Insight into the so-called spatial reciprocity

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Up to now, there have been a great number of studies that demonstrate the effect of spatial topology on the promotion of cooperation dynamics (namely, the so-called "spatial reciprocity"). However, most researchers probably attribute it to the positive assortment of strategies supported by spatial arrangement. In this paper, we analyze the time course of cooperation evolution under different evolution rules. Interestingly, a typical evolution process can be divided into two evident periods: the enduring (END) period and the expanding (EXP) period where the former features that cooperators try to endure defectors' invasion and the latter shows that perfect C clusters fast expand their area. We find that the final cooperation level relies on two key factors: the formation of the perfect C cluster at the end of the END period and the expanding fashion of the perfect C cluster during the EXP period. For deterministic rule, the smooth expansion of C cluster boundaries enables cooperators to reach a dominant state, whereas, the rough boundaries for stochastic rule cannot provide a sufficient beneficial environment for the evolution of cooperation. Moreover, we show that expansion of the perfect C cluster is closely related to the cluster coefficient of interaction topology. To some extent, we present a viable method for understanding the spatial reciprocity mechanism in nature and hope that it will inspire further studies to resolve social dilemmas.

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I. INTRODUCTION

In evolutionary biology and social science, a challenging problem is understanding the emergence of cooperative traits and their sustenance under the pressure of a free rider [1–3]. To explain the origin of this phenomenon, evolutionary game theory, which provides a theoretical framework, has been extensively investigated from different disciplines over the past decades [4–6]. The prisoner's dilemma game, in particular, illustrating the social conflict between cooperative and selfish behaviors, has attracted considerable attention both in theoretical as well as in experimental studies [7–19]. In this simple paradigmatic model, two individuals simultaneously decide to adopt one of two strategies: cooperation (C) and defection (D). If both cooperate (defect), they receive the reward R (the punishment P). If, however, one player chooses cooperation while the other defects, the latter gets the temptation T and the former is left with the sucker's payoff S. These payoffs satisfy $T > R > P > S$ and $2R > T + S$; thus, defection optimizes the individual payoff, despite the fact that mutual cooperation could yield a higher collective benefit. This is to say, when populations play a prisoner's dilemma game in the well-mixed case, this setup does not support the organization of cooperative dynamics.

To overcome this unfavorable outcome, a great number of scenarios have been identified for supporting the evolution of cooperation. Typical examples include effective strategies, such as the tit-for-tat [20] or win-stay-lose shift [21,22], voluntary participation [23,24], spatially structured populations [25–31], heterogeneity or diversity [32,33], the mobility of players [34–36], and co-evolutionary selection of dynamical rules [37–39], to name but a few. Whereas, Nowak recently attributed all these to five mechanisms: kin selection, direct reciprocity, indirect reciprocity, network reciprocity, and group selection [40], these mechanisms can be somewhat related to the reduction of an opposing player's anonymity relative to the so-called well-mixed situation. Among the five mechanisms, network reciprocity, where players are arranged on the spatially structured topology and interact only with their direct neighbors, has attracted the most notable attention. In this setup, cooperators can survive by means of forming compact clusters, which minimize the exploitation by defectors and protect those cooperators that are located in the interior of such clusters [7]. Following this discovery, various types of spatial topology have been introduced into this field to extend the scope of cooperation on complex networks [25,26,41–43]. For example, in a recent research paper [25,26] where a heterogeneous scale-free network was employed as the potential interaction topology, the state that cooperation completely dominates was reported. In Refs. [44–46], it was shown that, on an interdependent network, a high value of the fitness coefficient was beneficial for the evolution of cooperative traits and could lead to the long-term sustenance of cooperation.

Despite the relative large body of work that has been accumulated, there still is a situation of practical relevance that has received less attention until now. This is the case of why spatial topology can provide the beneficial environment for the evolution of cooperation (namely, how the network reciprocity works). Inspired by this interesting question, in the present paper, we plan to explore the potential and genuine mechanism of spatial reciprocity based on the elementary spatial populations. Through systematic study, we divide the time course of cooperation evolution into two sequential

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processes: enduring (END) and expanding (EXP) periods. In particular, we inspect that different update dynamics and the initial distribution of strategies can lead to diverse evolution periods, which directly affects the final level of cooperation in the spatial populations.

II. SPATIAL MODEL OF THE PRISONER'S DILEMMA

We consider an evolutionary two-strategy prisoner's dilemma game with players located on the sites of a network, whose size is N. Each player i can adopt one of the two strategies: cooperation ($s_i = C$) or defection ($s_i = D$). For simplicity, but without loss of generality, we use a standard parametrization of the game: reward for mutual cooperation $R = 1$, punishment for mutual defection $P = 0$, then the payoff matrix can be rescaled as follows:

$$\begin{pmatrix} R & S \\ T & P \end{pmatrix} = \begin{pmatrix} 1 & -D_r \\ 1 + D_g & 0 \end{pmatrix}, \tag{1}$$

where $D_r = P - S$ $(0 \leqslant D_r \leqslant 1)$ is the stag-hunt-type dilemma and $D_g = T - R$ $(0 \leqslant D_g \leqslant 1)$ denotes the chicken-type dilemma [47].

The game is iterated forward in accordance with the sequential simulation procedure comprising the following elementary steps. First, each player i acquires its payoff P_i by playing the game with all his neighbors. After the evaluation of payoff for the entire population, player i updates its strategy synchronously. Here, we mainly pay attention to two types of update rule. One is the deterministic rule: imitation max (IM), and the other is the stochastic rule: pairwise Fermi (PW-Fermi), both of which are the most universal rules.

The IM policy can be described in the following way: The strategy $s_i(t)$ of individual i at time step t will be

$$s_i(t) = s_j(t-1), \tag{2}$$

where j is one member among player i and all his neighbors Ω_i (namely, $j \in \{\Omega_i \cup i\}$) such that $j = \max\{P_\tau(t-1), \forall\, \tau \in \Omega_i \cup i\}$.

Another flexible update rule is the PW-Fermi rule where focal agent i randomly chooses one neighbor j and adopts the strategy s_j from the selected player j with the probability,

$$W(s_i \to s_j) = \frac{1}{1 + \exp[(P_i - P_j)/K]}, \tag{3}$$

where K denotes the amplitude of noise or the so-called intensity of selection [48].$K \to 0$ and $K \to \infty$ denote the completely deterministic and completely random selections of the neighbor's strategy, whereas, for any finite positive values, K incorporates the uncertainties in the strategy adoption. As a previous setting [11–15], we simply fix the value of K to be $K = 0.1$ in the present paper. Moreover, it is worth noting that, compared with IM, there are two stochastic processes: randomly choosing neighbor and probabilistic determination of updating strategy in PW-Fermi.

III. ENDURING AND EXPANDING PERIODS

For the sake of the following discussion, we define the terminology as the enduring (END) period and the expanding (EXP) period as shown in Fig. 1. In a typical evolution process where the initial value of the cooperation fraction is 0.5, there are usually two evident processes: The former period features the rapid downfall of cooperation, whereas, the following one is along with the increase in cooperation level unless the evolutionary trail is absorbed by the all-defectors state during the foregoing period. In our study, the first is the so-called END period because cooperators try to endure defectors' invasion (or cooperators avoid copying defection from neighbors). Correspondingly, we call the latter the EXP period since cooperators, who successfully survive through forming cooperator clusters (C clusters) in the END period, expand their area by converting defectors into cooperators.

To analyze how the network reciprocity works, we mainly pay attention to the evolution process on the most elementary (homogeneous) network: cycle graph and square lattice.

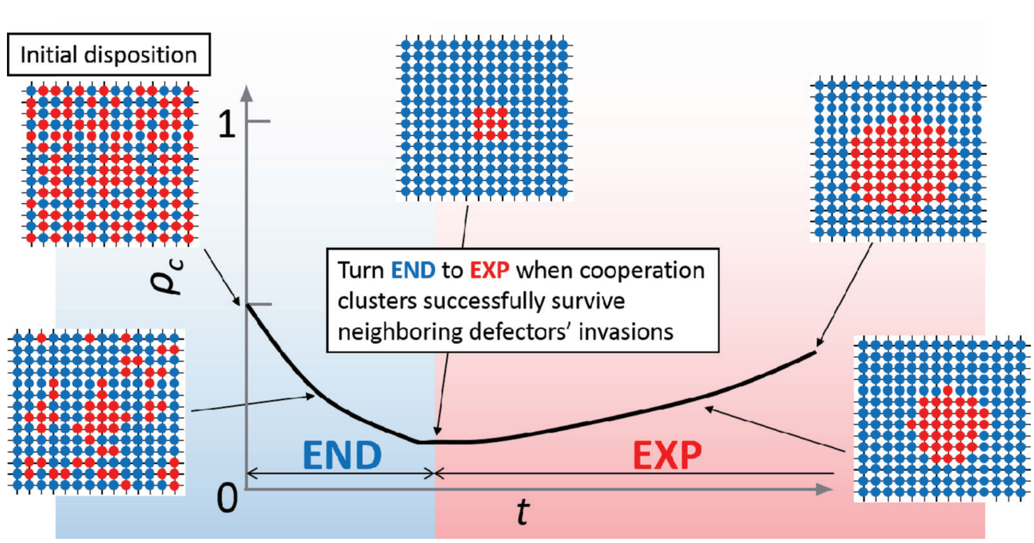

FIG. 1. (Color online) Schematic of a time evolution of a spatial prisoner's dilemma game with END and EXP periods. END period: initial cooperators (red) are rapidly plundered by defectors (blue), which cause only a few cooperators to be left through forming compact C clusters. EXP period: C clusters start to expand since a cooperator on the clusters' border can attract a neighboring defector into the cluster.

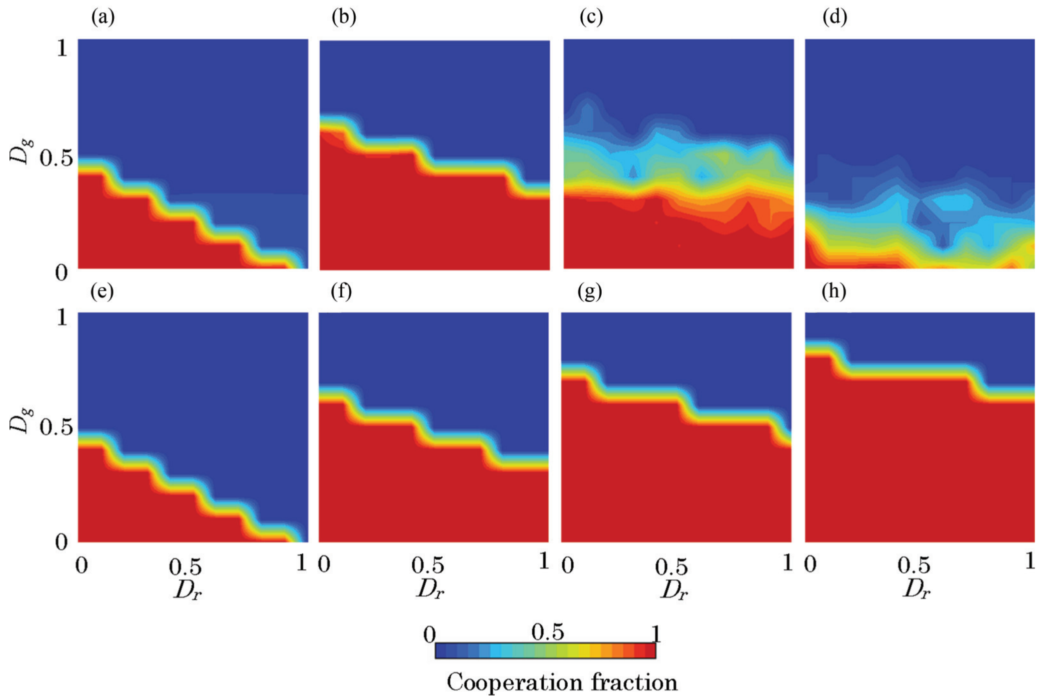

FIG. 2. (Color online) Fraction of cooperation on a $D_r - D_g$ plane with the IM rule on a cycle graph. From left to right, the neighborhood size is $k = 4$ [(a) and (e)], $k = 6$ [(b) and (f)], $k = 8$ [(c) and (g)], and 12 [(d) and (h)], respectively. In the upper panel, we show the result of the $\rho_C = 0.5$ case where, initially, cooperators and defectors are randomly distributed. Whereas, in the bottom panels, we assume a particular initial state (namely, the perfect C cluster case) where all agents are defectors except for $k + 1$ individuals, forming a perfect cooperative cluster in the center of the graph as shown in Fig. 3.

Interestingly, we find, under the stochastic rule PW-Fermi, that cooperator clusters (C clusters) usually possess rough (concavo-convex) boundaries at the end of the END period, which finally leads to the coexistence of cooperators and defectors. However, the deterministic rule could produce smooth boundaries of C clusters, which provides a more effective condition for the expansion of cooperative behaviors during the EXP period. In the following part, we will give more details.

IV. RESULTS AND DISCUSSION

We have performed extensive numerical simulations for the population comprising $N = 100^2$ individuals. The fraction of cooperation ρ_C is determined within the last 5000 out of the total 10^5 generations. To guarantee validity and statistical robustness of the data, the final results are averaged over up to 100 independent runs for each set of parameter values. During one time step, the agents update their strategies synchronously. Moreover, we have also checked that the qualitative results do not change for asynchronous updating.

A. Cycle graph

In this section, we discuss the results when the cycle graph is assumed as the underlying interaction network. Figure 2 shows cooperation fraction ρ_C versus different neighborhood size k and initial distribution of strategies on the $D_r - D_g$ plane, whereas, for the upper panel, each player is initially designated either as a cooperator or as a defector with equal probability, namely, the $\rho_C = 0.5$ case. To better explain the network reciprocity, we also consider another prepared initial state (bottom panel): except for $k + 1$ cooperators that form compact center on the center of graph, other agents are defectors, namely, the perfect C cluster case (see Fig. 3).

As shown in Fig. 2, the perfect C cluster case presents better performance of cooperation traits than the other case, although its initial cooperation level approaches 0 (such as the initial $\rho_C = 5 \times 10^{-4}$ for $k = 4$). This is particularly evident for the large neighborhood size. To explain the promotion impact, we need to examine time courses of cooperation (see Fig. 4 for a schematic representation). From the viewpoints of END and EXP periods, the perfect C cluster case directly starts the evolution from the EXP period since there hardly are any cooperators left to be exploited by defectors (Fig. 5). Under IM rule, this perfect cluster of cooperators, who possess

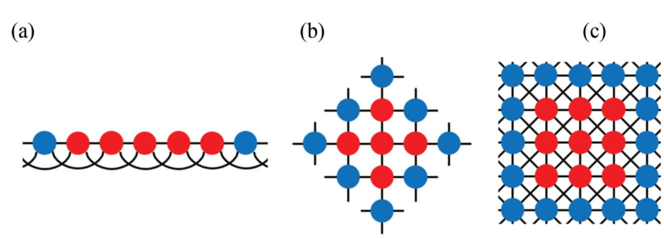

FIG. 3. (Color online) Examples of the perfect C cluster on (a) a cycle graph with $k = 4$, (b) a square lattice with $k = 4$, and (c) a square lattice with $k = 8$. Red and blue nodes denote cooperators and defectors, respectively.

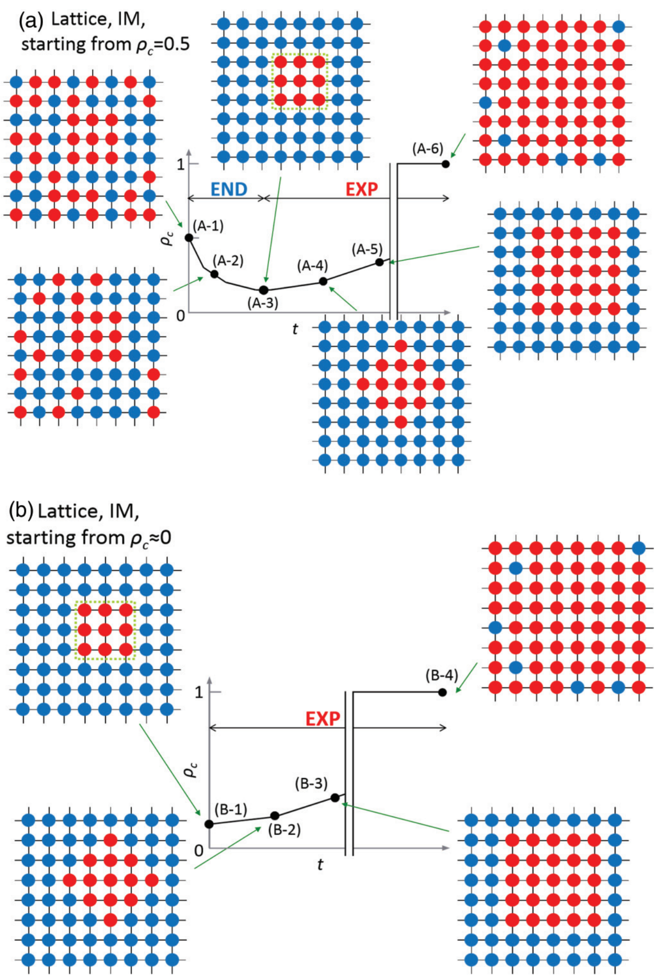

FIG. 4. (Color online) Schematic to explain the difference among END and EXP periods for (a) the $\rho_C = 0.5$ case and (b) the perfect C cluster case. For the sake of discussion, we assumed a lattice graph and an IM rule. Red and blue nodes denote cooperators and defectors, respectively.

higher payoffs, starts expanding its ground against weakened defectors. Crucial, thereby, is the fact that the clusters formed by these cooperators are impervious to defector attacks, which attracts more defectors transferring to cooperators and penetrate into the cluster. This ultimately results in widespread cooperation. For the small neighborhood size, tradition setup

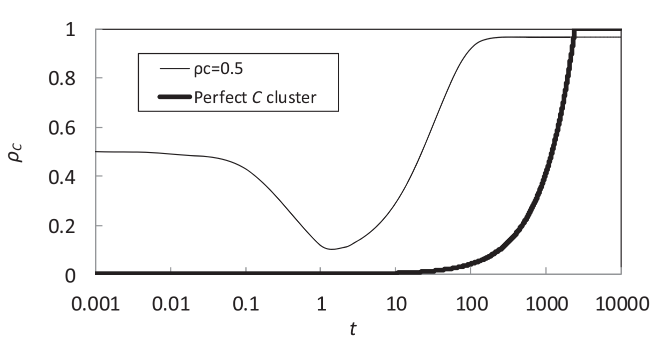

FIG. 5. Time course depicting the evolution of cooperation in the case of $D_g = D_r = 0.3$. The thin gray line indicates the $\rho_C = 0.5$ case, and the thick black line is the perfect C cluster case.

(the $\rho_C = 0.5$ case) can also get the similar perfect C clusters, which only needs longer enduring time. However, with the increment of neighborhood size, it becomes more and more difficult to organize the perfect C cluster for the traditional setup since cooperators have been invaded heavily before the formation of perfect clusters. Whereas, in the perfect C cluster case, large neighborhood size means that the enhanced cluster coefficient (CC) of a cycle graph enables an initially prepared perfect C cluster to expand its domain more effectively, which can still lead to a high cooperation level. Thus, these results suggest whether forming a perfect C cluster before the EXP period is the key for the final cooperation level in the system.

Moreover, another interesting question is why the perfect C cluster case possesses immediate transmission from the full C phase to the full D phase (see bottom panel of Fig. 2), whereas, there is a fluctuation coexistence region in the $\rho_C = 0.5$ case [see Figs. 2(c) and 2(d)]. As is known, when the IM update rule and the homogeneous cycle graph are assumed, there is no stochastic influence, which can guarantee the smooth expansion of the perfect C cluster under the weak dilemma or can destroy its evolution for the strong dilemma. However, the initial random distribution of strategies inevitably introduces stochastic factor, which directly affects the formation of perfect C clusters. If the perfect C cluster is constructed by the end of the END period, cooperators can coexist with a certain number of defectors; otherwise, the system is absorbed by the full D state (namely, fast diffusion of defectors inhibits the formation of effective cooperator clusters). This stochastic impact becomes particularly evident for large neighborhood sizes. In this sense, the fluctuation behaviors validate the importance of the perfect C cluster on the evolution of cooperation.

Next, it becomes meaningful to examine the evolution of cooperation for the PW-Fermi rule. Results presented in Fig. 6 feature how cooperators fare with the same condition of Fig. 2. It can be observed, irrespective of which case, that PW-Fermi cannot promote a high cooperation level and even leads to the disappearance of cooperators under the weak dilemma. This is because PW-Fermi is a stochastic rule, under which many evolutionary trails (such as, C clusters) can be absorbed by the full defector state. Along this line, the survival environment of only the C cluster becomes more difficult. Thus, it is easy to understand why less cooperation is observed in the perfect C cluster case than the traditional case. Additionally, through the comparison of both update rules (especially for the perfect C cluster case), we can see that the deterministic IM rule is more beneficial for the expanding of cooperation. In fact, this is closely related to the expanding fashion of different update rules: the IM rule enables C clusters to synchronously extend along all the boundaries (namely, always possessing the smooth boundary), which, thus, effectively resists invasion of defectors and recovers the lost ground; whereas, for the PW-Fermi rule, the rough (concavo-convex) boundary is the only expanding way. As schematically shown in Fig. 7, a stochastic update inevitably introduces rough (concavo-convex) parts on the boundary of a C cluster, which usually takes place in the process of the original cooperators transferring to defectors during the END period. This might finally make all C clusters perish because cooperators

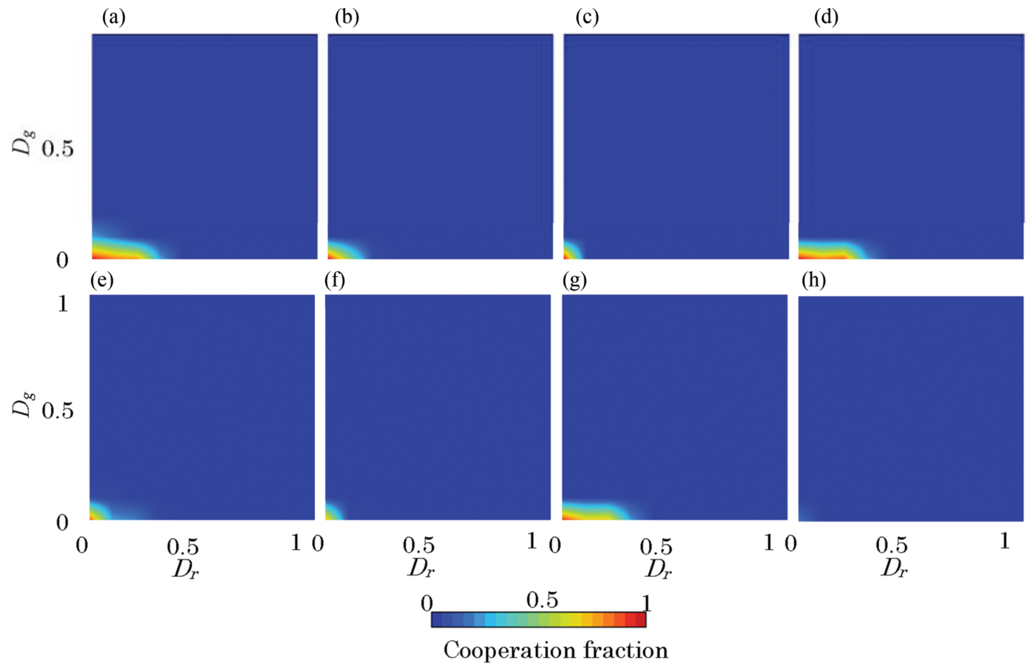

FIG. 6. (Color online) Fraction of cooperation on a $D_r - D_g$ plane with the PW-Fermi rule on a cycle graph. From left to right, the neighborhood size is $k = 4$ [(a) and (e)], $k = 6$ [(b) and (f)], $k = 8$ [(c) and (g)], and 12 [(d) and (h)], respectively. In the upper panel, we show the result of the $\rho_C = 0.5$ case; whereas, in the bottom panels, we assume the perfect C cluster case.

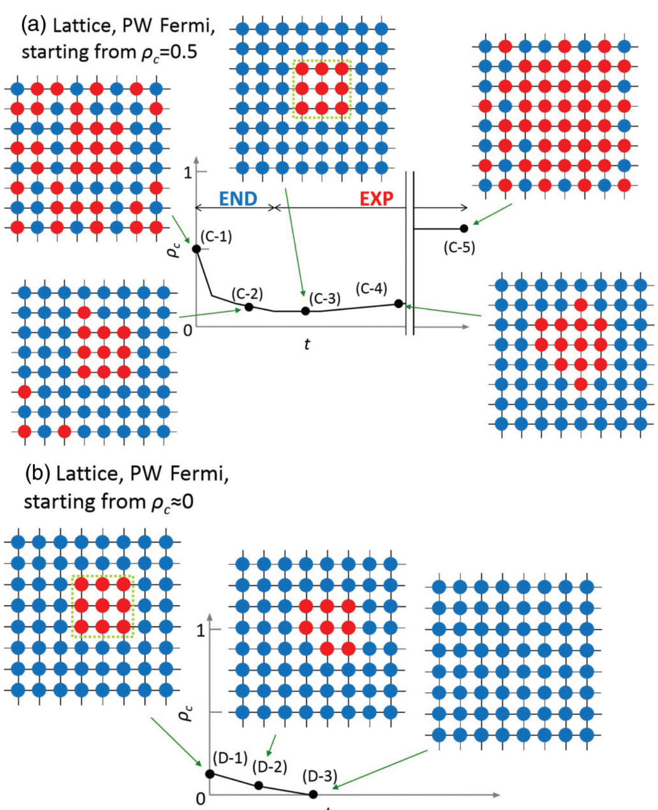

FIG. 7. (Color online) Schematic to explain the differences among END and EXP periods for the (a) $\rho_C = 0.5$ case and (b) the perfect C cluster case. For the sake of discussion, we assumed a lattice graph and PW-Fermi rule. Red and blue nodes denote cooperators and defectors, respectively.

could not form stable C clusters to resist the invasion of neighboring defectors.

Up to now, we have analyzed the evolution behavior of the system for different update rules and prepared initial conditions. We find that the final cooperation level depends on two key factors: one is whether perfect C cluster forms or not at the end of the END period; the other is the expanding fashion of the C cluster during the EXP period. Compared with the traditional version, the perfect C cluster case provides better guarantee for the cluster formation. Whereas, in the EXP period, the synchronous expanding along the smooth boundaries of the C clusters enables cooperation to dominate the system more effectively (to support our argument, we also validate the influence of stochastic factor or mutation in the evolution process [49]). Moreover, if any stochastic factor is introduced (such as, the random distribution of strategies or the nondeterministic updating rule), it becomes rather difficult to quantitatively depict the evolution characters of C clusters [50].

B. Square lattice

To further validate our argument, it is necessary to explore the evolution of cooperation on the square lattice where we expect to observe the similar traits. Figures 8 and 9 show the evolution of cooperation under the IM and PW Fermi rules. Clearly, except for the case of Fig. 8(c), a similar tendency is observed on the cycle graph, which attests to the fact that the deterministic IM rule on homogeneous topology guarantees a higher cooperative level due to its particular expansion fashion of clusters. Whereas, for the perfect C cluster case under the IM rule [namely, Fig. 8(c)], it is caused by its value of the cluster coefficient (CC), which equals zero for $k = 4$.

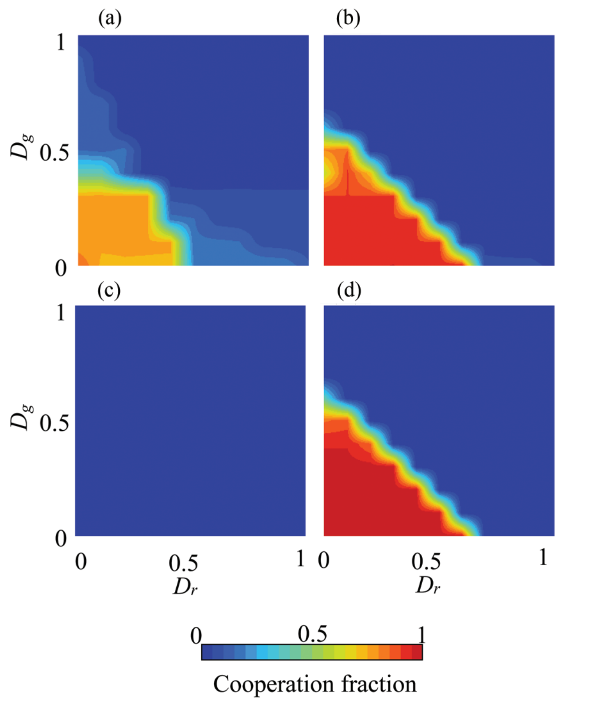

FIG. 8. (Color online) Fraction of cooperation on a $D_r - D_g$ plane with the IM rule on a square lattice. From left to right, the neighborhood size is $k = 4$ [(a) and (c)] and $k = 8$ [(b) and (d)], respectively. In the upper panel, we show the result of the $\rho_C = 0.5$ case; whereas, in the bottom panels, we assume the perfect C cluster case.

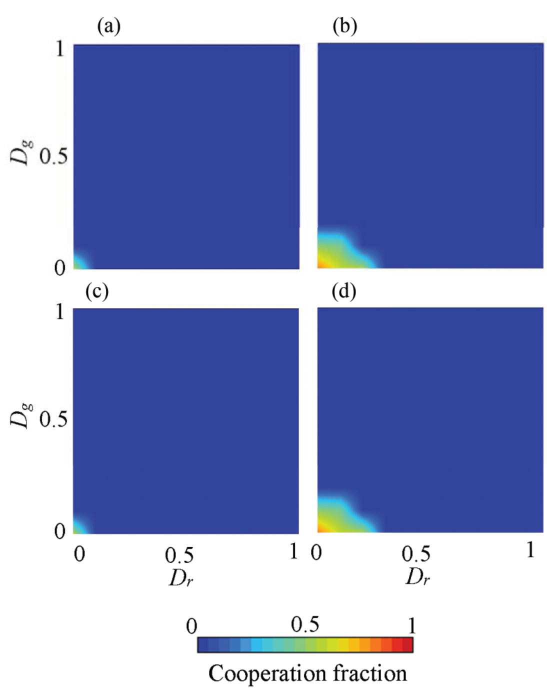

FIG. 9. (Color online) Fraction of cooperation on a $D_r - D_g$ plane with the PW-Fermi rule on a square lattice. From left to right, the neighborhood size is $k = 4$ [(a) and (c)] and $k = 8$ [(b) and (d)], respectively. In the upper panel, we show the result of the $\rho_C = 0.5$ case; whereas, in the bottom panels, we assume the perfect C cluster case.

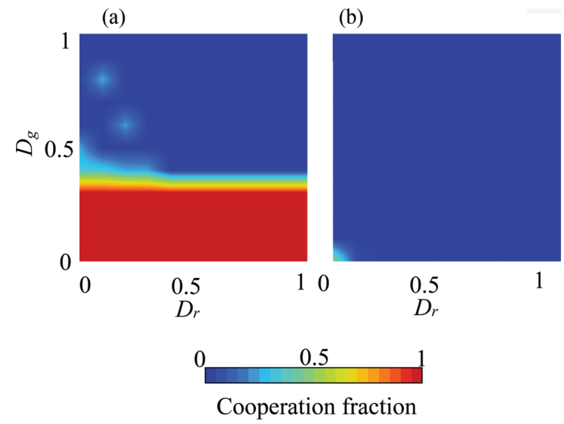

FIG. 10. (Color online) Fraction of cooperation on a $D_r - D_g$ plane with the (a) IM and (b) PW-Fermi rules on a square lattice. We assume a $k = 4$ neighborhood for the playing game, whereas, we assume a $k = 12$ neighborhood for strategy updating. We use the perfect C cluster case where all agents are defectors except for $k + 1 = 5$ agents that form a perfect cooperative cluster.

Interestingly, to improve the evolution environment of the perfect C cluster, Fig. 10 features the results of such a case: agents still play games within the first von Neumann neighborhood (namely, $k = 4$) but update strategies among the second von Neumann neighborhood (namely, $k = 12$). It is obvious that cooperation is greatly promoted under the IM rule (but there is no change under the rule), which can be attributed to the fact that the CC value of the network of strategy updating is not zero again (for a similar phenomenon, see Ref. [51]). Combined with the results of the perfect C cluster on a cycle graph [Figs. 2(e) and 6(e)], we can get that the expansion of the perfect C cluster is closely related to the CC of the strategy updating network under deterministic rule. In fact, the larger the value of the CC, the higher the final level of cooperation. Whereas, for the rule (due to stochastic impact), its improvement for cooperation evolution is limited on the perfect C cluster case.

V. DISCUSSION

We have elucidated that the so-called spatial reciprocity can be analyzed through dividing the time evolution trail into two typical periods. The END period is featured as the decline of cooperation due to the fast invasion of defectors, whereas, the EXP period demonstrates that the survival C clusters start to expand their territory by attracting defectors into their own partners. Interestingly, our results show that the evolution of cooperation depends on two factors: the formation of the perfect C cluster at the end of the END period and the expanding fashion of the C cluster during the EXP period. The IM rule can guarantee more robust EXP than the stochastic PW-Fermi rule; this is because the IM rule possesses the smooth expansion of C clusters on the homogeneous network, whereas, the PW-Fermi rule organizes the rough boundaries for C clusters, which impedes the effective expansion (this point can also get further support from the evolution of cooperation on heterogeneous networks [52]).

Notably, a slower strategy update for cooperators and a faster strategy update for defectors are helpful for END and

EXP periods, respectively. A slower strategy update gives cooperators more chances to survive during the END period. Whereas, a faster update can accelerate the conversion (from defection to cooperation), which is more meaningful for the EXP period. Along this line, it becomes easy to understand why cooperation is greatly enhanced in Refs. [53,54] where a richer neighbor is more likely to be chosen as the pairwise opponent. Under this case, an agent lying on the borders of clusters is apt to regard a cooperator inside the C clusters as the potential strategy donor. This declines the learning of defection behavior and boosts the expansion of cooperators as mentioned above. A similar phenomenon was observed recently in a spatial game where the social performance was refereed to denote the opponent's payoff and, thus, altered the impact of the EXP and END periods on the evolution of cooperation [55]. We hope that this paper will inspire future studies, especially in terms of understanding the emergence of cooperation traits in societies via a co-evolutionary process [9].

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[49] We find that, when a stochastic factor or mutation is introduced into the deterministic IM rule, the final level of cooperation is affected via two aspects. On one hand, it directly affects the formation of a perfect C cluster at the end of the END period. On the other hand, the expansion pattern of the C cluster also varies: The smooth expansion of the C cluster soon vanished. In this sense, it cannot guarantee the absolute dominance of a cooperation trait.

[50] In the same way, we also prove that the minimal C cluster capable of survival is, to a large extent, related with update rules and topology: If any stochastic effect is introduced, the quantitative prediction of cooperation seems impractical.

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[52] We find that, when the IM rule is implemented on the heterogeneous networks, the expansion patterns of the C clusters change: Smooth expansion of the C clusters vanishes. For the perfect C cluster case, the cooperation level soon becomes 0. Thus, we conclude that smooth expansion of the C clusters is the outcome for the combination of the deterministic update rule and homogeneous topology, which can guarantee the prosperity of cooperation.

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