Entropic memory erasure

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We have considered a Brownian particle confined in a two-dimensional bilobal enclosure where the state of the particle represents a bit of information having binary value 0 (left lobe) or 1 (right lobe). A time linear force is applied on the particle, driving it selectively to a particular lobe, and thus erasing one bit of information. We explore the statistics of heat and work associated with memory erasure to realize the Landauer limit in the entropic domain. Our results suggest that the mean value of work done associated with the complete erasure procedure satisfies the Landauer bound even when the memory is purely entropic in nature.

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I. INTRODUCTION

Logic gates form the basis of general purpose computation [1]. In digital computation, the binary digits 0 and 1 represent the two states of the inputs as well as the outputs. All computations are performed in terms of these two binary digits. In the case of basic logic operations (OR and AND) at least two inputs represented by 0 or 1 are converted to a single binary digit, depending upon the nature of the operation. This incident can be regarded as an erasing process; two bits converted to a single bit with assistance of the nonlinearity of the system. Erasure of information is very important for all three important steps of information processing: computation, measurement, and communication [2]. The idea of associating information processing with thermodynamics [2–11] has been an area of active research for many years. Landauer [2,3] showed that a minimal quantity of heat, $k_B T \ln 2$, known as the Landauer bound, is essentially generated when a classical bit of information is erased [10]. When the erasing process is very slow, the mean dissipated heat approaches this bound [2]. This fact has also been established by experiment. For details we refer to [10].

The two states of the binary information are generally similar in energy because the computing device should have the same energy irrespective of its information content. So one can think of a bistable potential with two wells representing two states (say, the left well represents the 0 state and the right well represents the 1 state). As noise is inherently present in all computational devices, we can consider the motion of an overdamped Brownian particle in the bistable potential mimicking the computational procedure. An external bias is required to transfer the particle selectively to the desired well, i.e., essentially to erase a bit of information. This setup has been used to verify Landauer's principle by numerical computation in small systems [11]. The nonlinearity of the bistable potential is exploited by the external bias in the erasing procedure. In the present work, we intend to investigate the process of information erasing in the absence of any intrinsic potential field but in the presence of geometrical confinement [12–16]. It is now well established that when a Brownian particle moves in a channel or tube of varying cross section, the confinement in a higher dimension gives rise to an entropic potential [13–26] in a reduced dimension. An effective entropic potential is encountered in the free energy expression of the Fick-Jacobs [12] equation, which is equivalent to a one-dimensional Smoluchowski equation. Our aim here is to explore the energy requirements when the information has an entropic origin. Since Landauer's principle concerns the two states separated by a potential energy barrier and calculations are done in terms of energy dissipation and entropy production, it would seem that an entropic barrier may play an important role in the calculation of these thermodynamic quantities related to the erasure process. Keeping this in mind we consider an overdamped Brownian particle confined in a two-dimensional bilobal enclosure. The two lobes of the enclosure represent the two binary states; 0 and 1. The interesting question is the statistics of the heat dissipated during an erasing process where the potential barrier is of entropic origin. We investigate the validity of the Landauer bound appearing as a boundary effect.

II. THE MODEL AND THE STOCHASTIC DYNAMICS

We consider the two-dimensional overdamped dynamics of a Brownian particle confined in a bilobal enclosure as shown in Fig. 1(a). The Langevin dynamics of the Brownian particle is described by the following equation:

$$\gamma \frac{d\vec{r}}{dt} = -G\widehat{e_y} + F(t)\widehat{e_x} + \sqrt{\gamma k_B T}\,\overrightarrow{\eta(t)}. \tag{1}$$

Here $\vec{r}$ represents the position vector of the particle and $\widehat{e_x}$ and $\widehat{e_y}$ are the unit vectors along the x and y directions, respectively. γ is the frictional coefficient of the system, and k_B and T denote the Boltzmann constant and temperature of the bath, respectively. G represents a very weak constant bias acting along the transverse direction of the system. $\overrightarrow{\eta(t)} = (\eta_x(t), \eta_y(t))$ is a zero-mean Gaussian white noise which obeys the fluctuation-dissipation relationship. The properties of the noise are described by the following equations:

$$\begin{aligned} \langle \overrightarrow{\eta(t)} \rangle &= 0, \\ \langle \eta_i(t)\eta_j(t') \rangle &= 2\delta_{ij}\delta(t-t'), \end{aligned} \tag{2}$$

for $i, j = x, y$.

Apart from the usual forces, the particle is subjected to an additional bias $F(t)\widehat{e_x}$ which is linear in time. The confinement is imposed on the particle by using the following boundary condition. The walls of the confinement [18], as shown in Fig. 1(a), can be represented by the equation

$$\begin{aligned} y_l(x) = -y_u(x) = \omega_l(x) = -\omega_u(x) \\ = L_y(x/L_x)^4 - 2L_y(x/L_x)^2 - c/2, \end{aligned} \tag{3}$$

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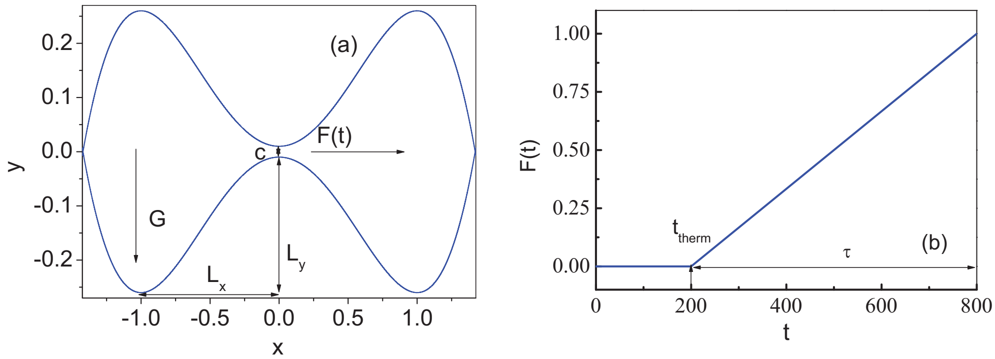

FIG. 1. (Color online) (a) The bilobal enclosure with its geometric parameters. (b) Time series plot of the external force $F(t)$ with the parameter set $F_{\max} = 1.0$, $t_{\text{therm}} = 200$, $\tau = 600$.

where $\omega_l(x)$ and $\omega_u(x)$ correspond to the lower and upper boundaries of the system, $y_l(x)$ and $y_u(x)$ represent the lower bound and upper bound of the y value at position x, respectively, L_x represents the distance between the middle point of the bottleneck and the position of the maximal width, L_y describes the narrowing of the boundary functions, and c denotes the remaining width of the bottleneck. Consequently,

$$\omega(x) = [\omega_u(x) - \omega_l(x)]/2 \tag{4}$$

corresponds to the local half-width of the bilobal structure. These wall functions cause the particle to move in a bilobal enclosure.

In order to simplify further analysis, we will use the dimensionless description [16–20,26] of the dynamical system. We scale the lengths with the characteristic length scale L_x, i.e., $\tilde{x} = x/L_x$ and $\tilde{y} = y/L_x$, implying $\tilde{c} = c/L_x$. This scaling ensures that the scaled boundary functions and the local half-width are $\tilde{\omega}_l(\tilde{x}) = \omega_l(x)/L_x = -\tilde{\omega}_u(\tilde{x})$ and $\tilde{\omega}(\tilde{x}) = \omega(x)/L_x$. The time t is scaled by a characteristic time t_{ref} as $\tilde{t} = t/t_{\text{ref}}$, where $t_{\text{ref}} = \gamma L_x^2/k_B T_R$, with T_R a reference temperature. t_{ref} is essentially twice the time required for a particle to diffuse a distance L_x at temperature T_R. The forces are scaled by $F_R = \gamma L_x/t_{\text{ref}}$, i.e., $\tilde{G} = G t_{\text{ref}}/\gamma L_x$ and $\tilde{F}(\tilde{t}) = F(t) t_{\text{ref}}/\gamma L_x$. In order to keep brevity and notational convenience, we shall omit the tilde from now on. In dimensionless form the Langevin equation can be written as

$$\frac{d\vec{r}}{dt} = -G\hat{e_y} + F(t)\hat{e_x} + \sqrt{D}\overrightarrow{\eta(t)}, \tag{5}$$

where D is the rescaled temperature and is given by T/T_R. The above Langevin dynamics can be decomposed into two equations along two mutually perpendicular directions (x and y) as

$$\begin{aligned} \frac{dx}{dt} &= F(t) + \sqrt{D}\eta_x(t), \\ \frac{dy}{dt} &= -G + \sqrt{D}\eta_y(t). \end{aligned} \tag{6}$$

Here $\eta_x(t)$ and $\eta_y(t)$ are the components of the Langevin force $\eta(t)$ along the x and y directions, respectively. The boundary function is represented by

$$\omega(x) = [\omega_u(x) - \omega_l(x)]/2 = -ax^4 + bx^2 + c/2. \tag{7}$$

In the above equation, we have defined the aspect ratio as $a = L_y/L_x$ and $b = 2a$, i.e., a and b are appropriately scaled constants.

The driving force $F(t)$ has the form

$$F(t) = 0$$

for $0 < t \leqslant t_{\text{therm}}$, and

$$F(t) = F_{\max}(t - t_{\text{therm}})/\tau \tag{8}$$

for $t_{\text{therm}} < t \leqslant t_{\text{therm}} + \tau$. Here t_{therm} is the thermalization time, the time interval during which the system is allowed to become thermalized initially with the bath in the absence of any bias; i.e., after the time t_{therm} the external force $F(t)$ is switched on. τ is the forcing time period and $F_{\max}$ is the amplitude of the driving force. The time series of the external bias force has been plotted in Fig. 1(b). Due to the presence of noise in the system, the thermodynamic quantities like work or heat corresponding to the erasure procedure are stochastic variables. Therefore we calculate the average quantities. We consider an ensemble of particles. Each particle is placed at the position of the bottleneck (0,0) at the initial time. As expected, during the thermalization time t_{therm} the particles become equally distributed in the two lobes, i.e., they contain both kinds of binary information. The left lobe is assigned the logical value 0 and the right lobe is assigned the logical value 1. It must be kept in mind that the two states of information should be well separated in order to ensure thermal stability of the information in the absence of any perturbation. The external bias $F(t)$ is switched on after the initial thermalization period and the particles are directed selectively to the desired lobe, thus erasing one kind of bit of information.

The Fokker-Planck equation [27] corresponding to the Langevin dynamics [Eq. (6)] in the absence of any external bias is presented as

$$\begin{aligned} &\frac{\partial P(x,y,t)}{\partial t} \\ &= D\frac{\partial}{\partial x}\exp\left[\frac{-u(x,y)}{D}\right]\frac{\partial}{\partial x}\exp\left[\frac{u(x,y)}{D}\right]P(x,y,t) \\ &\quad + D\frac{\partial}{\partial y}\exp\left[\frac{-u(x,y)}{D}\right]\frac{\partial}{\partial y}\exp\left[\frac{u(x,y)}{D}\right]P(x,y,t), \end{aligned} \tag{9}$$

where the potential function can be written as $u(x,y) = Gy$. To capture the effect of confinement, we use a reflecting boundary

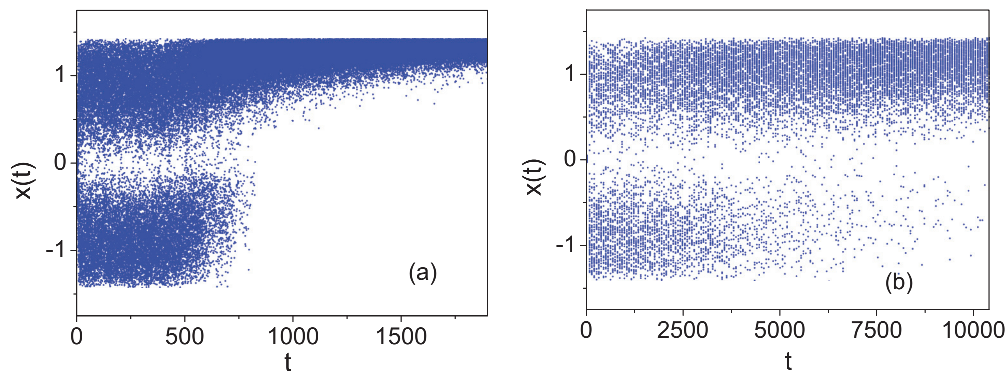

FIG. 2. (Color online) Time evolution of an ensemble of 100 trajectories during the erasure process. During the thermalization time interval ($t_{\text{therm}} = 400$), the state of memory 0 or 1 is equally probable. The parameter set used is $D = 0.02$, $F_{\max}\tau = 600$, and $G = 0$. (a) Complete erasure to memory state 1 with $F_{\max} = 0.4$ and (b) incomplete erasure to memory state 1 with $F_{\max} = 0.06$.

condition at the wall. The dimensional reduction (i.e., studying the dynamics only along the direction of interest) can be achieved by involving a marginal probability distribution $P(x,t)$ along the x direction [i.e., $P(x,t) = \int dy P(x,y,t)$] and a conditional local equilibrium probability density of y at a given x, $\rho(y;x)$, and assuming the condition $P(x,y,t) \cong P(x,t)\rho(y;x)$. Thus, after reducing the transverse direction the kinetic equation for the marginal probability distribution takes the form [from Eq. (9)]

$$\frac{\partial P(x,t)}{\partial t} = \frac{\partial}{\partial x}\left[D\frac{\partial}{\partial x}P(x,t) + A'(x,D,G)P(x,t)\right]. \quad (10)$$

In the present case with a constant force acting along the negative y direction, the potential function $A(x)$ reads as

$$A(x,D,G) = -D\ln\left[\frac{2D}{G}\sinh\left(\frac{G\omega(x)}{D}\right)\right]. \quad (11)$$

$A(x,D,G)$ represents the potential which is related to the varying cross-sectional width of the system. As a consequence, this potential is entropic rather than energetic in origin. Equations (9)–(11) are necessary to realize the appearance of an entropic potential in a reduced dimension when a Brownian particle is confined in a higher-dimensional space having varying cross section [14–16]. That is, the motion of the particle is retarded due to the variation of the width of the confinement even when there is no conventional potential barrier present. This dimension reduction formulation is still valid [18] in the presence of external bias [28] when the force is not too high.

III. NUMERICAL SIMULATIONS: RESULTS AND DISCUSSION

We have numerically simulated the overdamped two-dimensional dynamics of the Brownian particle [Eq. (6)] along with the boundary conditions Eq. (7) using an improved Euler algorithm. The time step has been chosen to be equal to 10^{-3}. Noise terms have been generated using the Box-Muller algorithm. We have numerically checked that the fluctuation-dissipation relation is satisfied in our study. The values of a, b, and c are set as 0.25, 0.5, and 0.02 for the entire study. We have kept the value of the transverse force G equal to zero throughout our study. For analytical consideration, there is a problem in making G exactly equal to zero because it leads to an unrealistic value of the entropic potential [Eq. (11)]. An entropy-dominated situation [18] is realized when the transverse force G tends to zero. This problem does not arise for numerical simulations and one may set the value of G equal to zero as we solve the exact two-dimensional Langevin dynamics [Eq. (6)] of the particle subjected to reflecting boundary conditions [Eq. (7)]. Although in practice, a numerical study using $G \to 0$ and $G = 0$ leads to the same result, to make sure of the purely entropic nature of the memory, we have set $G = 0$ for our entire numerical study. This ensures that we are actually analyzing the statistics of work and heat associated with an entropic memory erasure procedure. There appears no contribution from any field derived from a potential. The whole work and heat effect arises as a nontrivial boundary phenomenon. The erasing process has been demonstrated in the x vs t plot for 10^2 particles for two different values of forcing amplitude $F_{\max}$ [Figs. 2(a) and 2(b)]. As expected, during the thermalization time, the x value concentrates around $\pm x_m$ (the two positions of the maximal width of the structure), reflecting the equal occupancy of the two binary states 0 and 1. But after switching on the bias force, one of the states (here the 0 state) is completely erased for the first case and partially for the second case.

The quantifier, i.e., the success rate accounts for the success of an erasure process, by measuring the number of cycles at the end of which the particle stays in the desired well, out of the total number of trajectories. In the present context a total number 3×10^3 of trajectories have been considered. From Figs. 2(a) and 2(b), it is evident that the success rate of the erasure process is dependent on the amplitude of the bias force. A very small value of the forcing amplitude cannot make the particle cross the barrier (i.e., erase one bit of information). A plot of the percent success rate against $F_{\max}$ shows that the success rate is very small for low amplitude and saturates at the maximum value (100%) after crossing a threshold value of $F_{\max}$ (Fig. 3). It is observed that the saturation occurs faster for a lower value of noise strength D. The product $F_{\max}\tau$ has been kept constant throughout the calculation of the plot of percent success rate vs $F_{\max}$ for definiteness [10].

The quantity of primary importance is the work done corresponding to the erasing of a bit of information. We

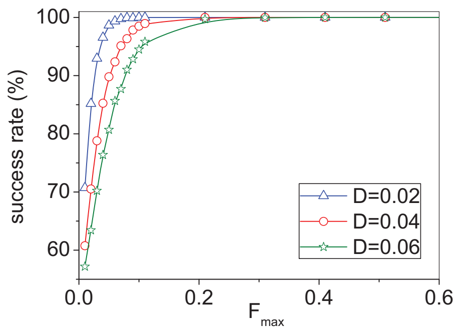

FIG. 3. (Color online) Percent success rate vs $F_{\max}$ plot for three different values of D. The parameter set used is $t_{\text{therm}} = 400$, $F_{\max}\tau = 600$, and $G = 0$.

calculate the work done using the following equation:

$$W = \int_{t_{\text{therm}}}^{t_{\text{therm}}+\tau} F(t)\dot{x}\,dt. \tag{12}$$

The above expression follows from the definition of work itself: the force acting on a particle multiplied by the displacement caused by the application of this force. And as we intend to capture the history of the entire time interval, we multiply force by velocity and integrate over the time period of action of the force. As erasing occurs due to the application of the external force and is associated with the displacement of the particle from one memory state to another (i.e., one lobe to the other), the above expression represents the work corresponding to the memory erasure procedure. Equations (6), (7), and (8) along with Eq. (12) are used for computing the work value related to erasure. It is clear from the expression Eq. (12) that when the particle moves along the direction of application of the force, work is absorbed by the particle and when it moves against the force work is produced. When we calculate $\langle W\rangle$, it is observed that the average value of work done as per the erasure protocol increases with an increasing value of $F_{\max}$. But the value of $\langle W\rangle$ can never reach a value less than that of the Landauer limit in any circumstances for a complete erasure process. However, for incomplete erasure $\langle W\rangle$ can have a value less than the limiting value. We demonstrate this as follows: As mentioned earlier, $\langle W\rangle$ increases monotonically with $F_{\max}$. So, the minimum value of $\langle W\rangle$ for a complete erasure process is achieved at the minimum $F_{\max}$ value at which full erasing occurs. From the minimum values of $F_{\max}$ for which complete erasure takes place for the given values of D (as obtained from Fig. 3), we designate the threshold values of the forcing amplitude. From the plot of $\langle W\rangle$ against $F_{\max}$ for three different values of D (same as those used in Fig. 3) shown in Fig. 4(a), it is evident that the value $\langle W\rangle$ is always above the level k_BTln2, i.e., the Landauer limit, for the $F_{\max}$ values which lie above the threshold. (W actually represents dimensionless work value. So, we would compare this with $Dln2$ containing the dimensionless temperature D.) We have plotted $\langle W\rangle$ for the $F_{\max}$ values which correspond to full memory erasure in Fig. 4(a) and scan the region near the threshold values of $F_{\max}$ and the Landauer limits for three different values of temperatures in Fig. 4(b) for better understanding. In Fig. 4(c) we have plotted $\langle W\rangle$ against $F_{\max}$ for a single temperature to

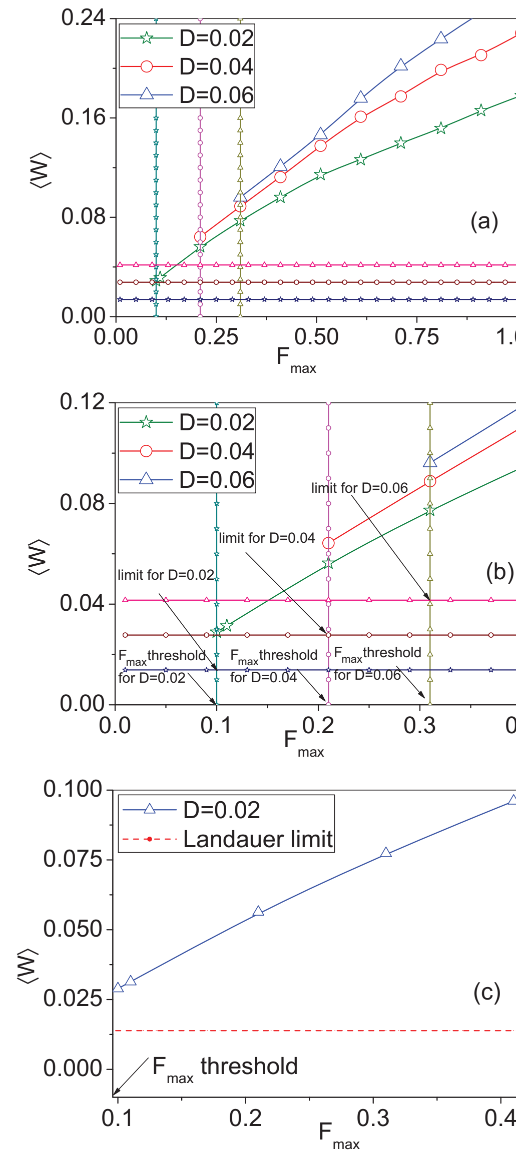

FIG. 4. (Color online) (a) $\langle W\rangle$ vs $F_{\max}$ plot for three different values of : $D = 0.02$ (stars), $D = 0.04$ (circles), and $D = 0.06$ (up triangles). The parameter set used is $t_{\text{therm}} = 400$, $F_{\max}\tau = 600$, and $G = 0$. The vertical lines represent the threshold values of $F_{\max}$ for three different values of D (line with small stars for $D = 0.02$, line with small circles for $D = 0.04$, and line with small up triangles for $D = 0.06$). The horizontal lines represent the limiting value of $\langle W\rangle$, $D\ln 2$, for three different values of D (line with small stars for $D = 0.02$, line with small circles for $D = 0.04$, and line with small up triangles for $D = 0.06$). (b) The region near the thresholds of $F_{\max}$ and the limiting values of $\langle W\rangle$ of (a) is scanned. (c) $\langle W\rangle$ vs $F_{\max}$ for $D = 0.02$.

clearly show that $\langle W\rangle$ always lies above the Landauer limit for a full erasure of a bit of information. But for $F_{\max}$ values below the threshold, i.e., for incomplete erasure, $\langle W\rangle$ can have a value smaller than that of the limiting value. We analyze the above findings as follows. The Landauer principle gives the limiting value of $\langle W\rangle$ for the complete erasure of one bit of information in a system with thermal fluctuations [10,11]. In the present study, this corresponds to the average value of

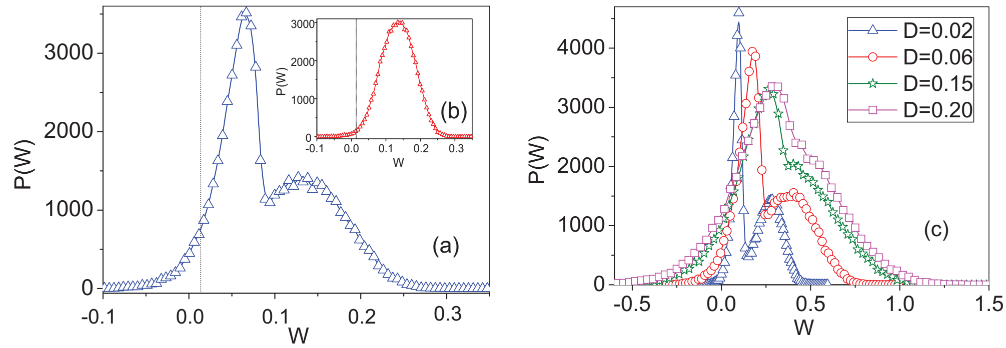

FIG. 5. (Color online) (a) $P(W)$ vs W plot for a system having an equally probable two-state distribution initially. (b) $P(W)$ vs W plot for a system having only one occupied state (left lobe) initially. (c) Bimodal $P(W)$ vs W distribution for four different values of D. The parameter set used is $t_{\text{therm}} = 400$ for (a) and (c); $\tau = 1500$ for (a) and (b) and 600 for (c); $F_{\max} = 0.4$ for (a) and (b) and 1.0 for (c); $D = 0.02$ for (a) and (b); and $G = 0$.

work associated with the complete transfer of an ensemble of particles from one lobe (say, the left lobe or the binary state 0) to the other lobe (say, the right lobe or the binary state 1). In our study, it has been found that the value of $\langle W \rangle$ satisfies the Landauer limit even when the memory has an entropic origin. Our result supports the recent development in information thermodynamics [6,7]. These studies [6,7] reveal that the lower bound of thermodynamic energy associated with the erasure procedure obeys the Landauer principle for symmetric memory. In the present study, we deal with symmetric memory as both the memory states (i.e., the left lobe and the right lobe of the enclosure) are occupied with equal probability in the absence of any external bias. And the limiting value of $\langle W \rangle$ validates the Landauer bound. For an incomplete erasure process, the value of $\langle W \rangle$ might lie below the Landauer limit as this is associated with partial transfer of particles from one lobe to the other, giving rise to an average value of W lower than that of the Landauer limit. This is because a lesser number of transfer of particles in an ensemble would require less work as the particles which remain in the starting lobe contribute almost no work to the total work value and consequently to its average. For a success rate r, the generalized Landauer bound [10] is realized as $k_B T[\ln 2 + r\ln(r) + (1-r)\ln(1-r)]$. So it is evident that for complete erasure, i.e., $r = 1$, the lower bound of the average work done reaches the Landauer bound, and for any incomplete erasure, i.e., $r < 1$, the limiting value lies below the Landauer limit. But this does not mean that the Landauer principle is being violated in our study. If we concentrate on the region of complete erasure (where one should apply the Landauer principle) in the $F_{\max}$ parameter space, it is evident that the Landauer principle is valid for all circumstances.

We have calculated the work distribution for 6×10^4 trajectories. Figure 5(a) represents the plot of the probability distribution for erasure work allowing the particles first to become thermalized for a considerable time before the application of the bias force. In the inset Fig. 5(b), we have demonstrated the distribution of erasure work, conditionally keeping the particles initially in the left lobe. In the first case, transitions occur to state 1 (desired state), i.e., to the right lobe from both the states 0 and 1 (left and right lobe). The distribution is bimodal [Fig. 5(a)]; the left peak corresponds to the work done by half the particles which were already located at the right lobe, whereas the right peak corresponds to the erasure work done by the other half of the particles which were initially present at the left lobe. We obtain a unimodal distribution as depicted in Fig. 5(b) when we initially keep all the particles in the left lobe, i.e., at state 0. The peak displays the $0 \rightarrow 1$ transition of particles representing bits of information. The important thing to note here is that for individual realizations, W can have values lower than that of the Landauer limit (can even have values less than zero) [10,11]. But the average value of W is always greater than this limit for a full memory erasure. This observation deserves explanation. Thermodynamic quantities, such as W, are stochastic variables at the microscopic level. As W is being calculated using direct Langevin dynamics for a single trajectory, the effect of thermal fluctuation is reflected in the individual values of W. Due to the thermal fluctuation induced by the heat bath, W can have a value lower than the Landauer limit for an individual realization. This is demonstrated in Figs. 5(a) and 5(b). The vertical dotted lines present the Landauer limit. It is observed that W can lie below the Landauer limit for a very small fraction of realizations. Again, this does not imply any departure from the Landauer principle. Actually, when one is looking for a thermodynamic description of a system with thermal fluctuation, it is advisable to concentrate on the average quantities as they contain the entire information of the system as a whole. As a consequence, in the presence of thermal fluctuation it is better to compare $\langle W \rangle$ with the Landauer limit [10,11] rather than comparing W obtained from a single realization of the trajectory. $\langle W \rangle$ never crosses the Landauer limit for a full erasure process, thus satisfying the principle for a system with thermal fluctuation. We have plotted $P(W)$ vs W for different noise strengths in Fig. 5(c). It has been observed that with increasing noise strength, the two peaks come close together and at very high values of noise strength, we obtain a unimodal distribution.

Another important observation is that for a large erasure cycle $\langle W \rangle$ approaches a limiting value. This is shown in Fig. 6, where the averaging has been done over 10^4 trajectories. One important thing to note here is that the limiting value of $\langle W \rangle$ for large values of τ lies above the Landauer limit, whereas in the case of a similar study in the presence of an energetic barrier, $\langle W \rangle$ approaches the Landauer limit. The reason behind this

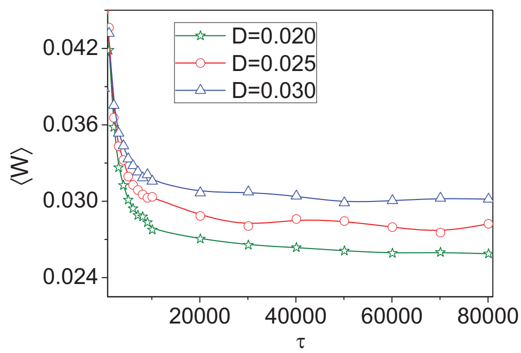

FIG. 6. (Color online) $\langle W \rangle$ vs τ plot for three different values of D. The parameter set used is $t_{\text{therm}} = 400$, $F_{\text{max}} = 0.1$, and $G = 0$.

observation may be explained in this way. In the energetic case, there is always an intrinsic force field present in the system. This force field derived from the potential assists the externally applied force directly to transfer the particles (i.e., information) from one potential well (i.e., state) to another, and the average work done approaches the Landauer limit in the case of a very slow erasure process. But particles subjected to an entropic barrier in a reduced dimension are essentially free when we consider their movement in the actual two-dimensional system. The only constraint is that the particles cannot move out of the given confinement. As there is no intrinsic force field present in the system with varying cross section, the external force does not get direct assistance from the system nonlinearity in the case of directed transfer of particles from one lobe to another, i.e., while erasing one kind of bit of information. It is true that the shape of the system plays a role in this process but that cooperation may be rather indirect and less effective compared to the memory erasing process when the two memory states are separated by an energetic barrier. The erasing process requires extra work done on the particle when the memory states are entropic in nature. As a consequence, the value of $\langle W \rangle$ saturates to a value higher than that of the Landauer limit for a very large erasure cycle.

We now emphasize a pertinent point. In previous studies on the Landauer limit in the energetic domain, the erasure protocol is accompanied by a cycle involving two steps: symmetric lowering of the potential energy barrier and tilting of the potential. The lowering of the barrier does not contribute to the calculation of the work done [10]. This is even more obvious in systems having physical confinement with varying width. To manipulate the depth or size of the lobes, one has to modulate the wall of the system. As a result, the barrier lowering part does not contribute to the direct numerical calculation of the work done. It is therefore appropriate to omit this step and use only the external forcing which drives the particle to the desired lobe. As there is no symmetric modulation of the wall function (equivalent to the symmetric lowering of the potential energy barrier) in our study, we do not keep the entropic barrier too high so that barrier crossing becomes too improbable. In a true sense, we assume that barrier lowering has already occurred when we start our time of observation, and terminate the cycle instantly after the bias force is switched off. Thus the system regains its initial state as soon as the bias force is turned off.

IV. CONCLUSION

We have explained an entropic analog of the information erasing procedure. It is observed that individual realizations may turn up with work values lower than that of the Landauer limit due to thermal fluctuation. But the average value of work done is always greater than the limiting value even when the information has an entropic representation. The Landauer bound can be realized for an entropic potential. The lower limit of the average work associated with entropic memory erasure does not cross the Landauer bound in any circumstances.

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