Robustness of cooperation on scale-free networks under continuous topological change

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In this paper, we numerically investigate the robustness of cooperation clusters in prisoner's dilemma played on scale-free networks, where the network topologies change by continuous removal and addition of nodes. Each removal and addition can be either random or intentional. We therefore have four different strategies in changing network topology: random removal and random addition (RR), random removal and preferential addition (RP), targeted removal and random addition (TR), and targeted removal and preferential addition (TP). We find that cooperation clusters are most fragile against TR, while they are most robust against RP, even for large values of the temptation coefficient for defection. The effect of the degree mixing pattern of the network is not the primary factor for the robustness of cooperation under continuous change in network topology, which is quite different from the cases observed in static networks. Cooperation clusters become more robust as the number of links of hubs occupied by cooperators increase. Our results might infer the fact that a huge variety of individuals is needed for maintaining global cooperation in social networks in the real world where each node representing an individual is constantly removed and added.

I. INTRODUCTION

The emergence of cooperation is one of the challenging problems in both social and biological sciences. Cooperators benefit others by incurring some costs to the actor while defectors do not pay any costs. Thus, under a well-mixed population, cooperation cannot evolve because defectors are always better off than cooperators. This relationship between cooperators and defectors is well parametrized by the prisoner's dilemma (PD) game [1]. In PD, two individuals decide whether to cooperate or defect simultaneously. They both obtain R for mutual cooperation and P for mutual defection. If one selects cooperation and the other selects defection, the former gets S for being the sucker of the defector, and the latter gets T as a reward for the temptation to defect. The order of the four payoffs is $T > R > P > S$ in PD. Nowak and May revealed that spatial structures are required for the evolution of cooperation [2]. Recently, it has been possible to map any given spatial structure on a suitable network topology, and the evolution of cooperation has been investigated through the analysis of PD played on the corresponding complex network [3–13].

In this context, the spatial structure required for the emergence of cooperation is referred to as network reciprocity and becomes one of the most important factors for the emergence of cooperation [14]. If the network reciprocity and some other mechanisms are combined, cooperation is promoted more [15]. For instance, teaching activity [16], social diversity [17], an ability to infer the reputation of others [18], a sparse environment [19,20], selecting high

fitness individuals in adopting strategies [21,22], age structure [23], and incorporating environmental factors as the fitness of the focal individual [24] promote cooperation when they are combined with the network reciprocity. In these studies, various "heterogeneities" enhancing cooperation combined with network reciprocity are considered. On the other hand, networks can generate structural heterogeneity by themselves, which we refer to here as "network heterogeneity." For scale-free networks, this network heterogeneity is the key for promoting cooperation [8]. If cooperators occupy hubs in a scale-free network surrounded by other cooperators, the payoffs for these cooperators are considerably higher than for other individuals. Thus, they can spread their cooperative strategy. In contrast, if defectors occupy hubs surrounded by other defectors, this cluster of defectors is quite vulnerable and is easily replaced by cooperators. These two effects contribute to the evolution of cooperation on scale-free networks.

For scale-free networks, the robustness of cooperation has been examined on growing [25] or reducing [26] networks. Poncela *et al.* [25] have proposed an evolutionary preferential attachment, in which high payoff nodes attract more links from new nodes. They introduced a control parameter, $\epsilon \in [0,1)$, for the preference weight. In the limit, $\epsilon \to 1$, each existing node can get a new link in proportion to its payoff. If $\epsilon = 0$, a new link is connected to any node at random, which means the payoffs are completely ignored. They have shown that cooperation is most promoted at the limit, $\epsilon \to 1$, resulting in a scale-free network because a center cooperator in the cooperative group has a high preference weight and tends to get a new link more easily. Moreover, this causes positive feedback of the increment of the degree of the center cooperator. The rich get richer. It should be noted, however, that the network

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growth in this model is only in the direction of increasing the number of nodes and that the opposite possibility of decreasing the number of nodes is totally ignored.

On the other hand, Perc [26] has studied the evolution of cooperation in the direction of decreasing the number of nodes. He has implemented two ways of node removal from the Barabási and Albert (BA) network model [27]. One is random removal of a fraction of nodes Λ, and the other is targeted removal of nodes from the largest degree up to a fraction Λ. He has shown that the cooperation on scale-free networks is extremely robust against random node removal, while it declines rapidly against targeted attack. Notice that, in his model, removed nodes are never restored [26]. However, in artificial networks, there are many cases in which the restoration of removed nodes immediately occurs. Likewise, in ecological networks, a vacant site due to the death of an individual is often filled with a new individual immediately. Therefore, it is plausible that a node removal is followed by an addition of another new node. The present paper deals with such a bidirectional network topological change and investigates the effects of continuous removal and addition of nodes in the evolution of cooperation.

One of the other factors that potentially affects the robustness of cooperation is the degree correlation between nodes represented by degree mixing patterns, which was investigated by Rong *et al.* [9]. In their model, a network is referred to as assortative (disassortative) according to the tendency of highly connected nodes (hubs) to choose nodes with similar (dissimilar) degrees as neighbors.

Rong *et al.* [9] have shown that the assortative network favors defection because the hubs tend to connect closely, which allows defectors to invade cooperators. In contrast, cooperation is maintained in the disassortative networks because the isolation of hubs due to disassortativity enables them to keep their initial strategy. At the same time, however, the influence of the hubs becomes weaker as the disassortativity increases because the tendency of the isolation also increases. Therefore, uncorrelated networks promote cooperation to the maximum extent by spreading the strategy of hubs most effectively.

This conclusion, nevertheless, only applies to networks with a static topology. Once we allow the change of network topology by removal and addition of nodes or links, the mixing patterns change accordingly, and the conclusion observed in static networks might fail to apply. It is therefore also worth investigating the effects of the alteration of degree correlation caused by continuous node removal and addition on the evolution of cooperation.

In this paper, we perform evolutionary simulations under such topological changes of networks and find that cooperation is decreased to the greatest extent when targeted removal and random addition of nodes are combined. In contrast, cooperation is maintained even at a high temptation to defect when random removal and preferential addition are combined. We also show that the degree variance, which measures the network heterogeneity, directly controls the robustness of cooperation. We find that the effect of the degree mixing pattern of the network is not the primary factor for the robustness of cooperation under a continuous change of network topology due to consequential removal and addition

of nodes, which is quite different from the cases observed in static networks.

This paper is organized as follows. In Sec. II, we introduce a model in which removal and addition processes are considered on scale-free networks. In Sec. III, we present the numerical results for the robustness of cooperation under such topological changes and an analysis of the results from the view point of network heterogeneity defined by the degree variance. We also investigate the effect of the degree mixing pattern on the evolution of cooperation. The summary and conclusion are given in Sec. IV.

II. MODEL

To incorporate the network heterogeneity in the degree distribution observed in real networks, we employ the Barabási-Albert method for generating initial networks in numerical experiments [27]. Starting from a complete graph with a given small number of nodes m_0, a new node with $m \leqslant m_0$ links is added at every time step. This new node is connected to m existing nodes selected according to the probability $p_i = k_i / \sum k_i$, where k_i is the degree of node i of each selected node. Thus, nodes with a larger degree are more likely to be selected, hence the "preferential attachment." After t discrete time steps, the resulting network consists of $N = t + m_0$ nodes and mt links according to a power-law degree distribution with an exponent of about 3 [27].

We investigate the PD game on this initially scale-free network. Let N be the total number of nodes occupied by individuals; each of the nodes has its strategy classified as either C (cooperator) or D (defector). Initially, both strategies C and D are randomly and equally distributed among the nodes of the network.

Each node i plays PD with all of its k_i neighbors. The payoffs of the game are the following. Both individuals obtain R for mutual cooperation and P for mutual defection. If one selects cooperation and the other selects defection, the cooperator obtains S as the sucker of the defector, and the defector obtains T as the reward for temptation to defect. The order of the four payoffs is $T > R > P > S$ in PD. The sum of the payoff of individual i against its k_i neighbors is denoted by P_i. Following Nowak and May [2], we set $P = 0$, $T = b > 1$, $R = 1$, and $S = 0$, where b is the temptation to defect. Next, one randomly chosen neighbor of i, denoted by j, also plays PD with its neighbors and obtains the payoff P_j. If $P_i < P_j$, individual i imitates individual j's strategy with probability $(P_j - P_i)/[(T - S)k_{\max}]$, where $k_{\max}$ is the largest degree between i and j. This update principle of strategy has been adopted in various studies [8,9,26]. All individuals update their strategies simultaneously at each time step. After this update, the network topology is altered by one removal and one addition of nodes. Here we consider the following four combinations of node removal and addition. First, an existing node is removed in two different ways, namely, *random* removal and *targeted* removal. In the random removal, one randomly selected node is removed. In the targeted removal, a node with the largest degree is removed. In both cases, the links connected to the removed node are also removed from the network. After the removal, a new node is added in two different ways, namely, *random* addition

and *preferential* addition. In the random addition, a new node connects to m randomly selected existing nodes. In the preferential addition, a new node of m degree connects to each existing node with probability $p_i = k_i / \sum_i k_i$. Because the number of removed links is preserved, the remaining links other than m are also connected in each manner. We classify these four different combinations of node removal and addition, which cause continuous alteration of the network topology, into the following four models.

(1) *Random removal and random addition (RR)*. After the removal of one randomly selected node of degree n, a new node of degree m is added and connected to m randomly selected existing nodes. If $m < n$, each remaining $n - m$ link is connected from a randomly selected node (referred to as the source) to a randomly selected node (referred to as the target). If $m \geqslant n$, only n links of the added node are connected to n existing nodes. For $n = 0$, a new node immediately becomes an isolated node after it is added. This linking principle is also applied to the other three models.

(2) *Random removal and preferential addition (RP)*. After the removal of one randomly selected node of degree n, a new node of degree m is added and connected to each existing node with probability $p_i = k_i / \sum_i k_i$, which is proportional to the degree of node i. If $m < n$, each remaining $n - m$ link is connected from a randomly selected source node to a target node with probability p_i. If $m \geqslant n$, only n links of the added node are connected to n existing nodes.

(3) *Targeted removal and random addition (TR)*. After the removal of the node with the largest degree n among the existing nodes, a new node of degree m is added and connected to m randomly selected existing nodes. If $m < n$, each remaining $n - m$ link is connected from a randomly selected source node to a randomly selected target node. If $m \geqslant n$, only n links of the added node are connected to n existing nodes.

(4) *Targeted removal and preferential addition (TP)*. After the removal of the node with the largest degree n among the existing nodes, a new node of degree m is added and connected to each existing node with probability $p_i = k_i / \sum_i k_i$. If $m < n$, each remaining $n - m$ link is connected from a randomly selected source node to a target node with probability p_i. If $m \geqslant n$, only n links of the added node are connected to n existing nodes.

Note that for all four models, both the total number of nodes and the total number of links remain unchanged. In contrast, the network topologies do change. The strategy of a newly added node is randomly chosen from strategies C and D. The PD game of all existing nodes, updating their strategies, and the node removal and addition procedure make up one entire process in a numerical experiment, which we refer to as "generation." This generation is repeated up to a given number of steps.

Figure 1 shows a schematic picture of one generation.

III. RESULTS AND DISCUSSIONS

To generate the initial networks according to the BA method, we took $m = m_0 = 2$ and added nodes up to $N = 5000$.
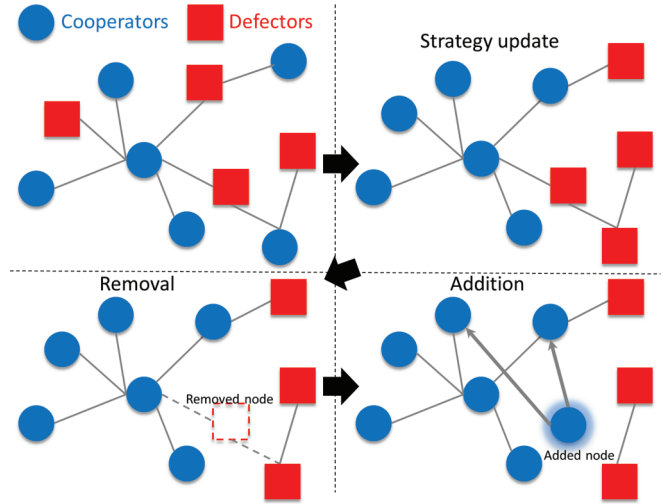

FIG. 1. (Color online) An example of one "generation" of the evolutionary game considered in this paper. First, all individuals update their strategies simultaneously. Then, one defector with two links is selected for removal in this case. Finally, one cooperator (the strategy is randomly selected) is added and connected by two links to the existing nodes. This "generation" is repeated up to a given number of steps. We consider four models (RR, RP, TR, TP) for the removal and addition of nodes.

In Fig. 2, we plot the fraction of cooperators as a function of the temptation to defect b for the four models. The results for each sample are obtained by averaging over 1000 generations after a transient time of 10 000 generations. The final results are obtained by averaging over 20 independent samples for each set of parameters. The fraction of cooperators shows quite different profiles depending on the model.

We also plot the case of the original BA model. This case always shows the highest level of cooperation because its hub structures, which benefit cooperation, are not altered.

The evolution of cooperation in models containing targeted node removal (TR and TP) is considerably suppressed even
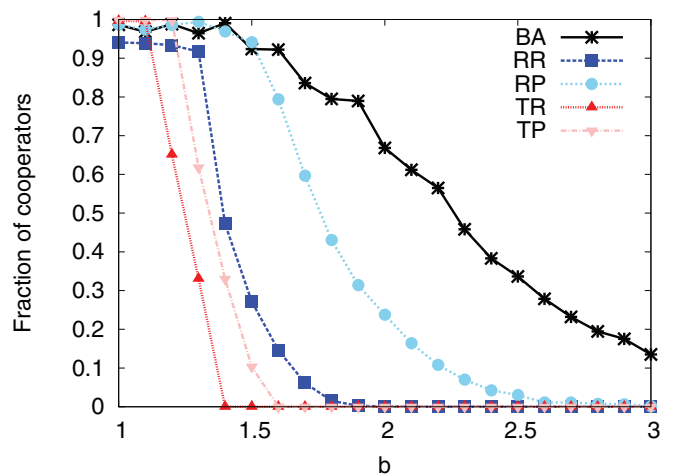

FIG. 2. (Color online) Fraction of cooperators as a function of the temptation to defect b in our four different models and the original BA model averaged over 20 independent samples. In the BA model, the structure is kept unchanged during all generations.

for small values of the temptation to defect b. The evolution of cooperation is also suppressed in models containing random addition of nodes (TR and RR). Thus, the fraction of cooperators is the most fragile in the TR model, which is the combination of targeted removal and random addition. In contrast, the fraction of cooperator has relatively large values in the RP model, which is the opposite combination of TR, even in the region with rather large values for the temptation to defect.

The qualitative reason for this difference in the profiles of the fraction of cooperators is the following. It is commonly known that the network heterogeneity determines the fate of cooperation [26]. If cooperators occupy the hubs of a network surrounded by other cooperators, their payoffs are considerably higher than other individuals. Cooperative hubs can therefore easily spread their strategy to the surrounding nodes. Since the fraction of hubs is extremely small even in a scale-free network, it is rare that a choice for random node removal hits a hub. Thus, cooperative hubs are maintained in random node removal. Moreover, the preferential addition tends to increase the degree of the hubs, which contributes to the resiliency of cooperation by expanding the network heterogeneity. This is the reason of the resiliency of the fraction of cooperators in the RP model. The reason of the fragility in the TR model is completely opposite to the case of the RP model. For quantitative support for this reasoning, we next examine the network characteristics relating to the network heterogeneity corresponding to the four models.

A. Network heterogeneity

The network heterogeneity is represented by the degree variance $V = [(1/N) \sum k_i^2 - \bar{k}^2]/\bar{k}$, where $\bar{k} = (1/N) \sum k_i$. This value becomes zero if the all nodes have the same degree, while it takes a larger value if some nodes have an extremely large degree, such as hubs.

Figure 3 shows the degree variance as a function of generation in the four models. In all models, the degree variance decreases as the generation increases from the largest degree variance of the initial BA network.
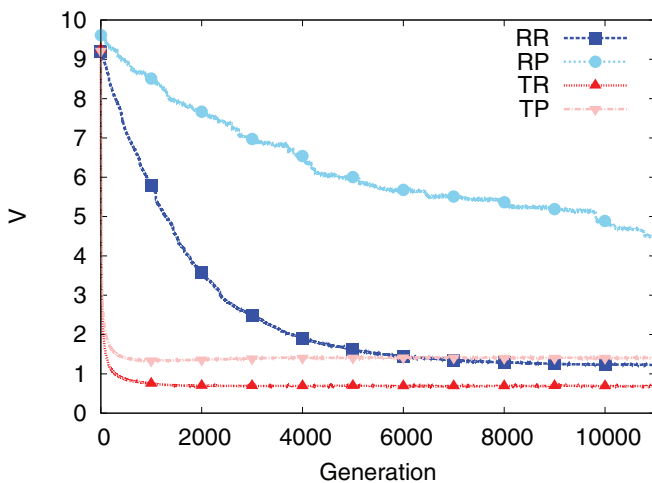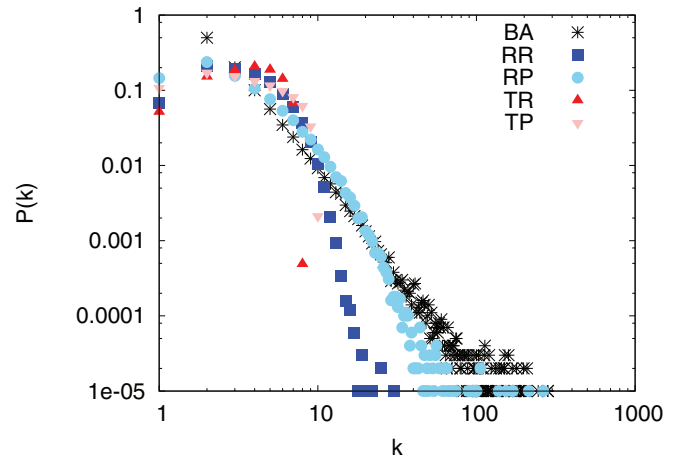

FIG. 4. (Color online) The degree distribution at the final generation in our four models along with the degree distribution of the initial BA model averaged over 20 independent runs.

The details of the collapse of the degree variance, however, are different in the four models. The degree variances in TR and TP show drastic decrease in the early stages of generation because the largest hub is always removed in the targeted models. On the other hand, the values of the degree variance in RR and RP do not show such a drastic decrease. In RP, in particular, a node is randomly removed without paying attention to its degree, and the preferential addition of node introduces new heterogeneity. The network heterogeneity that supports the fraction of cooperators is thus mostly maintained in the RP model.

In Fig. 4, we compare the final degree distribution of the four models to the initial BA model. The RP model maintains some hubs, while the other three models do not. By conducting further network analysis, we find that the four network topologies are completely altered from that of the original BA model (Supplemental Fig. S1), but the hubs in RP, which cause the network heterogeneity, are still maintained. This supports the result that cooperation is robust in RP [28].

B. Effect of the degree mixing pattern

It has been realized that the degree correlation between the node connection represented by the degree mixing pattern sometimes considerably modifies the results obtained from the mean field analysis based only on the degree distribution [29–33]. In static networks, it is known that an uncorrelated network promotes cooperation [9]. Here we investigate the effects of degree correlation on the resilience of the clusters of cooperators in the present cases in which the network topologies are constantly changed.

According to Newman, we measure the degree correlation of a network by the Pearson coefficient r_k [34]. If r_k is positive, nodes with almost the same degree tend to be connected; the correlation is denoted as "assortative." In assortative networks, hubs tend to be connected to other hubs. If r_k is negative, nodes with different degrees tend to be connected; the correlation is denoted as "disassortative." In disassortative networks, hubs tend to be connected to nodes with small degrees. Newman pointed out that the Pearson coefficient of the BA model takes
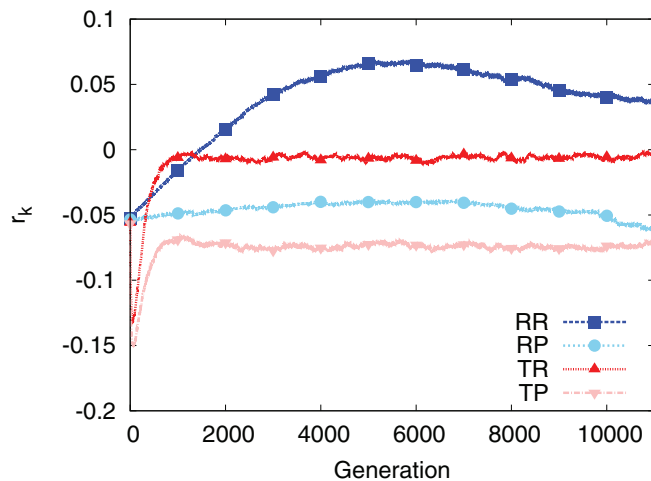

FIG. 3. (Color online) The degree variance as a function of the generation in the four models averaged over 20 independent runs.

FIG. 5. (Color online) The correlation coefficient r_k as a function of the generation in the four models averaged over 20 independent runs.

a very small value, $r_k \approx 0$, which means that the BA networks are almost uncorrelated [34].

Figure 5 shows the variation of the correlation coefficient with respect to the generation. We see three different correlation regimes: disassortative (RP and TP), uncorrelated (TR), and assortative (RR). By reexamining the results for the fraction of cooperators (Fig. 2) in terms of degree correlation (Fig. 5), the RP model, in which the fraction of cooperators is most robust, falls in the disassortative regime. It does not seem, however, that the degree correlation plays a key role in the robustness of the cooperation since the fraction of cooperators in the TP model, which also falls in the disassortative regime, is rather fragile. For the robustness of cooperation, the resiliency of the hubs with the largest degree controlling the stability of cooperation is most important. In this regard, elimination of the hubs due to targeted attack is most fatal to the robustness of the cooperation. On the contrary, random node removal rarely hits the hubs in elimination. This is the main reason for the difference between RP and TP in the PD game on networks with continuously changing network topology. The fate of cooperation is thus dominated by the network heterogeneity, and the degree correlation seems to be a secondary factor in the dynamic network.

It should be noted that the fraction of cooperators is most fragile in the TR model, which falls in the uncorrelated regime. This result is different from the analysis of static networks, where uncorrelated networks have an advantage in terms of the robustness of the clusters of cooperators [9].

IV. SUMMARY

The evolution of cooperation is still an open question in various fields. It is commonly accepted that the network structure is one of the main controlling factors for the promotion of cooperation. In social or ecological systems in the real world, it is plausible to assume that an individual or a species represented by a node is constantly replaced by or added to another, which introduces a continuous topological change in the network structure.

It is therefore important to know whether cooperation is maintained under such circumstances. Based on these motivations, we numerically investigated the robustness of cooperation on scale-free networks under continuous topological changes due to the removal and addition of nodes in a network.

We have found that cooperation is most robust against random removal and preferential addition of nodes, while cooperation is most vulnerable against targeted attack. The damage caused by the targeted attack is not fully compensated by either random or preferential addition. By calculating several network characteristics, we have revealed that the network heterogeneity dominates the fate of cooperation. If the degree variance is large, cooperation is maintained. We have also shown that the degree correlation does not affect the cooperation much on dynamical networks because cooperation mainly depends on the existence of cooperative hubs, which shows a sharp distinction from the cases observed in static networks. These results might explain the fact that a vast variety of individuals is needed in a society where many individuals independently join and leave because hubs are actually important for maintaining cooperation on an online friendship network [35].

[1] R. Axelrod, *The Evolution of Cooperation* (Basic Books, New York, 1984).

[2] M. A. Nowak and R. M. May, Nature (London) **359**, 826 (1992).

[3] G. Abramson and M. N. Kuperman, Phys. Rev. E **63**, 030901(R) (2001).

[4] N. Masuda and K. Aihara, Phys. Lett. A **313**, 55 (2003).

[5] N. Masuda, Proc. R. Soc. London, Ser. B **274**, 1815 (2007).

[6] H. Ohtsuki, C. Hauert, E. Lieberman, and M. A. Nowak, Nature (London) **441**, 502 (2006).

[7] D. Ashlock, in *Proceedings of the 2007 IEEE Symposium on Computational Intelligence in Games* (IEEE Press, Piscataway, NJ, 2007), pp. 48–55.

[8] F. C. Santos and J. M. Pacheco, Phys. Rev. Lett. **95**, 098104 (2005).

[9] Z. Rong, X. Li, and X. Wang, Phys. Rev. E **76**, 027101 (2007).

[10] M. G. Zimmermann and V. M. Eguíluz, Phys. Rev. E **72**, 056118 (2005).

[11] J. Tanimoto, Phys. Rev. E **76**, 021126 (2007).

[12] R. Suzuki, M. Kato, and T. Arita, Phys. Rev. E **77**, 021911 (2008).

[13] G. Ichinose and M. Kobayashi, BioSystems **105**, 1 (2011).

[14] M. A. Nowak, Science **314**, 1560 (2006).

[15] M. Perc and A. Szolnoki, BioSystems **99**, 109 (2010).

[16] A. Szolnoki and M. Perc, New J. Phys. **10**, 043036 (2008).

[17] M. Perc and A. Szolnoki, Phys. Rev. E **77**, 011904 (2008).

[18] Z. Wang, L. Wang, Z. Y. Yin, and C. Y. Xia, PLoS ONE **7**, e40218 (2012).

[19] Z. Wang, A. Szolnoki, and M. Perc, Sci. Rep. **2**, 369 (2012).

[20] Z. Wang, A. Szolnoki, and M. Perc, Phys. Rev. E **85**, 037101 (2012).

[21] Z. Wang and M. Perc, Phys. Rev. E **82**, 021115 (2010).

[22] M. Perc and Z. Wang, PLoS ONE **5**, e15117 (2010).

[23] Z. Wang, Z. Wang, X. Zhu, and J. J. Arenzon, Phys. Rev. E **85**, 011149 (2012).

[24] Z. Wang, A. Murks, W. B. Du, Z. H. Rong, and M. Perc, J. Theor. Biol. **277**, 19 (2011).

[25] J. Poncela, J. Gómez-Gardenes, L. M. Floria, A. Sanchez, and Y. Moreno, PLoS ONE **3**, e2449 (2008).

[26] M. Perc, New J. Phys. **11**, 033027 (2009).

[27] A. Barabási and R. Albert, Science **286**, 509 (1999).

[28] See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevE.88.052808 for the network analysis.

[29] M. A. Serrano, M. Boguñá, and R. Pastor-Satorras, Phys. Rev. E **74**, 055101(R) (2006).

[30] A. V. Goltsev, S. N. Dorogovtsev, and J. F. F. Mendes, Phys. Rev. E **78**, 051105 (2008).

[31] Y. Shiraki and Y. Kabashima, Phys. Rev. E **82**, 036101 (2010).

[32] M. Ostilli, A. L. Ferreira, and J. F. F. Mendes, Phys. Rev. E **83**, 061149 (2011).

[33] T. Tanizawa, S. Havlin, and H. E. Stanley, Phys. Rev. E **85**, 046109 (2012).

[34] M. E. J. Newman, Phys. Rev. Lett. **89**, 208701 (2002).

[35] F. Fu, X. Chen, L. Liu, and L. Wang, Phys. Lett. A **371**, 58 (2007).