Quenched disorder forbids discontinuous transitions in nonequilibrium low-dimensional systems

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Quenched disorder affects significantly the behavior of phase transitions. The Imry-Ma-Aizenman-Wehr-Berker argument prohibits first-order or discontinuous transitions and their concomitant phase coexistence in low-dimensional equilibrium systems in the presence of random fields. Instead, discontinuous transitions become rounded or even continuous once disorder is introduced. Here we show that phase coexistence and first-order phase transitions are also precluded in nonequilibrium low-dimensional systems with quenched disorder: discontinuous transitions in two-dimensional systems with absorbing states become continuous in the presence of quenched disorder. We also study the universal features of this disorder-induced criticality and find them to be compatible with the universality class of the directed percolation with quenched disorder. Thus, we conclude that first-order transitions do not exist in low-dimensional disordered systems, not even in genuinely nonequilibrium systems with absorbing states.

I. INTRODUCTION

Quenched disorder has a dramatic effect on both the statics and the dynamics of phase transitions [1–3]. A time-honored argument by Imry and Ma explains in a simple and parsimonious way why symmetries cannot be spontaneously broken in low-dimensional systems in the presence of quenched random fields [4]. In a nutshell, the argument is as follows. Suppose a discrete symmetry (e.g., Z_2 or up-down) was actually spontaneously broken in a d-dimensional system and imagine a region of linear size L with a majority of random fields opposing the broken-symmetry state. As a direct consequence of the central limit theorem, by reversing the state of such a region the bulk-free energy would decrease proportionally to $L^{d/2}$, but this inversion would also lead to an interfacial energy cost proportional to L^{d-1}. Comparing these two opposing contributions for large region sizes, it follows that for $d \leqslant 2$ the first dominates, making the broken-symmetry state unstable. If the distinct phases are related by a continuous symmetry, soft modes reduce the effect of the boundary conditions to L^{d-2} and the marginal dimension is $d = 4$ [5]. Thus, the energetics of low-dimensional systems is controlled by the random field, which is symmetric, thus preventing symmetries from being spontaneously broken and continuous phase transitions from existing. Instead, in higher dimensional systems, the situation is reversed, symmetries can be spontaneously broken, and phase transitions do exist.

The Imry-Ma argument (1) holds for equilibrium systems (where the free energy is well defined), (2) is backed by more rigorous renormalization group calculations, which prove that no symmetry breaking occurs even at the marginal case $d = 2$ (where rough interfaces could potentially break the argument above [5]), (3) has been verified in countless examples both

experimentally and numerically, and (4) has been extended to quantum phase transitions [6,7].

In contrast with the equilibrium case, recent work by Barghathi and Vojta [8] shows that second-order phase transitions may survive to the introduction of random fields even in one-dimensional cases [9,10] in genuine nonequilibrium systems with absorbing states for which there is not such a thing as free energy [11–14]. Therefore, *the Imry-Ma argument does not apply to these nonequilibrium systems owing to the presence of absorbing states, and, in consequence, states of broken symmetry can exist in the presence of random fields.*

Let us now shift the discussion to first-order phase transitions, for which system properties such as the magnetization, energy, density, etc., change abruptly as a control parameter crosses a threshold value at which two distinct phases coexist. As shown by Aizenman and Wehr, first-order phase transitions in low-dimensional equilibrium systems are rounded (made less sharp) by disorder, and, even more remarkably, the rounding may result into a critical point; i.e., first-order or discontinuous phase transitions become second-order or continuous ones upon introducing (random-field) disorder [5]. A similar conclusion applies to the case of random interactions [5,15,16]; indeed, a random distribution of interactions (e.g., bonds) locally favors one of the two phases, and thus, it has the same effects as random fields. Different Monte Carlo results support this conclusion; furthermore they suggest that the disorder-induced continuous transition exhibits critical exponents which are consistent with those of the corresponding pure model. An argument explaining these findings was put forward by Kardar *et al.* [17].

In close analogy with the argument above for the absence of symmetry breaking, in the case of phase coexistence as well, regions (or "islands") of arbitrary size of one of the phases

TABLE I. Random fields in low-dimensional disordered systems. Summary of the effects of quenched random fields on the existence of continuous or second-order transitions (with spontaneously symmetry breaking) and discontinuous or first-order (with associated phase coexistence) phase transitions in $d \leqslant 2$ systems. Both the equilibrium and nonequilibrium cases are considered, the latter including the possibility of one or more absorbing states.

System with random fields $d \leqslant 2$	Second-order (spontaneous symmetry breaking)	First-order (phase coexistence)
Equilibrium	No [4]	No[4,5,15,16]
Nonequilibrium (absorbing states)	Yes [8]	?

appear in a stable way within the other. Therefore, islands exist within islands in any of the two phases in a nested way, leading always to hybrid states. Hence, two distinct phases cannot possibly coexist, and first-order transitions are precluded in disordered low-dimensional equilibrium systems.

Thus, the question arises as to whether shifting to the nonequilibrium realm entails the shattering of a fundamental cornerstone of equilibrium statistical mechanics as it happens for continuous phase transitions (see Table I for a synthetic summary); do first-order phase transitions, and hence phase coexistence, exist in low-dimensional nonequilibrium disordered systems?

Aimed at shedding some light on this issue, we study nonequilibrium models with absorbing states in the presence of disorder. More specifically, we study a variant of the well-known "contact process," sometimes called the "quadratic contact process," in which two particles are needed to generate an offspring while isolated particles can spontaneously disappear [11–13,18,19]. As a first step, we verify that the pure version of the model exhibits a first-order transition separating an active phase from an absorbing one. Then we introduce disorder in the form of a site-dependent transition rates and investigate whether the discontinuous character of the transition survives.

II. TWO POSSIBLE SCENARIOS

Two alternative scenarios might be expected *a priori* for the impure or disordered model:

(1) The Imry-Ma argument breaks down in this nonequilibrium case and a *first-order phase transition* is observed or

(2) The Imry-Ma prediction holds even if the system is a nonequilibrium one, and a disorder-induced *second-order phase transition* emerges.

If the latter were true, we could then ask what universality class such a continuous transition belongs to. *A priori,* it could share universality class with other already-known critical phase transitions in disordered systems with absorbing states [20–24] or, instead, belong to a new universality class defined by this disorder-induced criticality.

If no novel universal behavior emerges, then it is expected for the model to behave as a standard two-dimensional contact process (or directed percolation) with quenched disorder with the following main features [20,22–24]:

(1) There should be a critical point separating the active from the absorbing phase.

(2) At criticality, a logarithmic or *activated* type of scaling (rather than algebraic) should be observed. For instance, for quantities related to activity spreading such as the survival probability, averaged number of particles, and radius from a localized initial seed, we expect $P_s(t) \sim [\ln(t/t_0)]^{-\bar{\delta}}$, $N(t) \sim [\ln(t/t_0)]^{\bar{\theta}}$, and $R(t) \sim [\ln(t/t_0)]^{1/\Psi}$, respectively; t_0 is some crossover time, and $\bar{\delta}$, $\bar{\theta}$, and Ψ should take the values already reported in the literature [23].

(3) There should be a subregion of the absorbing phase, right below the critical point, exhibiting generic algebraic scaling with continuously varying exponents, i.e., a Griffiths phase [25]. Griffiths phases stem from the existence of rare regions where the disorder takes values significantly different from its average [24].

These features follow from a strong-disorder renormalization group approach for the disordered contact process, which concludes that this anomalous critical behavior can be related to the random transverse-field Ising model for sufficiently strong disorder [20], and have been confirmed in computational studies which suggested that this behavior is universal regardless of disorder strength [23,24,26].

III. MODEL AND RESULTS

We study the simplest nonequilibrium model with absorbing states exhibiting a first-order or discontinuous transition. Given that, owing to different reasons, one-dimensional systems with absorbing states rarely exhibit first-order phase transitions (even in pure systems) [27,28], here we focus on the physically more relevant two-dimensional case. In this two-dimensional reaction-diffusion contact-process-like model [11–14], individual particles disappear at a fixed rate, μ, while a pair of nearest-neighbor particles is required to create an offspring at some rate λ:

$$A \xrightarrow{\mu} \emptyset, \quad 2A \xrightarrow{\lambda} 3A, \tag{1}$$
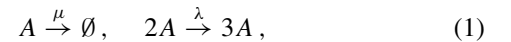
with the additional ("hard-core" or "Fermionic") constraint preventing sites from housing more than one particle. This restriction can be relaxed at the cost of introducing a reaction

$$3A \xrightarrow{\lambda} 2A, \tag{2}$$

which keeps the number of particles bounded. In either case, the corresponding rate or mean-field equations are (see the Appendix)

$$\dot{\rho}(t) = -\mu\rho(t) + \lambda\rho(t)^2[1 - \rho(t)], \tag{3}$$

where ρ represents the density of active sites or particles. This equation has the trivial stationary solution, $\rho = 0$, and an additional one at $\rho^* = \frac{1}{2}(1 + \sqrt{1 - 4\mu/\lambda})$ for $\lambda > 4\mu$, with an associated discontinuous transition at $\lambda = 4\mu$.

A. Pure model

Among the many possible ways in which the above particle system can be implemented [29–31], we employ the model proposed in Ref. [31], which was numerically studied in two dimensions and verified to exhibit a first-order phase transition separating an active from an absorbing phase [31].

We consider a two-dimensional square lattice and define a binary occupation variable $s = 0, 1$ (empty or occupied) at each site. We consider some initial conditions and perform a sequential updating following the standard procedure [11–14]: (1) an active site is randomly selected (from a list including all active ones); (2) with probability p_d (death) the particle is annihilated, otherwise, with complementary probability $1 - p_d$ a nearest neighbor site is chosen; (3) if this latter is empty, the selected particle diffuses to it, and otherwise an offspring particle is created at a randomly chosen neighboring site with probability p_b (birth) provided it was empty; otherwise nothing happens. We keep $p_b = 0.5$ fixed and use p_d as the control parameter.

As customarily done, we perform two types of experiments [11–14], considering as initial condition either a homogeneous state, i.e., a fully occupied lattice of linear size L, or a localized seed, consisting in this case of a few, at least a couple, neighboring particles in an otherwise empty lattice.

1. Homogeneous initial conditions

Figure 1 shows results of computer simulations for the temporal decay of the particle density from $\rho(t = 0) = 1$. The upper panel shows an abrupt change of behavior at a threshold value $p_{d_{\text{thr}}} \approx 0.0747$; activity survives indefinitely for $p_d < p_{d_{\text{thr}}}$ (at least up to the considered maximum time) and the particle density converges to relatively large steady state values ($\rho \approx 0.6$), while activity dies off exponentially for
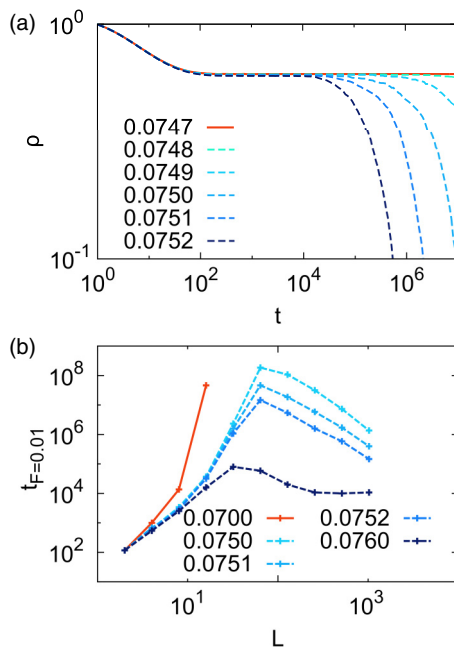
any $p_d > p_{d_{\text{thr}}}$. This behavior is compatible with a first-order phase transition, but the location of the threshold value has to be considered as a rough estimate.

To better locate the transition point, we study the mean survival time (MST) as a function of system size. Figure 1(b) shows a nonstandard nonmonotonous dependence of the MST as a function of size $N = L^2$. As we see, there are two regimes: (1) for $L < L_c$ there is an exponential increase of the MST with system size, and (2) for $L_c < L$, and quite counterintuitively, the MST decreases with increasing system size. This behavior can be rationalized following recent work where a particle system very similar to ours is studied by employing a semiclassical approach [32] (see also Ref. [33]). Following this study, the first regime corresponds to the standard Arrhenius law, i.e., the fact that a quasistationary state with a finite particle density experiences a large fluctuation extinguishing the activity in a characteristic time which grows exponentially with system size [34]. On the other hand, there is a "critical system size" above which the most likely route to "extinction" consists on the formation of a critical nucleus that then expands in a ballistic way, destabilizing the quasistationary state. Obviously the larger the system size the most likely that a critical nucleus is spontaneously formed by fluctuations. Finally, for sufficiently large system sizes there is a last "multidroplet" regime in which many nuclei are formed and the MST ceases to depend on system size, reaching an asymptotic value [32]. This picture fits perfectly well with our numerical findings.

From this analysis, we conclude that, with the present computational resolution, we can just give a rough estimation for the location of the transition point $0.070 < p_{d_{\text{thr}}} < 0.075$.

To show further evidence of the discontinuous nature of the phase transition, Fig. 2 illustrates the system bistability around the transition point: depending on the density of the initial configuration, a homogeneous steady state may converge either to a stationary state of large density (active) or to the absorbing state. A separatrix marks the distinction between the two different basins of attraction. Let us remark that systems exhibiting a first-order transition are bistable only at exactly

FIG. 1. (Color online) Decay from a homogeneous initial condition in the pure model . (a) Time evolution of the total averaged particle density for $N = L^2 = 256^2$; a first-order phase transition can be observed near $p_{d_{\text{thr}}} \approx 0.0747$. (b) Mean survival time, t_F, required to reach an arbitrarily small density value here fixed to 0.01 (results are robust against variations of this choice) as a function of system size. Up to 10^3 realizations have been used to average these results. From this finite size analysis, the threshold point can be bounded to lie in the interval [0.070, 0.075].
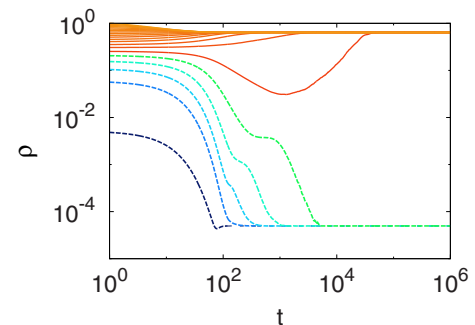

FIG. 2. (Color online) Bistability at the transition point of the pure model. Log-log plot of the averaged particle density as a function of time for different initial conditions in the neighborhood of the transition point (results here are for $p_d = 0.07315$). Depending on the initial density, the system stabilizes in the active or in the absorbing phase. The selected initial densities are equi-spaced in the interval [0.005, 1] with constant increments 0.05; system size $N = 256^2$ and averages performed over up to 10^6 realizations.
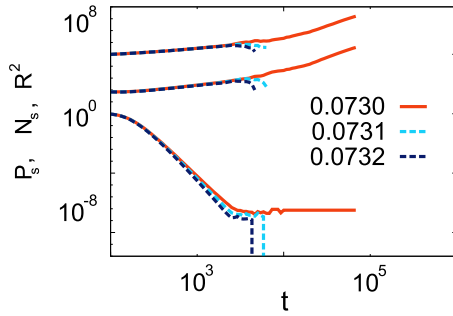
FIG. 3. (Color online) Spreading experiments for the pure model. Double-logarithmic plot of the (from bottom to top), (i) the survival probability $P_s(t)$, (ii) the averaged number of particles $N_s(t)$ (averaged over surviving runs), and (iii) the averaged squared radius (averaged over of all runs), as a function of time, for $N = 1024^2$ using up to 4×10^{10} experiments. Curves for $R^2(t)$ have been shifted upwards for clarity. In spite of the large number of runs used to average, curves are still noisy. This is due to the fact that, being very close to the transition point, a large fluctuation is needed for the system to "jump" to the active phase from the vicinity of the absorbing one and only a few runs reach large times.

the transition point but for finite system sizes the coexistence region has some nonvanishing thickness. The existence of bistability makes a strong case for the discontinuous character of the transition.

2. Spreading experiments from a localized seed

We consider a few (at least two) neighboring particles at the center of an otherwise empty lattice and monitor how activity spreads from that seed. Each simulation run ends whenever the absorbing state is reached or when activity first touches the boundary of the system. We monitor the averaged squared radius from the origin $R^2(t)$, the averaged number of particles over surviving trials, $N_s(t)$, and the survival probability, $P_s(t)$ [11]. Figure 3 shows log-log plots of these three quantities as a function of time. In all cases we find a threshold value $p_{d_{thr}} \approx 0.073$ that marks a change of tendency, signaling the frontier between the absorbing and active phases. In the active phase ($p_d < p_{d_{thr}}$) and for large values of t, both $N_s(t)$ and R^2 grow approximately as t^2 (as expected for ballistic expansion), while $P_s(t)$ converges to a constant (i.e., some runs do survive indefinitely). On the other hand, in the absorbing phase all three quantities curve downwards, indicating exponential extinction.

Thus, the pure model exhibits a discontinuous transition at some value of $p_{d_{thr}} \approx 0.073$, which separates a phase of high activity from an absorbing one. Observe that the estimation of the transition point is compatible with the interval obtained above.

B. Disordered model

In the disordered version of the model, each lattice site has a random uncorrelated (death) probability. In particular, we take $p_d(\mathbf{x}) = p_d r$ where p_d is a constant and r is a homogeneously distributed random number $r \in [0,2]$ (and, thus, the mean value is p_d). Spatial disorder is refreshed for each run, to ensure that averages are independent of any specific realization of the disorder.
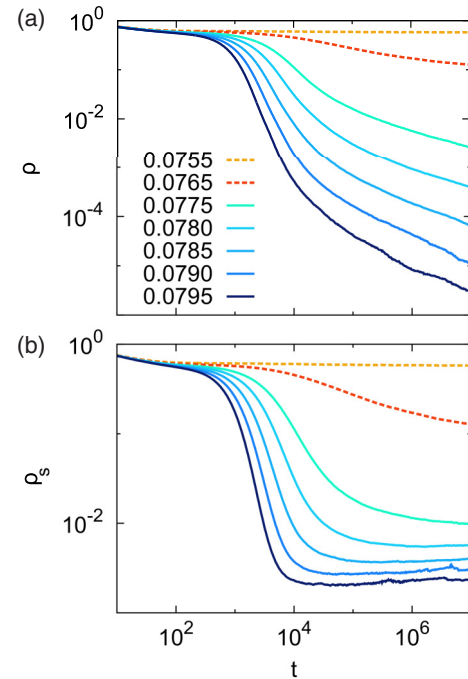

FIG. 4. (Color online) Density decay from a homogeneous initial conditions in the disordered model. (a) Particle density averaged over all trials in a lattice of size 256^2 and up to 2×10^4 realizations (curves in the active phase are plotted with dashed lines). Observe the presence of a broad region with generic power-law behavior, i.e., a Griffiths phase which starts roughly at $p_d = 0.0775$. (b) As (a) but averaging only over surviving trials. Note the nonmonotonic behavior in the Griffiths phase (see main text for details).

1. Homogeneous initial conditions

We have computed time series for (1) the mean particle density averaged over all runs and (2) the mean particle density for surviving runs (i.e., those which have not reached the absorbing state). Figure 4 shows time evolution after up to 2×10^4 realizations. Results are strikingly different from those of the pure model.

For values below threshold, $p_d < p_{d_c} \approx 0.077$, the particle density converges to a constant value for asymptotically large times, while for $p_{d_c} > 0.077$ curves decay as power laws (a much more precise estimation of the critical point will be computed below). The generic algebraic decay is observed for a wide range of p_d; however, the transient before the power-law regime increases with p_d, which makes it difficult to determine the exact boundaries of the mentioned range. The presence of generic algebraic scaling in an extended region is the trademark of Griffiths phases.

Plotting the activity over the surviving trials [Fig. 4(b)], we observe that the evolution is nonmonotonous in the absorbing (Griffiths) phase: the curves decrease up to a minimum value and then increase. This stems from the fact that realizations with large rare active regions remain active for longer times than those with smaller ones; as realizations with only relatively small rare regions progressively die out, those with larger and larger rare regions are filtered through, and, thus, the overall average density grows as a function of time, being limited only by system size.
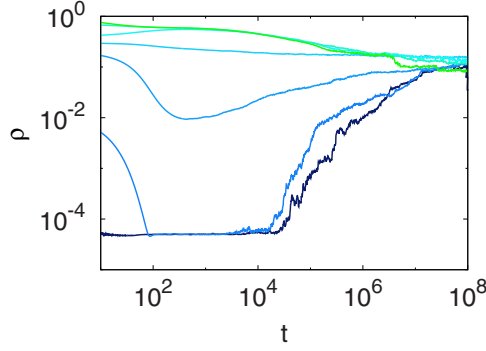
FIG. 5. (Color online) Absence of bistability in the disordered model. Double-logarithmic plot of the averaged particle density as a function of time for $p_d = 0.0765$, with $N = 256^2$, and up to 10^6 realizations. Initial densities are $\rho_0 = 0.00006, 0.01, 0.2, 0.3, 0.4, 0.7, 1$. Regardless of the initial condition, the system stabilizes to a constant small value of the density, as expected for a second-order phase transition.

In addition, we observe that, contrarily to the pure case, there is no bistability around the transition point (Fig. 5). Indeed, very near to the transition point ($p_d = 0.07650$), all curves regardless of their initial value converge to a unique well-defined stationary density close to zero, as appropriate for a continuous transition to an absorbing state.

2. Spreading experiments from a localized seed

Figure 6 shows results for three spreading observables as a function of time; for all of them, we clearly observe generic asymptotic power laws with continuously varying exponents. These spreading quantities also allow us to scrutinize the behavior at the critical point. As discussed in the Introduction, in a disordered system as the one under study, we expect logarithmic (activated scaling) at criticality. Indeed, Fig. 7 shows results for the usual spreading quantities represented in a double logarithmic plot of the different quantities as a function of $\ln(t/t_0)$. The value of t_0 is in principle unknown and constitutes a significant error source [23]. We fix it as the value of t such that it gives the best straight lines at the transition point for all three quantities [23]). Right at the critical point ($p_c \approx 0.07652$ to be obtained with more accuracy below) a straight asymptotic behavior indicates that results are compatible with logarithmic (i.e., activated) type of scaling. The best estimates for the (pseudo-) exponents listed in Sec. II are $\bar{\delta} \approx 1.90$, $\bar{\theta} \approx 2.09$, and $\Psi \approx 0.43$, which are compatible with the values reported in the literature for the universality class of directed percolation with quenched disorder [i.e., $\bar{\delta} = 1.9(2)$, $\bar{\theta} = 2.05(20)$, $\Psi = 0.51(6)$].

Similarly, following the work of Vojta and collaborators [23], we represent in Fig. 8 one of the spreading quantities as a function of another one, e.g., $N(t)$ as a function of $P_s(t)$ to eliminate the free variable t_0 from the plot. This type of plot allows for the identification of power-law dependencies rather than logarithmic ones, i.e., $N(t) \sim P_s(t)^{-\bar{\theta}/\bar{\delta}}$. If the second-order phase transition belongs to the universality class of the directed percolation with quenched disorder (see Sec. II), we should have $N(t) \sim P_s(t)^{-1.08(15)}$, using as a reference the values in the literature [23]. Indeed, as shown in Fig. 8 we
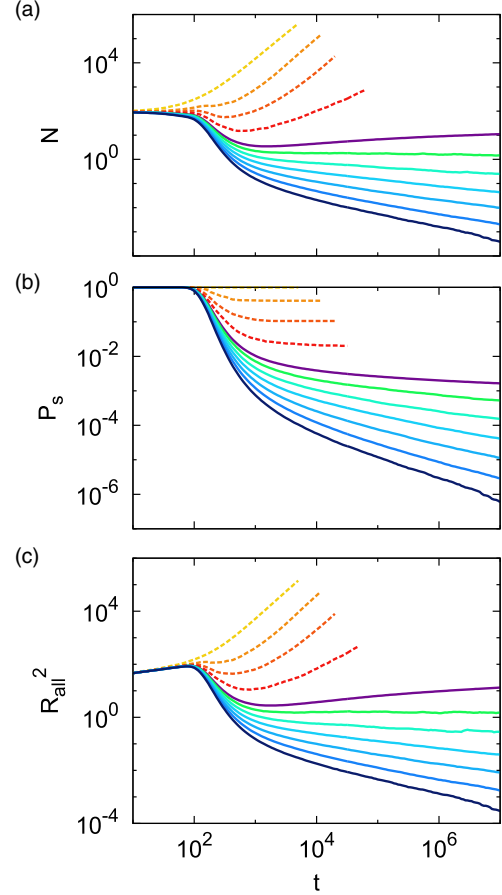

FIG. 6. (Color online) Spreading experiments in the disordered model. Double logarithmic plot of the three usual spreading quantities showing the presence of generic power laws with continuously varying exponents all along the Griffiths phase ($p_d \lesssim 0.07652$). Parameter values (from top to bottom) 0.06, 0.07, 0.073, 0.075, 0.07652, 0.077, 0.0775, 0.078, 0.0785, 0.079, 0.0795 (curves in the active phase are plotted with dashed lines), up to 5×10^7 realizations.

obtain $N(t) \sim P_s(t)^{-1.10(2)}$, in very good agreement with the expected value [23], and this is the method by which the critical point location, $p_d \approx 0.07652$, is obtained with best accuracy.

IV. CONCLUSIONS AND DISCUSSION

In contrast with the pure model, in the disordered case we have found a Griffiths phase and a second-order phase transition with an activated type of scaling. Therefore, in this nonequilibrium system with one absorbing state the situation remains much as in equilibrium situations: disorder annihilates discontinuous transitions and induces criticality.

Results are rather similar to those reported for the standard contact process with quenched disorder. Indeed, results are fully compatible (up to numerical precision) with the standard strong-disorder fixed point of the universality class of the directed percolation with quenched disorder [20,23,24]. We believe that our results are robust upon considering other types of (weaker) disorders [26]. Thus, two different models with significantly different pure versions, i.e., one with a first-order and one with a second-order transition, become very similar
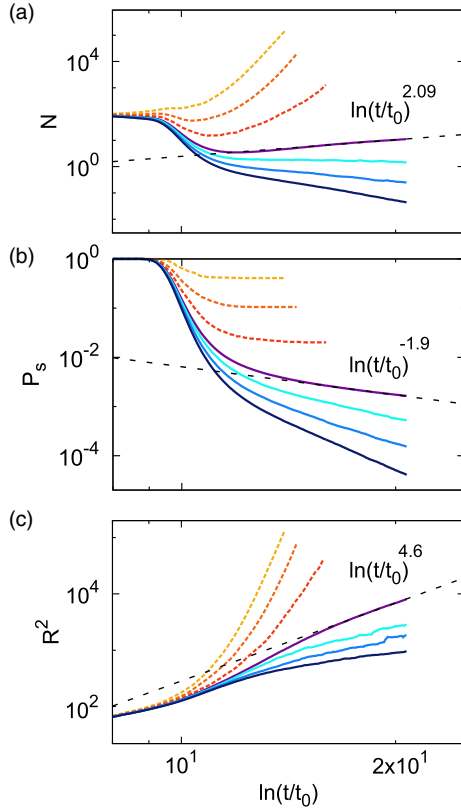
FIG. 7. (Color online) Spreading experiments in the disordered model. Double logarithmic plot of the three usual spreading quantities as a function of $\ln(t/t_0)$ for different parameter values (from top to bottom, 0.07, 0.073, 0.075, 0.07652, 0.077, 0.0775, 0.078; curves in the active phase are plotted with dashed lines). Same network sizes as in the homogeneous case and up to 10^6 realizations. By conveniently choosing $t_0 = 0.01$ (see main text) we observe straight lines at the critical point, $p_d \approx 0.07652$. From their corresponding slopes we measure the associated (pseudo-) exponents: $\bar{\theta} \approx 2.09$, $\bar{\delta} \approx 1,9$, and $2/\Psi \approx 4.6$ (slopes marked by dashed lines). These values have a large uncertainty, as changes in the value of t_0 severely affect them. Estimating these exponents with larger precision is computationally very demanding.

once quenched disorder is introduced. Both exhibit Griffiths phases and activated scaling at the transition point.

From a more general perspective, deciding whether novel universal behavior emerges in disorder-induced criticality is still an open problem in statistical mechanics. For illustration, let us point out that recent work suggests that disorder-induced second-order phase transitions in an Ising-like system with up-down symmetry does *not* coincide with Ising transition [35]. Similarly, in Ref. [36] a novel type of critical behavior is found for disorder-induced criticality. In the case studied here, the disorder-induced criticality does not seem to lead to novel behavior (up to numerical precision); indeed, all evidences suggest that it behavior coincides with the universality class of the directed percolation with quenched disorder.

After a careful inspection of the literature in search of discontinuous transitions in disordered nonequilibrium low-dimensional systems, we found a very recent work in which the authors study the popular (two-dimensional) Zipf-Gulari-Barshad (ZGB) model for catalytic oxidation of
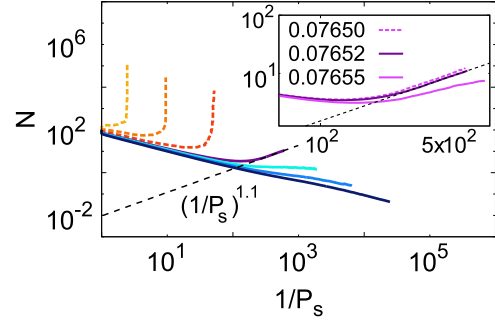

FIG. 8. (Color online) Double logarithmic plot of $N(t)$ as a function of $1/P_s(t)$ for spreading experiments at criticality in the disordered model [24]. Our best estimate for the slope of at the critical point (separatrix of the curves; see main panel) is compatible with the value reported in the literature $N(t) \sim P_s(t)^{-\bar{\theta}/\bar{\delta} \approx -1.08}$ corresponding to the universality class of the directed percolation with quenched disorder. The inset shows a zoom around the critical point. Lattice size $N = 1024^2$; averages up to 5×10^7 realizations, and same parameters as in Fig. 7.

carbon monoxide [37] in the presence of catalytic impurities (a fraction of inert sites) [38]. The pure ZGB model is known to exhibit, among many other relevant features, a discontinuous transition into an absorbing state. However, after introducing quenched disorder, no matter how small its proportion, the discontinuous transition is replaced by a continuous one [38], similar to our findings here.

In conclusion, we conjecture that first-order phase transitions cannot appear in low-dimensional disordered systems with an absorbing state. In other words, the Imry-Ma-Aizenman-Wehr-Berker argument for equilibrium systems can be extended to nonequilibrium situations including absorbing states. The underlying reason for this is that, even if the absorbing phase is fluctuationless and hence is free from the destabilizing effects the Imry-Ma argument relies on, the other phase is active and subject to fluctuation effects. Therefore, intrinsic fluctuations destabilize it as predicted by the Imry-Ma-Aizenman-Wehr-Berker argument, precluding phase coexistence.

Remarkably, in the case studied by Barghathi and Vojta, in which the Imry-Ma argument is violated in favor of a second-order phase transition, the two broken-symmetry states are absorbing ones: once the symmetry is broken in any of the two possible ways, the system becomes completely frozen, i.e., free from fluctuation effects, and, consequently, the Imry-Ma argument breaks down. Thus, the only possibility to have first-order phase transitions in low-dimensional disordered systems would be to have (in its pure version counterpart) a discontinuous phase transition between two different fluctuationless states, and we are not aware of any such transition. Therefore, we conclude that quenched disorder forbids discontinuous phase transitions in low-dimensional nonequilibrium systems with absorbing states.

APPENDIX: SIMPLE MEAN-FIELD APPROACH

In order to obtain a simple mean-field approach to the present problem, expected to hold in the high-dimensional limit, we consider a fully connected lattice in which each node is nearest neighbor to any other one. $n(t)$ is the total number of particles at a given time, and $\rho(t) = n(t)/N$ the corresponding density. Allowed changes at any time step are of the magnitude $\pm 1/N$. We can thus write the overall transition rates as $W^-(\rho \to \rho - 1/N) = \mu\rho$ and $W^+(\rho \to \rho + 1/N) = \lambda\rho^2(1 - \rho)$ for creation and annihilation processes, respectively. Expanding the associated master equation in power series, and keeping only the first two leading terms, we obtain the Fokker-Planck equation:

$$\partial_t P(\rho,t) = -\partial_\rho[(-\lambda\rho^3 + \lambda\rho^2 - \mu\rho)P]$$
$$+ \frac{1}{2N}\partial_\rho^2[(-\lambda\rho^3 + \lambda\rho^2 + \mu\rho)P],$$

equivalent to the (Itǒ)Langevin equation [34]:

$$\partial_t \rho = -\mu\rho + \lambda\rho^2 - \lambda\rho^3 + \sqrt{\frac{\mu\rho - \lambda\rho^3 + \lambda\rho^2}{N}}\xi(t),$$

where $\xi(t)$ is a Gaussian white noise. In the $N \to \infty$ (mean-field) limit one recovers the rate equation [Eq. (3)] with its associated discontinuous transitions. Using the noise term it is possible to derive (using the theory of mean first passage times [32–34]) the scaling of the escape time as a function of the system size.

In general, one-component reaction diffusion systems with l-particle creation and k-particle annihilation [39],

$$kA \to (k - n)A, \quad lA \to (l + m)A, \quad (A1)$$
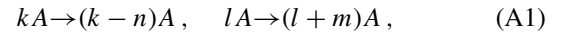
always exhibit a first-order transition if $l > k$ if the particles are fermionic. Instead, if the hard-core constraint is excluded, an additional reaction $iA \to jA$ with $i > j$ and $i > l$ is needed to stabilize the system.

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