Influence of stochastic perturbation of both action updating and strategy updating in mixed-strategy 2 × 2 games on evolution of cooperation

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In a mixed-strategy game framework, each agent's strategy is defined by a real number; on the other hand, in a discrete strategy game framework, only binary strategies, either cooperation or defection, are allowed. In a spatial mixed-strategy game, with respect to the process for updating action (offer), either a synchronous or an asynchronous strategy update should be presumed. This study elucidates how stochastic perturbation that results from a synchronous or an asynchronous process for updating action significantly affects the enhancement of cooperation in an evolutionary process. Especially, when a *synchronous process for updating action* is assumed, the extent of cooperation increases with an increase in degree.

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I. INTRODUCTION

The mechanism of emergence and maintenance of cooperation has attracted significant attention in various sciences [1,2]. Evolutionary game theory sheds some light on this long-standing issue, allowing exploration of this ubiquitous cooperation [3]. The theory was first introduced as a tool for studying animal behavior [4], but it has become a general theoretical framework for the study of ecosystems, economic problems, and human social evolution [5]. In particular, a simple and paradigmatic model, the prisoner's dilemma (PD) game, wherein two agents simultaneously decide to adopt one of two actions, cooperation (C) or defection (D), has been studied extensively within both theoretical and experimental studies as a plausible social metaphor of real interactions. When a PD game is played by an infinite well-mixed population, the organization of cooperative dynamics is not supported. In past decades, a great number of scenarios have been identified to offset such an unfavorable outcome of social dilemmas, wherein cooperators are condemned to extinction, thereby leading to the evolution of cooperation [6–11]. As part of this trend, Nowak has attributed all these to five scenarios: kin selection, direct reciprocity, indirect reciprocity, network reciprocity, and group selection, which, compared with the so-called well-mixed population, can be somewhat related to the reduction of an opposing agent's anonymity for maintaining the cooperative trait [12]. As Nowak explains, the five scenarios add "social viscosity" to the original well-mixed population by oppressing anonymity when two agents play a game.

Since the appearance of the innovative five scenarios, network reciprocity, in which agents are arranged on a spatially structured topology and interact only with their immediate neighbors, who are specified by edges and nodes, has drawn the greatest interest. This is because in network reciprocity, cooperators can survive by forming compact clusters, which minimize exploitation by defectors. This seminal idea, the role of spatial structure, and its various underlying variances in evolutionary games have been keenly explored [13–23] (see [24] for a recent review). In networks, scientists have found that strategy update rules and dynamics significantly impact the evolution of cooperation [25–39]. Let us mention two notable examples. In recent studies [40–45], where agents were allowed to adjust their strategy based on diverse learning ability or aspiration to the fittest opponent, the prevalence of cooperative behavior, even under significant temptation to defect, was observed. In [46], it was reported in detail that several specific evolutionary dynamics presumed that PD games on heterogeneous networks, such as scale-free networks, can lead to an outbreak of cooperation, even if inherent dilemma strength does not favor the spreading of cooperators. It was promising, furthermore, that both strategy update and update dynamics, which regulate how the time step advances in a simulation and which considers either synchronicity or asynchronicity in a simulation, were more influential on the evolution of cooperation than network topology (overviewed in [47,48]).

Going back to the original concept of strategy definition itself, some researchers altered it to answer the question of why agents are restricted to act using either C or D in traditional models [49]. In real systems, individual behavior (or traits in evolutionary ecology) can hardly be expected to have such a patently discrete nature. From this perspective, evolutionary dynamics in mixed-strategy or continuous strategy games have been studied, strategies that permit agents to act with more diverse behavior than simply to cooperate or defect [50]. A public goods (PG) game, which can be classified as a multiplayer PD game, often presumes continuous rather than discrete strategy, because, unlike binary action, a continuous range of action is more appropriate to the inherent nature of this dilemma game [51–53]. Similar to a PD game, a PG game is thought to be able to model irrational human behavior; game participants are required to offer a certain cost burden for their support of sustainable PG. Enhanced cooperation is observed if an assumed model is efficiently implemented for adding social viscosity in the society [54–58] (recently reviewed by [59]).

Despite the reports of several precursors, the manner in which the different strategy systems produce different cooperative phases remains ambiguous. A deeper study of both continuous strategy and mixed-strategy games—in which an agent decides a subsequent action with probability specified

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by a strategy value—would be especially valuable. As Zhong *et al.* [49] found, games on a spatial structure with these strategies have equilibria that are quite different from those on well-mixed and infinite populations. In particular, they showed that a continuous strategy in a spatial PD game with a relatively larger chicken-type dilemma than stag-hunt-type dilemma [60], where interactions instinctively tend toward the so-called internal equilibrium, realizes a much higher fraction of cooperators than the discrete strategy game. The reason is that the offer of a middle cooperative action by a real strategy value between 0 and 1 more efficiently and flexibly leads to a more cooperative equilibrium. Also, in a situation in which games feature a relatively stronger stag-hunt-type dilemma than a chicken-type dilemma, games with a somewhat bistablelike equilibrium, in which all agents are either defectors or cooperators, the mixed-strategy game shows a more cooperative equilibrium.

Meanwhile, in almost all previous studies, a mixed-strategy game of spatial PD implicitly assumes an asynchronous action update, in which focal agents stochastically decide the subsequent action, either C or D, based on their strategy values to all their neighbors independently. Even though a mixed strategy by a synchronous action update (wherein focal agents offer consistent C or D, stochastically determined by their strategy, to all their neighbors in a single time step) is as likely as an asynchronous strategy update, it is unclear what the synchronous process for updating likes and how it differs from the case of the asynchronous process. In a conventional discrete strategy game, it is well known that the so-called "stochasticity" in a game process produced by features such as underlying networks and strategy adaptation rules remarkably affects the evolution of cooperation [61–66]. Therefore, evolutionary dynamics in a mixed strategy, where the decision-making process accompanying the strategy system instinctively has large stochasticity, might merit study for understanding the role of stochasticity in the evolution of cooperation. This study, motivated by all the above mentioned factors, thoroughly investigates evolutionary dynamics in the mixed-strategy 2×2 game on a network.

The remaining part of the paper is structured as follows. Section II is devoted to the description of a 2×2 game, underlying network, and game dynamics by a mixed strategy. Section III is devoted to a discussion of a series of numerical simulations highlighting differences that result from different update dynamics, which determine the moment at which the agent's action is updated in a mixed-strategy game. Finally, conclusions are discussed in Sec. IV.

II. MODEL

A. 2 × 2 game

We consider a 2×2 game as an archetype. For a mixed-strategy definition, agent i has strategy s_i, which is defined by a real number in range [0,1] instead of discrete strategy, $s_i = 0$ or 1. In this framework, s_i is the probability that agent i will cooperate with a neighbor. Namely, agent i always cooperates if $s_i = 1$ (complete cooperator, C) or always defects if $s_i = 0$ (complete defector, D).

Agents are rewarded (R) for mutual cooperation and punished (P) for mutual defection. If one agent chooses C and the other chooses D, the latter receives a temptation payoff (T), while the former receives a payoff labeled as sucker (S).

According to the seminal idea [60], we define $D_r = P - S$ and $D_g = T - R$. We should say there is a dilemma to some extent in a presumed game unless both are negative. D_g indicates the dilemma intensity of how two equal players are inclined to exploit each other, which is called the chicken-type dilemma. While D_r implies the dilemma intensity of how equal players try never to be exploited by each other, which is called the stag-hunt-type dilemma. Thus, a PD where both D_g and D_r are positive has both the chicken- and the stag-hunt-type dilemmas at the same time. In this study, rescaling the payoffs such that $R = 1$ and $P = 0$ without loss of mathematical generality, the payoff matrix can be given as

$$\begin{pmatrix} R & S \\ T & P \end{pmatrix} = \begin{pmatrix} 1 & -D_r \\ 1 + D_g & 0 \end{pmatrix}, \tag{1}$$

where the elements are assumed to satisfy $T > R > P > S$ and $2R > T + S$. This means that in the following discussion, we limit the PD game class by assuming $0 \leqslant D_g \leqslant 1$ and $0 \leqslant D_r \leqslant 1$.

B. Network

As the underlying network on which each agent plays games with immediate agents connected with edges, we employ a regular lattice with a periodic boundary condition with various degrees k. The total number of population N is set to be $70 \times 70 = 4900$, which is sufficiently large to draw almost insensitive results to the influence of social size.

C. Strategy updating and action updating

In classic evolutionary 2×2 game models with the discrete strategy definition, the updating process for strategy is presumed to model the concept of strategy adaptation. In addition to this, the action updating must be presumed before the strategy updating in the present simulation framework in which mixed strategy is adopted since each agent's action can be switched according to the probability determined by the strategy during an evolutionary scenario (see Sec. II A). Here a strategy can be paraphrased as the behavioral profile for an agent, and action means the actual behavior, either C or D derived from the strategy. Thus, we inevitably divide the updating process into two parts in our simulation: *strategy updating* and *action updating*.

First, we explain the part of action updating. Agents must decide the subsequent action, either C or D, according to the probability of their own strategy s_i when they play a game with opponents specified by the underlying network. The presumed timing for updating action in mixed-strategy games can be identified as one of two types: *synchronous process for updating action* or *asynchronous process for updating action*.

In the case of synchronous process for updating action, all agents update their own action simultaneously according to the probability, which relies on the strategy s_i at each time step. The decided action must be fixed during a time step. After doing so, using the decided action, each agent accumulates the payoff while gaming with one's immediate neighbors.

In the case of asynchronous process for updating action, agents sequentially update their action at every moment they play a game with an opponent. Let us take an example assuming agent i having two neighbors: agent j_1 and agent j_2. When agent i plays a game with agent j_1, he or she updates one's action according to the probability s_i. Following to this, agent j_1 also updates the action. Subsequently, when agent i plays a game with agent j_2, agent i reupdates the action according to the probability s_i. It is followed by agent j_2 updating his or her action. Thus, if the asynchronous process is used for updating action, each one's action is refreshed at each gaming event with each opponent to accumulate payoffs.

As is known well in the field, after accumulating payoffs, there are also two types of processes, synchronous or asynchronous strategy updating, to determine the dynamics of strategy adaptation. If strategy s is updated synchronously, the game is iterated forward in accordance with the sequential simulation procedure comprising the following elementary steps. First, agent i acquires his or her payoff π_i by playing games with all of his or her neighbors. Then, we similarly evaluate the payoffs of all agents. In each game, each agent's action is determined by the rule defined in the previous paragraph. Last, agents synchronously update their strategy at every step based on the accumulated payoffs with all neighbors. If an asynchronous process is used, agent i is chosen randomly from the whole population consisting of N agents. Next, agent i and all immediate opponents acquire their own respective payoffs by playing games with their respective opponents, and then, each agent's action is determined by the rule defined in the previous paragraph. Finally, agent i updates the strategy based on the accumulated payoff. In the same manner, by choosing the next focal agent, the procedure is iterated forward for all agents.

In this study, we adopt imitation max (IM) as the strategy update rule, in which a focal agent unconditionally imitates the strategy that acquires the largest payoff among all the strategies available to both focal agent and immediate neighbors. For emulating a realistic human decision-making process, one might think a pairwise process or other stochastic method would be appropriate, rather than a deterministic method such as IM. Although we agree, we do, in fact, presume IM this time, because our primary concern is how stochasticity caused by a different updating process for action in mixed-strategy games affects the evolution of cooperation. Thus, other assumed conditions in simulations such as strategy updating method and network topology should be fixed to deterministic or homogeneous.

D. Simulation setting

Each simulation runs as follows. Initially, to initiate the evolutionary process, each agent is assigned the strategy s_i drawn from a uniform distribution of range $[0,1]$. The action, either C or D, is determined by the rule mentioned above. Then, several time steps are run until strategy values arrive at an equilibrium state. Finally, we obtain the averaged frequency of cooperators and values of s for the final 100 time steps. In this framework, an agent can employ a different action for each neighbor if the action is determined asynchronously, because it is probabilistically defined. Therefore, the average fraction of cooperators, ρ_c, is calculated on the basis of all actions that actually happened, either C or D, which is observed in the entire population during the final 100 time steps. From the perspective of statistical robustness, we take an ensemble average, which is evaluated on the basis of 100 independent realizations for each dilemma strength point.

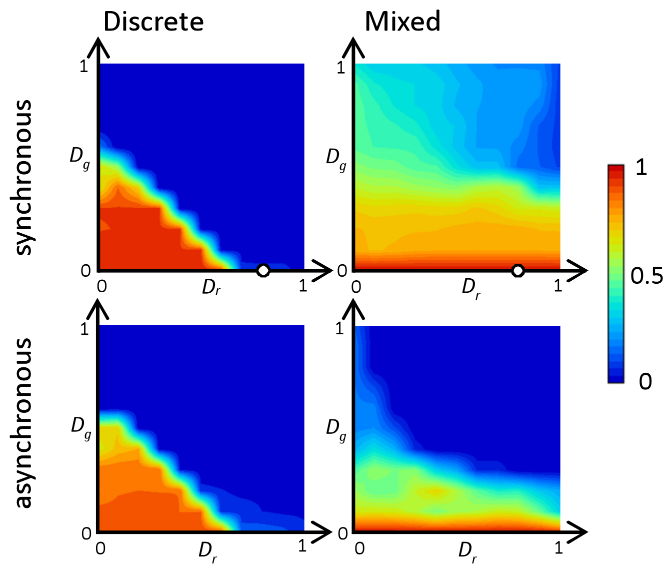

FIG. 1. (Color online) Comparison of fraction of cooperators for discrete strategy definition and mixed-strategy definition in the PD area on a regular lattice with $k = 8$. The top panels show results when the strategy is updated synchronously, while the bottom panels show when it is updated asynchronously. In this case, for the moment to update the action for mixed-strategy definition, an asynchronous process for updating action is used.

III. RESULTS AND DISCUSSION

Before we discuss the complicated story, to make things simple and to grasp the entire picture, we first review evolutionary outcome in the discrete strategy versus the mixed-strategy 2×2 game. Figure 1 shows the fraction of cooperators in the discrete strategy setting, where $s_i = 0$ or 1 only is allowed and in the mixed-strategy setting, where s_i is initially defined in the range from 0 to 1 by random uniformity. Here we employ a square lattice with $k = 8$ and an asynchronous process for updating action in the mixed-strategy game. Apparently, mutual cooperation is enhanced in a mixed strategy than in a discrete strategy if we are concerned only with a game that features a strong stag-hunt-type dilemma. In the conventional discrete strategy setting, the variety of payoffs that a cooperator obtains is either R or S, and a defector can obtain either T or P. On the other hand, in the mixed-strategy setting, it would be possible to acquire all four payoffs, R, S, T, and P, irrespective of strategy value (except for $s_i = 0$ and 1), because each action is a result of the stochastic process arising from the focal agent's strategy; thus, even a relatively cooperative (defective) agent may act as a defector (cooperator). Because of this, a mixed-strategy game prevents a C-type agent (a relatively

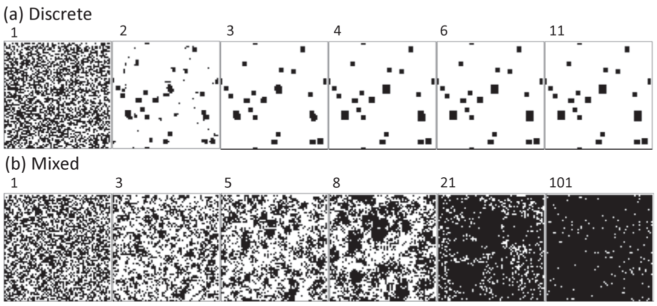

FIG. 2. Snapshots for action C (black solid square) and D (white square) in $D_g = 0$ and $D_r = 0.8$ (shown in Fig. 1 by the white circle on the horizontal axis). Panel (a) shows the action in a discrete strategy setting, where either $s_i = 0$ or 1 is permitted as the strategy parameter. Panel (b) shows the action in a mixed-strategy setting with an asynchronous process for updating action. IM is employed for both simulations as a strategy update rule.

cooperative agent equipped with $s_i > 0.9$ [67]) from being exploited, and it decreases the payoff of a D-type agent who imposes S on a C-type opponent and acquires T. Such an effect sustains cooperation in the long run under the setting for a mixed strategy. Figure 1 also indicates that a synchronous process for updating strategy surpasses an asynchronous one.

To verify the above observations, Fig. 2 shows snapshots depicting each agent's actual action, C or D, with a corresponding color. In the case of a discrete strategy setting, where only a pure strategy is allowed, in the early period of an evolutionary scenario, cooperators are invaded by surrounding defectors; thus, they are exploited by neighboring defectors who impose S on them; thus, the cooperators hardly form a C cluster. Therefore, cooperation does not evolve when dilemma strength becomes strong [panel (a) of Fig. 2]. In contrast, in the case of a mixed-strategy setting, D-type agents that have a low s_i value do not always have an advantage over C-type agents, because all agents act stochastically, not always offering a time constant C or D, even under the same strategy value. Hence, once cooperators survive the initial invasion by defectors, some germs of C clusters can survive, leading to the maintenance of cooperation to some extent [panel (b) of Fig. 2]. To paraphrase, cooperators who would become defectors at the beginning of the evolutionary dynamics in a discrete strategy game framework are allowed to remain as cooperators in a mixed-strategy game framework, although other cooperators, who would *not* become defectors, might convert to defectors in the discrete strategy game framework.

Incidentally, we may ask why the choice of strategy adaptation process—synchronous or asynchronous accumulation of payoff after playing games with neighbors—greatly affects the outcome of the evolution of cooperation even under a mixed strategy, which is inherently stochastic. The presumed framework of how an agent accumulates payoff strongly controls the amount of stochastic randomness, or say stochasticity, which the model has as a whole. As mentioned above, in mixed-strategy games, to a great extent, in the decision-making process of an agent whose action depends on a strategy value ranging from 0 to 1, stochastic nature is already embedded. Despite this fact, the further stochastic mechanism by which an agent's payoff is accumulated, or says whether strategy update dynamics are synchronous or asynchronous, still significantly impacts the final outcome (right panels of Fig. 1). Meanwhile, in the discrete strategy setting, cooperation is slightly destroyed when payoff is calculated asynchronously (left panels of Fig. 1). In both discrete and mixed-strategy cases, since the asynchronous update rule adds stochastic perturbation, it is difficult to accomplish the perfect cooperative phase, although cooperation can be slightly maintained even in some part of strong dilemma areas. One possible reason for the phenomenon is that when randomness that exists in the

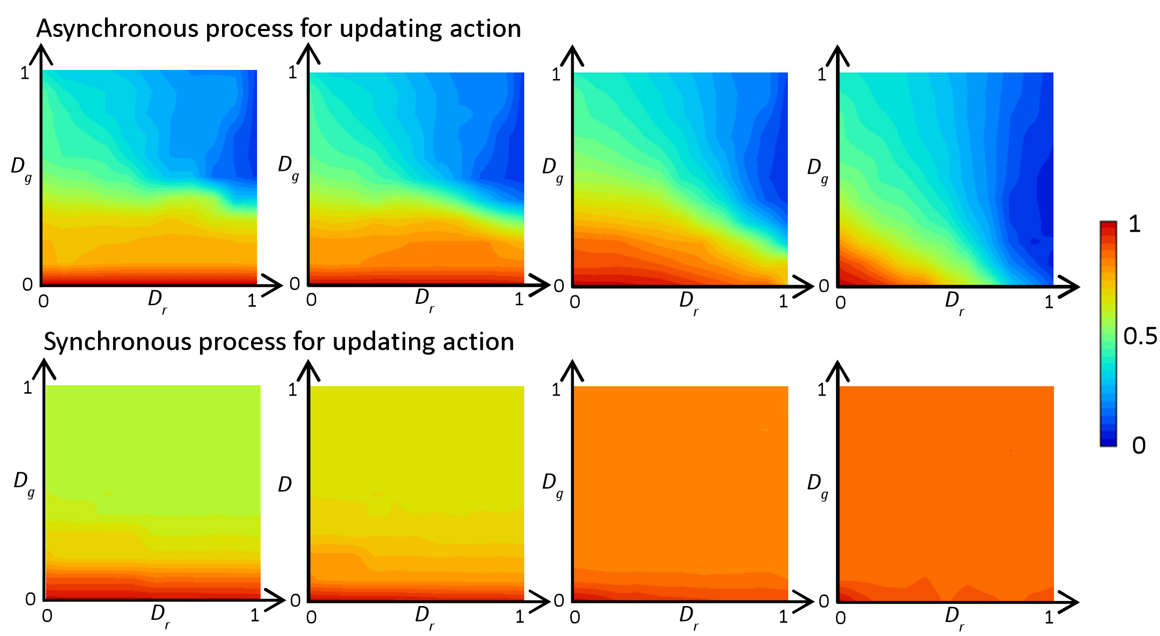

FIG. 3. (Color online) From left to right, the final fraction of cooperators on a regular lattice with $k = 8$, 12, 24, and 48 with the mixed-strategy definition for agents' strategy. The panels in the top row are results in the case of an asynchronous process for updating action, and the panels in the bottom row are results in case of a synchronous process for updating action.

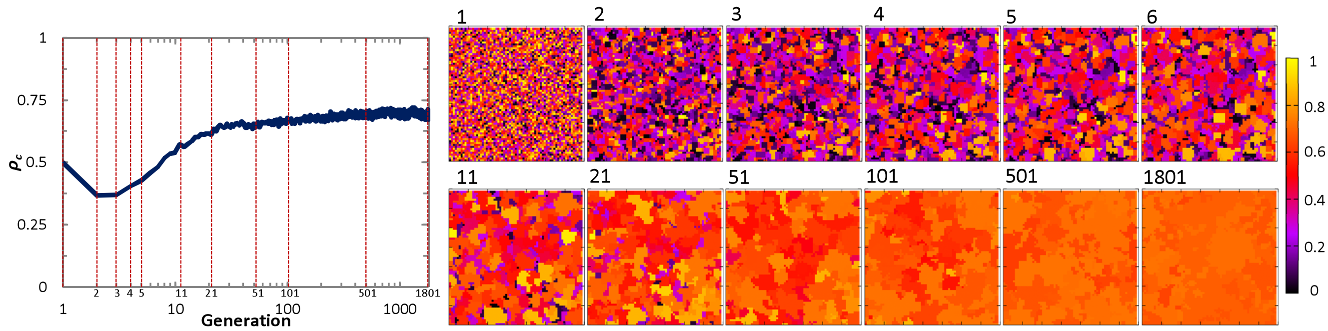

FIG. 4. (Color online) Snapshot for the average of each strategy value s_i and its evolutionary course, which is defined by a real number in range [0,1] on a regular lattice with $k = 8$, for $D_g = D_r = 0.3$, respectively. The synchronous process for updating action is employed. The number shown in the upper part of each snapshot corresponds to the generation of the evolutionary scenario. The red dotted lines also correspond to the generations for the snapshot. In the first step, D-type agents having low s_i invade C-type agents who possess a relatively high value of s_i. However, as time advances, surviving C-type clusters can expand gradually and finally reach a frozen state.

game process increases, the survival of cooperators will not be always guaranteed, whereas cooperators have a chance to survive under a strong dilemma structure [68]. As explained above, we find that the stochastic nature of the game process tremendously influences the evolution of cooperation.

As the next step, we discuss the result of the mixed-strategy game in more detail. For mixed-strategy games, two types of update timing can be assumed to determine the moment when agents update the subsequent action based on the probability s_i. We verify the effect of the rule for updating action on the evolution of cooperation in the mixed strategy. In the above discussion, although we found that the setting of the strategy adaptation process—the calculation method for payoff, whether synchronously or asynchronously—significantly affects the final state in the mixed-strategy games, we briefly introduce the following discussion: We set a synchronous update for the strategy adaptation process (we confirm that the same conclusion would be drawn if an asynchronous strategy update was presumed). We now show the result. Figure 3 indicates the fraction of cooperators when either a synchronous or an asynchronous process for updating action is used for different degrees in terms of the underlying network. Obviously, a different way of updating action brings about an entirely different final result. This is especially true in the case of a synchronous process, where mutual cooperation is remarkably promoted. Of particular note is that the prosperity of a cooperator is proportional to the number of degrees of the underlying network. As previous studies highlight, despite the fact that a discrete strategy system is assumed, it is well known that increasing anonymity on a network by increasing average degree means approximately closing it to a well-mixed population, driving the population to a defective state in PD games. This is consistent with the result for an asynchronous process (top panels in Fig. 3), where we see that cooperation weakens as degree increases. However, the result for a synchronous process for updating action (bottom panels in Fig. 3) surprisingly contradicts this. This means that the influence of reducing social viscosity, which usually induces defective behavior, might work positively in the present framework.

Our next question is, "Why does the choice of synchronous versus asynchronous updating of a process for action have a large impact on the evolution of cooperation in mixed-strategy games?" To answer this in detail, we look at a typical snapshot of a small-degree case and a large-degree case. Figures 4 and 5 illustrate a time series of average strategy value s for all

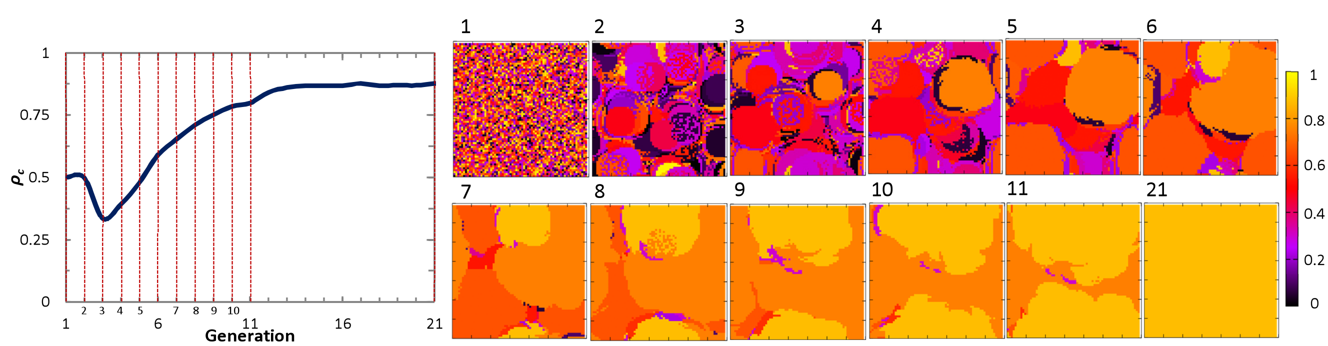

FIG. 5. (Color online) Snapshot for the average of each strategy value s_i and its evolutionary course, which is defined by a real number in range [0,1] on a regular lattice with $k = 192$, for $D_g = D_r = 0.3$, respectively. The synchronous process for updating action is employed. The number shown in the upper part of each snapshot corresponds to the generation of the evolutionary scenario. The red dotted lines also correspond to the generations for the snapshot. In the first step, D-type (or those possessing a half-baked value of s_i) agents invade fiercely. However, in the second step, the surviving C-type cluster can expand rapidly.

agents on a regular lattice network with $k = 8$ and $k = 192$, a synchronous process for updating action. By observing cooperator fraction ρ_c, which decreases at first (END, shown in Fig. 8) and increases afterward (EXP, shown in Fig. 8), we conjecture what happens as follows: D-type agents attack C-type agents during the early period of the episode (END), and surviving C-type agents successfully form several C-type clusters after this early period of ordeal (EXP). Comparing both cases of different k, we can confirm that a higher degree facilitates a higher cooperation at the equilibrium, which is realized by a single cluster composed of C-type agents with a single uniformed cooperative strategy. The stability of a C-type cluster generally depends on the amount of obtained payoff. Let us observe the payoff difference in the two different k cases taken in Figs. 4 and 5. Figure 6(a1) shows the time series of average payoffs for five agent groups classified by the level of strategy, while Fig. 6(a2) shows the time series of maximum payoff among agents in each of the five different strategy groups. These are drawn from the same trial observed in Fig. 4 snapshot, which is the case for $k = 8$. Figures 6(b1) and 6(b2) indicate the counterpart of Figs. 6(a1) and 6(a2) for the $k = 192$ case observed in Fig. 5. Note that none of these five strategy groups necessarily implies a single agent cluster, except for the more cooperative strategy groups after the seventh generation in case of $k = 192$.

We start the discussion with the case of $k = 8$ shown in Figs. 6(a1) and (a2). We should note that two strategy groups, 0.60–0.65 (dotted red line) and 0.70–0.75 (dotted purple line with square), survive until the end of the episode, although they are neither the most cooperative (0.95–1.00, dashed sky-blue line) nor the most defective (0.00–0.05, dash-dotted line) group. This is the way that these two strategy groups generate a higher maximum payoff than other groups for most of the period except for the initial chaotic time [Fig. 6(a2)]. This inhibits the follower agents from belonging to the same cluster, sharing the same strategy and having the highest maximum payoff agent similar to their neighbors, and copying a more defective strategy from neighboring strategy groups. This implies that this particular strategy group has less incentive to change strategy, and it can maintain its relatively cooperative strategy despite neighboring agents having a less cooperative strategy. Furthermore, in a PD game, a less cooperative strategy can always earn more than a more cooperative strategy. If this is so, why does the event mentioned before happen, and can cooperative clusters survive stably? To elucidate the answer, we consider a relatively cooperative cluster. Even

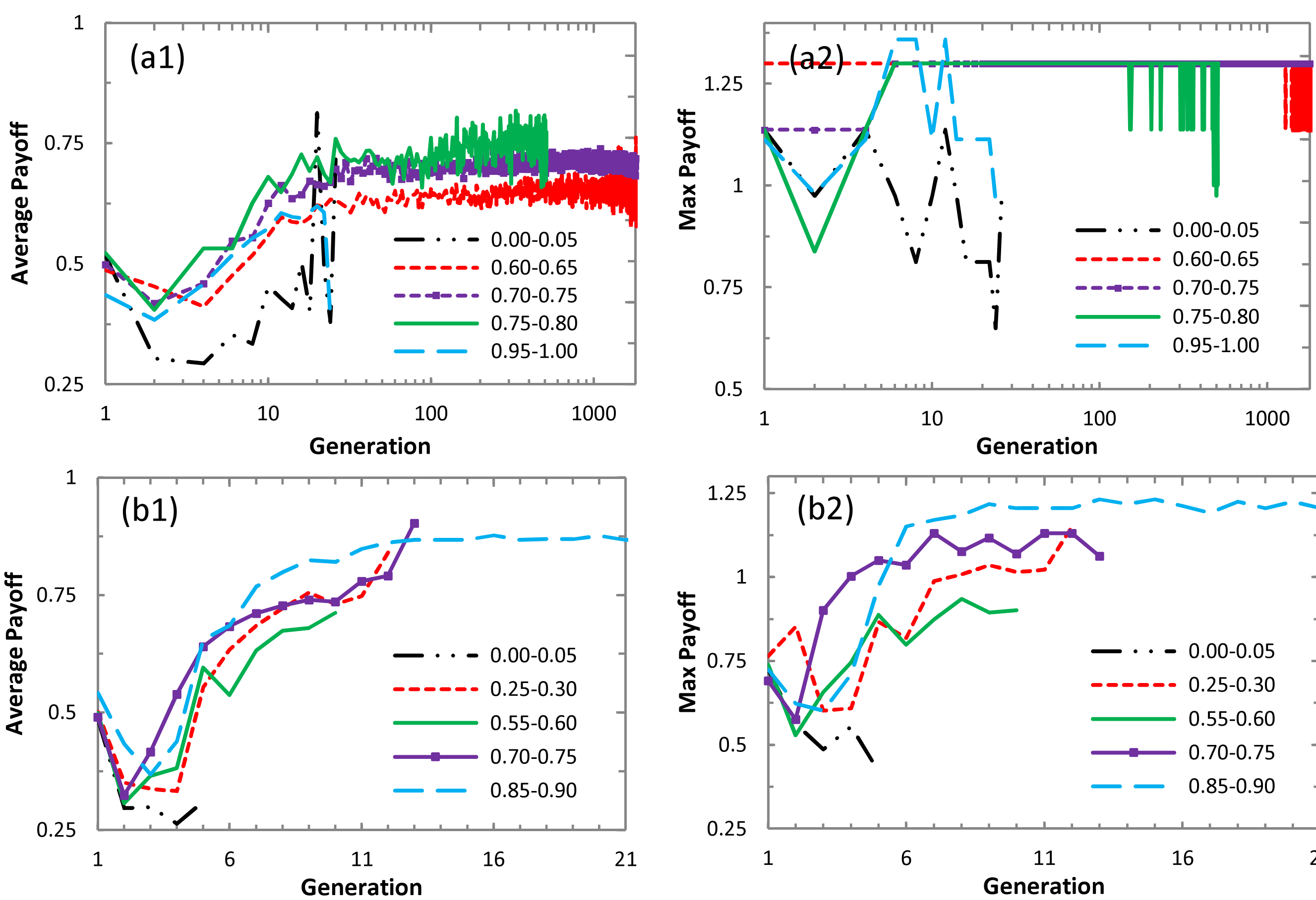

FIG. 6. (Color online) Time series for the average payoff among agents, which is classified by strategy values possessed (interval depicted shown in legend) and maximum payoff among them. Panels (a1) and (a2) show the average payoff and maximum payoff for $k = 8$, for $D_g = D_r = 0.3$, and panels (b1) and (b2) show them for $k = 192$, for $D_g = D_r = 0.3$, respectively. Payoffs in these figures are normalized by the number of degrees of the network, k. A logarithmic axis is used when $k = 8$ for simple visual information because we need long time steps to equilibrium. The synchronous process is used as a process for updating action.

in a cooperative cluster, by the dint of the mixed strategy with a synchronous process for updating action, agents may offer D's to all of their neighbors in spite of their relatively cooperative strategy, and most of their neighbors perhaps offer C because of their relatively cooperative strategy. Thus, the focus agent can temporarily obtain several T's from neighboring agents in the cluster (belonging to the same strategy group, of course). This pushes the agent to be the max-payoff agent among those in the same strategy group and in comparison with other strategy groups. In the case of a large-degree network, namely, results for $k = 192$ [Fig. 6(b2)], such an effect becomes more prominent. The most cooperative strategy group among those from which the agent draws, 0.85–0.90 (dashed sky-blue line) is able to survive until the end [Fig. 6(b1)] and entirely dominates the system (Fig. 5). In spatial PD games on homogeneous networks, it is evident that a cluster formed with a high value of s can always acquire more payoff than that with a low value if the cluster size is sufficiently large (see Appendix A). Therefore, it is natural that a cluster with a relatively high value of s can stably exist once its members successfully form a sufficiently large cluster. Meanwhile, in case of a large-degree network, agents with a relatively cooperative strategy can temporarily obtain many T's by offering D against their many neighbors who are honestly offering C. This transient event enables them to acquire the extremely high payoff at a certain moment. In addition, an IM of a large degree helps to enhance this, because the speed of strategy diffusion in the case of IM is much larger than in other pairwise ways. For all of these reasons, the case of a large network degree realizes a more cooperative strategy group surviving by constructing an extremely large C-type cluster, almost dominating the system, as we observed in Fig. 5. Likewise, one might expect that games on a network with a small degree might cause the same phenomenon. However, it is not noticeable, as we confirmed in Fig. 4. This is because the probability that this situation actually happens, i.e., one in which an agent standing at the center of the cluster offers D, while all neighbors offer C, is much higher in a small-degree case than it is in a large-degree case (see Appendix B). This means that in a small-degree case, many more agents who can obtain the maximum payoff by this special situation simultaneously exist. Among those agents' strategy that will be copied by their immediate neighbors, some of their strategy values might be high and some might be low. Thus, the clear difference of maximum payoffs among several groups is difficult to establish throughout an episode in the small-degree case, as compared in Figs. 6(a2) and 6(b2). Consequently, the evolution of cooperation is far inferior to that of an interaction with a large degree.

In the case of an asynchronous process for updating action, since focal agents update the action each time they play a game with each of their neighbors in accordance with the probability, the payoff of the focal agent does not have drastic ups and downs as in the synchronous process we discussed above. This indicates that there is no special advantage for enhancing cooperation other than the fact that a mixed strategy is implemented. In the end, the asynchronous process for updating action promotes less cooperation than the synchronous process. Looking back at the ample stock of precursors, it has been proven that stochastic perturbation (introducing a stochastic nature into evolutionary dynamics, e.g., adding noise to the payoff matrix, establishing a heterogeneous network, employing probabilistic updating) helps to enhance network reciprocity [69–71]. It is worthwhile to note that what we have found above, namely the difference between the synchronous and asynchronous update setting for an agent's action in the framework of a mixed-strategy game, should be understood in the context of the amount of stochastic perturbation that is realized in the model. It should be addressed that stochastic perturbation realized by updating actions in a synchronous manner, not an asynchronous manner, markedly increases the evolution of cooperation. An asynchronous updating action in the mixed-strategy system inherently entails probabilistic offering actions by gaming agents more often than a synchronous one, which leads the event frequency of C or D approximating to the average value expressed by strategy s_i. This obviously suppresses stochastic perturbation. This is the point that seems crucially important for why synchronous updating action in a mixed strategy surges cooperation.

Next let us mention the role of the underlying network. In the above discussion we presumed a homogeneous network: a regular lattice. Since each agent has the same number of neighbors, their payoffs are determined only by the strategy. On the other hand, when we presume a heterogeneous network, a BA scale-free network [72] for example, the cooperation is not so phenomenally enhanced vis-á-vis the lattice case as the network average degree increases even in the case that synchronous process for updating action is presumed with the mixed strategy. This is because not only the strategy but also the difference of degree among agents determines the agents' payoff, which implies that a hub agent having large degree becomes influential for spreading his or her strategy irrespective of his or her strategy value. Due to that fact, the drastic enhancement of cooperation can be seen only on a homogeneous network, but not so much on a heterogeneous network.

Even though a synchronous process for updating action is used in mixed-strategy games on a homogeneous network, there is a limit to enhance cooperation. Figure 7 shows the fraction of cooperators ρ_c versus the degree of network k. The peak of cooperation appears around $k = 48$, which is quite large compared with the critical degree for the network reciprocity to work effectively in conventional static spatial games. After the peak of ρ_c, it starts to decrease gradually in a linear fashion. Thus, a relatively high cooperation phase can be still observed on a quite dense network like the case of $k = 192$ shown in Fig. 5 among episodes, but such an episode occurs less frequently as the degree increases.

As we have discussed above, the difference of process for updating action whether synchronous or asynchronous significantly affects the evolutionary outcome as long as the updating process for strategy was fixed as a synchronous one. Let us mention the case of asynchronous process for updating strategy. In such cases, we obtain a similar tendency, one in which cooperation is enhanced in the case of a synchronous process for updating action, while weakened in the case of an asynchronous process (not shown); therefore, we may say that our conclusion is qualitatively the same as what would

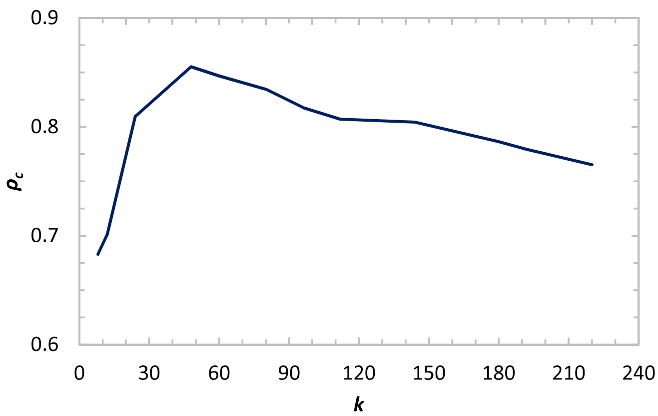

FIG. 7. (Color online) The fraction of cooperators ρ_c versus the degree of network k for $D_g = D_r = 0.3$. Each plot is obtained averaging over 100 independent realizations. The peak of ρ_c appears around $k = 48$. After that it starts to decrease in a linear fashion.

have been observed by implementing an synchronous strategy updating.

It is known that the cooperation is not significantly enhanced in PD games if the probabilistic strategy updating rule such as Fermi update rule (see Appendix C) is used in the traditional discrete strategy game. To explore the influence of the other strategy updating rule in our model, we adopted the Fermi update rule instead of IM. Despite presuming the mixed strategy, we confirmed that the cooperation was not enhanced on a regular lattice network, as in the case of the discrete strategy definition.

IV. CONCLUSION

In this study, we investigated the evolutionary mechanism of cooperation in a mixed-strategy 2×2 game and compared it with a discrete strategy setting. In a mixed-strategy game, an agent acts according to the probability defined by the strategy value. We found that the specification of a process for the agent's updating action, whether synchronously or asynchronously, significantly affects the final result. Especially, we showed a surprising result that cooperation is escalated as the degree of the underlying network increases in the case of a synchronous process for updating action, which opposes the general understanding that the higher the degree of an underlying network, the lesser the cooperation one is able to attain. The difference between a synchronous process and an asynchronous process for the updating offer is synonymous with the difference between how stochastic perturbation is considered in the game for the two processes. It is well known that the difference of stochastic perturbation derived from the choice of update rule for strategy adaptation that is used causes a relatively large influence in discrete strategy games. As shown in this study, such stochastic perturbation by the difference of the strategy adaptation rule also strongly impacts the final fraction of cooperators in mixed-strategy games, although the strategy definition itself is stochastic by nature. In addition, stochastic perturbation caused by the setting of the process for updating action plays a crucial role for the evolution of cooperation in the mixed-strategy game.

Shigaki *et al.* [73] elucidate one of the substantial mechanisms of network reciprocity to enhance cooperation, in which they insisted that the evolutionary process can be classified into two sequential periods: an enduring (END) period, where initial cooperators are rapidly plundered by defectors, leaving only a few cooperators forming compact C clusters, and an expanding (EXP) period, where C clusters start to expand (see the schematic view shown in Fig. 8). Evidently, such a tendency that they observed can be seen in the present model (time series shown in Figs. 4 and 5). This leads us to identify a universal principle in cases of network reciprocity, even those with superficial features: The different models that are assumed are observed to be quite different from our result showed, where a larger degree enables more cooperation, which contrasts common knowledge about what is observed in most models.

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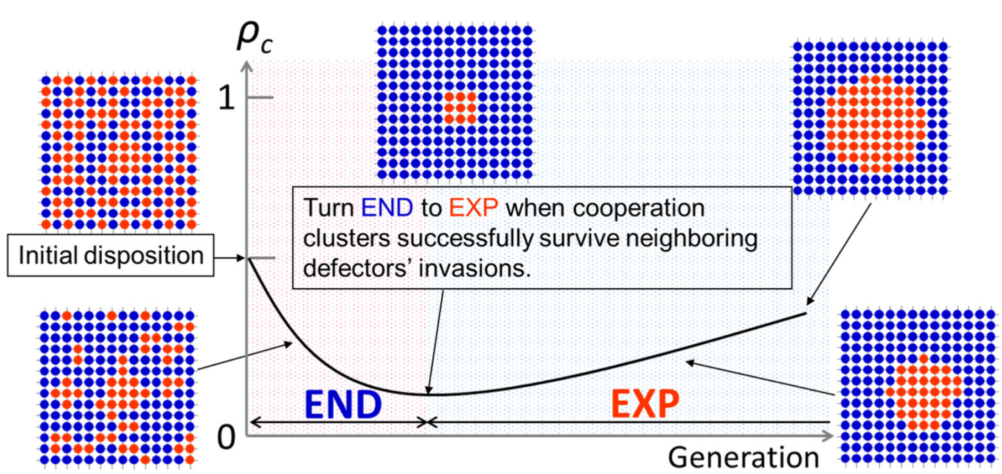

FIG. 8. (Color online) Schematic view for the evolution of spatial PD game with the concept of END and EXP. *Enduring (END) period*: Initial cooperators will be rapidly plundered by defectors, which leave only few cooperators by forming compact C clusters. *Expanding (EXP) period*: C clusters start to expand, since a cooperator on the clusters' border can attract a neighboring defector into the cluster. In the present framework, evolutionary dynamics can also be classified into these two types.

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APPENDIX A

When the size of a cluster is sufficiently large, the payoff acquired by agents who are placed near the center of a cluster, far from the boundary, can be predicted by applying mean field approximation (MFA). The average payoff per link in the cluster with a mixed strategy s_x within a range of $[0,1]$ is as follows:

$$\begin{aligned}\langle\pi\rangle &= s_x^2 R + s_x(1-s_x)S + (1-s_x)s_x T + (1-s_x)^2 P \\ &= (R-T-S+P)s_x^2 + (S+T-2P)s_x + P \\ &= (R-S-T+P)\left\{s_x + \frac{S+T-2P}{2(R-S-T+P)}\right\}^2 \\ &\quad - \frac{(S+T-2P)^2}{4(R-S-T+P} + P. \end{aligned} \tag{A1}$$

By taking the derivative with respect to s_x, we obtain

$$\frac{d\langle\pi\rangle}{ds_x} = 2(R-S-T+P)s_x + S + T - P. \tag{A2}$$

Depending on whether $R-S-T+P$ is positive, 0, or negative, we can evaluate whether $\frac{d\langle\pi\rangle}{ds_x}$ is positive by examining whether the minimum $\frac{d\langle\pi\rangle}{ds_x}$, or say, either $\frac{d\langle\pi\rangle}{ds_x}|_{s_x=0}$ or $\frac{d\langle\pi\rangle}{ds_x}|_{s_x=1}$, is positive, because Eq. (A2) is linear. Let us pursue the three cases as below:

(i) When $R-S-T+P>0$,

$$\begin{aligned}\left.\frac{d\langle\pi\rangle}{ds_x}\right|_{\min} &= \left.\frac{d\langle\pi\rangle}{ds_x}\right|_{s_x=0} \\ &= 2(R-S-T+P)0 + S + T - P \\ &= S + T - P; \end{aligned} \tag{A3}$$

(ii) when $R-S-T+P=0$,

$$\begin{aligned}\left.\frac{d\langle\pi\rangle}{ds_x}\right|_{\min} &= 2(0)s_x + S + T - P \\ &= S + T - P; \end{aligned} \tag{A4}$$

(iii) when $R-S-T+P<0$,

$$\begin{aligned}\left.\frac{d\langle\pi\rangle}{ds_x}\right|_{\min} &= 2(R-S-T+P)1 + S + T - P \\ &= 2R - S - T + P. \end{aligned} \tag{A5}$$

To the end, we obtain the necessary and sufficient condition to ensure that $\langle\pi\rangle$ is monotonically increasing as

$$2R > S + T - P > 0. \tag{A6}$$

As long as we presume archetype PD games, where $R=1$ and $P=0$ are assumed, and that R reciprocity, not ST reciprocity (see [74]), is satisfied, the condition above is always valid. As precisely noted in [74], R reciprocity means that the dilemma situation of mutual R's is meaningful for both players, as in typical PGs, whereas ST reciprocity means that the specific dilemma situation of alternating S and T is more beneficial than that of constantly obtaining mutual R's. The so-called leader game and hero game, which are both classified under the chicken game category, are typical dilemmas where ST reciprocity is meaningful.

In summary, when we presume typical PDs, where R reciprocity, not ST reciprocity, is satisfied, $\langle\pi\rangle$ is monotonically increasing with respect to the cooperation extent s_x.

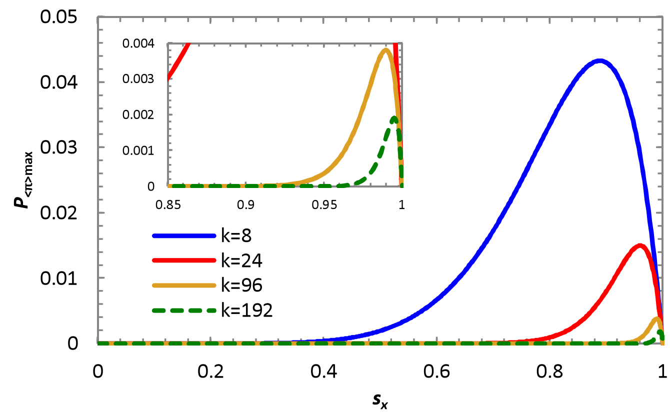

FIG. 9. (Color online) Probabilities that the special situation mentioned in Fig. 10 occurs in a cluster with degrees $k=8$, 24, 96, and 192. The inset shows magnification for the right bottom area. In the small-degree case, agents have a relatively high possibility of being the most profitable agent in the wide range of s_x.

APPENDIX B

We presume the special situation, one in which center-placed focal agents offer D, while all their neighbors offer C, as schematically shown in Fig. 10. Figure 9 shows the probability of the special situation above, in which the probability can be defined as $P_{\langle\pi\rangle\max} = (1-s_x)s_x^k$, where s_x means strategy shared by focal agents as well as all their neighbors. Obviously, the larger the degree, the rarer the special situation.

APPENDIX C

Imitation max is the deterministic strategy updating rule in which an agent unconditionally imitates the strategy which can produce the highest payoff among neighbors, while the Fermi

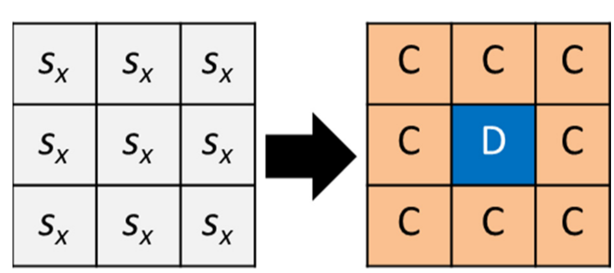

FIG. 10. (Color online) Schematic view of special situation in the case of $k=8$, where a central agent in a cluster offers D, while all his or her neighbors offer C, even though all agents belonging to the cluster have identical strategy s_x.

update rule is the one of probabilistic strategy updating rules. First, randomly selected agent i acquires his or her payoff π_i by playing the game with all his or her four neighbors. Next, one randomly chosen neighbor of agent i, denoted by j, also acquires his or her payoff π_j by playing games with his or her all neighbors. Then, focal agent i imitates the strategy of agent j by the pairwise comparison process that determines the probability of whether the agent would copy or not, depending on the payoff difference of those two,

$$P_{s_j \to s_i} = \frac{1}{1 + \exp\left(-\frac{\pi_j - \pi_i}{\kappa}\right)}, \quad \text{(C1)}$$

where κ denotes the uncertainty governing the strategy adaptation. In our study, κ is set to be 0.1.

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