig. 8(a), the slope for the temperature range between 100 and 110°C is much larger than that for the range between 130 and 160°C . As a result, the activation energy of thermal shrinkage of the PS sheets (≈ 253 kJ/mol) at 100 – 110°C is much higher than that (≈ 43 kJ/mol) at 130 – 160°C with a transition region ≈ 115 – 120°C . Thus, significant shrinkage occurs at elevated temperatures due to low activation energy. This result helps explain why the hinges in a self-folding sample need to reach 115 – 120°C to achieve self-folding. Self-folding relies on rapid and local delivery of energy to the hinge and therefore requires folding to occur quickly before the heat dissipates outside the hinge region. The critical temperature ≈ 115 – 120°C derived from the present study is consistent with our previous observations [7]. Our estimations for the activation energy of thermal shrinkage are within the reported range of activation energies for oriented PS in the literature [12], which range from 84 [21] to 502 kJ/mol [22].

The geometric model predicts that the maximum bending angle depends on the shrinkage of the polymer. Figure 8(b) shows the relationship between the ultimate shrinkage (S_∞) and temperature under isothermal conditions taken from Fig. 5. The manufacturing process defines the maximum shrinkage, whereas the ultimate shrinkage is the maximum value of S for a given set of conditions achieved on the time scale of our experiments. The ultimate shrinkage (S_∞) increases with increasing temperature and this trend can be fit

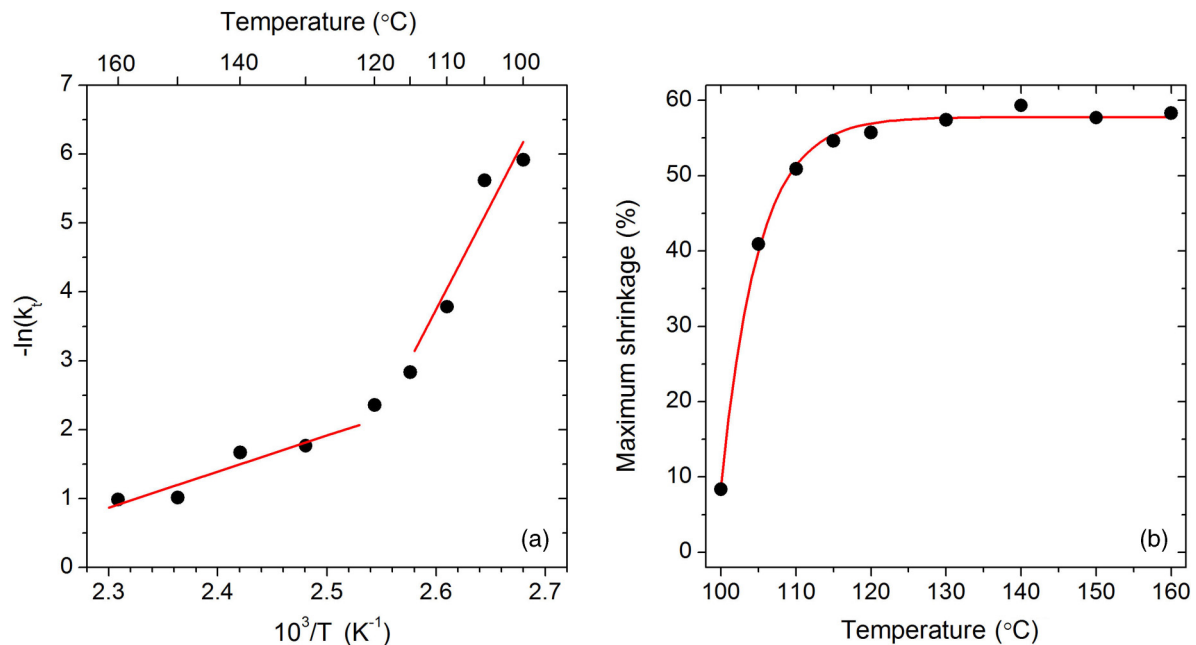


FIG. 8. (Color online) (a) Dependence of the shrinkage constant (k_t) on temperature during isothermal heating. The values of k_t have been obtained by fitting the data in Fig. 5 to Eq. (3). The slopes give activation energy for thermal shrinkage of PS above and below the critical temperature required for self-folding. (b) Maximum shrinkage (S_∞) of PS sheets as a function of temperature during isothermal heating. The line is the best fit to the experimental data using Eq. (5).

by Eq. (5):

$$S_{\infty} = S_m[1 - e^{-K_T(T-T^*)}]. \quad (5)$$

In Eq. (5), S_m , K_T , and T^* are the fitting parameters with the values of 57.8%, $0.204 \text{ }^{\circ}\text{C}^{-1}$, and $99.2 \text{ }^{\circ}\text{C}$, respectively. Each of these parameters has some physical significance. The value of S_m from the fit is consistent with the maximum shrinkage expected for the prestrained PS sheets ($\approx 55\%$). S_{∞} only reaches this maximum value when the temperature is at least $115\text{--}120 \text{ }^{\circ}\text{C}$ within the time scale of our experiments, which is consistent with the analysis given earlier. The reference temperature (T^*) in Eq. (5) ($99.2 \text{ }^{\circ}\text{C}$) is very close to the T_g ($\approx 103 \text{ }^{\circ}\text{C}$) of PS, as measured by DSC [7]. The reference temperature can be understood as a threshold temperature above which the PS sheet must be heated before any notable shrinkage occurs. It is therefore intuitive that this value should be close to the T_g of the PS sheet. Finally, based on the value of K_T , S_{∞} is a strong function of temperature in the range between 100 and $120 \text{ }^{\circ}\text{C}$, which is also in agreement with intuition. Taken in sum, Fig. 8 shows why $120 \text{ }^{\circ}\text{C}$ is a threshold temperature that the hinge must reach to achieve the maximum folding angle.

The results of this paper suggest that the temporal temperature of the hinge is an important predictor of folding behavior and that the model here works best when heat is delivered rapidly to the hinges. In a subsequent work we show that folding takes place when the power intensity of the incoming light is $>1 \text{ W/cm}^2$, which is the value used here, although some of that energy is lost via thermal dissipation. We evaluate the amount of heat dissipated from the edges of the inked regions into the polymer sheet to be $\approx 60 \text{ mW}$ and the heat loss from the hinge to the air to be $\approx 350 \text{ mW/cm}^2$ [23].

IV. CONCLUSIONS

This work offers a simple geometric model that successfully predicts the kinetics of self-folding of prestrained polymer (i.e., PS) sheets as a function of hinge width when the heat is delivered rapidly and locally to the hinge. To develop the model, we measured the thermal shrinkage of these prestrained PS sheets as a function of time under two separate thermal conditions: (1) constant temperature, and (2) constant heating rate. We mathematically fit the resulting macroscopic shrinkage behaviors of these PS sheets and used the resulting information to predict the folding dynamics using a simple geometric model. The appeal of the model is that it accurately predicts the angle of self-folding vs time by knowing only the temperature of the hinge with time and without the need for finite element analysis or measurement of the intrinsic thermomechanical properties of the polymer. The model

explains and predicts why wider hinges produce larger folding angles with all other conditions held constant.

There are several important findings from the study relevant to thermal shrinkage of prestrained polymer sheets. (1) The increase of shrinkage with time at constant temperature or at constant heating rate follows a relationship typically found in the first-order reaction kinetics. (2) Significant shrinkage takes place only at elevated temperatures ($>115 \text{ }^{\circ}\text{C}$ for the prestrained PS sheets investigated here) due to the lowered activation energy of thermal shrinkage. This result is consistent with the fact that self-folding in prestrained PS sheets occurs at temperatures $115\text{--}120 \text{ }^{\circ}\text{C}$, which is well above the T_g of PS. These high temperatures ensure that the polymer folds rapidly. (3) The dependence of the onset temperatures on heating rates follows the variation of T_g with heating rates reported earlier by others; thus extrapolation of T_0 to a zero heating rate provides an estimate of T_g .

Experiments involving self-folding help elucidate the heat management and strain relaxation in prestrained polymer sheets heated above the glass transition of the polymer. To that end, we established that the folding follows a two-step process. Specifically, sheet relaxation commences with nonisothermal shrinkage that takes place under constant heating rates. Shortly thereafter (2 to 3 s in our case), isothermal shrinkage ensues, which relaxes the strain further.

In spite of its simplicity, the model successfully predicts the folding angles measured experimentally as a function of the hinge width and time. Although we focused on predicting the effect of hinge width on self-folding, the model may also be useful for predicting the effect of thickness and degree of prestrain. In developing the model, we assumed that the strain profile across the film changes linearly with thickness and that the thickness is uniform. As we will show elsewhere [14], a more complex model involving temperature- and frequency-dependent dynamic mechanical measurement incorporated into a finite element analysis can be built to offer a more detailed insight into the strain relaxation through the film. Overall, this work offers a set of necessary guidelines for localized heat management in prestrained polymer sheets that facilitates preprogrammed sheet folding by a desired angle.

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