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Interplay between spin-glass clusters and geometrical frustration

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The presence of spin-glass (SG) order in highly geometrically frustrated systems is analyzed in a cluster SG model. The model considers infinite-range disordered interactions among cluster magnetic moments and the J_1-J_2 model couplings between Ising spins of the same cluster. This model can introduce two sources of frustration: one coming from the disordered interactions and another coming from the J_1-J_2 intracluster interactions (intrinsic frustration). The framework of one-step replica symmetry breaking is adopted to obtain a one-cluster problem that is exactly solved. As a main result we create phase diagrams of the temperature T versus intensity of the disorder J, where the paramagnetic-SG phase transition occurs at T_f when T decreases for high-J values. For low-J values, the SG order is absent for antiferromagnetic clusters without intrinsic frustration. However, the SG order can be observed within the intracluster intrinsic frustration regime even for lower intensity of disorder. In particular, the results indicate that the presence of small clusters in geometrically frustrated antiferromagnetic systems can help stabilize the SG order within a weak disorder.

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I. INTRODUCTION

The interplay between geometrical frustration (GF) and disorder is an interesting problem with many open questions. For instance, there are several geometrically frustrated magnets that exhibit spin-glass (SG) -like behavior such as $Y_2Mo_2O_7$ [1,2], $Zn_{1-x}Cd_xCr_2O_4$ [3], and $ZnCr_{2(1-x)}Ga_{2x}O_4$ [4,5]. Nevertheless, the level of disorder present in some of these geometrically frustrated systems has been estimated to be extremely small [1,4]. In fact, the amount of disorder to stabilize a glassy state in these real systems is much smaller than predicted by usual mean field theory for canonical SG systems [2,6]. Therefore, a very intriguing issue is how a SG-like state is stabilized in these geometrically frustrated systems with very small disorder. This issue is the main subject of the present work.

It should be remarked that there are few theoretical works to account the interplay between disorder and GF in systems presenting a SG-like state. For example, the classical Heisenberg antiferromagnet model with random variations in the exchange interactions has been studied to consider the SG state in geometrically frustrated systems [7,8]. This random bond model with weak disorder has estimated a critical temperature T_f proportional to the disorder strength. The pyrochlore compound $Y_2Mo_2O_7$ has been investigated by Monte Carlo simulations of this antiferromagnetic bond-disordered model with additional local lattice distortions [9]. In particular, a high T_f for the GF regime with very low disorder was found and attributed to the spin-lattice coupling [9]. The Heisenberg model on the pyrochlore lattice with random disorder was also simulated in Ref. [10], which indicated a possible cluster glass scenario to describe the SG state in geometrically frustrated systems. Nevertheless, no precise mechanism was proposed. Therefore, alternative approaches are required in order to understand the low-disorder SG phase in highly geometrically frustrated systems in which, for instance, magnetically correlated spin clusters can be present as in the case of $ZnCr_{2(1-x)}Ga_{2x}O_4$ [4,5].

In this work we present an approach in which the presence of spin clusters can be an essential element to account for the extreme sensitivity to the effects of disorder in geometrically frustrated systems. We consider clusters of spins with nondisordered intracluster spin interactions. The disorder appears only as a quenched random magnetic interaction between these spin clusters. For an appropriate choice of intracluster interactions, GF can arise, leading to a degeneracy of intracluster spins configurations. In that case, the intercluster random interaction could select from the manifold of spins configurations those that not only avoid, for instance, the full compensation of the total cluster magnetic moment but also greatly enhance the spin cluster sensitivity to the intercluster random interaction stabilizing a cluster SG order even when the intercluster random interaction is very weak.

Motivated by the above considerations, we study a simple cluster spin model in which it is possible to introduce intracluster frustration and disorder. This model considers clusters with a square lattice geometry that has two types of interactions: intercluster long-range disordered interactions and intracluster short-range interactions between first J_1 and second neighbors J_2, which can be an antiferromagnetic (AF) interaction or a ferromagnetic exchange (FE) one. The short-range interactions are between Ising spins on a square lattice that belong to the same cluster. Therefore, the intracluster interaction is given by the so-called J_1-J_2 model that can introduce frustration by adjusting the relation between J_1 and J_2 [11]. This we call intrinsic frustration.

The present cluster SG model with $J_2 = 0$ (without intracluster frustration) was used successfully to studied the inverse freezing (IF) [12]. Inverse freezing is characterized by a transition from the SG state to the paramagnetic phase when the temperature diminishes [13]. In this case, the simultaneous presence of short-range interactions with AF cluster spin compensation and disorder has been pointed out as being responsible for the IF phenomenon [12]. Therefore, the present work could also provide clarification of the mechanisms underlying IF, in which the interplay among disorder, AF short-range interactions, and intrinsic frustration is explored.

II. GENERAL FORMULATION

The model can be obtained from the Ising spin lattice that is divided into N_{cl} clusters with n_s sites in each cluster [14]. In the present approach the intercluster interactions are assumed to be of infinite range and disordered, while the intracluster interactions are short range [14]. This model can be described by the Hamiltonian

$$H = -\sum_{\nu\lambda}^{N_{cl}} J_{\nu\lambda} S_\nu S_\lambda - \sum_{\nu}^{N_{cl}} \sum_{(i,j)}^{n_s} J_{ij}^{\nu} s_{i\nu} s_{j\nu}, \tag{1}$$

where $J_{\nu\lambda}$ is a random variable that follows a Gaussian distribution with zero mean and variance J^2/N_{cl}. In Eq. (1), $J_{\nu\lambda}$ couples all distinct pairs of clusters, in which $S_\nu = \sum_i^{n_s} s_{i\nu}$ is the magnetic moment of cluster ν with $s_{i\nu}$ representing the Ising spin of site i of cluster ν. The intracluster interaction follows the J_1-J_2 model and is written as

$$\sum_{(i,j)}^{n_s} J_{i,j}^{\nu} s_{i\nu} s_{j\nu} = \sum_{(i,j)_1}^{n_s} J_1 s_{i\nu} s_{j\nu} + \sum_{(i,j)_2}^{n_s} J_2 s_{i\nu} s_{j\nu}, \tag{2}$$

where the sum $(i,j)_1$ $[(i,j)_2]$ runs over all nearest-neighbor (next-nearest-neighbor) sites of cluster ν with a square lattice geometry. In this work we study two types of couplings between nearest neighbor J_1 and next-nearest neighbor J_2. The first one considers J_1 and J_2 to be antiferromagnetic, while in the second J_1 is ferromagnetic and J_2 AF.

The replica method is used to carry out the disorder average. The average free energy per cluster is $\beta f = -\lim_{n\to 0}(\langle Z^n\rangle_{J_{\nu\lambda}} - 1)/N_{cl}n$, where $\langle\cdots\rangle_{J_{\nu\lambda}}$ means the average over the quenched disorder of $J_{\nu\lambda}$ and Z^n is the replicated partition function. This produces an effective replica Hamiltonian

$$H = -\frac{\beta J^2}{2N_{cl}} \sum_{\nu\lambda}^{N_{cl}} \sum_{\alpha,\gamma}^{n} S_\nu^\alpha S_\nu^\gamma S_\lambda^\alpha S_\lambda^\gamma - \sum_{\nu}^{N_{cl}} \sum_{\alpha}^{n} \sum_{(i,j)}^{n_s} J_{i,j}^{\nu} s_{i\nu}^\alpha s_{j\nu}^\alpha, \tag{3}$$

where $\beta = 1/T$ (T is the temperature) and α and γ are replica labels.

The four-spin cluster term is decoupled, introducing the replica matrix elements $\{Q\}$ via Hubbard-Stratonovitch transformations. The free energy per cluster is then obtained as

$$\beta f = \lim_{n\to 0} \frac{1}{n} \left\{ \frac{\beta^2 J^2}{2} \sum_{\alpha\gamma} Q_{\alpha\gamma}^2 - \frac{1}{N_{cl}} \ln \mathrm{Tr} \exp \right. \\ \left. \times \left[\beta \sum_{\nu} \left(\sum_{\alpha} \sum_{(i,j)}^{n_s} J_{i,j}^{\nu} s_{i\nu}^\alpha s_{j\nu}^\alpha + \sum_{\alpha\gamma} \beta J^2 Q_{\alpha\gamma} S_\nu^\alpha S_\nu^\gamma \right) \right] \right\}, \tag{4}$$

where the saddle-point equations are used, resulting in $Q^{\alpha\gamma} = \frac{1}{N_{cl}} \langle \sum_\nu S_\nu^\alpha S_\nu^\gamma \rangle$.

Parisi's scheme of one-step replica symmetry breaking (1S RSB) [15] is adopted to parametrize the replica matrix as $R = Q_{\alpha\alpha}$ and

$$Q_{\alpha,\gamma} = \begin{cases} Q_1 & \text{if} \quad I(\alpha/a) = I(\gamma/a) \\ Q_0 & \text{if} \quad I(\alpha/a) \neq I(\gamma/a), \end{cases} \tag{5}$$

where $I(x)$ gives the smallest integer that is greater than or equal to x. The order parameter $R = \langle S_\nu^\alpha S_\nu^\alpha \rangle$ represents the replica diagonal correlation of the cluster magnetic moment. Different from the canonical Ising SG models (where $R = 1$), here it can range from 0 to n_s^2. In addition, R can be interpreted as the intensity of the cluster magnetic moment [14]. In particular, R is strongly affected by AF intracluster interactions that can introduce compensated magnetic moments, decreasing the intensity of the cluster magnetic moment ($R \to 0$). The replica symmetry breaking is given by $Q_1 - Q_0$, which is the SG order parameter. The parameter a represents the size of diagonal blocks of the 1S RSB solution.

In this approximation, the free energy is obtained as

$$\beta f = \frac{\beta^2 J^2}{4}\left[R^2 + a\left(Q_1^2 - Q_0^2\right) - Q_1^2\right] \\ - \frac{1}{a} \int Dz \ln \int Dv [K(z,v)]^a, \tag{6}$$

where $K(z,v) = \int D\xi \; \mathrm{Tr} \; e^{-\beta H_{\text{eff}}}$, $\int Dx = \int_{-\infty}^{\infty} dx \frac{e^{-x^2/2}}{\sqrt{2\pi}}$ ($x = z$, v, or ξ), and

$$H_{\text{eff}} = -\sum_{(i,j)_1}^{n_s} J_1 s_{i\nu} s_{j\nu} - \sum_{(i,j)_2}^{n_s} J_2 s_{i\nu} s_{j\nu} - h S_\nu \tag{7}$$

represents the effective one-cluster model with the 1S RSB self-consistent field

$$h = J[\sqrt{Q_1 - Q_0}\, v + \sqrt{R - Q_1}\, \xi + \sqrt{Q_0}\, z]. \tag{8}$$

The parameters Q_1, Q_0, R, and a are obtained from the extreme of the free energy (6). The magnetic cluster susceptibility χ and specific heat c_v are derived from Eq. (6): $\chi = \beta[R - Q_1 + a(Q_1 - Q_0)]$ and $c_v = \frac{\partial U}{\partial T}$, where $U = -T^2 \frac{\partial(\beta f)}{\partial T}$.

III. RESULTS AND DISCUSSION

The effective one-cluster problem [Eqs. (6)–(8)] is now numerically solved by using exact diagonalization. We consider clusters with n_s Ising spins on a bidimensional lattice with short-range interactions with intensities J_1 and J_2. The analyses are done for two different intracluster J_1-J_2 couplings: one with both J_1 and J_2 antiferromagnetic (Sec. III A) and other with J_1 ferromagnetic and J_2 antiferromagnetic (Sec. III B). In particular, the intercluster disorder J results from the 1S RSB approximation for the effective field h [Eq. (8)]. Therefore, the SG order is characterized by the RSB solution ($Q_1 - Q_0 > 0$). In order to compare results for different clusters size, J is divided by n_s, becoming related to the intensity of the disorder per spin.

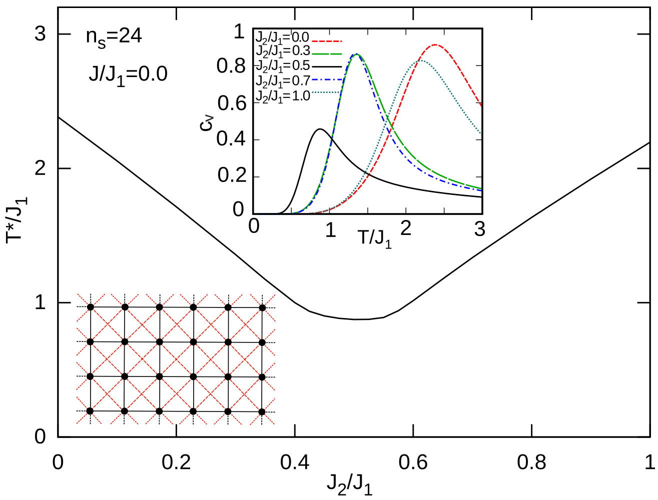

FIG. 1. (Color online) Temperature of the maximum value of the specific heat c_v, T^*, versus J_2/J_1 for an AF finite-size system with $n_s = 24$ and $J = 0$ (without disorder). The lattice geometry is shown at the bottom left and periodic boundary conditions are considered. The inset exhibits the c_v behavior for several J_2/J_1 values. The minimum of T^* appears at $J_2/J_1 = 0.5$ corresponding to the strongest intrinsic frustration.

A. Antiferromagnetic intracluster interactions

We present results for the classical antiferromagnetic J_1-J_2 model in a finite-size system without intercluster disorder (decoupled clusters with $J/J_1 = 0$). For example, the specific heat c_v for a square lattice with 24 spins and periodic boundary conditions (PBCs) is shown in the inset of Fig. 1. The c_v curve displays a maximum value at temperature T^*. The value of T^* for different ratios J_2/J_1 (the parameter that controls the intrinsic frustration) is used to create the phase diagram of Fig. 1, which presents the behavior expected for this model [16–18]. At temperatures below T^*, the finite-cluster results suggest that the Néel and collinear (lines of spins parallel are coupled antiparallel) antiferromagneticlike orders are found for values of J_2/J_1 lower and higher than 0.5, respectively [16]. It is important to observe that T^* is minimum at $J_2/J_1 = 0.5$, where the maximum intrinsic frustration occurs.

The effects of disorder are introduced by considering the intercluster disorder $J \neq 0$ for different cluster sizes with even numbers of spins and without PBCs. For instance, Fig. 2 shows that J is able to stabilize the SG phase and a second-order transition (solid lines) from the paramagnetic (PM) phase to the SG order occurs when the temperature decreases for high-J values. As J diminishes, the transition becomes first order (dotted lines) and reentrance can appear except for $J_2/J_1 = 0.5$ (intracluster frustration). This reentrance is associated with the interesting phenomenon of inverse freezing that was already discussed in Ref. [12], in which $J_2 = 0$. Here the phase diagrams exhibited in Fig. 2 with small-J_2/J_1 values are qualitatively the same as those obtained in Ref. [12]. The IF results from the interplay between intercluster disorder and short-range AF interactions. The intercluster disorder favors the SG phase that can present a higher entropic content than a low-temperature cluster PM phase. This PM phase appears as a result of the AF interactions, in which the intracluster spins become AF compensated, establishing clusters with low magnetic moments without long-range order. In other words, in the PM phase at low T, many nonmagnetic clusters with total moment $S = 0$ are found, where the intracluster spins freeze into perfect AF zero-moment states. This is the source of a small amount of magnetic entropy for the low-temperature cluster PM phase [12].

However, J_2 can introduce intracluster frustration effects on T_f. For instance, the maximum intracluster intrinsic frustration appears at $J_2/J_1 = 0.5$ for clusters with PBCs (see Fig. 1), where the number of intracluster J_1 and J_2 interactions are the same. Nevertheless, the PBCs are not used for the disordered intercluster problem. Instead, the cluster shapes of Fig. 2 are chosen such that the number of intracluster interactions between nearest neighbors and next-near neighbors are very close (see dashed red and black lines in the insets of Fig. 2) in order to explore the property of intrinsic frustration for J_2/J_1 very close to 0.5. In this case, the T^* dependence on J_2/J_1 for $J = 0$ (without disorder) exhibits behavior similar to that of Fig. 1 [see the inset of Fig. 2(b)]. However, the T^* is displaced to lower temperatures and its minimum at $J_2/J_1 = 0.5$ is hard locate because of the finite size of the cluster [16]. Nevertheless, the most important effect occurs when $J > 0$, in which case the T_f behavior for J_2/J_1 near the intracluster frustrated regime can be analyzed. For instance, Fig. 2 indicates that there is a minimum critical value of disorder to introduce the SG phase when $J_2/J_1 \neq 0.5$. In addition, T_f appears at lower temperatures when J_2 increases, but the most relevant result is obtained for $J_2/J_1 = 0.5$, where the SG phase occurs at lower intensities of disorder (see the black lines in Fig. 2). This result seems to be independent of the cluster size and shape. This means that the intracluster intrinsic frustration favors the SG phase that is stabilized for any infinitesimal value of disorder.

It is worth pointing out that T_f presents two different behaviors particularly around $J_2/J_1 = 0.5$: one for lower intensities of disorder, where the intrinsic frustration effects are relevant, and the other for high-J/J_1 values, in which there is no qualitative distinction between intracluster interaction regimes (see T_f for $J/J_1 > 4$ with different J_2/J_1 values in Fig. 2). In this regime T_f increases linearly with J. Furthermore, the intrinsic frustration affects the IF phenomenon by changing the T_f behavior. The reentrance disappears in the presence of intracluster geometrical frustration, which breaks one of the essential conditions for the occurrence of IF: the low entropy of the PM phase. The inset of Fig. 2(d) helps one understand the effects of the intrinsic frustration on the PM phase. It shows the correlation R as a function of J_2 for low temperatures in the PM region close to the PM-SG phase transition. The intracluster frustration causes an increment in the R curves, which has a direct impact on the intensity of the total cluster magnetic moment. This can contribute to the intercluster long-range disorder and at the same time it prevents the occurrence of the PM phase with a small-cluster magnetic moment. In other words, the cluster magnetic moment is maximized in the presence of intrinsic frustration and therefore the sensitivity to intercluster disordered interactions is enhanced.

The effects of J_2/J_1 on the intercluster disorder can also be analyzed in Fig. 3, which exhibits the minimal intensity of disorder $J_{\min}$ required to get the SG phase, as a function of

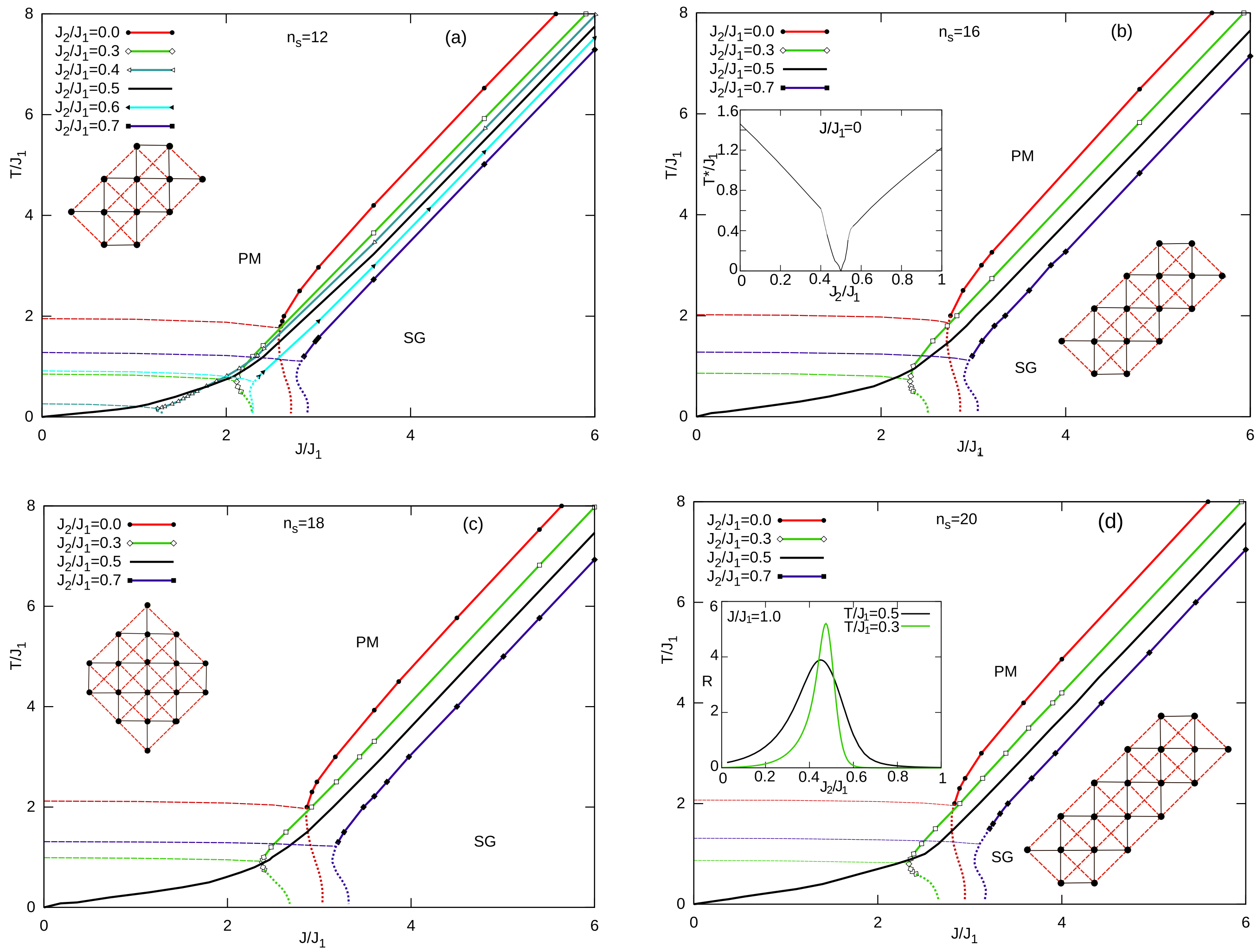

FIG. 2. (Color online) Phase diagrams T/J_1 versus J/J_1 for several values of J_2/J_1 with different cluster sizes and antiferromagnetic intracluster couplings. Cluster shapes without periodic boundary conditions are presented in each panel, where black and dashed red lines represent first- and second-neighbor interactions, respectively. Solid and dotted lines represent PM-SG second- and first-order transitions, respectively. Dashed lines correspond to the maximum of the susceptibility (discussed in Fig. 4). The inset in (b) shows the T^*/J_1 vs J_2/J_1 behavior for $J = 0$. The inset in (d) shows the order parameter R as a function of J_2/J_1 within the PM phase at low T ($J/J_1 = 1.0$ and $T/J_1 = 0.3$ and 0.5). The SG phase is always found when the disorder increases for an AF Ising cluster, but it is stabilized at lower intensities of disorder in the presence of intracluster frustration.

J_2/J_1. This phase diagram shows that the SG phase can always be obtained for a certain range of temperature when $J > J_{\min}$, while the PM phase is stable in the whole range of temperature for $J < J_{\min}$. The intensity of this minimum disorder is clearly reduced as the intrinsic frustration increases. For instance, the SG phase can be obtained for extremely small but nonzero J as $J_2/J_1 \to 0.5$. In other words, the intrinsic frustration potentiates the intercluster disordered couplings favoring the cluster SG phase. In addition, $J_{\min}/J_1$ increases faster when $J_2/J_1 > 0.5$ than when $J_2/J_1 < 0.5$. This behavior can also be attributed to the R dependence on J_2 that is not symmetric around $J_2/J_1 = 0.5$. As J_2/J_1 increases from 0.5, R decreases faster when compared with the case $J_2/J_1 < 0.5$ [see the inset of Fig. 2(d)], which affects the intercluster disordered interactions.

Figure 4 exhibits the 1S RSB order parameters for different ratios of J_2/J_1. It enforces the previous discussion related to the intensity of the cluster magnetic moment R, which depends on the temperature J and J_2. For $J_2 = 0$, R goes to zero as the temperature decreases for small-J values. This indicates that the cluster spins become AF compensated, affecting the intercluster coupling, which remains in the PM phase down to zero temperature. In contrast, the intracluster intrinsic frustration can introduce a different scenario, in which degenerate cluster spin configurations with uncompensated spins are thermodynamically favored. It can reflect on the increment of the magnetic moment of clusters as compared to other J_2/J_1 regimes. This increment of R favors the intercluster coupling that can generate the SG phase at lower disordered regimes (see $Q_1 - Q_0 > 0$ for $J_2/J_1 = 0.5$ in Fig. 4).

The cluster magnetic susceptibility χ can also be explored to clarify the low-disorder regime (see the inset of Fig. 4). Here χ shows a Curie-Weiss-like behavior at the high-temperature cluster PM phase. However, as previously discussed, one

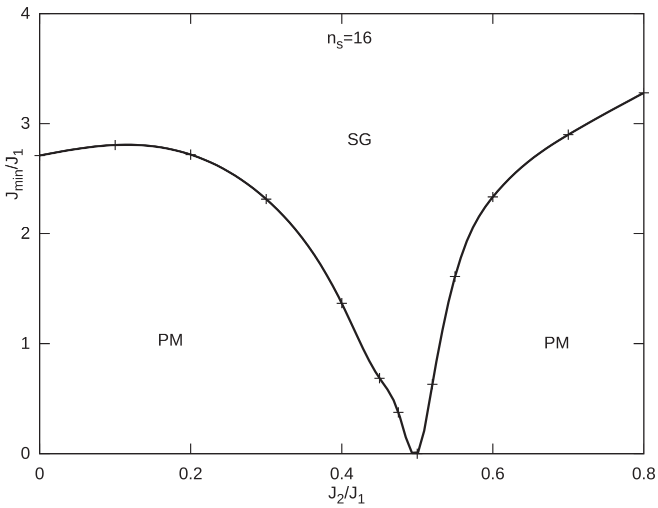

FIG. 3. Phase diagram $J_{\min}/J_1$ versus J_2/J_1 for $n_s = 16$, in which $J_{\min}$ is the minimum intensity of the intercluster disorder able to generate the SG phase with an AF cluster. The cluster shape is the same as in Fig. 2(b). At $J_2/J_1 = 0.5$, $J_{\min}$ is arbitrarily small but positive ($J_{\min}/J \to 0^+$).

can find a low-temperature PM phase, in which χ decreases when diminishing T from a certain value T^*_χ (temperature of the smooth maximum in χ). In this case, low magnetic moment clusters characterize the PM phase that arises as a consequence of the short-range AF interactions without long-range order. The location of T^*_χ in the phase diagrams of Fig. 2 is given by the dashed lines, which show clearly the dependence of T^*_χ on the short-range interactions. In particular, the intracluster frustration can eliminate the low-temperature PM and the system presents SG phase with a susceptibility

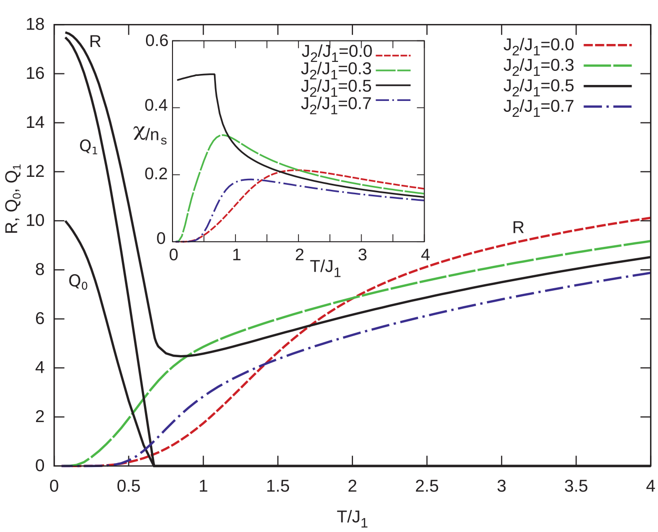

FIG. 4. (Color online) Order parameters in the low-disorder regime ($J/J_1 = 2.0$) for several J_2/J_1 values and $n_s = 16$ with the same AF cluster shape as in Fig. 2(b). The inset exhibits the susceptibility as a function of the temperature. The intrinsic frustration can increase the intensity of the the magnetic moment of clusters R favoring the occurrence of a cluster SG phase at lower disorder.

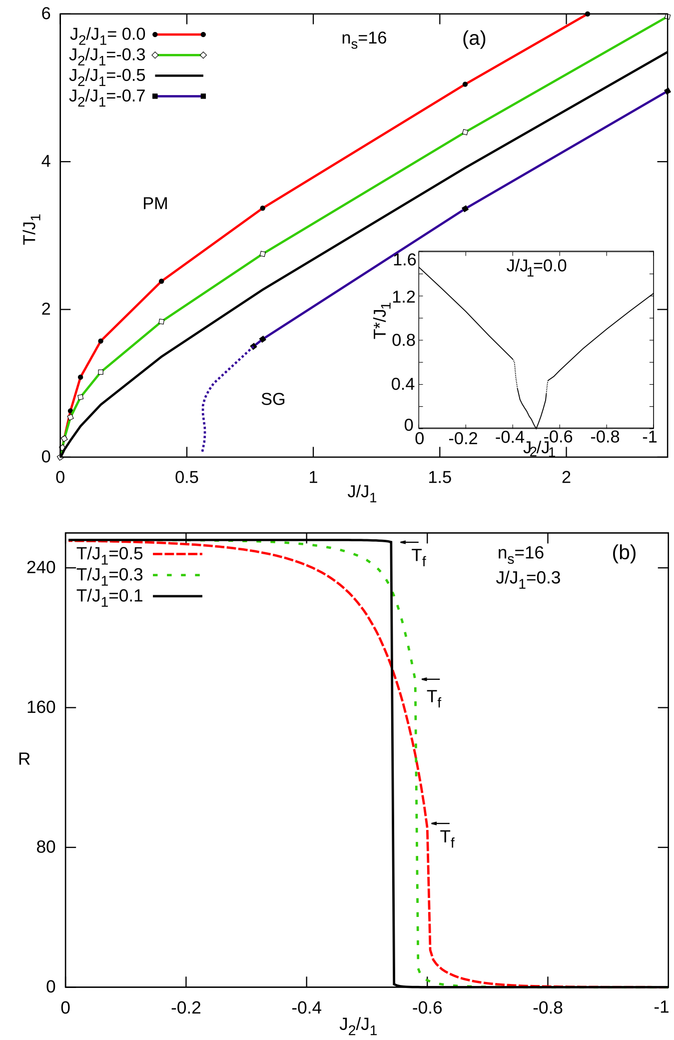

FIG. 5. (Color online) Results for FE and AF intracluster interactions with $n_s = 16$ and the same cluster shape as in Fig. 2(b). (a) Phase diagrams T/J_1 versus J/J_1 for several values of J_2/J_1. (b) Order parameter R as a function of J_2/J_1 for lower temperatures. The inset exhibits the behavior of T^*/J_1 as a function of J_2/J_1 for $J = 0$.

weakly dependent on temperature (see χ for $J_2/J_1 = 0.5$ in Fig. 4).

B. Ferromagnetic and antiferromagnetic intracluster interactions

Now, ferromagnetic and antiferromagnetic intracluster interactions are considered between first and second neighbors, respectively. Here we assume explicitly the sign of the AF interaction. For ferromagnetic clusters ($0 > J_2/J_1 > -0.5$), the phase diagrams exhibit SG order even for low values of disorder J/J_1 [see Fig. 5(a)]. When $-J_2/J_1$ increases ($J_2/J_1 < -0.5$), the antiferromagnetic short-range intracluster interactions become stronger and the SG phase is only observed for higher intensities of disorder. Furthermore, a reentrant first-order PM-SG transition appears in which the IF occurs. In particular, at J_2/J_1 very close to -0.5 [see inset of Fig. 5(a)], intracluster intrinsic frustration is strong

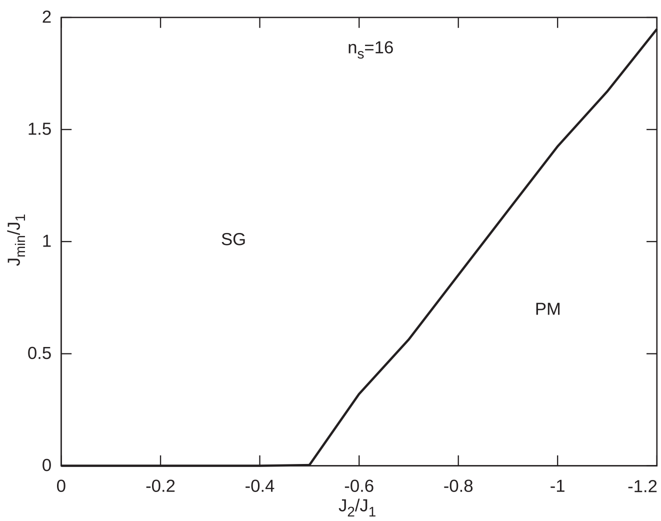

FIG. 6. Phase diagram $J_{\min}/J_1$ versus J_2/J_1 for $n_s = 16$ with FE (J_1) and AF (J_2) short-range intracluster interactions. The cluster shape is the same as in Fig. 2(b).

and the SG phase is found for $J \to 0$. As in the case of AF J_1-J_2 interactions, the results for other cluster sizes explored ($n_s = 12$, 18, and 20) are qualitatively the same.

The behavior of R in Fig. 5(b) shows two different intracluster regimes: one for low $-J_2/J_1$ (where R is maximum) and another for high $-J_2/J_1$ (R is minimum). It can help to explain the presence of the cluster SG phase for lower J when the ferromagnetic intracluster interactions are dominant. In this case, the intercluster disordered couplings are enhanced by the large cluster magnetic moments. In contrast, intracluster AF compensation makes the clusters less predisposed to the intercluster disordered interactions. Therefore, the SG phase cannot be found at lower intensities of J when $-J_2$ is higher. Furthermore, this AF compensation can introduce the low-temperature PM phase, which is an essential condition for IF occurrence (see Sec. III A).

Figure 6 also enforces that FE clusters are able to present the cluster SG state within a very small disordered regime, while AF clusters require higher disorder to show the SG phase. It suggests that FE clusters are able to enhance the SG phase, but AF clusters can suppress the SG phase. In addition, the effects of intrinsic frustration are less pronounced than in the case analyzed in Sec. III A.

IV. CONCLUSION

Summing up, we studied the interplay between intrinsic frustration and quenched disorder in a cluster spin-glass model that considers intracluster interactions following the (J_1-J_2)-like model for finite clusters and long-range disordered intercluster interactions. The mean-field replica method is used to obtain an effective one-cluster model, which is computed by exact diagonalization.

The results show that the intracluster AF compensation affects the disordered intercluster interactions. It suppresses the spin-glass phase and can introduce a reentrant SG-PM transition that is associated with the inverse freezing. On the contrary, FE clusters can reinforce the SG states. However, the more important result is obtained by considering AF clusters with intrinsic frustration. At the most frustrated point, there are some spins that can be returned freely without any energy cost. It results that the $S = 0$ ground state of the cluster becomes degenerate with nonzero spin states and then any infinitesimal value of intercluster interaction will induce a SG state. In other words, the intracluster frustration can increase the intensity of cluster magnetic moment favoring the intercluster SG order even with a very weak disorder. Although these conclusions are obtained for a particular model (Ising spins on a square lattice), they indicate that the presence of small clusters in geometrically frustrated systems can help stabilize the SG order within weakly disordered regimes.

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