Absence of exponential sensitivity to small perturbations in nonintegrable systems of spins 1/2

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We show that macroscopic nonintegrable lattices of spins 1/2, which are often considered to be chaotic, do not exhibit the basic property of classical chaotic systems, namely, exponential sensitivity to small perturbations. We compare chaotic lattices of classical spins and nonintegrable lattices of spins 1/2 in terms of their magnetization responses to an imperfect reversal of spin dynamics known as Loschmidt echo. In the classical case, magnetization is exponentially sensitive to small perturbations with a characteristic exponent equal to twice the value of the largest Lyapunov exponent of the system. In the case of spins 1/2, magnetization is only power-law sensitive to small perturbations.

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I. INTRODUCTION

Despite the successes of statistical physics, the notion of chaos in many-particle quantum systems is still not fully understood. A classical system is called chaotic if its phase space trajectories exhibit exponential growth of initially small deviations between them. This growth is characterized by Lyapunov exponents [1]. Chaos requires nonlinear dynamics. In contrast, quantum dynamics is fundamentally linear with respect to small perturbations of quantum amplitudes. Yet, many researchers know from experience that the notion of dynamical randomness has merit for quantum systems with sufficiently dense energy spectra. Hence the term chaos is frequently invoked in the foundations of quantum statistical physics [2,3].

Quantum chaos is often defined through a chaotic classical limit [4]. This definition, however, is problematic for many-particle systems, such as systems of many spins 1/2. Individual spins 1/2 are as far from the classical limit as an object can only be, but nonintegrable systems of many spins 1/2 exhibit [5,6] the Wigner-Dyson statistics [7] of spacings between adjacent energy levels, which is known to be a generic characteristic of quantum systems that do have chaotic classical limit [8].

It is commonly assumed that, in many-particle systems, the linearity of quantum dynamics is compensated by the exponentially large number of quantum eigenstates, so that, on experimentally observable time scales, appropriate superpositions of quantum eigenstates can mimic chaotic classical behavior of macroscopic variables. In this paper, however, we show that the above connection cannot be established for macroscopic systems of spins 1/2 precisely in the situation when the classical macroscopic response exhibits the quantitative signature of Lyapunov instability. We arrive to the above conclusion by analyzing the behavior of the total magnetization under an imperfect time reversal known as the Loschmidt echo.

The idea that chaos affects Loschmidt echo responses of macroscopic systems was first proposed in Ref. [9] in the context of nuclear magnetic resonance (NMR) echo experiments on a spin 1/2 system. The authors of Ref. [9] reported that, despite their best effort, they were not able to improve the echo response beyond a certain level. Similar observations were also reported earlier in Ref. [10]. Reference [9] suggested that chaos inhibits one's ability to implement perfect time reversal. This proposition motivated a significant body of research on Loschmidt echoes [11–19]. However, to the best of our knowledge, no quantitative connection between chaos characteristics of *many-spin* systems and their *observable* Loschmidt echo responses has yet been proposed.

In this paper, we first demonstrate that, for macroscopic systems of classical spins, one can extract the fundamental indicator of chaos, namely, the largest Lyapunov exponent, from the behavior of the total magnetization recovered by the Loschmidt echo. If real spins were classical, the above result would resolve one of the outstanding issues of statistical physics, namely, how to obtain experimental evidence of microscopic chaos in a many-particle system [20–23]. However, we also show that nonintegrable macroscopic systems of quantum spins 1/2 do not exhibit the above signature of chaos.

II. FORMULATION OF THE PROBLEM

We consider lattices of N_s classical spins or N_s quantum spins 1/2 at high temperatures governed by the nearest-neighbor (NN) Hamiltonian

$$\mathcal{H}_0 = \sum_{i<j}^{\mathrm{NN}} J_x S_{ix} S_{jx} + J_y S_{iy} S_{jy} + J_z S_{iz} S_{jz}, \tag{1}$$

where J_x, J_y, J_z are the nearest-neighbor coupling constants, and $(S_{ix}, S_{iy}, S_{iz}) \equiv \mathbf{S}_i$ either represent three projections of the classical spin on the ith lattice site normalized by condition $\mathbf{S}_i^2 = 1$, or denote operators of spins 1/2. Different lattice dimensions are considered—all with periodic boundary conditions.

We characterize Loschmidt echo response by the difference between the values of the total magnetization for perfectly and imperfectly reversed dynamics. The time reversal is achieved by reversing the sign of the interaction Hamiltonian as done, e.g., in NMR magic echo experiments [24–28]. We

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consider two kinds of perturbations to perfect time reversal: (i) small *instantaneous* rotations of spins at the moment of time reversal; and (ii) *continuously present* small perturbations to the Hamiltonian of the time-reversed evolution.

In the quantum case, we are primarily interested in the perturbations that are small at the level of individual spins, so that the total magnetization remains nearly the same, but, at the same time, sufficiently many spins are perturbed, so that the overlap of the perturbed and unperturbed many-spin wave functions is negligible. Although the macroscopic limit of this setting has not yet been addressed in the literature, various aspects of the results reported below were anticipated in Refs. [12,29–34].

III. CLASSICAL SPINS

A. Lyapunov exponents

We parametrize the phase spaces of a classical spin lattice by vector $\bar{\mathbf{X}} \equiv \{S_{1x}, S_{1y}, S_{1z}, S_{2x}, S_{2y}, S_{2z}, \dots\}$. The difference between two nearby phase-space trajectories is denoted by vector $\bar{\mathbf{D}} \equiv \{\delta S_{1x}, \delta S_{1y}, \delta S_{1z}, \delta S_{2x}, \delta S_{2y}, \delta S_{2z}, \dots\}$. It can also be expressed as

$$\bar{\mathbf{D}}(t) \equiv \bar{\mathbf{X}}\left(t, \bar{\mathbf{X}}_0 + \bar{\mathbf{D}}_0\right) - \bar{\mathbf{X}}\left(t, \bar{\mathbf{X}}_0\right), \tag{2}$$

where $\bar{\mathbf{X}}\left(t, \bar{\mathbf{X}}_0\right)$ is a phase space trajectory as a function of time t and initial position $\bar{\mathbf{X}}_0$, and $\bar{\mathbf{X}}\left(t, \bar{\mathbf{X}}_0 + \bar{\mathbf{D}}_0\right)$ is another trajectory initially separated from the first one by infinitesimal displacement $\bar{\mathbf{D}}_0 \equiv \bar{\mathbf{D}}(0)$.

A system of N_s classical spins is characterized by $2N_s$ Lyapunov exponents. We denote the maximum positive Lyapunov exponent as $\lambda_{\max}$ and the corresponding Lyapunov vector as $\bar{\mathbf{d}}_{\max}(t)$. A many-particle system is technically defined to be chaotic, when $\lambda_{\max} > 0$. In Refs. [35,36], we found that, for lattices of classical spins with nearest neighbor interaction, $\lambda_{\max}$ is an *intensive* quantity, i.e., its value is size independent for sufficiently large lattices. It can be roughly estimated as $\lambda_{\max} \approx 0.25\sqrt{N_{\mathrm{nn}}(J_x^2 + J_y^2 + J_z^2)}$, where N_{nn} is the number of nearest neighbors.

We exploit the idea of the standard numerical algorithm for computing $\lambda_{\max}$ [37]. Namely, we consider two phase-space trajectories $\bar{\mathbf{X}}\left(t, \bar{\mathbf{X}}_0\right)$ and $\bar{\mathbf{X}}\left(t, \bar{\mathbf{X}}_0 + \bar{\mathbf{D}}_0\right)$, where $\bar{\mathbf{D}}_0$ is a very small vector pointing in a randomly selected direction. This vector has random projections on each of the Lyapunov vectors including $\bar{\mathbf{d}}_{\max}(0)$. After sufficiently long time, the growth of $|\bar{\mathbf{D}}(t)|$ is entirely dominated by $\lambda_{\max}$, so that $\lambda_{\max}$ can be obtained as

$$\lambda_{\max} = \frac{1}{t} \lim_{t\to\infty; |\bar{\mathbf{D}}(0)|\to 0} \ln \frac{|\bar{\mathbf{D}}(t)|}{|\bar{\mathbf{D}}(0)|}. \tag{3}$$

In order to register this exponential growth, $|\bar{\mathbf{D}}_0|$ should be sufficiently small, so that the projections of $\bar{\mathbf{D}}(t)$ satisfy the inequality $\delta S_{k\mu} \ll 1$ for sufficiently long time.

In an ergodic system, the asymptotic exponential growth of $|\bar{\mathbf{D}}(t)|$ does not depend on the choice of $\bar{\mathbf{X}}_0$ and $\bar{\mathbf{D}}_0$ [38]. This means that, for an ensemble of $\bar{\mathbf{X}}_0$ and/or $\bar{\mathbf{D}}_0$, the ensemble-average, denoted as $\langle \cdots \rangle$, also exhibits asymptotic exponential growth $\langle |\bar{\mathbf{D}}(t)| \rangle \cong e^{\lambda_{\max} t}$. The time required to establish this growth is, typically, of the order of $1/\lambda_{\max}$ (see the Supplemental Material [39]).

B. Noise

Let us now consider the case of equilibrium noise at infinite temperature for the total x component of magnetization $M_x \equiv \sum_k S_{kx}$. We compare two magnetization time series: $M_{x0}(t)$, corresponding to the initial conditions $\bar{\mathbf{X}}_0$, and $M_{x1}(t)$, corresponding to slightly perturbed initial conditions $\bar{\mathbf{X}}_0 + \bar{\mathbf{D}}_0$. In the initial small-deviations regime, $M_{x1}(t) - M_{x0}(t)$ is determined by the projection of $\bar{\mathbf{D}}(t)$ on the direction in the phase space representing variable M_x and given by the vector $\bar{\mathbf{d}}_{M_x} \equiv (1,0,0,1,0,0,1,\dots)$. If $\bar{\mathbf{D}}_0$ is small enough, then there is a time interval when the growth of $\bar{\mathbf{D}}(t)$ is controlled by $\lambda_{\max}$, while its orientation is controlled by $\bar{\mathbf{d}}_{\max}(t)$. In this regime, the projection of $\bar{\mathbf{D}}(t)$ on $\bar{\mathbf{d}}_{M_x}$ fluctuates in time (and may change sign), but the amplitude of this fluctuating projection should grow exponentially as $e^{\lambda_{\max} t}$. As shown in Fig. 1, this is indeed what we observed numerically [38]. As also shown in the inset of Fig. 1, the fluctuations can be suppressed by averaging over a large number of independent noise realizations, which means that, in the asymptotic regime, $\langle |M_{x1}(t) - M_{x0}(t)| \rangle \cong e^{\lambda_{\max} t}$.

The above analysis can now be adapted to the imperfect time reversal of magnetization noise, when one observes $M_x(t)$, and then, at time $t = t_0$, changes the sign of the Hamiltonian and simultaneously rotates each spin by a small randomly chosen angle [40]. In this case, $M_x(t_0 - \tau)$ corresponds to $M_{x0}(\tau)$ in the previous example and represents a perfectly time reversed signal, while $M_x(t_0 + \tau)$ corresponds to $M_{x1}(\tau)$. Therefore, for small enough random rotations, there is a range of times τ where $\langle |M_x(t_0 - \tau) - M_x(t_0 + \tau)| \rangle \sim e^{\lambda_{\max} \tau}$, which, in turn, implies that $\langle [M_x(t_0 - \tau) - M_x(t_0 + \tau)]^2 \rangle \cong e^{2\lambda_{\max} \tau}$. The latter equation, together with the equilibrium relation

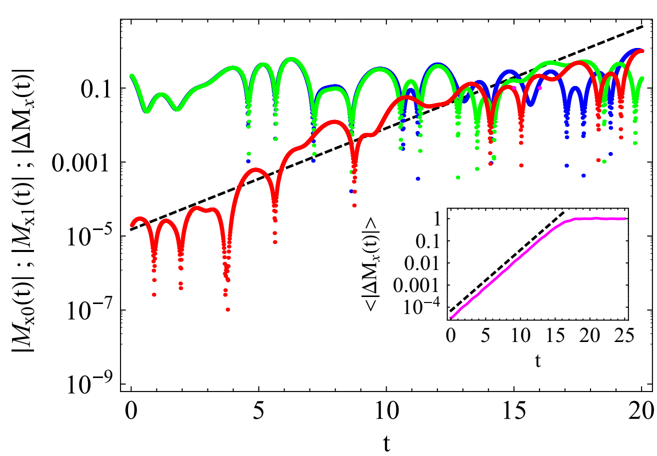

FIG. 1. (Color online) Sensitivity of classical magnetization noise to small perturbations for a cubic lattice of $16 \times 16 \times 16$ classical spins with $J_x = -0.41$, $J_y = -0.41$, $J_z = 0.82$. Blue line: magnetization noise $|M_{x0}(t)|$ for randomly chosen initial conditions $\bar{\mathbf{X}}_0$. Green line: magnetization noise $|M_{x1}(t)|$ for the initial conditions $\bar{\mathbf{X}}_0 + \bar{\mathbf{D}}_0$, where $\bar{\mathbf{D}}_0$ represents small rotations of each spin around a random axis by an angle randomly selected from $[-10^{-4}\pi, 10^{-4}\pi]$. Red line: $|\Delta M_x(t)| \equiv |M_{x1}(t) - M_{x0}(t)|$. Inset: ensemble average $\langle |\Delta M_x(t)| \rangle$ over 1000 random realizations of $\bar{\mathbf{X}}_0$ and $\bar{\mathbf{D}}_0$. Black dashed lines: constant $\times\, e^{\lambda_{\max} t}$ with $\lambda_{\max} = 0.63$ computed directly [35,37,39].

$\langle M_x^2(t_0-\tau)\rangle = \langle M_x^2(t_0+\tau)\rangle \equiv \langle M_x^2\rangle$, leads to

$$\frac{\langle M_x(t_0-\tau)M_x(t_0+\tau)\rangle}{\langle M_x^2\rangle} = 1 - Ce^{2\lambda_{\max}\tau}, \qquad (4)$$

where C is a proportionality constant.

C. Relaxation

Equation (4) for equilibrium noise can now be converted into the description of a Loschmidt echo for nonequilibrium relaxation in a setting similar to a NMR magic echo [25]. Namely, at $t = t_0 - \tau$, the system starts in a slightly x-polarized state with probability distribution $\rho_0 \cong e^{-\beta M_x}$, where β is a very small constant. For $t_0 - \tau < t < t_0$, the magnetization relaxes under the action of Hamiltonian $\mathcal{H}_0$. At $t = t_0$, the Hamiltonian switches sign, and, simultaneously, the spins are rotated by small random angles. Afterward, the magnetization is measured at $t = t_0 + \tau$. We define the normalized echo function as $F(\tau) \equiv \langle M_x\rangle_f / \langle M_x\rangle_0$, where $\langle M_x\rangle_0$ and $\langle M_x\rangle_f$ represent averages with respect to ρ_0 and $\rho_f = \hat{U}_{-\mathcal{H}_0}(\tau)\ \hat{U}_R\ \hat{U}_{\mathcal{H}_0}(\tau)\ \rho_0$, respectively. Here, ρ_f is the probability distribution at $t = t_0 + \tau$, while $\hat{U}_{\mathcal{H}_0}(\tau)$ and $\hat{U}_{-\mathcal{H}_0}(\tau)$ are the time evolution operators with Hamiltonians $\mathcal{H}_0$ and $-\mathcal{H}_0$, respectively, and $\hat{U}_R$ is the operator representing the effect of the small spin rotations. In the limit $\beta \ll 1$, $F(\tau)$ transforms into the left-hand side of Eq. (4). Therefore, its asymptotic behavior is

$$F(\tau) = 1 - C\ e^{2\lambda_{\max}\tau}. \qquad (5)$$

We have tested Eq. (5) numerically. The results are presented in Fig. 2(a). They clearly exhibit the expected $e^{2\lambda_{\max}\tau}$ dependence for $1 - F(\tau)$.

Let us now consider time reversal disturbed by term $\sum_k h_k S_{kz}$ added to the reversed Hamiltonian. Here h_k are small random magnetic fields. Such a perturbation continuously feeds the deviation of the imperfectly reversed trajectory from the perfectly reversed one. This deviation initially grows linearly in time, but then it is exponentially amplified by the intrinsic chaotic dynamics of $\mathcal{H}_0$ as in the preceding case. Therefore, the asymptotic behavior (5) is also expected here. This is, indeed, what we observed numerically [see Fig. 2(b)].

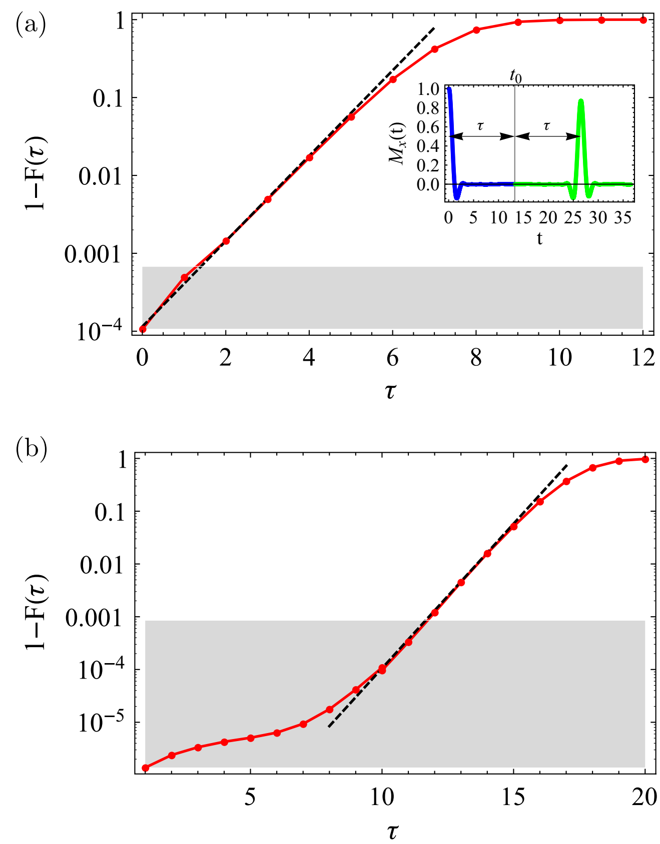

FIG. 2. (Color online) Loschmidt echoes for the same lattice of classical spins as in Fig. 1. (a) Echo disturbed by small random rotations of spins around randomly chosen axes by angles selected from the interval $[-10^{-2}\pi, 10^{-2}\pi]$. Inset: Relaxation and echo for one value of τ. (b) Echo disturbed by the perturbation to the reversed Hamiltonian of the form $\sum_k h_k S_{kz}$, where each h_k is randomly selected from the interval $[-2\times 10^{-4}, 2\times 10^{-4}]$. Solid red lines: averages over 2.8×10^5 and 1.7×10^5 independent time evolutions in (a) and (b), respectively. The initial polarization is 10%. Gray areas cover the values of $1 - F(\tau)$ below four root-mean-squared values of the statistical noise for $F(\tau)$. Dashed black lines: constant $\times\ e^{2\lambda_{\max} t}$ with $\lambda_{\max} = 0.63$.

IV. SPINS 1/2

Now we consider the Loschmidt echo for the relaxation of M_x in spin-1/2 lattices perturbed by small random rotations around the z axis at the moment of time reversal. The same linear-response relation as in the classical case allows us to express the echo function as an equilibrium correlation function [39]

$$F(\tau) = \frac{\mathrm{Tr}\{e^{i\mathcal{H}_0\tau}\ R^\dagger\ e^{-i\mathcal{H}_0\tau}\ M_x\ e^{i\mathcal{H}_0\tau}\ R\ e^{-i\mathcal{H}_0\tau}\ M_x\}}{\mathrm{Tr}\{M_x^2\}}, \qquad (6)$$

where

$$R = \prod_k e^{-i\delta\theta_k S_{kz}} = \prod_k [\mathbb{1}\cos(\delta\theta_k/2) - 2i\,S_{kz}\sin(\delta\theta_k/2)] \qquad (7)$$

is the quantum operator rotating each spin around the z axis by angles $\delta\theta_k$ randomly chosen in the interval $[-\delta\theta_{\max}, \delta\theta_{\max}]$ with $\delta\theta_{\max} \ll 1$. The discussion below deals with the evolution of a typical nonequilibrium wave function representing the above trace, thereby relying [39] on the typicality results of Refs. [41,42].

Each operator $\mathbb{1}\cos(\delta\theta_k/2) - 2i\,S_{kz}\sin(\delta\theta_k/2)$ in Eq. (7) creates a superposition of the original many-spin wave function with a small admixture of the wave function obtained from the original one by flipping the x projection of the kth spin. The probability of flipping any given spin by the action of operator R is, therefore, small, but, if it happens, the value of S_{kx} and hence its contribution to M_x switches completely between $1/2$ and $-1/2$. The overall effect of the operator R can be thought of as turning the wave function just before the time reversal, Ψ_-, into a superposition of wave functions $\Psi_+ = \sum_\nu c_\nu \Psi_\nu$, where each Ψ_ν is obtained from Ψ_- by flipping a small fraction of randomly selected spins of the order of $< \delta\theta_k^2 >$, and c_ν are the complex amplitudes [39].

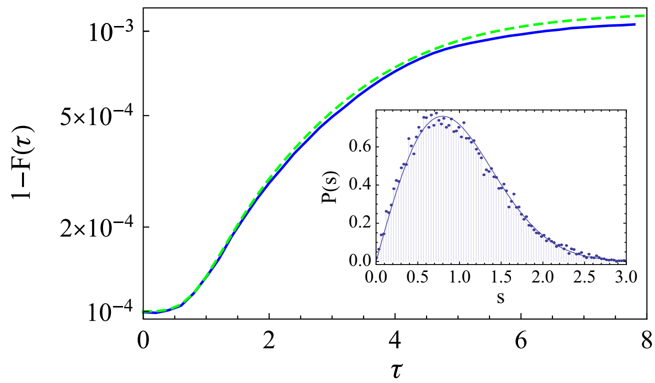

FIG. 3. (Color online) Loschmidt echoes for 5×5 square lattice of spins 1/2 with $J_x = -0.47$, $J_y = -0.47$, $J_z = 0.94$ and 5% initial polarization. Blue solid line: time reversal is disturbed by rotations around the z axis with angles $\delta\theta_k$ randomly chosen from $[-\pi/100,\pi/100]$. Green dashed line: time reversal is disturbed by flipping one spin 1/2, $S_{kx} \to -S_{kx}$ (plot rescaled). Inset: evidence of nonintegrability. Dots: distribution $P(s)$ of level spacings s for one irreducible block of $\mathcal{H}_0$. Solid line: Wigner-Dyson fit for the Gaussian orthogonal ensemble [7].

If a perfect time reversal were to be disturbed by flipping only one spin, the disturbance induced by this single spin would propagate to the neighbors as a perturbation bubble. The number of perturbed spins in this bubble [quantum equivalent of $|\bar{\mathbf{D}}(t)|$] would grow following a power law rather than an exponential. This kind of growth is not supposed to be exponential even in a chaotic classical system, because the initial perturbation is not small. Spin-1/2 lattices accessible to direct numerical simulations are not large enough to test the above conjecture, but Loschmidt echoes disturbed by complete flipping of only one spin can be simulated for large classical spin lattices, which indeed exhibit the power-law growth of $1 - F(\tau)$ [39].

Let us now assume that Ψ_+ is equal to one of Ψ_ν, which means that time reversal is disturbed by flipping a small randomly selected fraction of all spins. In this case, the initial power-law disturbance around each flipped spin should grow as the above perturbation bubble. When different bubbles start overlapping, the system enters the saturation regime $1 - F(\tau) \sim 1$ without $1 - F(\tau)$ ever exhibiting exponential growth.

The fact that Ψ_+ is a superposition of many Ψ_ν does not change the above conclusion. As we show in Ref. [39], the interference between different Ψ_ν averages to zero in the expression for $F(\tau)$, which implies that the absence of the exponential growth of $1 - F(\tau)$ for a typical Ψ_ν is representative of the entire superposition $\Psi_+ = \sum_\nu c_\nu \Psi_\nu$.

The above conclusion can, to a limited extent, be confirmed by direct quantum simulations [39,42] of a nonintegrable 5×5 cluster of spins 1/2 shown in Fig. 3. For this cluster, the interesting range of rotations $|\delta\theta_k| \gg 1/\sqrt{N_s}$ required to assure that $\langle\Psi_-|\Psi_+\rangle \approx 0$ does not leave any room for a possible Lyapunov growth. Instead, we simulated the limit $|\delta\theta_k| \ll 1/\sqrt{N_s}$, which leads to $\langle\Psi_-|\Psi_+\rangle \approx 1$ and, therefore, implies that $F(\tau)$ remains close to 1 for any τ. Nevertheless, if a Lyapunov exponent were definable, $1 - F(\tau)$ should have exhibited at least the first signs of the exponential growth $e^{2\lambda_{\max}\tau}$ before entering the saturation regime. However, as shown in Fig. 3, the initial interval of quadratic growth turns immediately into subexponential growth. In the same figure, we also include nearly the same Loschmidt echo shape for the case when time reversal is disturbed by flipping a single spin 1/2. In this case, the echo shape is, by definition, controlled by the growth of a single perturbation bubble.

Finally, we turn to a quantum Loschmidt echo disturbed by a small perturbation to the reversed Hamiltonian. This perturbation can be viewed, by analogy with the earlier discussion for classical spins, as feeding a seed deviation between perfectly and imperfectly reversed time evolutions, which is then amplified by the intrinsic dynamics of the perfectly reversed Hamiltonian. Since, in the quantum case, this intrinsic dynamics leads to a power-law amplification, the overall echo response should exhibit a power-law sensitivity to small perturbations in the reversed Hamiltonian. The above conclusion is consistent with our finite-size simulations [39], but it should be properly tested in NMR magic echo experiments [39].

V. CONCLUSIONS

To summarize, we have found that stationary nonintegrable systems of spins 1/2 do not exhibit exponential sensitivity to small perturbations of Loschmidt echoes while chaotic systems of classical spins do. This absence of exponential sensitivity in spin 1/2 systems is likely applicable beyond the Loschmidt echo setting, since it reflects the fact that extreme quantization of the projections of spins 1/2 does not leave room for the Lyapunov growth. Such a conclusion certainly represents good news for the efforts to create quantum simulators [43]. At the same time, our findings are not as disturbing for the foundations of statistical physics as they may appear at first sight. The notion of chaos defined as exponential sensitivity to small perturbations is a sufficient but not necessary condition for ergodicity, which is, in turn, required to justify Gibbs equilibrium. Also, the long-time exponential relaxation, which is known to be the same for chaotic classical and nonintegrable quantum spin systems [27,44–48], does not exclude the power-law sensitivity to small perturbations [44]. We finally remark that a recent investigation by one of us [49] indicated that classical and quantum spin systems exhibit other qualitative differences as far as the equilibration dynamics is concerned.

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[38] If $\bar{\mathbf{D}}_0$ has no projection on $\bar{\mathbf{d}}_{\max}(0)$, the growth of $|\bar{\mathbf{D}}(t)|$ is controlled by the second largest Lyapunov exponent, which is very close to $\lambda_{\max}$. In general, Lyapunov growth emerging over time interval Δt includes significant contributions from Lyapunov exponents λ in the range $\lambda_{\max} - 1/\Delta t \leqslant \lambda \leqslant \lambda_{\max}$. This fact does not change the conclusions of our analysis.
[39] See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevE.89.012923 for (i) details of the numerical simulations of classical spins and the videos illustrating these simulations; (ii) simulations of classical Loschmidt echoes perturbed by flipping of one spin; (iii) analytical considerations pertaining to Loschmidt echoes in spin-1/2 systems; (iv) details of numerical simulations of spin-1/2 systems; (v) simulations of Loschmidt echoes in spin-1/2 systems perturbed by a small change in the Hamiltonian; (vi) discussion of possible NMR Loschmidt echo experiments aimed at testing the prediction of this work.
[40] In principle, small random spin rotations slightly change the total energy of the system by the value of the order of $\epsilon_0\sqrt{\langle\delta_k^2\theta\rangle N_s}$, where ϵ_0 is a typical one-spin energy. This changes $\lambda_{\max}$ but only by the value of the order of $1/\sqrt{N_s}$, which can be neglected in the limit $N_s \to \infty$. See Ref. [36] for further explanation.
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