Dielectric function of a collisional plasma for arbitrary ionic charge

H. B. Nersisyan,[1,2,*] M. E. Veysman,[3] N. E. Andreev,[3,4] and H. H. Matevosyan[1]
[1]*Theoretical Physics Division, Institute of Radiophysics and Electronics, 0203 Ashtarak, Armenia*
[2]*Center of Strong Fields Physics, Yerevan State University, 0025 Yerevan, Armenia*
[3]*Joint Institute for High Temperatures RAS, Moscow 125412, Russia*
[4]*Moscow Institute of Physics and Technology (State University), Moscow 113303, Russia*
(Received 28 November 2013; published 6 March 2014)

A simple model for the dielectric function of a completely ionized plasma with an arbitrary ionic charge that is valid for long-wavelength high-frequency perturbations is derived using an approximate solution of a linearized Fokker-Planck kinetic equation for electrons with a Landau collision integral. The model accounts for both the electron-ion collisions and the collisions of the subthermal (cold) electrons with thermal ones. The relative contribution of the latter collisions to the dielectric function is treated phenomenologically, introducing some parameter $\varkappa$ that is chosen in such a way as to get a well-known expression for stationary electric conductivity in the low-frequency region and fulfill the requirement of a vanishing contribution of electron-electron collisions in the high-frequency region. This procedure ensures the applicability of our model in a wide range of plasma parameters as well as the frequency of the electromagnetic radiation. Unlike the interpolation formula proposed earlier by Brantov *et al*. [Brantov *et al*., JETP **106**, 983 (2008)], our model fulfills the Kramers-Kronig relations and permits a generalization for the cases of degenerate and strongly coupled plasmas. With this in mind, a generalization of the well-known Lee-More model [Y. T. Lee and R. M. More, Phys. Fluids **27**, 1273 (1984)] for stationary conductivity and its extension to dynamical conductivity [O. F. Kostenko and N. E. Andreev, GSI Annual Report No. GSI-2008-2, 2008 (unpublished), p. 44] is proposed for the case of plasmas with arbitrary ionic charge.

DOI: 10.1103/PhysRevE.89.033102 PACS number(s): 52.25.Dg, 52.25.Mq, 52.25.Fi, 52.38.−r

I. INTRODUCTION

The problem of interactions of intense laser pulses with solids and plasmas continues to be the subject of intense experimental and theoretical research. These interactions are associated with both the fundamental aspects of the behavior of matter in ultrastrong laser fields and various applications such as fast ignition [1], the development of new sources of x-ray radiation and warm dense matter production [2], particle acceleration [3], and the laser generation of shock waves. In most part of these studies the high-power laser pulse ionizes the matter, so one eventually has to deal with a partially or fully ionized plasma. In the past few decades much effort has been devoted to investigating the various aspects of laser-plasma interactions (see, e.g., Refs. [4–7]). Currently, various models of these interactions are widely discussed (see, e.g., Refs. [8–13] and references therein). The key quantity that characterizes laser-matter interactions as well as the optical properties of matter is the plasma dielectric function (permittivity) ε, which determines the electrodynamic response of the system on perturbations. Thus, the construction of theoretical models for plasma permittivity that are valid in a wide range of the plasma parameters is of fundamental and practical importance.

Plasma permittivity has been studied in detail and is well known in two limiting cases corresponding to the collisionless case based on the solution of the Vlasov kinetic equation [5–7,14] and to the strongly collisional hydrodynamic limit [15,16]. In the latter regime the ranges of applicability of the corresponding expressions for the permittivity of a collisional plasma are strongly restricted and cannot be used for arbitrary values of ω/ν_e and $k\lambda_{ei}$, where ν_e is the electron-ion collision frequency and λ_{ei} is the mean free path of electrons with respect to their collisions with ions. An important development in recent years is the weakly collisional theory proposed in Ref. [17], which extends the range of the analytical description of the permittivity for a collisional plasma compared to the collisionless case.

To obtain a qualitative description of the collisional regimes of a plasma the Bhatnagar-Gross-Krook (BGK) [18] collisional model in the kinetic equation for electrons has been widely used with or without a number-conservation procedure [7,19–24]. The appeal of this model is its simplicity, which in its original nonconserving form amounts to the replacement of $\omega \to \omega + i\nu$ in the argument of the plasma dispersion function, where ν is a model collision frequency. Furthermore, more advanced number- and energy-conserving BGK models as well as number-, momentum-, and energy-conserving BGK models have been presented in Refs. [25,26] and [27,28], respectively, which yield analytic expressions for the permittivities in terms of combinations of the plasma dispersion function. However, for a completely ionized plasma, the model permittivity within the BGK approximation and the corresponding Drude model for the transverse permittivity [7,22–24] lead to the significant deviations from the known limiting cases in the range of moderate and strong collisions [29–31]. For instance, it has been found that this model cannot reproduce the plasma permittivity in the strongly collisional hydrodynamic regime considered in Ref. [16]. A significant improvement of the theory has been achieved within the Lorentz plasma model [31–33]. However, the Lorentz plasma model cannot describe permittivity accurately in a wide range of parameters, even for a highly ionized plasma, as long as

*hrachya@irphe.am

the electron-electron collisions are neglected in this model. We also mention the model of Ref. [34] with a simplified Fokker-Planck kinetic equation, where the diffusion tensor and the friction coefficient are treated as given constants. The resulting dielectric function has been compared with the number-conserving Mermin dielectric function demonstrating that both functions are almost identical.

For the case of a plasma with a large ionic charge $Z \gg 1$, where the electron-electron collision integral is involved only in the equation for the isotropic part of the electron distribution function, the longitudinal and transverse permittivities have been obtained in Refs. [35,36] and [30], respectively. Generalization of the latter results to the case of an arbitrary ionic charge Z requires, in addition, the consideration of the electron-electron collision integral for the anisotropic part of the perturbed distribution function. This problem was considered in Ref. [37] without any constraints on the parameters under consideration. The model developed in Ref. [37] is based on the solution of a linearized kinetic equation for electrons with a Landau collision integral. In addition, the suggested method of solving the kinetic equation is valid for an arbitrary ionic charge Z, an arbitrary relation between the perturbation inhomogeneity scale length k^{-1} and the electron mean free path, and an arbitrary relation between the characteristic time scale ω^{-1}, electron collision time, and the time scale of collisionless electron motion $1/kv_{\rm th}$, where $v_{\rm th}$ is the thermal electron velocity.

However, the model proposed in Ref. [37] being accurate in a wide range of parameters is rather complicated and does not determine the permittivity in an explicit form expressed through the plasma parameters. Therefore, simplified but still accurate models for the plasma permittivity are highly desirable. Besides, the model of Ref. [37] considers the case of ideal nondegenerate plasmas only, which restricts its use for the description of laser-matter interactions in a wide range of parameters.

In the present study we propose an alternative and simplified solution of the kinetic equation for electrons with a Landau collision integral for an arbitrary charge of plasma ions. The model accounts for both electron-ion collisions and collisions of the subthermal (cold) electrons with thermal ones. As has been shown in Ref. [17], the latter collisions may contribute considerably in the common integral of collisions and one can derive an algebraic expression for the respective parts of the integral of electron-electron collisions containing, however, some free parameter. This parameter is then adjusted so as to ensure agreement of the present model with the respective expression for a stationary electric conductivity at low frequencies [37,38] and the proper behavior of high-frequency conductivity (or permittivity) at high frequencies. Moreover, the presented model permits simple extensions for the cases of degenerate and/or strongly coupled plasmas, which makes it possible to use it for the description of optical properties of plasmas in a wide range of temperatures and densities. Thus, this model represents a generalization of the well-known Lee-More model [39] for a stationary conductivity and its extension for a dynamical conductivity [40] (in the same relaxation-time approximation). It is valid for plasmas with arbitrary degeneracy and arbitrary ionic charge, where the electron-electron collisions play an essential role.

II. THEORETICAL MODEL

Within linear response approximation the evolution of small perturbations arising in a homogeneous, collisional, and unmagnetized plasma is considered below. The case of long-wavelength and high-frequency perturbations is considered for the electron component of the plasma. The dynamics of the plasma ions is neglected. More specifically, we assume that $kv_{\rm th} \ll \omega$, $k\lambda_{ei} \ll 1$, and $k\lambda_{ee} \ll 1$, where k^{-1} is the wavelength of the perturbations, ω^{-1} is the characteristic time, and λ_{ei} (λ_{ee}) is the mean free path of the electrons with respect to their collisions with ions (electrons).

The evolution of the electron component of the plasma is governed by the Fokker-Planck kinetic equation for the velocity distribution function $f(\mathbf{v},t)$ of the electrons. The distribution function of the ions is fixed and is given by $f_i(\mathbf{v},t) = \delta(\mathbf{v})$. Neglecting the spatial inhomogeneity of the electron distribution function in the case of the long-wavelength perturbations, the kinetic equation can be written as [5,6,14]

$$\frac{\partial f}{\partial t} - \frac{e}{m}\mathbf{E}\cdot\frac{\partial f}{\partial \mathbf{v}} = J[f] \equiv \frac{\partial}{\partial v_i}\left(D_{ij}\frac{\partial f}{\partial v_j} - F_i f\right), \tag{1}$$

where $J[f] = J_{ee}[f] + J_{ei}[f]$ is the collision term with the contributions of the electron-electron $J_{ee}[f]$ and electron-ion $J_{ei}[f]$ collisions, respectively, $\mathbf{E}$ is the self-consistent electric field strength, and D_{ij} and $\mathbf{F}$ are the diffusion tensor and the friction force in a velocity space, respectively.

Taking the collision term $J[f]$ in the form of Landau [5,6,14], the velocity diffusion tensor and the friction force are given by

$$D_{ij} = \frac{h}{2}\left[\frac{1}{Z}\int f(\mathbf{v}',t)g_{ij}(\mathbf{u})d\mathbf{v}' + g_{ij}(\mathbf{v})\right], \tag{2}$$

$$F_i = \frac{h}{2}\left[\frac{1}{Z}\int f(\mathbf{v}',t)\frac{\partial g_{ij}(\mathbf{u})}{\partial u_j}d\mathbf{v}' + \frac{m}{m_i}\frac{\partial g_{ij}(\mathbf{v})}{\partial v_j}\right], \tag{3}$$

where $\mathbf{u} = \mathbf{v} - \mathbf{v}'$,

$$g_{ij}(\mathbf{v}) = \frac{1}{v}\left(\delta_{ij} - \frac{v_i v_j}{v^2}\right), \tag{4}$$

$\partial g_{ij}(\mathbf{v})/\partial v_j = -2v_i/v^3$, δ_{ij} is the unit tensor of rank 3, $h = 3\sqrt{\pi/2}\nu_e v_{\rm th}^3$,

$$\nu_e = \frac{4\sqrt{2\pi}n_e Z e^4}{3(mT^3)^{1/2}}\Lambda \tag{5}$$

is the effective electron-ion collision frequency, and $v_{\rm th} = \sqrt{T/m}$. Here $-e,m,n_e$ and Ze,m_i,n_i are the electron and ion charges, masses, and equilibrium densities, respectively, T is the temperature of electron component, and Λ is the Coulomb logarithm, which is defined later. Charge neutrality of the plasma with $n_e = Zn_i$ and an arbitrary (and finite) ionic charge Z are assumed.

The first and the second terms in Eqs. (2) and (3) correspond to the electron-electron and electron-ion collisions, respectively. The last term in Eq. (3) describes the energy exchange between electrons and ions and is proportional to the small parameter $\sim m/m_i \ll 1$. This term will be neglected in the subsequent calculations. The electron-electron collisions terms in Eqs. (2) and (3) contain the inverse Z^{-1} of the ionic charge number Z. Hence, these terms vanish at the limit $Z \gg 1$ of the highly ionized ions and one arrives at the Lorentz plasma model [22] in this case, which is frequently used in hydrodynamic codes due to its simplicity [9–13].

The Lorentz model is justified only for plasma with highly ionized ions with $Z \gtrsim 10$. For plasmas with $Z < 10$ electron-electron collisions should be accounted for numerically more precise calculations, although, due to the momentum conservation (i.e., $\int \mathbf{v} J_{ee}[f] d\mathbf{v} = 0$) they do not directly contribute to the induced current density. Nevertheless, they modify the electron distribution function and thus influence the value of permittivity. A rigorous kinetic theory for the calculation of the permittivity of a plasma taking into account electron-electron collisions and nonlocal transport was proposed in Ref. [37].

In the present paper a simpler but physically motivated approach is considered, which makes it possible to derive a simple expression for the permittivity of plasmas that takes into account the contribution of electron-electron collisions and permits further generalizations for quantum plasmas and/or for strongly coupled plasmas. Unlike the interpolation formula proposed in Ref. [37], the present model fulfills Kramers-Kronig relations and permits further extension for the degenerate plasma case.

In order to derive this model we note that, in accordance with Ref. [17], the effective frequency for collisions of subthermal (cold) electrons (with velocities $v \ll v_{\rm th}$) with thermal ones (with $v \sim v_{\rm th}$) behaves as $\nu_{c,ee} \sim (v_{\rm th}/v)^3 \gg \nu_{ee}$, so it considerably exceeds the similar frequency ν_{ee} for the collisions of thermal electrons. Therefore, even in a weakly collisional plasma the cold electrons experience strong collisions with the thermal ones and may essentially contribute to the coefficients (2) and (3). With this in mind, we restrict the upper limits of the velocity integrations in Eqs. (2) and (3) by some value $v_m \lesssim v_{\rm th}$. Also, since $v \simeq v_{\rm th}$ in Eqs. (2) and (3), the tensor $g_{ij}(\mathbf{u})$ and the vector $\partial g_{ij}(\mathbf{u})/\partial u_j$ can be replaced by $g_{ij}(\mathbf{v})$ and $\partial g_{ij}(\mathbf{v})/\partial v_j$, respectively, removing them from the $\mathbf{v}'$ integrals in Eqs. (2) and (3).

Next, within linear response approach the distribution function $f(\mathbf{v}',t)$ in Eqs. (2) and (3) can be replaced by the equilibrium distribution function of the electrons $f_0(v')$ and, recalling the affirmations stated above, $f_0(v')$ can be replaced by $f_0(v') \simeq f_0(0)$. As a result, from Eqs. (2) and (3) we obtain

$$D_{ij} = \frac{h}{2}\left(1 + \frac{1}{Z_*}\right) g_{ij}(\mathbf{v}), \tag{6}$$

$$F_i = \frac{h}{2Z_*}\frac{\partial g_{ij}(\mathbf{v})}{\partial v_j}, \tag{7}$$

where $Z_* = Z/\varkappa$ with $\varkappa = \frac{4\pi}{3} v_m^3 f_0(0)$.

It is seen that the contribution of the electron-electron collisions (the terms containing the effective charge number Z_*) is not negligible in the coefficients (6) and (7). The parameter $\varkappa$ introduced above is the relative fraction of slow electrons contributing to the coefficients (6) and (7). Clearly $\varkappa \lesssim 1$, which results in $Z_* > Z$, i.e., a larger effective charge of the ions compared to Z.

To obtain an equation for perturbed distribution function one can substitute $f = f_0 + f_1$ (with $f_1 \ll f_0$) into (1) to get the equation

$$-i\omega f_{1\omega}(\mathbf{v}) - \frac{e}{m}(\mathbf{E}_\omega \cdot \mathbf{v})\frac{1}{v} f_0'(v) = J[f_{1\omega}(\mathbf{v})] \tag{8}$$

for the Fourier transform with respect to the time t of the perturbed distribution function f_1. Here $\mathbf{E}_\omega$ is the Fourier transform of the electric field and the prime indicates the derivative with respect to the argument. The equilibrium distribution function in the unperturbed state is assumed to be isotropic $f_0 = f_0(v)$.

In order to solve Eq. (8) it is convenient to introduce a new unknown and isotropic function $\Phi_\omega(v)$ via the relation

$$f_{1\omega}(\mathbf{v}) = \frac{e}{m\omega}(\mathbf{E}_\omega \cdot \mathbf{v})\Phi_\omega(v). \tag{9}$$

This relation (9) explicitly separates the isotropic [the term $\Phi_\omega(v)$] and anisotropic [the term $(\mathbf{E}_\omega \cdot \mathbf{v})$] parts of the distribution function $f_{1\omega}(\mathbf{v})$. Note that such a choice for the perturbed distribution function is stimulated by the structure of (8). Then inserting Eq. (9) into (8) and using the diffusion tensor (6) and the friction force (7) yields, after straightforward calculations, an ordinary differential equation for the unknown function $\Phi_\omega(v)$,

$$\frac{1}{\omega Z_*}\Phi_\omega'(v) + \frac{i}{hv}(v^3 + ih/\omega)\Phi_\omega(v) = -\frac{1}{h} v f_0'(v). \tag{10}$$

An expression similar to Eq. (10) was considered previously in Refs. [9–13,30], neglecting, however, the first term containing the derivative of the function $\Phi_\omega(v)$, which is justified for $Z \gg 1$. In this case the differential equation (10) is reduced to an algebraic one with a simple solution

$$\Phi_\omega^{(\rm L)}(v) = i\frac{v^2 f_0'(v)}{v^3 + ih/\omega}, \tag{11}$$

which eventually yields the Lorentz model for plasma permittivity [9–13,22,30]. For an arbitrary charge state Z of the plasma ions and for a finite parameter $\varkappa$, the solution of Eq. (10) is given by

$$\Phi_\omega(v) = \frac{Z_*\omega}{h}\int_v^\infty \exp\left[\frac{iZ_*\omega}{3h}(u^3 - v^3)\right]\left(\frac{v}{u}\right)^{Z_*} f_0'(u) u\, du. \tag{12}$$

The perturbations of the current induced in the plasma by the electric field $\mathbf{E}$ are determined by $\mathbf{j}_1 = -n_e e \int \mathbf{v} f_1(\mathbf{v},t) d\mathbf{v}$. The Fourier transform of this quantity is then given by

$$\mathbf{j}_{1\omega} = -\frac{\omega_p^2}{4\pi\omega}\int \mathbf{v}(\mathbf{E}_\omega \cdot \mathbf{v})\Phi_\omega(v) d\mathbf{v}, \tag{13}$$

where $\omega_p^2 = 4\pi n_e e^2/m$ is the plasma frequency. Using this relation one can calculate the conductivity tensor and hence the permittivity tensor of the collisional electron plasma, which can be represented in the form $\varepsilon_{ij}(\omega) = \varepsilon(\omega)\delta_{ij}$ with

$$\varepsilon(\omega) = 1 - \frac{\omega_p^2}{\omega^2} K_0(\omega),$$

$$K_0(\omega) = \frac{4\pi i}{3}\int_0^\infty \Phi_\omega(v) v^4 dv. \tag{14}$$

The obtained expression together with the distribution function (12) determines the high-frequency dielectric function of the collisional plasma for an arbitrary effective charge Z_* of the ions. The expression (14) can be further simplified if Eq. (12) is inserted into it and one performs an integration by parts. This yields

$$K_0(\omega) = \frac{i\chi_{z_2}}{\xi_\omega}\frac{8\sqrt{2}\pi}{3}v_{\rm th}^3\int_0^\infty F(1;\alpha_z;i\beta_z\xi^3)\xi^6 f_0'(\xi)d\xi, \tag{15}$$

where $f_0'(\xi)$ denotes the derivative of $f_0(\xi)$ over ξ,

$$\xi = \frac{v}{\sqrt{2}v_{\rm th}},\quad \xi_\omega = \frac{3\sqrt{\pi}}{4}\frac{\nu_e}{\omega},\quad \alpha_z = \frac{Z_*+8}{3},\quad \beta_z = \frac{Z_*}{3\xi_\omega}, \tag{16}$$

and $F(a;b;z)$ is the confluent hypergeometric function. Using the properties of the confluent hypergeometric functions (see, e.g., Ref. [41]) one can write the series expansion for F over its third argument for the case $\beta_z\xi^3 \ll 1$,

$$F(1;\alpha_z;i\beta_z\xi^3) = 1 + i\frac{\beta_z\xi^3}{\alpha_z} - \frac{\beta_z^2\xi^6}{\alpha_z(\alpha_z+1)} + \cdots, \tag{17}$$

and the asymptotic expression for F over the value of Z_*^{-1}, $Z_* \gg 1$:

$$F(1;\alpha_z;i\beta_z\xi^3) = \frac{1}{1-\widetilde{\beta}_z} + \sum_{n\geq 1}\frac{1}{Z_*^n}\frac{\widetilde{\beta}_z P_n(\widetilde{\beta}_z)}{(1-\widetilde{\beta}_z)^{2n+1}}, \tag{18}$$

where $\widetilde{\beta}_z = i\xi^3/\xi_\omega$ and $P_n(\widetilde{\beta}_z)$ are polynomials of $\widetilde{\beta}_z$ of the power n. The first three have the following values:

$$P_1 = 5\widetilde{\beta}_z - 8, \tag{19}$$

$$P_2 = 10\widetilde{\beta}_z^2 - 47\widetilde{\beta}_z + 64, \tag{20}$$

$$P_3 = -10\widetilde{\beta}_z^3 + 48\widetilde{\beta}_z^2 + 69\widetilde{\beta}_z - 512. \tag{21}$$

Considering Eq. (17), one can derive from Eq. (15) the following expression for the function $K_0(\omega)$ in the limiting case of low frequencies $\omega \ll \nu_e$:

$$K_0(\omega) = \frac{3\chi_{z_1}}{\xi_\omega^2}\langle\xi^6\rangle - \frac{2i\chi_{z_2}}{\xi_\omega}\langle\xi^3\rangle, \tag{22}$$

where $\langle\xi^n\rangle$ indicates an average of the value ξ^n over the unperturbed distribution function $f_0(\xi)$ and the two parameters χ_{Z_1} and χ_{Z_2} depend on the effective charge Z_* as follows:

$$\chi_{z_1} = \frac{1}{(1+5/Z_*)(1+8/Z_*)},\quad \chi_{z_2} = \frac{1}{1+5/Z_*}. \tag{23}$$

Considering Eq. (18), one can derive from Eq. (15) the expression for the function $K_0(\omega)$ in the opposite limiting case of high frequencies $\omega \gg \nu_e$:

$$K_0(\omega) = 1 - i\chi_{z_3}\frac{8\pi\sqrt{2}}{3}v_{\rm th}^3\xi_\omega f_0(\xi=0), \tag{24}$$

where the parameter

$$\chi_{z_3} = 1 + 2/Z_* \tag{25}$$

contains the dependence on the effective charge Z_*. Equations (22) and (24) represent well-known cases for the normal low-frequency and normal high-frequency skin effects, respectively. It should be emphasized that they depend essentially on the ion effective charge Z_* and they are valid for an arbitrary equilibrium distribution function f_0, including one for the degenerate electron plasma. Below, these limiting cases will be used for the determination of the unknown parameter $\varkappa = Z/Z_*$.

A. Nondegenerate electron plasma

For the Maxwell equilibrium distribution function $f_0(\xi) = (2\pi v_{\rm th}^2)^{-3/2}e^{-\xi^2}$ one has from Eq. (15) the following expression:

$$K_0(\omega) = \frac{-8i\chi_{z_2}}{3\xi_\omega\sqrt{\pi}}\int_0^\infty F(1;\alpha_z;i\beta_z\xi^3)\xi^7 e^{-\xi^2}d\xi. \tag{26}$$

The limiting cases (22) and (24) for the case of the Maxwell distribution function give, respectively,

$$K_0(\omega) = \frac{315}{8}\frac{\chi_{z_1}}{\xi_\omega^2} - \frac{8i}{\sqrt{\pi}}\frac{\chi_{z_2}}{\xi_\omega} \tag{27}$$

and

$$K_0(\omega) = 1 - i\frac{4}{3\sqrt{\pi}}\xi_\omega\chi_{z_3}, \tag{28}$$

which completely agree with the standard forms of the corresponding expressions [5,6,9–14] in the case $\chi_{Z_1} = \chi_{Z_2} = \chi_{Z_3} = 1$, which follows from Eqs. (23) and (25) in the formal limit $Z \to \infty$. Inserting the first term of Eq. (18) into Eq. (26), one gets the Lorentz model for optical properties of plasmas:

$$K_0(\omega) = \frac{8\chi_{z_2}}{3\sqrt{\pi}}\int_0^\infty \frac{\xi^7 e^{-\xi^2}}{\xi^3 + i\xi_\omega}d\xi, \tag{29}$$

considered previously (for $\chi_{Z_2} = 1$) in Refs. [9–13,30].

In order to use Eq. (15) or (26), one has to derive an expression for the relative fraction $\varkappa$ of electron-electron collisions with subthermal electrons. This can be done if one takes into account the above limiting cases. (i) For $\omega \to \infty$ the permittivity does not depend on electron-electron collisions [5,6,22,37], which means that it should not contain a dependence on Z_*. Recalling Eqs. (25) and (24), this means that

$$Z_* \to \infty \quad \text{for } \omega \to \infty. \tag{30}$$

(ii) For $\omega \to 0$ one has the respective interpolation formula for stationary conductivity

$$\sigma_0 = \gamma_\sigma(Z)\sigma_{\rm sh},\quad \sigma_{\rm sh} = \frac{2}{\pi^{3/2}}\frac{\omega_p^2}{\omega\xi_\omega},\quad \gamma_\sigma = \frac{a+Z}{b+Z}, \tag{31}$$

where $a = 0.87$ and $b = 2.2$ (see Refs. [37,38]). Considering the connection

$$\sigma' = -\frac{\omega_p^2}{4\pi\omega}\mathrm{Im}[K_0(\omega)] \tag{32}$$

of the real part of conductivity and the function $K_0(\omega)$, one can write the following expression for the imaginary part of $K_0(\omega)$ in the stationary case: $\mathrm{Im}[K_0(\omega)]|_{\omega\to 0} = -8i\gamma_\sigma/\sqrt{\pi}\xi_\omega$. Comparing this expression with Eqs. (27) and (23), one gets

$$\varkappa(Z,\omega\to 0) = \frac{Z}{Z_*(\omega\to 0)} = \frac{Z(b-a)}{5(Z+a)}. \tag{33}$$

Taking into account Eqs. (30) and (33), one can propose the following interpolation for $\varkappa(\omega)$ in the whole frequency range:

$$\varkappa(\omega) = \varkappa_0[1 + (C/\xi_\omega)^s]^{-1}, \quad (34)$$

where $\varkappa_0 = \varkappa(\omega \to 0)$ is given by Eq. (33) and $C > 0$ and $s > 0$ are positive numerical constants, which can be withdrawn, for example, from the comparison with the exact calculations.

B. Degenerate electron plasma

In this section we generalize the permittivity (15) obtained for a nondegenerate electron plasma to the cases of a partially or fully degenerate plasma. Strictly speaking, the starting point in this case should be the quantum kinetic equation. However, arguments below show that a simple generalization of Eq. (15) is possible in the manner analogous to that done for the case of a Lorentz plasma with arbitrary degeneracy in Refs. [39,40].

First, it has been shown previously (see, e.g., Ref. [42]) that the calculation of a velocity-dependent electron-ion collision frequency $\nu(v)$ [$\nu(v) \sim h/v^3$, where h has been introduced in Sec. II], on the basis of the quantum kinetic equation, yields the same result as if one had started from the classical kinetic equation, where, however, the classical Coulomb logarithm has to be replaced by the quantum one. Second, the electron-electron collisions in a degenerate plasma have been investigated in detail in Refs. [43–46] using the quantum kinetic equation approach. However, starting from the quantum kinetic equation and following the same steps that led to Eqs. (6) and (7), we now get similar expressions. Finally, it is well known (see, e.g., Refs. [6,7]) that at vanishing quantum recoil with $\hbar k^2/2m \ll \omega$, the dielectric function that follows from the collisionless quantum kinetic equation in a random-phase approximation [47] is identical to the corresponding classical expression. Thus, in the case of a degenerate plasma Eq. (15) is applicable, assuming that $\hbar k^2/2m \ll \omega$ in addition to the conditions introduced at the beginning of Sec. II.

In the case of a partially degenerate electron plasma the equilibrium distribution function $f_0(\xi)$ in Eq. (15) is given by the Fermi-Dirac distribution

$$f_0(\xi) = B_0 f_{\mathcal{F}}(\xi), \quad f_{\mathcal{F}}(\xi) = [1 + \exp(\xi^2 - \varepsilon_\mu)]^{-1}, \quad (35)$$

where $B_0 = (3/4\pi)(m/2E_{\rm F})^{3/2}$ is the normalization constant, $E_{\rm F} = \frac{\hbar^2}{2m}(3\pi^2 n_e)^{2/3}$ is the Fermi energy, $\varepsilon_\mu = \mu/T$, and μ is the chemical potential. Inserting the distribution (35) into Eq. (15), we arrive at

$$K_0(\omega) = \frac{-2i\chi_{z_2}}{\xi_\omega \varepsilon_{\rm F}^{3/2}} \int_0^\infty F(1; \alpha_z; i\beta_z\xi^3) f_{\mathcal{F}}(\xi)[1 - f_{\mathcal{F}}(\xi)]\xi^7 d\xi \quad (36)$$

for a partially degenerate electron plasma with $\varepsilon_{\rm F} = E_{\rm F}/T$. It should be emphasized that the definitions of the dimensionless quantities ξ_ω and β_z [see Eq. (16)] in Eq. (36) should now contain a quantum expression for the Coulomb logarithm Λ in the expression for collision frequency [Eq. (5)].

The dimensionless chemical potential in the expression for $f_{\mathcal{F}}$ is calculated from

$$\varepsilon_\mu = X_{1/2}\left(\tfrac{2}{3}\varepsilon_{\rm F}^{3/2}\right), \quad (37)$$

where $X_{1/2}$ is the function inverse to the Fermi integral $F_{1/2}(x)$, $X_{1/2}(F_{1/2}(x)) = x$, where $F_\alpha(x) = \int_0^\infty t^\alpha (1 + e^{t-x})^{-1} dt$. For the numerical evaluation of Eq. (37) it is useful to use the highly accurate rational function approximations for the Fermi integrals and their inverse functions derived in Ref. [48].

To compare the present approach with the previously known models it is also constructive to consider some particular cases of the general expression (36). In the case of a highly degenerate electron plasma with $T \ll E_{\rm F}$ the function (36) is simplified and is given by

$$K_0(\omega) = -\frac{i\chi_{z_2}}{\eta_\omega} F(1; \alpha_Z; i\gamma_Z). \quad (38)$$

Here $\gamma_Z = Z_*/3\eta_\omega$ and $\eta_\omega = \xi_\omega/\varepsilon_{\rm F}^{3/2} = \nu_d/\omega$, where $\nu_d = (4Zme^4/3\pi\hbar^3)\Lambda_d$ is the electron-ion collision frequency in the case of a fully degenerate electron plasma derived by Flowers and Itoh [45] and more recently by Shternin and Yakovlev [46] and Λ_d is the corresponding Coulomb logarithm.

Noting that for $E_F > T$ one has $Z_* \gg 1$ (see below), one can use the expansion (18) to calculate the confluent hypergeometric function in Eq. (38). With only the first term in this expansion one gets from Eq. (38)

$$K_0(\omega) = \frac{1}{1 + i\eta_\omega}, \quad (39)$$

i.e., the Drude expression for the function $K_0(\omega)$.

In the limit of low frequencies $\omega \ll \nu_e$ one can obtain from Eq. (22) the expression for a degenerate plasma similar to that for the nondegenerate one (27):

$$K_0(\omega) = \frac{3\chi_{z_1}}{\xi_\omega^2}\frac{F_{7/2}(\varepsilon_\mu)}{F_{1/2}(\varepsilon_\mu)} - \frac{2i\chi_{z_2}}{\xi_\omega}\frac{F_2(\varepsilon_\mu)}{F_{1/2}(\varepsilon_\mu)}, \quad (40)$$

which in the limit $T \ll E_{\rm F}$ turns into

$$K_0(\omega) = \chi_{z_1}/\eta_\omega^2 - i\chi_{z_2}/\eta_\omega. \quad (41)$$

Note that this result follows also from Eq. (38).

From Eqs. (41) and (32) one can obtain the following expression for the real part of the stationary electric conductivity $\sigma'(\omega \to 0)$ of a highly degenerate plasma (at $T \ll E_{\rm F}$):

$$\sigma' = \frac{\chi_{z_2}}{\hbar}\frac{\sqrt{E_{\rm F}^3/E_{\rm H}}}{\sqrt{2}\pi Z}\frac{1}{\Lambda_d}, \quad (42)$$

where $E_{\rm H} = me^4/\hbar^2 \simeq 27.2$ eV is the Hartree energy. This expression coincides with the generalization of the well-known Ziman formula [49] for the partially degenerate case [26] if one uses the expression

$$\Lambda_d = \int_0^\infty \frac{S(k)}{k}\frac{f_{\mathcal{F}}(k\lambda\!\!\!^{-})dk}{|\varepsilon_{\rm L}(k,0)|^2} \quad (43)$$

for the Coulomb logarithm Λ_d and sets $\chi_{Z_2} = 1$ in Eq. (42). In Eq. (43) $\lambda\!\!\!^{-} = \hbar/(2mT)^{1/2}$ is the thermal wavelength, $S(k)$ is the static structure factor, and $\varepsilon_{\rm L}$ is the Lindhard dielectric function [47] for a partially degenerate electron gas [50,51]. In the opposite limiting case of high frequencies $\omega \gg \nu_e$, from Eq. (24) one can obtain the expression

$$K_0(\omega) = 1 - i\chi_{z_3}\xi_\omega\varepsilon_{\rm F}^{-3/2}(1 + e^{-\varepsilon_\mu})^{-1}, \quad (44)$$

which in the case of high degeneracy with $E_F \gg T$ becomes

$$K_0(\omega) = 1 - i\chi_{z_3}\eta_\omega. \tag{45}$$

Next, in the limit $Z_* \gg 1$, taking the first term of Eq. (18), in leading order one gets from Eq. (36) the following expression:

$$K_0(\omega) = \frac{2\chi_{z_2}}{\varepsilon_F^{3/2}} \int_0^\infty \frac{f_{\mathcal{F}}(\xi)[1 - f_{\mathcal{F}}(\xi)]}{\xi^3 + i\xi_\omega} \xi^7 d\xi, \tag{46}$$

which in the particular case $\chi_{Z_2} = 1$ coincides with a result obtained in Refs. [39,40] for the electron conductivity of a Lorentz plasma.

As mentioned above, for an accurate numerical treatment of the permittivity of degenerate plasmas one should use a proper expression for the Coulomb logarithm in Eq. (5) [and hence in Eqs. (16) and (36)]. For moderate values of the degeneracy parameter $\Theta = \varepsilon_F^{-1} = T/E_F \gtrsim 1$ a wide-range formula for stationary electric conductivity for hydrogenlike plasmas ($Z = 1$) was proposed in Ref. [52]. Comparing the expression for σ' obtained in Ref. [52] and Eq. (31) for $Z = 1$ and for a weakly degenerate plasma ($\Theta \gg 1$), one can use the following interpolation expression for Λ in a wide range of density and temperature:

$$\Lambda(\Gamma,\Theta) = \frac{1/2}{1 + b_1/\Theta^{3/2}} \left[D\ln(1+A+B) - C - \frac{b_2}{b_2 + \Gamma\Theta} \right], \tag{47}$$

where $\Gamma = (4\pi n_e/3)^{1/3} Ze^2/T$ is the coupling parameter. The quantities A, B, C, and D are functions of the parameters Γ and Θ and are given by

$$A = \frac{\Gamma^{-3}(1 + a_4/\Gamma^2\Theta)}{1 + a_2/\Gamma^2\Theta + a_3/(\Gamma^2\Theta)^2} [a_1 + c_1 \ln(c_2\Gamma^{3/2} + 1)]^2,$$

$$B = \frac{b_3(1 + c_3\Theta)}{\Gamma\Theta(1 + c_3\Theta^{4/5})}, \quad C = \frac{c_4}{\ln(1 + \Gamma^{-1}) + c_5\Gamma^2\Theta},$$

$$D = \frac{\Gamma^3 + a_5(1 + a_6\Gamma^{3/2})}{\Gamma^3 + a_5},$$

with a set of numerical constants $a_0 = 0.03064$, $a_1 = 1.1590$, $a_2 = 0.698$, $a_3 = 0.4876$, $a_4 = 0.1748$, $a_5 = 0.1$, $a_6 = 0.258$, $b_1 = 1.95$, $b_2 = 2.88$, $b_3 = 3.6$, $c_1 = 1.5$, $c_2 = 6.2$, $c_3 = 0.3$, $c_4 = 0.35$, and $c_5 = 0.1$ (see Ref. [52] for details). The expression (36) with the Coulomb logarithm given by Eq. (47) gives an accurate description of the permittivity of plasmas for $Z = 1$ and for $Z \gg 1$, where it applies to the Lorentz model of Lee and More [39] for stationary conductivity and its extension for dynamical conductivity [40].

For highly and moderately degenerate plasmas the influence of electron-electron collisions will be decreased due to Pauli blocking [52]. This effect can be taken into account if one uses the expression for the Spitzer factor in a degenerate electron plasma [53,54],

$$\widetilde{\gamma}_\sigma(Z) = \gamma_\sigma(Z) + \frac{1 - \gamma_\sigma(Z)}{1 + 0.6\ln(1 + \Theta/20)}, \tag{48}$$

instead of the respective expression for the nondegenerate Spitzer factor $\gamma_\sigma(Z)$ [Eq. (31)]. In Ref. [53] it was demonstrated that the interpolation formula (48) gives results very similar to those obtained by a rigorous quantum statistical approach.

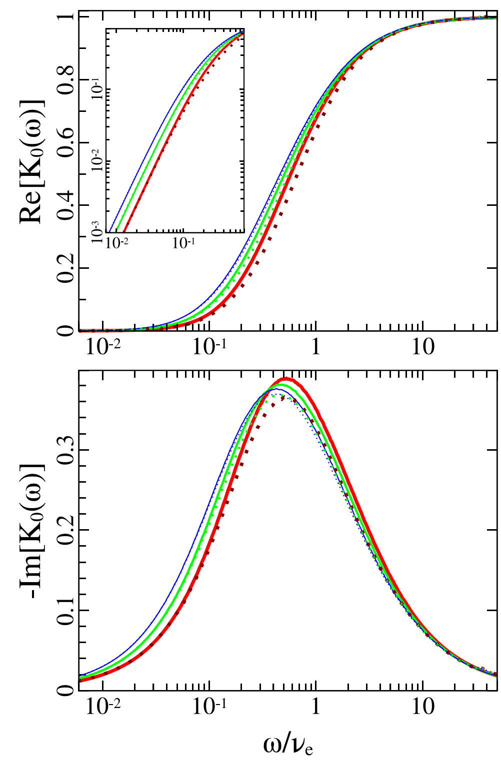

FIG. 1. (Color online) Real (top) and imaginary (with minus sign) (bottom) parts of $K_0(\omega)$ for the nondegenerate electron plasma with different ionic charges $Z = 1$ (thick lines), $Z = 3$ (thinner lines), and $Z = 10$ (thinnest lines), calculated by Eqs. (26) and (34) with $C = s = 1$ (solid lines) and by the interpolation formula of Brantov *et. al.* [37] (dotted lines).

Using the same arguments that were used for the derivation of the expression (33), one can obtain the following expression for the value of $\varkappa_0 = Z/Z_*(\omega \to 0)$ for the case of partially or fully degenerate plasmas:

$$\varkappa_0 = Z\left[\widetilde{\gamma}_\sigma^{-1}(Z) - 1\right]/5, \tag{49}$$

where $\widetilde{\gamma}_\sigma$ is given by Eq. (48). The frequency dependence of $\varkappa$ is given by the same equation (34) as in the case of a degenerate plasma.

It should also be mentioned that the theoretical model described above is valid for frequencies $\omega \lesssim \omega_p$. For frequencies higher than the plasma frequency the value of the real part of the function $K_0(\omega)$ will be considerably decreased in comparison with the one for $\omega < \omega_p$ [55–57] as long as a charged particle screening at the plasma frequency is replaced by a screening at the laser frequency for $\omega > \omega_p$. This can be approximately accounted for by replacing ω_p by ω in the Coulomb logarithm for the case $\omega > \omega_p$ [56].

III. NUMERICAL RESULTS

In Fig. 1 the results of the numerical calculations of the real $\mathrm{Re}[K_0(\omega)]$ and imaginary (with a minus sign) $-\mathrm{Im}[K_0(\omega)]$ parts of the function $K_0(\omega)$ for nondegenerate plasmas by Eqs. (26), (33), and (34) are presented for different ionic charges $Z = 1,3,10$ as functions of the scaled frequency ω/ν_e of the electromagnetic radiation. The case of highly charged plasma ions with $Z = 10$ is almost identical to the Lorentz model. The parameters C and s in Eq. (34) are equal to 1. For comparison, the results of the calculation by the interpolation formula suggested by Brantov *et al.* [37] are also shown by dotted lines. For the considered long-wavelength perturbations ($k \to 0$) this interpolation formula consists of Eq. (29) with $\chi_{Z_2} = 1$ and the dimensionless quantity ξ_ω is replaced by $\xi_\omega G_Z(\omega)$, where

$$G_Z(\omega) = \frac{1 + C_0\xi_\omega/\gamma_\sigma(Z)}{1 + C_0\xi_\omega}, \quad C_0 = \frac{4}{15\sqrt{\pi}}(1 + 2i). \tag{50}$$

Here the factor $\gamma_\sigma(Z)$ is given by Eq. (31). In the limit $Z \gg 1$ the factor $\gamma_\sigma(Z) \to 1$ and therefore $G_Z(\omega) \to 1$, which gives the Lorentz model.

It can be seen that our results shown in Fig. 1 are very close to the interpolation results obtained in Ref. [37]. The largest difference between both models occurs for the imaginary part of the function $K_0(\omega)$ at $\omega/\nu_e \sim 0.5$ and $Z = 1$ and the relative deviation is within 5%. However, the interpolation formula of Ref. [37] has an accuracy of about 7% compared to the more rigorous fully kinetic treatment [37].

It should be noted that both the models (26), (33), and (34) and the interpolation formula suggested in Ref. [37] lead to the correct asymptotic expressions for the permittivity in the low- and high-frequency limits, although the interpolation formula [37] does not satisfy the fundamental property $\varepsilon(-\omega) = \varepsilon^*(\omega)$ and the Kramers-Kronig relations [58]. This is because the function $G_Z(\omega)$ given by Eq. (50) does not satisfy the relation $G_Z(-\omega) = G_Z^*(\omega)$. Contrarily, our model satisfies the equality $\varepsilon(-\omega) = \varepsilon^*(\omega)$ and the Kramers-Kronig relations.

It should also be emphasized that the model presented here only weakly depends on the actual choice of the fitting parameters C and s in the expression (34). More specifically, the results are only slightly changed in the interval $C,s \in [0.5;2]$.

In Fig. 2 the function $K_0(\omega)$, obtained by Eqs. (36), (49), and (34), is shown for the cases of partially degenerate plasmas with different degeneracy parameters $\varepsilon_F = E_F/T = 10^{-5}, 10^{-2}, 1.5, 10$ and different ionic charges $Z = 1,10$. The results for a weakly degenerate case with $\varepsilon_F = 10^{-5}$ coincide for all Z (thick solid and dashed lines in Fig. 2) with those calculated by Eqs. (26), (33), and (34) obtained for a nondegenerate plasma. For $Z \geq 10$ the results of calculations by Eqs. (36), (49), and (34) are close to those obtained for the nondegenerate case if $\varepsilon_F \lesssim 0.3$.

For $\varepsilon_F \gtrsim 0.1$ the Spitzer factors (48) for a degenerate plasma are very close to 1. That is, for moderately and highly degenerate plasmas the electron-electron collisions do not play a significant role and $K_0(\omega)$ does not depend on Z. For this case and for $\varepsilon_F < 1$ (i.e., at $0.1 \lesssim \varepsilon_F < 1$) the dependence of $K_0(\omega)$ on the frequency is the same as in the nondegenerate case with $Z \geq 10$.

As shown in Fig. 2, a substantial difference between the nondegenerate and degenerate regimes occurs at $E_F/T \gtrsim 1$. For $E_F/T \gg 1$ the difference is dramatic: The function $K_0(\omega)$ is shifted to the left along the ω/ν_e axis while increasing E_F/T. This is stipulated by the fact that, in accordance with Eq. (38), the function $K_0(\omega)$ for a degenerate plasma depends on $\eta_\omega = \xi_\omega/\varepsilon_F^{3/2}$ rather than on the parameter ξ_ω as in the nondegenerate

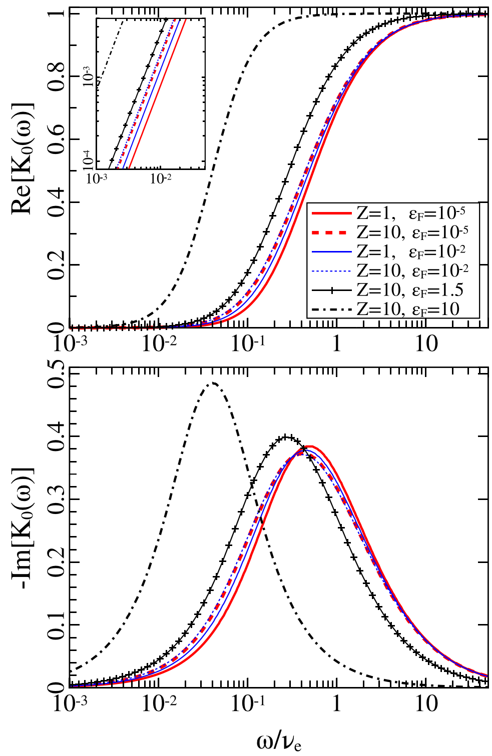

FIG. 2. (Color online) Real (top) and imaginary (with minus sign) (bottom) parts of $K_0(\omega)$, calculated by Eqs. (36), (49), and (34) with $C = s = 1$, for the degenerate electron plasma with different ionic charges and different degeneracy parameters: $Z = 1, \varepsilon_F = 10^{-5}$ (thick solid lines); $Z = 10, \varepsilon_F = 10^{-5}$ (thick dashed lines); $Z = 1, \varepsilon_F = 10^{-2}$ (thin solid lines); $Z = 10, \varepsilon_F = 10^{-2}$ (thin dashed lines); $Z = 10, \varepsilon_F = 1.5$ (marked lines); and $Z = 10, \varepsilon_F = 10$ (dash-dotted lines).

case. This means that the displacement of the maximum of the function $K_0(\omega)$ along the ω/ν_e axis is proportional to $\varepsilon_F^{3/2}$ for $\varepsilon_F \gg 1$. Therefore, to gain more insight we plot in Fig. 3 the function $K_0(\omega)$ versus the quantity η_ω^{-1}, i.e., excluding the factor $\varepsilon_F^{3/2}$ in the scaled frequency. One can easily see that for $\varepsilon_F \geq 5$ all curves are similar and centered near $\eta_\omega = 1$ and for $\varepsilon_F > 10$ one can use the Drude formula (39) to calculate the permittivity.

IV. SUMMARY

In this paper we have obtained an analytical solution of the linearized Fokker-Planck kinetic equation with a Landau collision integral and for a completely ionized and unmagnetized electron plasma with an arbitrary ionic charge. This solution accounts for both electron-ion collisions and the collisions of the subthermal (cold) electrons with thermal ones. The latter collisions have been treated phenomenologically by introducing a parameter $\varkappa$ related to the relative contribution of the subthermal electrons to the friction force and diffusion coefficient in velocity space (the limit $\varkappa \to 0$ corresponds to the vanishing contribution of the electron-electron collisions).

Using the obtained solution of the Fokker-Planck kinetic equation, we have proposed an analytical model for the high-frequency ($\omega \gg kv_{\text{th}}$) dielectric function of the collisional

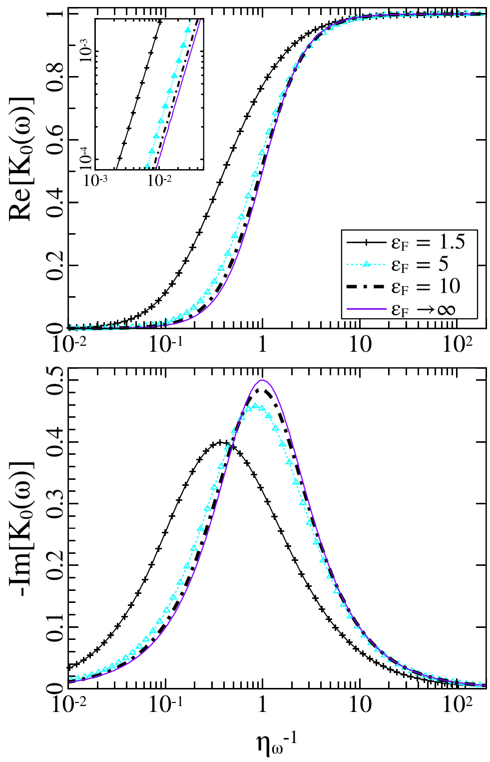

FIG. 3. (Color online) Real (top) and imaginary (with minus sign) (bottom) parts of $K_0(\omega)$ as functions of the parameter η_ω^{-1}, calculated by Eqs. (36), (49), and (34) with $C = s = 1$, for the degenerate electron plasma with $\varepsilon_F = 1.5$ (lines with crosses), $\varepsilon_F = 5$ (lines with triangles), $\varepsilon_F = 10$ (dash-dotted lines), and for $\varepsilon_F \to \infty$ (solid lines). In the latter case the function $K_0(\omega)$ is given by Eq. (38), which, however, in the limit $\varepsilon_F \to \infty$ coincides with the Drude model (39). The results do not depend on the value of Z (for $Z > 1$ and $\varepsilon_F > 1$).

electron plasma with an arbitrary ionic charge. More precisely, the validity of the model is restricted to the long-wavelength high-frequency perturbations when k^{-1} is the largest length scale of the problem with $kv_{\rm th} \ll \omega$, $k\lambda_{ei} \ll 1$, and $k\lambda_{ee} \ll 1$, where λ_{ei} and λ_{ee} are the electron-ion and electron-electron mean free paths, respectively.

In our model the dielectric function contains the contribution of the electron-electron collisions through an unknown parameter $\varkappa(\omega)$, which has been treated as a function of the frequency ω. Then $\varkappa(\omega)$ is adjusted by considering the low-frequency ($\omega \to 0$) limit of the dielectric function, where it should agree with the well-known expression for the stationary electric conductivity. On the other hand, at high frequencies ($\omega \to \infty$) it behaves as $\varkappa(\omega) \to 0$ to fulfill the requirement of a vanishing contribution of the electron-electron collisions. One important feature of the outlined model is the possibility of generalization of the results to the cases of partially degenerate and/or strongly coupled plasmas. Making such a generalization, we have assumed an additional limitation $\hbar k^2/2m \ll \omega$ on the wavelength of the excitations.

In a further step we have considered a number of limiting cases: (a) the limit of a highly degenerate ($T \ll E_F$) plasma, (b) the limit of low frequencies, (c) the limit of high frequencies, and (d) the asymptotic behavior of the dielectric function at large ionic charge $Z \gg 1$ when our model coincides with the Lorentz plasma model derived for either nondegenerate [22] or partially degenerate plasmas [39,40]. These limiting cases facilitate the systematic comparison of our analytical results with previous theoretical models.

In particular, the present model has been compared both analytically and numerically with the interpolation formula suggested by Brantov *et al.* [37]. It has been demonstrated that our results agree satisfactorily with those obtained in Ref. [37], showing relative deviations of less than 5% in an unfavorable case of lowest ionic charge $Z = 1$. It should be noted, however, that the interpolation formula by Brantov *et al.* has an accuracy of about 7% compared to the more rigorous fully kinetic treatment of Ref. [37].

As the main goal of this paper we suggested a simple but more advanced analytical model for calculations of the dielectric function and related quantities in a wide range of parameters that is appropriate for modeling many experiments with laser-matter interactions. In addition, further improvement of the present model can be achieved by considering the spatial inhomogeneity of the perturbations (i.e., finite wavelengths k^{-1}) in the Fokker-Planck kinetic equation (1). This can be done using the method of Ref. [37] for the solution of the kinetic equation and, to treat the electron-electron collisions, following the same steps that led to the approximate coefficients (6) and (7). Systematic investigation of this problem is left for future work.

ACKNOWLEDGMENTS

The work of H.B.N. and H.H.M. was supported by the State Committee of Science of the Armenian Ministry of Higher Education and Science (Project No. 13-1C200). The work of M.E.V. and N.E.A. was supported in part by the programs on fundamental research of the Russian Academy of Sciences.

[1] Y. Kitagawa, H. Fujita, R. Kodama, H. Yoshida, S. Matsuo, T. Jitsuno, T. Kawasaki, H. Kitamura, T. Kanabe, S. Sakabe *et al.*, IEEE J. Quantum Electron. **40**, 281 (2004).

[2] U. Zastrau, P. Audebert, V. Bernshtam, E. Brambrink, T. Kämpfer, E. Kroupp, R. Loetzsch, Y. Maron, Y. Ralchenko, H. Reinholz *et al.*, Phys. Rev. E **81**, 026406 (2010).

[3] D. C. Carroll, O. Tresca, R. Prasad, L. Romagnani, P. S. Foster, P. Gallegos, S. Ter-Avetisyan, J. S. Green, M. J. V. Streeter, N. Dover *et al.*, New J. Phys. **12**, 045020 (2010).

[4] V. P. Silin, *Parametric Effect of High-Intensity Radiation on Plasmas* (Nauka, Moscow, 1973) [in Russian].

[5] V. P. Silin and A. A. Rukhadze, *Electromagnetic Properties of Plasma and Plasma-like Media*, 3rd ed. (URSS, Moscow, 2013) [in Russian].

[6] V. P. Silin, *Introduction to the Kinetic Theory of Gases*, 3rd ed. (URSS, Moscow, 2013) [in Russian].

[7] A. F. Alexandrov, L. S. Bogdankevich, and A. A. Rukhadze, *Principles of Plasma Electrodynamics* (Springer-Verlag, Berlin, 1984).

[8] A. Pukhov, Rep. Prog. Phys. **66**, 47 (2003).

[9] N. E. Andreev, M. E. Veysman, V. P. Efremov, and V. E. Fortov, Teplofiz. Vys. Temp. **41**, 679 (2003) [High Temp. **41**, 594 (2003)].

[10] M. E. Veysman, B. Cros, N. E. Andreev, and G. Maynard, Phys. Plasmas **13**, 053114 (2006).

[11] M. E. Veysman, M. B. Agranat, N. E. Andreev, S. I. Ashitkov, V. E. Fortov, K. V. Khishchenko, O. F. Kostenko, P. R. Levashov, A. V. Ovchinnikov, and D. S. Sitnikov, J. Phys. B **41**, 125704 (2008).

[12] M. E. Povarnitsyn, N. E. Andreev, E. M. Apfelbaum, T. E. Itina, K. V. Khishchenko, O. F. Kostenko, P. R. Levashov, and M. E. Veysman, Appl. Surf. Sci. **258**, 9480 (2012).

[13] M. E. Povarnitsyn, N. E. Andreev, P. R. Levashov, and O. N. Rosmej, Phys. Plasmas **19**, 023110 (2012).

[14] S. Ichimaru, *Basic Principles of Plasma Physics* (Benjamin, Reading, MA, 1973).

[15] S. I. Braginskii, in *Review of Plasma Physics*, edited by M. A. Leontovich (Consultants Bureau, New York, 1965), Vol. 1.

[16] I. P. Shkarofsky, T. W. Johnston, and M. P. Bachynski, *The Particle Kinetics of Plasmas* (Addison-Wesley, Reading, MA, 1966).

[17] V. P. Silin, Phys. Usp. **45**, 955 (2002).

[18] P. L. Bhatnagar, E. P. Gross, and M. Krook, Phys. Rev. **94**, 511 (1954).

[19] N. D. Mermin, Phys. Rev. B **1**, 2362 (1970).

[20] H. B. Nersisyan and A. K. Das, Phys. Rev. E **69**, 046404 (2004).

[21] H. B. Nersisyan and A. K. Das, in *Advances in Plasma Physics Researches*, edited by F. Gerard (Nova Science, New York, 2008), Vol. 6, Chap. 2, p. 81.

[22] E. M. Lifshitz and L. P. Pitaevskii, *Physical Kinetics* (Butterworth-Heinemann, Oxford, 1981).

[23] P. C. Clemmow and J. P. Dougherty, *Electrodynamics of Particles and Plasmas* (Addison-Wesley, Redwood City, CA, 1990).

[24] M. Opher, G. J. Morales, and J. N. Leboeuf, Phys. Rev. E **66**, 016407 (2002).

[25] B. D. Fried, A. N. Kaufman, and D. L. Sachs, Phys. Fluids **9**, 292 (1966).

[26] A. Selchow, G. Röpke, and K. Morawetz, Nucl. Instrum. Methods Phys. Res. Sect. A **441**, 40 (2000).

[27] K. Morawetz and U. Fuhrmann, Phys. Rev. E **61**, 2272 (2000).

[28] G. S. Atwal and N. W. Ashcroft, Phys. Rev. B **65**, 115109 (2002).

[29] A. V. Brantov, V. Y. Bychenkov, W. Rozmus, and C. E. Capjack, IEEE Trans. Plasma Sci. **34**, 738 (2006).

[30] V. Y. Bychenkov, V. T. Tikhonchuk, and W. Rozmus, Phys. Plasmas **4**, 4205 (1997).

[31] V. Y. Bychenkov, Plasma Phys. Rep. **24**, 801 (1998).

[32] R. A. Koch and W. Horton, Jr., Phys. Fluids **18**, 861 (1975).

[33] J. R. Peñano, G. J. Morales, and J. E. Maggs, Phys. Plasmas **4**, 555 (1997).

[34] A. Selchow and K. Morawetz, Phys. Rev. E **59**, 1015 (1999); **69**, 039902 (2004).

[35] A. V. Brantov, V. Y. Bychenkov, W. Rozmus, and C. E. Capjack, Phys. Rev. Lett **93**, 125002 (2004).

[36] A. V. Brantov, V. Y. Bychenkov, W. Rozmus, and C. E. Capjack, JETP **100**, 1159 (2005).

[37] A. V. Brantov, V. Y. Bychenkov, and W. Rozmus, JETP **106**, 983 (2008).

[38] R. Balescu, Phys. Fluids **3**, 52 (1960).

[39] Y. T. Lee and R. M. More, Phys. Fluids **27**, 1273 (1984).

[40] O. F. Kostenko and N. E. Andreev, GSI Annual Report No., 2008 (unpublished).

[41] I. S. Gradshteyn and I. M. Ryzhik, *Tables of Integrals, Series and Products*, 2nd ed. (Academic, New York, 1980).

[42] H. Reinholz and G. Röpke, Phys. Rev. E **85**, 036401 (2012).

[43] M. Lampe, Phys. Rev. **170**, 306 (1968).

[44] M. Lampe, Phys. Rev. **174**, 276 (1968).

[45] E. Flowers and N. Itoh, Astrophys. J. **206**, 218 (1976).

[46] P. S. Shternin and D. G. Yakovlev, Phys. Rev. D **74**, 043004 (2006).

[47] J. Lindhard, K. Dan. Vidensk. Selsk. Mat. Fys. Medd. **28**, 1 (1954).

[48] H. M. Antia, Astrophys. J. Suppl. Ser. **84**, 101 (1993).

[49] J. M. Ziman, Philos. Mag. **6**, 1013 (1961).

[50] C. Gouedard and C. Deutsch, J. Math. Phys. **19**, 32 (1978).

[51] N. R. Arista and W. Brandt, Phys. Rev. A **29**, 1471 (1984).

[52] A. Esser, R. Redmer, and G. Röpke, Contrib. Plasma Phys. **43**, 33 (2003).

[53] J. R. Adams, N. S. Shilkin, V. E. Fortov, V. K. Gryaznov, V. B. Mintsev, R. Redmer, H. Reinholz, and G. Röpke, Phys. Plasmas **14**, 062303 (2007).

[54] W. A. Stygar, G. A. Gerdin, and D. L. Fehl, Phys. Rev. E **66**, 046417 (2002).

[55] J. Dawson and C. Oberman, Phys. Fluids **5**, 517 (1962).

[56] C. D. Decker, W. B. Mori, J. M. Dawson, and T. Katsouleas, Phys. Plasmas **1**, 4043 (1994).

[57] H. Reinholz, R. Redmer, G. Röpke, and A. Wierling, Phys. Rev. E **62**, 5648 (2000).

[58] L. D. Landau and E. M. Lifshitz, *Electrodynamics of Continuous Media* (Pergamon, Oxford, 1984).