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Elasticity in drift-wave–zonal-flow turbulence

Z. B. Guo,[1,*] P. H. Diamond,[1,2] Y. Kosuga,[3] and Ö. D. Gürcan[4]
[1]*WCI Center for Fusion Theory, NFRI, Daejeon 305-333, South Korea*
[2]*CMTFO and CASS, University of California, San Diego, La Jolla, California 92093, USA*
[3]*IAS and RIAM, Kyushu University, Kasuga 816-8580, Japan*
[4]*LPP, Ecole Polytechnique, CNRS, Palaiseau Cedex 91128, France*
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We present a theory of turbulent elasticity, a property of drift-wave–zonal-flow (DW-ZF) turbulence, which follows from the time delay in the response of DWs to ZF shears. An emergent dimensionless parameter $|\langle v\rangle'|/\Delta\omega_k$ is found to be a measure of the degree of Fickian flux-gradient relation breaking, where $|\langle v\rangle'|$ is the ZF shearing rate and $\Delta\omega_k$ is the turbulence decorrelation rate. For $|\langle v\rangle'|/\Delta\omega_k > 1$, we show that the ZF evolution equation is converted from a diffusion equation, usually assumed, to a telegraph equation, i.e., the turbulent momentum transport changes from a diffusive process to wavelike propagation. This scenario corresponds to a state very close to the marginal instability of the DW-ZF system, e.g., the Dimits shift regime. The frequency of the ZF wave is $\Omega_{\rm ZF} = \pm\gamma_d^{1/2}\gamma_{\rm modu}^{1/2}$, where γ_d is the ZF friction coefficient and $\gamma_{\rm modu}$ is the net ZF growth rate for the case of the Fickian flux-gradient relation. This insight provides a natural framework for understanding temporally periodic ZF structures in the Dimits shift regime and in the transition from low confined mode to high confined mode in confined plasmas.

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Spatiotemporal pattern formation is ubiquitous in turbulence [1]. The study of flow patterns propagating in turbulence dates back to Kelvin [2], who hypothesized that a mesoscale flow pattern can resemble a wave, where turbulence acts as the elastic force. This is an example of a second sound phenomenon and an intuitive precursor to turbulent elasticity. In magnetized plasmas, the self-organization of zonal flows from the bath of drift waves is well known. Though considerable effort has been devoted to the study of zonal-flow (ZF) generation mechanisms [3–5], most studies assume an instantaneous response of turbulent momentum flux to ZF gradient and do not address any time-history dependence [6]. More recently, a spatially regular ZF pattern, the ZF staircase, was discovered [7]. An analogy with jam formation in traffic flow [8] was proposed as an explanation, but theoretical study of the ZF temporal response is still underdeveloped. There is accumulated experimental evidence [9,10] pointing to the appearance of a temporally periodic ZF structure, i.e., especially ZF propagation and limit-cycle oscillations during the process of the transition from the low confined mode to the high confined mode (LH transition) in the edge regions of confined plasmas. These facts show that the frequently employed Fickian turbulent transport model is incomplete. A general and enlightening way to understand the wavelike dynamics of mesoscale zonal structures is by introducing the concept of turbulent elasticity [11]. Just as for the elastic property of ordinary materials, turbulence can also exhibit elastic behavior when responding to a large-scale deformation (e.g., ZF). The critical questions are what the physics foundation of turbulent elasticity is and when elastic behavior can appear and how strong it is.

In this Rapid Communication we present a theory of elastic dynamics in the drift-wave (DW)–ZF system. Central to turbulent elasticity is the role of a finite delay time in the response of the zonal flow to the Reynolds force drive. In the wave turbulence picture, the turbulent relaxation of the DW-ZF system is driven by both the local DW-DW interaction and the nonlocal DW-ZF interaction, which can be understood as direct and indirect collision processes between DW packets. Thus, the delay time is necessarily set by the turbulence decorrelation time and the ZF shearing time. For $|\langle v\rangle'|/\Delta\omega_k < 1$, the delay time is determined mainly by the local DW-DW scattering process and the back-of-the-envelope estimate of the turbulent momentum flux (i.e., the Fickian flux) works. However, for $|\langle v\rangle'|/\Delta\omega_k > 1$ (which tends to occur when the system is very close to its marginal instability, e.g., the Dimits shift regime [12]), the strength of the nonlocal interaction exceeds that of the local interaction, so the DW-DW collision is mainly mediated by the ZF and, correspondingly, the delay time is set by the ZF shearing time $|\langle v\rangle'|^{-1}$. In this scenario, as is shown below, the so-called mean free path of the DW packet will be longer than the characteristic width of the ZF band, so the Fickian flux-gradient relation fails, i.e., $|\langle v\rangle'|/\Delta\omega_k$ is a measure of the degree of Fickian flux-gradient relation breaking. Then the ZF evolution equation is converted from a diffusion equation to a telegraph equation. The above-mentioned processes can be easily extended to other two-dimensional wave turbulence systems such as Rossby wave–zonal-flow turbulence. Turbulent elasticity is an important idea in ZF research and it expands the understanding of turbulence–self-organization phenomena, such as how the patterns (e.g., ZF shear layers) form in space and how they evolve in time. The most visible applications include, but are not limited to, wavelike propagation of ZF, superdiffusive spreading of turbulence intensity, and limit-cycle oscillation among ZF and turbulence intensities. In this Rapid Communication we first investigate how the structure of ZF evolution equation changes with the introduction of time delay between turbulent momentum flux and the ZF gradient. Since the structure of the

*guozhipku@gmail.com

delay time is crucial to understanding the physical essence of turbulent elasticity, we also give a heuristic discussion on the scaling of the delay Time.

Zonal-flow wave induced by turbulent elasticity. The nonzero response time is a clue that the Fickian flux-gradient relation (which assumes an instant response) for the ZF is incomplete, so we must deal with evolution equations for momentum and momentum flux simultaneously. The constitutive equations of the DW-ZF system are the momentum balance equation for ZF

$$\frac{\partial}{\partial t}\langle v\rangle = -\frac{\partial}{\partial x}\Pi - \gamma_d\langle v\rangle \quad (1)$$

and the wave kinetic equation [4]

$$\frac{\partial}{\partial t}N_k + v_{g,k}\frac{\partial}{\partial x}N_k - k_y\langle v\rangle'\frac{\partial}{\partial k_x}N_k = -\gamma_{N_k}(N_k - N_{0,k}), \quad (2)$$

where $\Pi = \langle\tilde{v}_x\tilde{v}_y\rangle$ is the poloidally averaged turbulent momentum flux, $\tilde{v}_x$ and $\tilde{v}_y$ are velocity fluctuations in the radial and poloidal directions, and γ_d is the friction coefficient for ZF. Here $v_{g,k} = \partial\omega_k^l/\partial k_x$ is the linear group velocity of the DW, $N_k = E_k/\omega_k^l = (1 + k_\perp^2\rho_s^2)^2|\phi_k|^2/2\omega_*$ is the wave action density, E_k is the wave energy density, $\omega_k^l = \omega_*/(1 + k_\perp^2\rho_s^2)$ is the linear DW frequency, ϕ_k is electrostatic potential, ω_* is the diamagnetic drift frequency, and ρ_s is the ion-sound Larmor radius. Equation (2) includes relaxation modeled by a Krook operator, i.e., $-\gamma_{N_k}(N_k - N_{0,k})$. Here $\gamma_{N_k}^{-1}$ is the relaxation time of the wave action density and $N_{0,k}$ is the wave action density at the equilibrium state. The wave action density is equivalent to the potential enstrophy density, so this collision term also accounts for the forward cascade of the potential enstrophy. Since the forward potential enstrophy cascade is a consequence of potential vorticity mixing [13], γ_{N_k} can also be interpreted as the rate of local potential vorticity mixing.

Multiplying by $k_y v_{g,k}$ on both sides of Eq. (2) yields the evolution equation for the wave momentum flux

$$\frac{\partial}{\partial t}\Pi + \frac{\partial}{\partial x}\Gamma_\Pi + \alpha\frac{\partial}{\partial x}\langle v\rangle = -\gamma_N(\Pi - \Pi_0), \quad (3)$$

where $\Pi = \sum_k \Pi_k = \sum_k k_y v_{g,k} N_k = \langle\tilde{v}_x\tilde{v}_y\rangle$ is the total wave momentum flux. Since the momentum of the electrostatic field is zero, Π is also the turbulent flow (i.e., nonresonant particles in kinetic picture) momentum flux in Eq. (1) [14,15]. This follows since for fluidlike dynamics, the total momentum equals the nonresonant particle momentum, which equals the wave momentum. Here $\gamma_N \equiv \sum_k \gamma_{N,k}(N_k - N_{0,k})/\sum_k(N_k - N_{0,k})$ accounts for the characteristic response time of DW turbulence. Then $\Gamma_\Pi = \Gamma_{\Pi,l} = \sum_k k_y v_{g,k} v_{g,k} N_k$ is the transport of wave momentum flux. Without the first two terms on the left-hand side, Eq. (3) reduces to the familiar Fickian relation $\tilde{\Pi} = \Pi - \Pi_0 = -\alpha/\gamma_N\partial_x\langle v\rangle$. Here

$$\alpha/\gamma_N = \sum_k \frac{2\omega_* k_x k_y^2\rho_s^2}{(1 + k_\perp^2\rho_s^2)^2}\frac{\partial N_k}{\partial k_x}\Big/\gamma_N < 0$$

is the negative viscosity, which describes local growth of the ZF through modulational instability.

Here Γ_Π may be thought of as a flux of momentum flux of the DW gas. The nonzero divergence of Γ_Π is critical in exciting zonal flow, as a second sound wave. A consistent derivation of the momentum flux requires the proper closure of Eq. (3), i.e., we need to find the relation between Γ_Π and Π. In general, the flux-gradient relation can be expressed as

$$\Gamma_\Pi = -\int^x dx'\mathcal{K}(x,x')\frac{\partial}{\partial x'}\Pi(x'), \quad (4)$$

where $\mathcal{K}(x,x')$ is a kernel function, representing the generalized diffusivity. The form of Γ_Π is sensitive to whether the transport of Π is local or nonlocal. If the mean free path $l_{\rm MFP}$ of the DW is much shorter than the characteristic scale length L_Π of Π, then the transport is driven via strong local DW-DW scattering. This scenario corresponds to a diffusive flux and $\mathcal{K}(x,x')$ can be taken to be a delta function, i.e., $\mathcal{K}(x,x') = D\delta(x - x')$, where D is the Fickian diffusion coefficient. From Eq. (4) the usual flux-gradient relation $\Gamma_\Pi = -D\partial_x\Pi$ is recovered. In the weak local scattering scenario, DW packets can propagate a longer distance, so $l_{\rm MFP}$ is comparable to L_Π and the Fickian diffusion ansatz fails. In other words, we need a nonlocal integral kernel in the flux equation. In this Rapid Communication we focus on in the flux-gradient relation when $l_{\rm MFP} \gg L_\Pi$, which is equivalent to the nonlocal interaction exceeding the local interaction. Given the continuity of the content, we delay the discussion of the physical meaning of this limit to the next section. In this scenario, each scatterer position makes an equal contribution to $\mathcal{K}(x,x')$ and hence $\mathcal{K}(x,x')$ can be expressed as a step function, i.e., $\mathcal{K}(x,x') = v_c\Theta(x - x')$, with $\Theta = 1$ for $x \geqslant x'$ and $\Theta = 0$ for $x < x'$. Here v_c is the characteristic transport speed. Putting the step function into Eq. (4), Γ_Π is readily derived as

$$\Gamma_\Pi = -v_c\Pi, \quad (5)$$

which gives a convective flux-gradient relation. A more general method of calculating Γ_Π is via flux-limited diffusion theory [16]. Similar to heat transfer in radiation hydrodynamics [17], the limited-flux–gradient relation can be cast in the phenomenological form [18]

$$\Gamma_\Pi = -\frac{D\nabla\Pi}{\sqrt{1 + (l_{\rm MFP}\nabla\ln\Pi)^2}}, \quad (6)$$

where $1/\sqrt{1 + (l_{\rm MFP}\nabla\ln\Pi)^2}$ is a flux-limiting factor. For small values of mean free path, Eq. (6) reduces to the diffusion form. In the strong nonlocality scenario, the factor is approximated as

$$1/\sqrt{1 + (l_{\rm MFP}\nabla\ln\Pi)^2} \simeq \mathrm{sgn}(\nabla\ln\Pi)1/(l_{\rm MFP}\nabla\ln\Pi).$$

Putting it into Eq. (6), one obtains $\Gamma_\Pi \simeq -\mathrm{sgn}(\nabla\ln\Pi)D/l_{\rm MFP}\Pi = -v_c\Pi$ [$D = \mathrm{sgn}(\nabla\ln\Pi)v_c l_{\rm MFP}$ is used]. This result is the same as Eq. (5) and hence our choice of the step function Θ as the integral kernel in Eq. (4) appears proper.

From Eqs. (3) and (5) the evolution equation for the perturbed momentum flux is

$$\frac{\partial}{\partial t}\tilde{\Pi} - v_c\frac{\partial}{\partial x}\tilde{\Pi} + \alpha\frac{\partial}{\partial x}\langle v\rangle = -\gamma_N\tilde{\Pi}. \quad (7)$$

Thus, combining Eqs. (1) and (7) yields at last a telegraph evolution equation for the ZF

$$\frac{\partial^2}{\partial t^2}\langle v\rangle + \left(\gamma_N + \gamma_d - v_c\frac{\partial}{\partial x}\right)\frac{\partial}{\partial t}\langle v\rangle + \left(-\gamma_d v_c\frac{\partial}{\partial x} - \alpha\frac{\partial^2}{\partial x^2} + \gamma_N\gamma_d\right)\langle v\rangle = 0, \tag{8}$$

which immediately suggests wavelike solutions. In the limit of short delay time, when the term with the second time derivative in Eq. (8) is negligible, that equation reduces to the familiar parabolic momentum equation. We linearize Eq. (8) when the deviation of N_k from its equilibrium $N_{0,k}$ is small. To obtain the dispersion relation of the ZF wave, using transformations $\partial_t \to -i\Omega_{\rm ZF} + \gamma_{\rm ZF}$ and $\partial_x \to iq$ (q is the radial wave number of the ZF), the real and imaginary parts of Eq. (8) are readily obtained as

$$-\Omega_{\rm ZF}^2 + \gamma_{\rm ZF}^2 + \gamma_{\rm ZF}(\gamma_N + \gamma_d) - q\Omega_{\rm ZF}v_c + \alpha_0 q^2 + \gamma_N\gamma_d = 0, \tag{9a}$$

$$(\gamma_N + \gamma_d)\Omega_{\rm ZF} + q\gamma_d v_c + 2\Omega_{\rm ZF}\gamma_{\rm ZF} + qv_c\gamma_{\rm ZF} = 0. \tag{9b}$$

Here

$$\alpha_0 = \sum_k \frac{2\omega_* k_x k_y^2\rho_s^2}{\left(1 + k_\perp^2\rho_s^2\right)^2}\frac{\partial N_{0,k}}{\partial k_x}.$$

Equation (9b) shows that $v_c \neq 0$ implies the existence of a steady ZF wave solution. It should be emphasized that Eqs. (9a) and (9b) are obtained in the laboratory frame, therefore their wave solutions are also in the laboratory frame; the wave is *not* a propagating localized pulse. Since we seek a stationary solution, by setting $\gamma_{\rm ZF} = 0$ the dispersion relation follows as

$$\Omega_{\rm ZF} \simeq \pm\left(\frac{\gamma_d|\alpha_0|q^2}{\gamma_N} - \gamma_d^2\right)^{1/2}. \tag{10}$$

A necessary but not sufficient condition for the existence of a ZF wave is $|\alpha_0|q^2/\gamma_N > \gamma_d$, which states that the growth rate $|\alpha_0|q^2/\gamma_N$ of the modulational instability must overcome the frictional damping of the ZF. Equation (10) also gives a critical ZF wave number $q_c = (\gamma_d\gamma_N/|\alpha_0|)^{1/2}$ and $|q| \geqslant q_c$ is a necessary condition for the existence of a ZF wave. In the large-wave-number regime $|q| \gg q_c$, one has $\Omega_{\rm ZF} \simeq \pm(\gamma_d|\alpha_0|/\gamma_N)^{1/2}q$, which is just the second sound dispersion relation with $(\gamma_d|\alpha_0|/\gamma_N)^{1/2}$ being the phase velocity. We can rewrite $\Omega_{\rm ZF}$ as the geometric mean of $\gamma_{\rm modu}$ and γ_d, i.e., $\Omega_{\rm ZF} = \sqrt{\gamma_{\rm modu}\gamma_d}$, where $\gamma_{\rm modu} = |\alpha_0|q^2/\gamma_N - \gamma_d$ is the net ZF growth rate for the case of the Fickian flux-gradient relation.

Here we give a physical picture of the propagation mechanism for the ZF wave (Fig. 1). For the initial ZF pattern, the response of the ZF friction force is transient, but the divergence of turbulent momentum flux (i.e., Reynolds force) at this moment is zero because of the delayed response of the DW turbulence to the ZF pattern. Thus, the amplitude of the ZF pattern tends toward the value zero. In this sense, we can view the friction force as a kind of restoring force. As it approaches the value zero, the restoring force becomes weaker and weaker, but the Reynolds force gradually increases, which drives the ZF pattern away from zero. Thus, the Reynolds force acts as kind of repulsive force. Once the ZF reaches zero, the restoring force disappears, but the repulsive force is maximal. In other words, the momentum carried by the initial ZF pattern is totally converted into pseudomomentum carried by the DW packets. Like classical molecules, these wave packets can scatter into the no-flow region by mutual collisions, i.e., nonlinear interactions that mix potential vorticity. Because of this spatial mixing of the momentum, a new ZF pattern can be created in the no-flow region. When the new ZF pattern attains its peak value, the restoring force will again assert itself. By repeating this process, a sequence of spatiotemporal structures will occur. The characteristic time scales of the restoring and repulsive forces are γ_d^{-1} and $\gamma_{\rm modu}^{-1}$, respectively, so one might expect that the period of the ZF wave will scale as $\sim\gamma_d^{-\beta}\gamma_{\rm modu}^{-\delta}$, with $\beta + \delta = 1$. This heuristic argument is consistent with our analytical result (10), where $\beta = \delta = 1/2$. The propagation direction of the ZF wave is affected by the boundary conditions and mean flow profiles. For example, at the edge of tokamaks, there exist strong mean shear flows that can reflect an outward propagating ZF wave back inward. Hence one can expect that most ZF waves propagate inward from the edge. Indeed, two-way pulse propagation, reflection from the edge, and the ultimate predominance of the inward propagating population have been observed in a recent experiment [10].

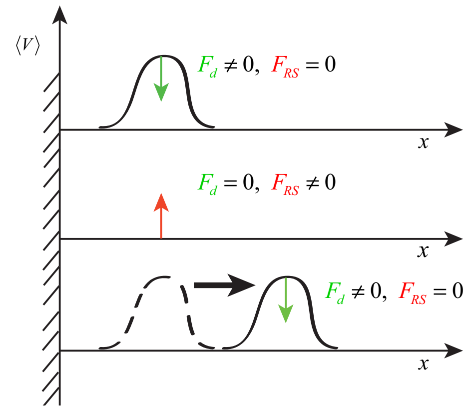

FIG. 1. (Color online) Sketch of the propagation mechanism for the ZF wave. Here F_d is the restoring force (friction force), $F_{\rm RS}$ is the repulsive force (Reynolds force), and the black arrow denotes the direction of the DW momentum flux.

Structure of the delay time. From the above discussion, we know that the delay time τ_N ($\tau_N = \gamma_N^{-1}$) is a fundamental quantity and measures the elastic strength of DW turbulence. Also, a more complete understanding of the ZF wave requires knowing the structure of τ_N. Thus, we here seek a deeper understanding and characterization of τ_N. This is an important element of the theory of turbulent elasticity. Physically, τ_N originates from the finite collision time during the turbulent relaxation of the DW-ZF system. There are two types of collision processes: direct DW-DW collisions, which are local interactions, and indirect DW-DW collisions, i.e., DW-DW scattering mediated by ZF, which are nonlocal interactions.

In this work, we mainly focus on wave momentum transport in a quasistationary, near marginal state, where both the growth rate and the amplitude of the DW are small. In this respect, the collision via the direct DW-DW interaction [Fig. 2(a)] is weaker than that via the indirect DW-DW interaction

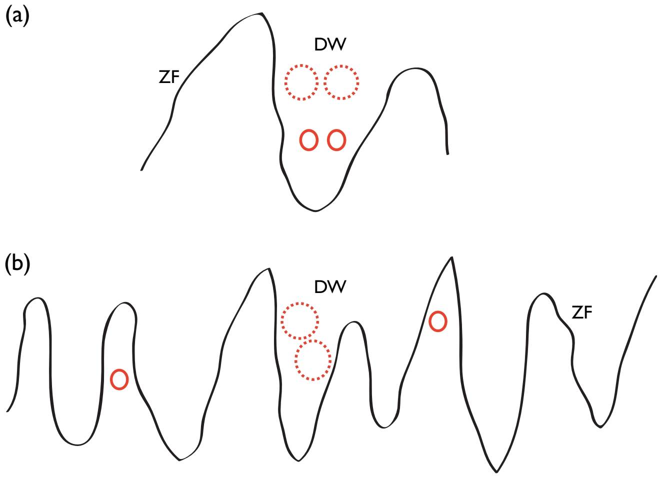

FIG. 2. (Color online) (a) The direct DW-DW collision is dominant: $\tau_N \simeq \Delta\omega_k^{-1}$. (b) The indirect DW-DW collision is dominant: $\tau_N \simeq |\langle v\rangle'|^{-1}$.

[Fig. 2(b)], i.e., $\Delta\omega_k < |\langle v\rangle'|$, where $\Delta\omega$ is proportional to the DW intensity. When the indirect collision is dominant over the direct collision, one can expect that the delay time is determined mainly by the ZF shearing time, i.e.,

$$\tau_N \simeq |\langle v\rangle'|^{-1}. \qquad (11)$$

With this scaling, one has $l_{\rm MFP}|q| = |v_g/\langle v\rangle||\tau_N\langle v\rangle'| \simeq |v_g/\langle v\rangle|$, with $l_{\rm MFP} = v_g\tau_N$, $|q| = L_\Pi^{-1} = |\langle v\rangle'/\langle v\rangle|$, and $v_g = \sum_k v_{g,k}N_k/\sum_k N_k$ the characteristic group velocity of the DW packet. In general, v_g is much larger than $|\langle v\rangle|$ in DW-ZF turbulence and hence the long excursion condition $l_{\rm MFP} \gg |q|^{-1}$ is satisfied, i.e., a DW packet can retain its identity while propagating through many ZF bands [Fig. 2(b)] and the Fickian flux-gradient fails. A physical setting of this scenario is the Dimits shift regime [12], where the DW turbulence is nearly quenched by the ZFs and $\Delta\omega_k < |\langle v\rangle'|$ tends to occur.

With Eqs. (9b) and (11), v_c scales as $v_c \sim |\langle v\rangle'|^{1/2}\gamma_d^{-1/2}\varepsilon^{1/2}$, which can be interpreted as the scaling of turbulence intensity spreading associated with the ZF wave. With Eq. (11), the scaling of $\Omega_{\rm ZF}$ is derived as $\Omega_{\rm ZF} \sim |\langle v\rangle'|^{-1/2}\gamma_d^{1/2}\varepsilon^{1/2}q$. Since turbulent relaxation occurs by spatial scattering of the DW packets, τ_N is necessarily set by the relative dispersion of the DW packets. Thus, the structure of τ_N can also be analyzed in a more rigorous way, i.e., studying the relative dispersion of two DW packets by using the two-point correlation function of the wave action density [19]. It is straightforward to show that the more rigorous way gives the same scaling with Eq. (11); details are beyond the scope of the present paper.

Turbulent elasticity, as proposed in this Rapid Communication, may be tested in the following three ways. (i) A ZF wave is detected. Wavelike propagation of a ZF pattern provides a dissipationless means for momentum transport. In fact, there is numerical and experimental evidence for the inward radial propagation of ZF during LH transition experiments [9,10]. (ii) Wavelike turbulence spreading is observed. This type of turbulence spreading can be very important near the marginal state, as the characteristic scale (i.e., $l_{\rm MFP}$) of elastic spreading is much larger than that of viscous spreading (i.e., $l_{\rm eddy}$) [20]. (iii) The turbulent elasticity can alter the dynamical structure of the usual predator-prey model [4], so access to an alternative limit cycle solution become possible. Simulation tests of this theory seem most viable at least initially. Such simulation tests could focus on (a) identifying and quantifying fast turbulence spreading, where "spreading" refers to the expansion of an ensemble of coupled turbulent eddies and zonal flows and "fast" refers to a superdiffusive process, and (b) identifying zonal waves in the laboratory frame and demonstrating consistency with the predicted dispersion characteristics.

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