Inequity aversion and the evolution of cooperation

Asrar Ahmed[*] and Kamalakar Karlapalem[†]

International Institute of Information Technology-Hyderabad, Centre for Data Engineering, Hyderabad, India
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Evolution of cooperation is a widely studied problem in biology, social science, economics, and artificial intelligence. Most of the existing approaches that explain cooperation rely on some notion of direct or indirect reciprocity. These reciprocity based models assume agents recognize their partner and know their previous interactions, which requires advanced cognitive abilities. In this paper we are interested in developing a model that produces cooperation without requiring any explicit memory of previous game plays. Our model is based on the notion of *inequity aversion*, a concept introduced within behavioral economics, whereby individuals care about payoff equality in outcomes. Here we explore the effect of using income inequality to guide partner selection and interaction. We study our model by considering both the well-mixed and the spatially structured population and present the conditions under which cooperation becomes dominant. Our results support the hypothesis that inequity aversion promotes cooperative relationship among nonkin.

I. INTRODUCTION

The evolution of cooperative behavior among seemingly competing organisms presents a challenge to the theory of natural selection [1]. An action that benefits others and comes at a cost to oneself should eventually disappear. On the contrary, there are numerous biological and social settings in animal and human societies where cooperation is ubiquitous [2–4]. Evolutionary game theory (EGT) [5,6] provides a simple framework to study this puzzle. In each generation agents play a finite number of games and receive reward according to a specified payoff matrix, usually the prisoner's dilemma game [7]. At the end of each generation, players are reproduced proportional to their relative fitness, subject to mutation. Thus EGT mimics natural selection by increasing the relative abundance of better performing (in terms of accumulated reward) individuals.

The prisoner's dilemma (PD) is one of the most widely used games to study the evolution of cooperation [8–11]. PD is a simultaneous two player game where each player decides whether to cooperate (C) or defect (D). Mutual cooperation gives higher payoff than mutual defection. But if one player cooperates, the other player is better off defecting. Thus each player has an incentive to "free ride" at a cost to the cooperative player. In the absence of any special mechanism defecting players would outperform cooperative players and (by natural selection) become dominant [12]. The game serves as a metaphor for various real-world settings where there is a conflict between individual and group interest [13].

Although many solutions have been put forth to resolve this puzzle, most of these models rely on some notion of direct or indirect reciprocity. In direct reciprocity [14,15] agents interact repeatedly, and after each interaction they update their response based on how cooperative (kind) or noncooperative (unkind) the other agent was. Indirect reciprocity [16–18] relies on the notion that individuals are kind to those who are kind to others. Both models assume agents recognize their

partner and know their previous interactions. In large dynamic settings such an assumption would require advanced cognitive abilities.

Approaches that do not require reciprocity include kin selection and group selection models. In kin selection [19,20] agents are favorably biased towards their genetic relatives. It requires agents to differentiate between kin and nonkin agents and does not explain cooperative outcomes observed in unrelated individuals. Group selection [21,22] addresses these issues by proposing that selection works not only on individuals but also on groups. Agents form groups and cooperate with other agents in the same group.

An interesting addition to the above reciprocity-free models of cooperation is the *tag-mediated* partner selection [23,24]. Tags are simple observable traits or cultural artifacts which agents use to favorably bias their interactions with agents having similar tags. Experimental results [24] show that a tag based model can produce stable cooperation even in single round prisoner's dilemma game.

In this paper, we are interested in developing a model of cooperation that does not require an agent's type or its past interaction to be known explicitly. Our model is based on the notion that individuals are averse to income inequalities. This concept has been formalized and widely studied within behavioral economics as *inequity aversion* [25]. It was introduced to account for outcome anomalies between experimental results and theoretical models. In the inequity aversion model, as proposed by Fehr and Schmidt [25], individuals enforce income equality by forgoing monetary payoff. If the material payoffs of players i, j are x_i, x_j, respectively, the *experienced utility* (which players maximize) of player i is given by

$$U_i(x_i,x_j) = x_i - k_1 \max(x_j - x_i, 0) - k_2 \max(x_i - x_j, 0),$$

where $k_1 < k_2$ and $0 \leqslant k_2 \leqslant 1$ are inequity aversion sensitivity parameters. It follows from the above equation that the experienced utility is maximized when inequity is zero. The condition $k_1 < k_2$ implies that the decrease in a player's utility is higher when it is behind (in terms of payoff) than when it is ahead. Fehr and Schmidt [25] show that the above

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[*]asrar.ahmed@research.iiit.ac.in
[†]kamal@iiit.ac.in

model can account for cooperation observed in various human interactions.

Here we study the effect of using the social paradigm of inequity aversion as a criterion for partner selection. Instead of defining experienced utility we simply allow agents to use accumulated payoff to select their game partner. Thus inequity aversion in our context implies that agents avoid interacting with other agents whose accumulated payoff is higher or lower than their own payoff. Our experimental results show that cooperation can emerge even if individuals receive only rudimentary environmental signals about others' well-being (and not their type or specific behavior). It supports the hypotheses that inequity aversion promotes cooperation among nonkin [26]. We also observe a strong correlation between cooperation and inequity aversion that indicates a possible coevolution of these two behaviors [26].

The rest of the paper is organized as follows. We first present the model and define the partner selection bias as a function of income inequality. We then evaluate the model by considering both the well-mixed and the spatially structured population and highlight the evolutionary dynamics of cooperation and inequity aversion. We also study the effects of various parameters on the fraction of cooperative players in the environment and then conclude.

II. MODEL

In this section we first introduce the general model where agents interact in a well-mixed population. We later present the model with spatial constraints.

Agents. The model consists of a fixed number of agents N. Each agent is either the cooperative or defecting type. The cooperative type always plays cooperate, and the defecting type always plays defect. Agents have an associated parameter λ which gives a measure of how sensitive or tolerant they are to payoff inequality. A player's type and sensitivity parameter are subjected to evolutionary changes (see evolutionary step below). Players also have an accumulated reward which is initialized to zero at the start of each generation and updated according to the payoff received in each game.

Payoff matrix. The payoff received by each player is specified by the standard prisoner's dilemma game:

$$\begin{pmatrix} b-c & -c \\ b & 0 \end{pmatrix}.$$

Cooperative players provide benefit b to their opponent and incur a cost c ($b > c > 0$). Defecting players neither provide benefit nor incur any cost. Given the payoff structure, the dominant strategy for each player, irrespective of what the other player does, is to defect. Thus players end up with a payoff of zero instead of the mutually beneficial payoff $b-c$. This characterizes the dilemma between individual interest and group well-being. Without loss of generality, we use the following normalized payoff matrix:

$$\begin{pmatrix} 1 & 0 \\ 1+c/b & c/b \end{pmatrix}.$$

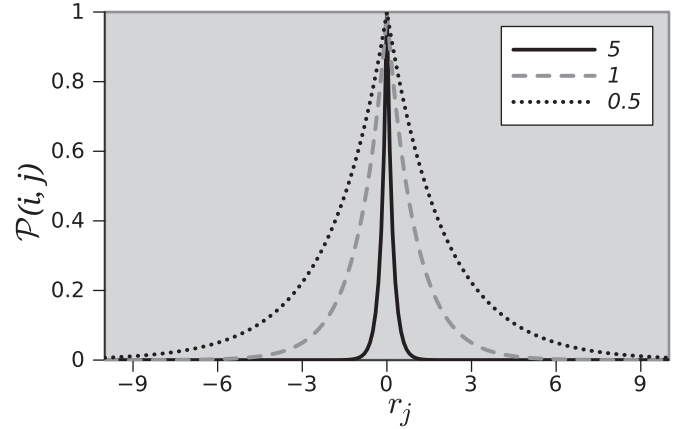This allows us to study the game as a function of a single cost-to-benefit parameter, $0 < c/b < 1$ [27,28].

FIG. 1. $\mathcal{P}(i,j)$ for different values of r_j and $\lambda_i = \{5,1,0.5\}$ when $r_i = 0$ and $\lambda_j = 0$.

Partner selection and interaction. In each generation, agents are selected sequentially (random order) and play a single round of the prisoner's dilemma game. Players use their accumulated payoff and sensitivity parameter to determine their game partner. The search space variable $N_s \in N$ gives the subset of agents from which players choose their partner. Denote r_i, r_j as the accumulated payoff and λ_i, λ_j as the sensitivity parameters of players i, j, respectively. The probability that player i would accept player j is given by

$$\phi(i,j) = e^{-\lambda_i |r_i - r_j|}.$$

Given the search space N_s, player i selects player j such that the above probability is maximized, $j = \arg\max_{j \in N_s} \phi(i,j)$. We similarly define $\phi(j,i)$ as the probability that player j would accept player i as its partner. The probability of interaction between players i and j is given by

$$\mathcal{P}(i,j) = \phi(i,j)\phi(j,i) = e^{-(\lambda_i + \lambda_j)|r_i - r_j|}. \tag{1}$$

The interaction is bilateral or with mutual consent since the probability of interaction depends on both players' sensitivity parameters. We set the range of the sensitivity parameter $\lambda \in [0,5]$. If $\lambda_i = \lambda_j = 0$, players are indifferent to inequity, and as λ increases, they become increasingly inequity averse. The upper limit of 5 ensures that the probability $\mathcal{P}(i,j)$ is close to zero, even for small payoff differences. To illustrate, Fig. 1 gives the probability of interaction between players i and j for different values of λ_i when i's accumulated reward $r_i = 0$ and $\lambda_j = 0$. For $r_j = 1$ and $\lambda_i = 0.5$, the probability of interaction $\mathcal{P}(i,j) = 0.6$, which decreases to $\mathcal{P}(i,j) = 0.006$ for $\lambda_i = 5$.

Once the partner is selected, the PD game is played with the probability given in Eq. (1). If both players are cooperative, their accumulated payoff increases by 1. If only one of the players is cooperative, the defecting player's accumulated payoff increases by $1 + c/b$, and the cooperative player's accumulated payoff remains unchanged.

We note that, unlike tag-mediated models of cooperation [24] where an agent's tags remain fixed in a given generation, in our model the accumulated payoff serves as a dynamic tag which changes after each interaction.

Evolutionary step. At the end of each generation agents are reproduced using the *binary tournament* procedure [29,30]:
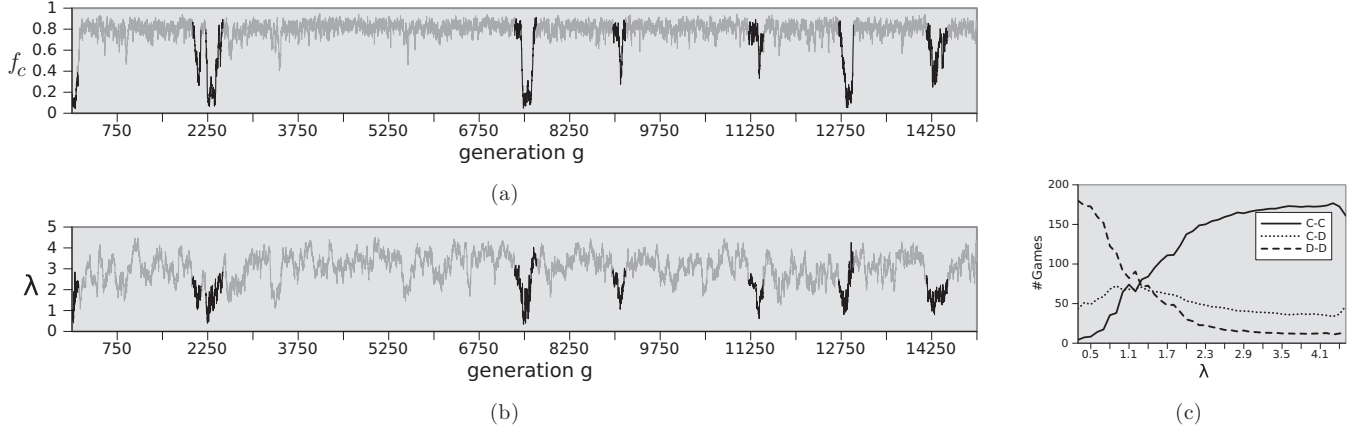
FIG. 2. (a) The fraction of cooperative players f_c across generations for a typical run for $N = 250$, $N_s = 8$, and $c/b = 0.45$. (b) The corresponding average λ value of cooperative players. (c) The corresponding number of games with C-C, C-D, and D-D interactions vs λ.

(1) Two distinct agents are randomly selected for a tournament, and the agent with the higher fitness value is declared the winner (we use the accumulated reward as the fitness value).

(2) A copy of the winner, called the *offspring*, is added to the new generation, and the above procedure is repeated N times.

Additionally, a mutation is applied to each offspring. The mutation value gives the probability with which the offspring's type is randomly reset to either the cooperative or defecting type. Since the sensitivity parameter is a continuous variable, we apply mutation by adding, with probability μ, Gaussian noise with mean 0 and deviation 1 to the inherited λ value [23]. The accumulated reward of the offspring is set to zero. The algorithm in Table I provides the pseudo code of the model.

Algorithm 1 Algorithm for agent interaction.
Require: μ, N, N_s.
  while generation $g \leqslant g^{\max}$ **do**
  for all agents $i \in N$ **do**
  select $N_s \in N$ agents randomly
  $j = \arg\max_{j \in N_s} \phi(i, j)$
  With Probability $\mathcal{P}(i, j)$, PLAY(i, j)
  Update accumulated payoff r_i, r_j
  end for
  New Set Of Players $N = $ EvolutionaryStep(N, μ)
  end while

III. RESULTS AND DISCUSSION

We recall that N is the number of agents, N_s is the search space, and c/b is the cost-to-benefit ratio. We denote the fraction of cooperative players with f_c. C-C denotes the cooperative-cooperative player interaction. We similarly define C-D and D-D interactions. For all experiments, we set $\mu = 0.1$ and initialize λ to a uniform value between $[0,5]$. The initial fraction of cooperative players is set to 10%. We report the results by averaging across 20 runs, with each run consisting of 15 000 generations. Our results cover the following aspects: (1) the evolutionary dynamics of f_c, λ, and

the correlation between them, (2) the change in the fraction of games with C-C, C-D, and D-D interactions as λ value changes, (3) the effect of the number of agents and search space on f_c as the cost-to-benefit ratio c/b increases, and (4) the effect of spatial constraints on f_c.

A. Mixed population

Figure 2(a) shows the fraction of cooperative players f_c across generations for a typical run (with $N = 250$, $N_s = 8$, and $c/b = 0.45$). Figure 2(b) gives the corresponding average λ value of cooperative players. We observe high levels of cooperation $f_c \geqslant 0.75$ interrupted by brief periods of defection. The generations when the fraction of cooperative players falls significantly are highlighted to show the close correlation between f_c and λ. Across all simulations we observe that an increase or decrease in the fraction of cooperative players is preceded by an increase or decrease in the average λ value. For example, as shown in Figs. 2(a) and 2(b), from generation 7100 to 7500, the average λ value decreases gradually from 4.2 to 0.3. We see a corresponding decrease in the f_c value from 0.84 to 0.08.

At the start of each generation, as the accumulated payoff of all players is 0, the probability of interaction between any two players is 1, irrespective of λ. So in the initial stages, defecting players have a slight advantage (since the cooperative players would not reject them as a game partner), and their accumulated payoff increases. But after the first few games, the cooperative agents involved in C-D interactions start forming temporary clusters sharing a common payoff. If these cooperative players are sufficiently inequity averse, in the subsequent iterations, they are more likely to interact within these clusters, and their payoff increases. The defecting players who initially exploit the cooperative players face a form of "social exclusion" from these groups. Thus inequity aversion allows cooperative players to seek new partners and form groups with whom they can share more equitable payoffs. As the interactions proceed, cooperative players that are more tolerant (small λ value) to income inequality continue to interact with defecting agents. They are outperformed by cooperative players with relatively high λ value and die out. Over
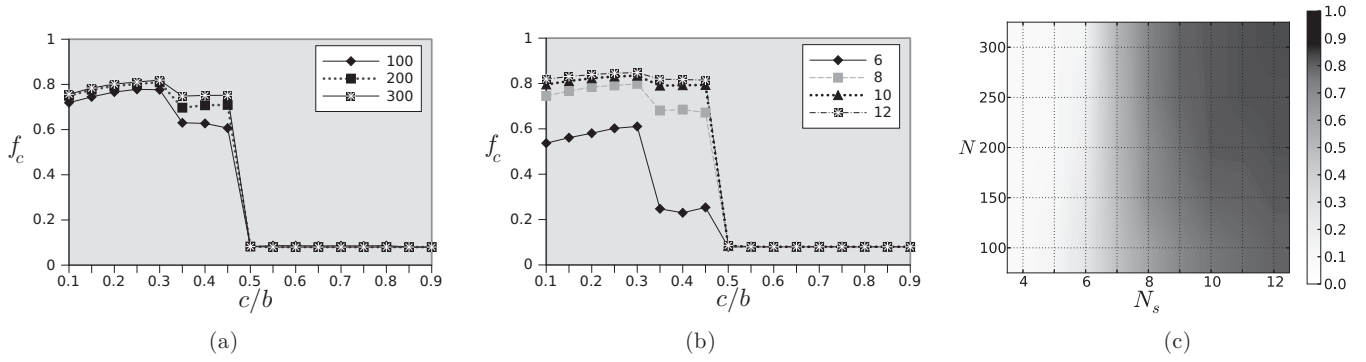
FIG. 3. (a) f_c vs c/b for $N_s = 8$ and $N = \{100, 200, 300\}$. (b) f_c vs c/b for $N = 150$ and $N_s = \{6, 8, 10, 12\}$. (c) Color map depicting f_c on the N_s-N plane for $c/b = 0.4$.

generations the average λ of cooperative players increases, which further reduces the likelihood of C-D interactions until cooperation becomes dominant.

However, this cooperation due to the temporary clustering effect is not permanent. When cooperation is established, agents have a high λ value. At this stage, the cooperative agents with a relatively smaller λ value have a slight advantage as they are more likely to tolerate inequality and play the game. And since cooperative agents are dominant, with high probability these agents interact with other cooperative agents, and their accumulated payoff increases. The environment faces a slight selection pressure towards higher tolerance levels (lower λ value). As this happens gradually over generations, at some threshold λ, agents become vulnerable to invasion by defectors, and the cooperation levels fall sharply. The system remains in this state until by chance, due to mutation, a few cooperative players with a relatively high λ value emerge and reestablish cooperation.

To further validate the correlation between f_c and λ, Fig. 2(c) shows the number of games with C-C, C-D, and D-D interactions as a function of λ. For a low λ value C-D and D-D interactions dominate. As the λ value increases, cooperative clusters emerge, and C-C becomes dominant.

The cycles of cooperation and defection are not regular or periodic, and how often the system goes into the defection state and how quickly it recovers depends on the parameters N, N_s, and c/b. In general we observe that as the cost-to-benefit ratio c/b increases or N_s decreases, it takes longer for the system to recover from the defection state.

Figures 3(a) and 3(b) show the fraction of cooperative players vs the cost-to-benefit ratio for different values of N and N_s, respectively. Across both simulations, as c/b increases, f_c decreases, and for $c/b \geqslant 0.5$, cooperation disappears. With respect to N, we observe the f_c value decreases marginally for $0.3 \leqslant c/b \leqslant 0.5$ as N decreases. The change in f_c is considerably higher with respect to change in N_s values. As the search space increases, f_c increases. The search space affects the probability of selecting an agent from within the temporary clusters that emerge. A higher search space translates to a higher probability of interaction within the cluster. We also observe a "thresholding" effect; that is, for a fixed change in N_s the increase in the f_c value is higher for smaller values of N_s. Figure 3(c) shows a color map of f_c for different values of N_s (along the X axis) and N (along the Y axis) with $c/b = 0.45$. For a high ($N_s \geqslant 12$) or low search space ($N_s \leqslant 4$), f_c does not change with N.

B. Two-dimensional lattice

We now consider the spatial prisoner's dilemma game [31,32] where each player occupies a cell in a square lattice. Similar to the well-mixed population model, agents are either the cooperative or defecting type and use their accumulated payoff to select their game partner and interact. But due to spatial constraints an agent's search space is restricted to its four neighboring cells. We also change the *binary tournament* procedure to reflect an agent's fixed position. For each offspring cell to be added to the new generation, two
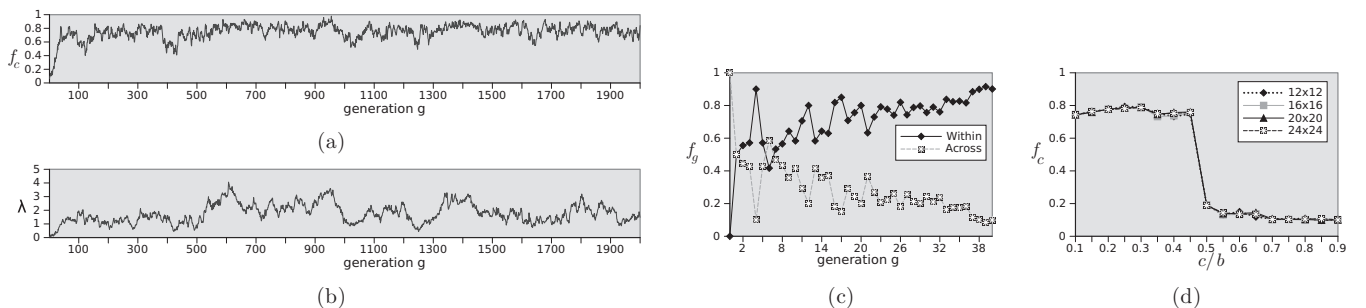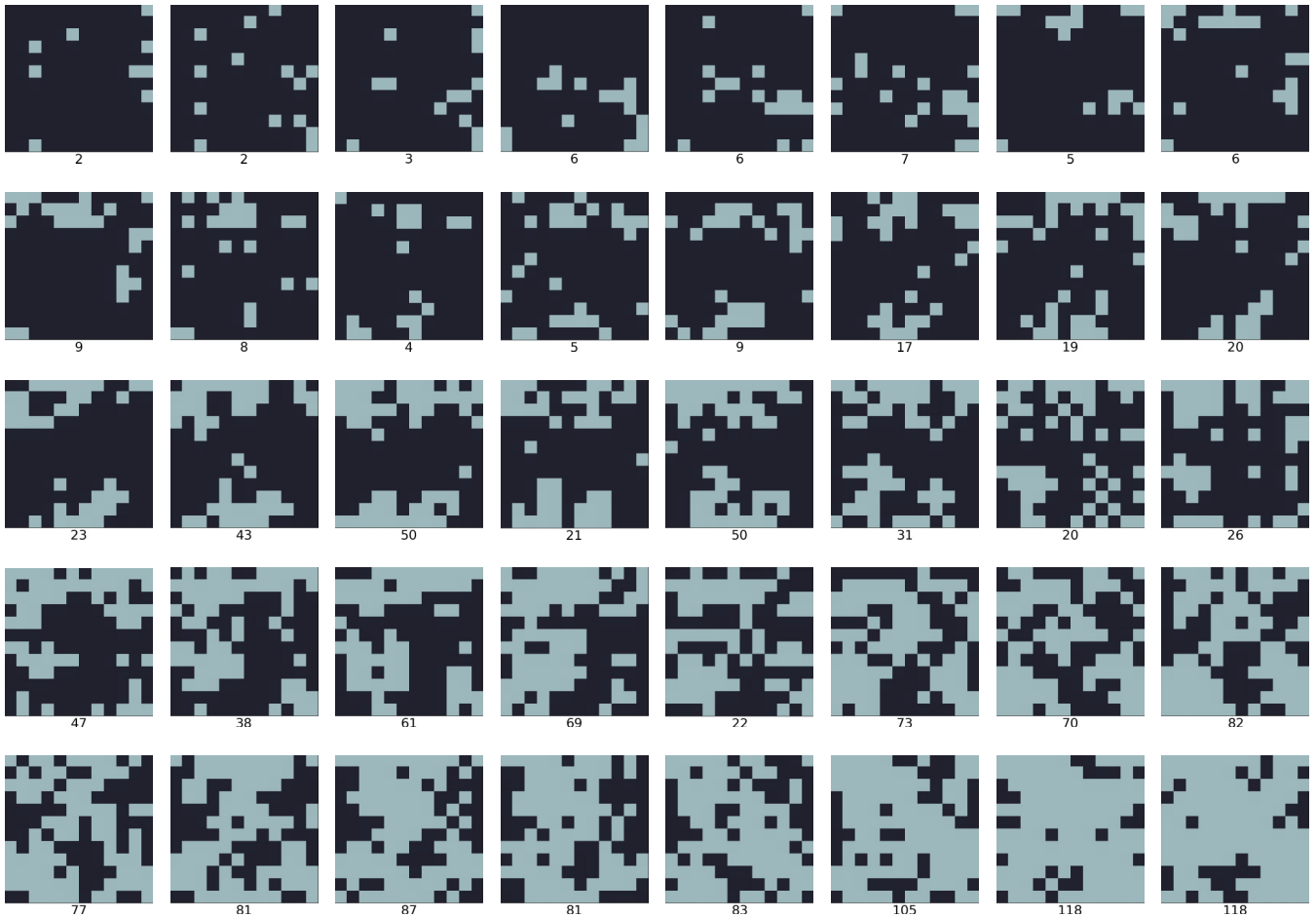

FIG. 4. (a) The fraction of cooperative players f_c across generations for a typical run with $N = 12 \times 12$ and $c/b = 0.45$. (b) The corresponding average λ value of cooperative players. (c) The corresponding fraction of games (f_g) within and outside the clusters for the first few generations. (d) f_c vs c/b for different grid sizes.

FIG. 5. (Color online) Snapshots for first 40 generations on 12×12 grid and $c/b = 0.45$. Defecting players are in dark. The labels give the size of the largest cooperative cluster.

distinct agents are randomly selected from the neighborhood of the cell. The agent with the higher fitness values is declared the winner, and the offspring inherits the winner's type and λ value. Mutation is applied as discussed in the previous model.

Figures 4(a) and 4(b) show the evolutionary dynamics of f_c and λ for a typical run with $N = 12 \times 12$ and $c/b = 0.45$ for the first 2000 generations (similar behavior is seen in subsequent generations). Across all simulations, we observe that the cooperation in the spatial prisoner's dilemma is more "robust" (i.e., unlike the previous model where cooperation almost disappears before recovering, here we see stable cooperation). The increase and decrease in the f_c value are marginal but are still closely correlated with the increase and decrease in the λ value.

While a single cooperative agent surrounded by defectors is always outperformed, cooperative players that are sufficiently inequity averse and adjacent to each other have two advantages: (1) the temporary clustering that arises due to sharing a common payoff and (2) the clustering provided by spatial constraints. Like the previous model, in the initial games of a generation, if cooperative and defecting agents interact, the inequity in accumulated payoff ensures that in the subsequent iterations the probability of C-D interactions decreases. And as is traditionally the case, the spatial constraint further improves the performance of cooperative players since

players in the interior of the cluster enjoy the benefit of mutual cooperation. Figure 4(c) shows the fraction of interactions (for the above sample run) within and outside the clusters for the first few generations. The corresponding snapshots for the first 40 generations along with the size of the largest cooperative cluster (given below each plot) are shown in Fig. 5. Defecting players are in black. We observe the cooperative players become dominant by generation 32.

Figure 4(d) shows the fraction of cooperators vs c/b for different grid sizes. We observe f_c does not change with N, and similar to the well-mixed model, cooperation disappears for $c/b \geqslant 0.5$.

IV. CONCLUSION

We developed a model of agent interaction that is motivated by the social paradigm that individuals are inequity averse and prefer to interact with others within the same social strata. We presented the results by considering both the well-mixed and the spatially structured populations across different parameter values. In general, cooperation becomes dominant when the cost of cooperation is low and is more robust for a structured population.

Our results support the hypotheses that inequity aversion promotes cooperation among nonkin [26]. It allows individuals

to seek new partners with whom they can share more equitable payoffs. And if the equitable payoff increases the relative fitness of such individuals (as is the case with the prisoner's dilemma), natural selection would guarantee that inequity-averse cooperative agents emerge. We also observe a strong correlation between inequity aversion and cooperation that points to the coevolution of these behaviors. Brosnan [26] provides an extensive discussion of inequity aversion observed in other species [33,34], including capuchin monkeys

[35], which "elucidate evolutionary precursors to inequity aversion" [36].

We believe the model presented in this paper is an important step towards better understanding coevolution of cooperation and inequity aversion. In the future, we intend to evaluate the model by considering generic random networks and to incorporate other social factors like group membership, dominance rank, context of interaction, etc., which have been shown to effect the overall response to inequity [26].

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