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Supratransmission induced by waves collisions in a discrete electrical lattice

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We numerically performed a way to produce a supratransmission phenomenon in the Salerno equation describing the dynamics of modulated waves in a discrete nonlinear transmission lattice. For the natural supratransmission phenomenon, there exists a threshold of amplitude for which energy can flow in the line. We show that gap transmission is possible with driven amplitude below the threshold due to the collision of different plane waves coming from both edges of the line. One of the two plane waves has a frequency in the forbidden gap, and another has a frequency in the allowed phonon band. During collision, the wave in the allowed band is considered as a perturbation of the ones in the forbidden gap.

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I. INTRODUCTION

Energy excitation with a driving frequency in the allowed band of the discrete system propagates through the chain naturally because of the system's dispersion relation. On the other hand, a plane wave scattered onto the chain by the periodic driving force with the frequency, which falls in the system's band gap, becomes evanescent. Recently, Geniet and Leon [1] discovered that, by harmonically and continuously driving one end of the lattice with frequencies within a band gap and amplitude above a defined driving threshold, a sudden energy flow takes place. This universal phenomenon, called nonlinear supratransmission by the authors, has been shown to be present in different models, such as mechanical systems of oscillators [2,3], superconductors [4], a birefringent quadratic medium [5], Josephson junction parallel arrays [2,6], the Fermi-Pasta-Ulam model [7], coupled optical waveguide arrays [8–10], and a discrete electrical transmission line [11,12]. The common result of all these studies is a simple and explicit expression for the threshold intensity above which transmission occurs.

In addition to these studies, certain authors have generated a gap transmission taking a driving amplitude that is not supposed to allow any information transmission and that shows that transmission is possible either by adding noise on the driving source [13] or by adding impurity in the line [14]. All these studies have been performed by harmonically driving one end of the lattice. Therefore, it seems natural to see what happens when the other edge of the lattice is also excited. Otherwise, supratransmission occurs due to an instability in the plane wave [15]; the collision of the plane waves induces an instability in the waves [16]. Can the collision of waves create gap transmission? The answer to this question is the aim of the present Rapid Communication.

The outline of this Rapid Communication is the following: In Sec. II, we first present a mathematical model and the nonlinear Salerno equation governing the modulated waves of this model. In Sec. III, we perform our method by integrating the full Salerno equation irradiating at the two edges. Finally, in Sec. IV, the present Rapid Communication is concluded.

II. MODEL DESCRIPTION

The lattice shown in Fig. 1 is a schematic of the boundary driven bi-inductance dispersive nonlinear transmission line under investigation. Both ends of the electrical lattice ($n = 0$ and $n = N$), respectively, have been submitted to a periodic driving source $V_L(t)$ and $V_R(t)$ with constant amplitude and frequency slightly above the cutoff frequency. For $n \in [1; N]$, the line can be considered as a set of elementary cells where each cell contains a series of linear inductance L_1 and a linear inductance L_2 in parallel with the nonlinear capacitor $C(V_n)$.

Applying Kirchhoff's laws and charge relationship in a varicap diode [17], we obtain the following system of nonlinear equations:

$$A\frac{\partial^2}{\partial t^2}\ln\left(1+\frac{V_n}{A}\right) = u_0^2(V_{n+1}+V_{n-1}-2V_n) - \omega_0^2 V_n, \quad (1)$$

with $u_0^2 = \frac{1}{L_1 C_0}$, $\omega_0^2 = \frac{1}{L_2 C_0}$, $A = 4.9$ V, and $C_0 = 470$ pF.

Linearizing Eq. (1) with respect to V_n and assuming a sinusoidal wave in which V_n is proportional to $\exp[i(kn - \omega t)]$, where ω and k, respectively, are the angular frequency and wave number, we derive the phonon spectrum given by the following linear dispersion law:

$$\omega^2 = \omega_0^2 + 4u_0^2 \sin^2\frac{k}{2}. \quad (2)$$

The above relation admits a lower cutoff mode frequency $\omega = \omega_0$ at $k = 0$ and an upper frequency $\omega = \omega_{\max} = \sqrt{(\omega_0^2 + 4u_0^2)}$ at $k = \pi$. $\omega_{\max}$ is a consequence of the discretization of the lattice, this means that it does not exist in the continuum limit. ω_0 is due to the presence of L_2, the inductance in parallel.

As depicted in Fig. 2, the linear dispersion relation has two forbidden band gaps (shaded areas), which will be the subject of special interest in the following.

We now turn our attention to the nonlinear behavior of the lattice and seek nonlinear modulated waves. In order to fully take into account the lattice discreteness, we assume that the gap angular frequency ω_0 is large with respect to the other frequencies of the system. Applying the rotating wave approximation, we then restrict our analysis to slow temporal

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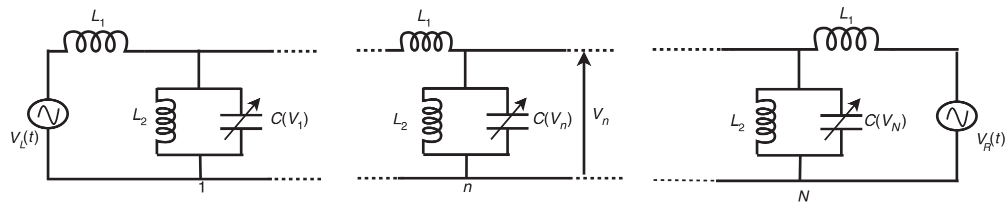

FIG. 1. Schematic of the boundary driven dispersive nonlinear transmission line. The left and right boundaries are connected to different voltages $V_L(t)$ and $V_R(t)$.

variations of the envelope and look for a solution to Eq. (1) in the form

$$V_n(t) = \psi_n e^{-i(\omega t - kn)} + \text{c.c.}, \tag{3}$$

where c.c. stands for complex conjugate. Following the same approximation as in Refs. [18,19], we obtain the nonlinear Salerno equation,

$$i\frac{d\phi_n}{d\tau} + (1+\mu|\phi_n|^2)(\phi_{n+1}e^{ik} + \phi_{n-1}e^{-ik}) - \nu|\phi_n|^2\phi_n = 0, \tag{4}$$

with $\psi_n = \phi_n \exp[i\tau(\omega^2 - \omega_0^2 - 2u_0^2)/u_0^2]$, $\tau = u_0^2 t/2\omega$, $\mu \equiv \frac{1}{A^2}$, $\nu \equiv \frac{2\omega^2+\omega_0^2+2u_0^2}{u_0^2 A^2}$, and $n \in [1; N[$. ν and μ, respectively, are the nonlinear cubic and nonlinear dispersion coefficients.

It is well established [12] that the supratransmission phenomenon does not exist in the lower forbidden band. Here, we will pay attention to the carrier wave in the upper forbidden gap; the equation governed by this mode is

$$i\frac{d\phi_n}{d\tau} - (1+\mu|\phi_n|^2)(\phi_{n+1} + \phi_{n-1}) - \nu_1|\phi_n|^2\phi_n = 0, \tag{5}$$

with $\nu_1 \equiv \frac{2\omega_{\max}^2+\omega_0^2+2u_0^2}{u_0^2 A^2}$. Contrary to the natural supratransmission phenomenon where one end of the line is driving, Eq. (5) will be driven at two ends. The left and right edges will be driven with the respective boundary condition,

$$\phi_0 = B_L e^{-i\vartheta_L \tau}, \quad \phi_{N+1} = B_R e^{-i\vartheta_R \tau}, \tag{6}$$

where B_L (B_R) is a driving amplitude and ϑ_L (ϑ_R) is a dimensionless frequency of the left (right) edge. When the driving voltage is applied at the end of the lattice, the real frequency is modified; by considering that, at site n, the dimensionless frequency is ϑ, the real frequency in the upper forbidden band is

$$\Omega = \omega_{\max} + \frac{u_0^2}{2\omega_{\max}}(\vartheta - 2). \tag{7}$$

The real frequency Ω is within the upper forbidden gap of the dispersion curve if $\vartheta > 2$ and is in the allowed phonon band for $2 - 2\omega_{\max}(\frac{\omega_{\max}-\omega_0}{u_0^2}) < \vartheta < 2$. In the linear approximation, the wave with frequency Ω_1 can propagate in the lattice if $\omega_0 < \Omega_1 < \omega_{\max}$; otherwise, they decay exponentially. Without the dimensional frequency ($\vartheta_L = \vartheta_R = 0$), the real frequency is within the allowed phonon band, and the boundary data are similar to those used to produce bistability in the nonlinear waveguide array [20]. There exists an amplitude threshold above which the energy supratransmission occurs for the Salerno equation. This threshold was first given by Togueu Motcheyo *et al.* [12],

$$B_{\text{thr}} = \sqrt{\frac{\vartheta - 2}{\frac{\nu_1}{2} + \frac{8\mu - \vartheta\mu}{6}}}. \tag{8}$$

Up to now, we know that the wave with an amplitude above this threshold can give rise to energy supratransmission in the lattice. Can the driving amplitude with a value below the threshold produce supratransmission phenomena in the forbidden band gap? The answer to this question will be found numerically in the next section.

III. NUMERICAL EXPERIMENT

In this section, the numerical study will be performed by using a set of boundary conditions given by (6). These conditions are different from Khomeriki's [8] ones where one end of the lattice (left) is driven with a periodic driving function and the other (right) is submitted to a dissipation for reduced edge reflection. The common element here is that the external driving amplitude B is given gradually from zero to its maximum value so as to avoid initial shock. Taking the dimensionless frequency $\vartheta_L = 2.06$, the real frequency Ω is within the upper forbidden band; from (8), we deduced the threshold of the amplitude given by $B_{\text{thr}} = 0.33$. For the natural supratransmission, every amplitude below this threshold only

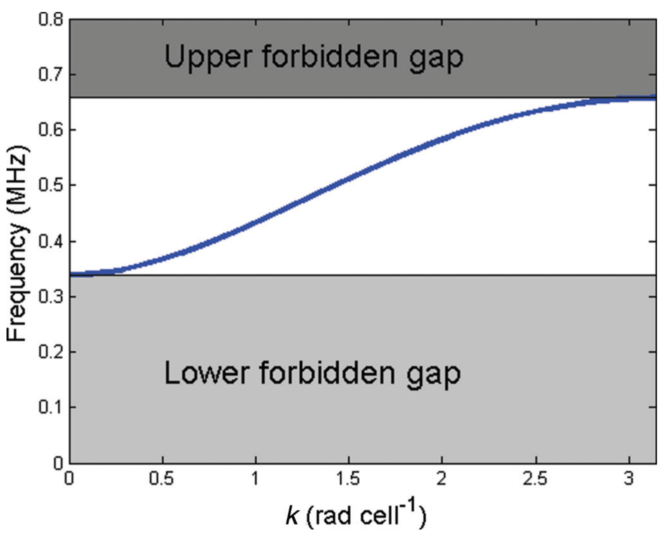

FIG. 2. (Color online) Curve of the linear dispersion relation. The forbidden band gaps correspond to the shaded areas of the figure. The characteristic frequencies of the network are $u_0 = 1.768 \times 10^6$ and $\omega_0 = 2.127 \times 10^6$ rad/s.

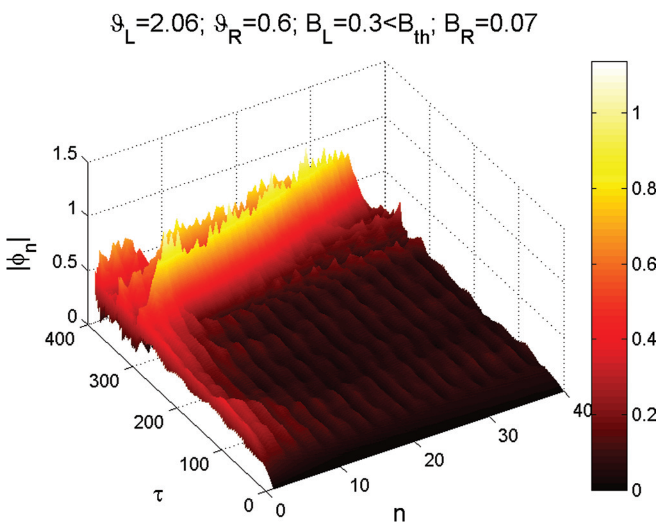

FIG. 3. (Color online) Typical numerical simulation of (5) submitted to boundary conditions (6).

excites several neighboring sites. In the allowed phonon band, any driving amplitude B will excite all the sites.

As follows from Fig. 3, when one boundary (left hand) of the lattice is driven with a dimensionless frequency ($\vartheta_L = 2.06$) in the forbidden band, the amplitude ($B_L = 0.3 < B_{\rm thr}$) below the threshold and the other boundary (right) is driven with the dimensionless frequency in the allowed phonon band, an energy flows in the lattice as if the driving amplitude (B_L) had been taken above the threshold. The plane wave in the allowed band and the other wave in the gap are moving in the opposite direction. As the wave in the gap has an amplitude below the threshold, it cannot excite several neighboring sites; the wave in the phonon allowed band moves to the left edge and collides. The collision is inelastic: as the two waves are moving in opposite directions, at the collision time, the amplitude increases (see Ref. [21] for details on the collision) and becomes above the threshold so supratransmission phenomenon takes place. This is our main result.

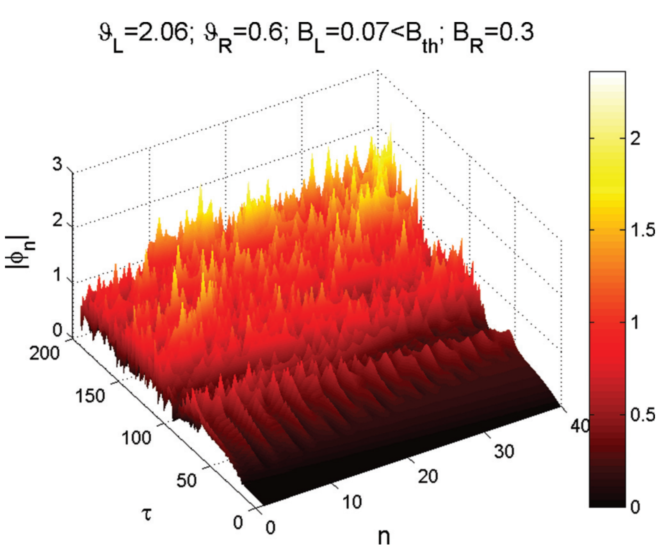

FIG. 4. (Color online) Typical numerical simulation of (5) submitted to boundary conditions (6).

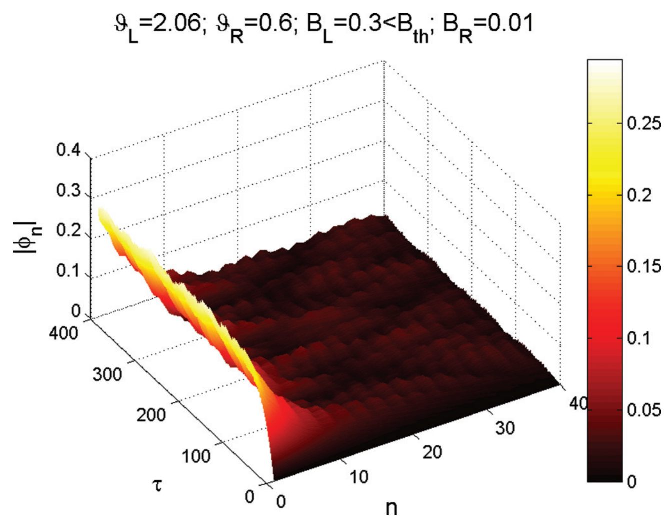

FIG. 5. (Color online) Typical numerical simulation of (5) submitted to boundary conditions (6).

Here, B_L is higher than B_R and is near the threshold. What happens if we keep the same frequency of the two waves but permute the amplitudes ($B_L = 0.07,\ B_R = 0.3$)?

In Fig. 4, we observe that the form of the plane wave in the allowed band dominates over the wave in the gap. This fact implies that, as the plane wave in the gap has a small amplitude, it is considered to be a perturbation of the wave in the phonon band. For Fig. 3, this result allows concluding that the plane wave (small driving amplitude) in the allowed band is the perturbation of the wave in the band gap, which undergoes an increase in amplitude causing the supratransmission. Contrary to Ref. [14] where the transmission in the gap is created from a resonant localized wave induced by an impurity, here, the gap transmission is due to the collision of the two waves.

The sum of B_L and B_R in Figs. 3 and 4 is above the threshold $B_{\rm thr}$. Can the phenomenon occur when this sum is below $B_{\rm thr}$? The time evolution of the Salerno equation driven at two edges, depicted in Figs. 5 and 6, shows the rapid decay of the wave

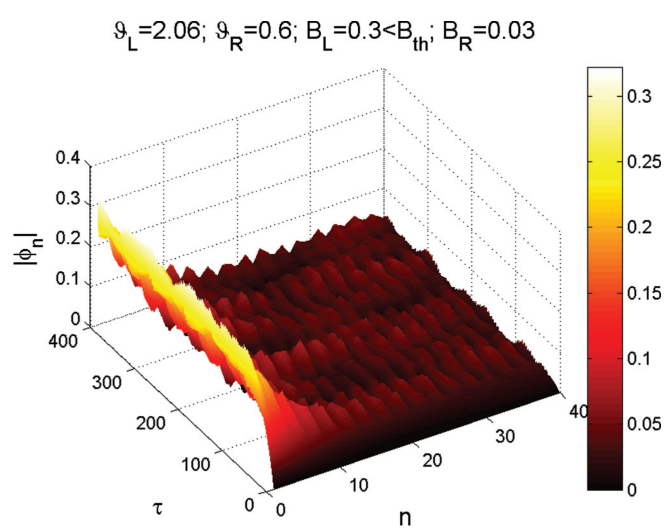

FIG. 6. (Color online) Typical numerical simulation of (5) submitted to boundary conditions (6).

for a couple of driving amplitudes $B_L = 0.3$, $B_R = 0.01$ (sum below the threshold) and $B_L = 0.3$, $B_R = 0.03$ (limit case for the classical gap transmission). This fact proves that, to achieve a supratransmission, the sum of the driving amplitudes will be above the threshold.

IV. CONCLUSION

To summarize, we offered, numerically, a way to create supratransmission in the upper forbidden gap in a discrete transmission line. We found that energy suddenly flows though the lattice for the harmonic driving amplitude below the maximum amplitude. This observation, which is contrary to the natural supratransmission, arises due to the collision of the plane waves coming from two edges of the line. One end of the line is submitted to monochromatic irradiation at a frequency in the gap and the other end of the line at a frequency in the allowed phonon band. The gap transmission is possible if the sum of two driving amplitudes exceeds the maximum amplitude; otherwise, the amplitude vanished. The explicit demonstration of this way is an open problem and would be a natural extension of this Rapid Communication.

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