Time-delay autosynchronization control of defect turbulence in the cubic-quintic complex Ginzburg-Landau equation

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We investigate the effectiveness of a Global time-delay autosynchronization control scheme aimed at stabilizing traveling wave solutions of the cubic-quintic Ginzburg-Landau equation in the Benjamin-Feir-Newell unstable regime. Numerical simulations show that a global control can be efficient and also can create other patterns such as spatiotemporal intermittency regimes, standing waves, or uniform oscillations.

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I. INTRODUCTION

During the past three decades, considerable progress has been made in our understanding of the spontaneous emergence of patterns in spatially extended nonequilibrium systems. The existence of simple spatial or spatiotemporal patterns has been rigorously established on the basis of equivariant bifurcation theory [1]. However, these simple patterns are often unstable in a given system, which evolves instead to a state of spatiotemporal chaos [2–5]. Spatiotemporal chaos (STC) exists abundantly in nature and is a crucial influence on the behavior of various systems. The investigations of the STC have attracted continual attention for more than a century in a large variety of fields of natural science, e.g., hydrodynamic turbulence [6], chemical turbulence [7,8], electrical turbulence in the cardiac muscle [9], and wave propagation in nonlinear optical fibers with gain and spectral filtering [10]. In some cases, STC arises in the proximity of the threshold and can be described within the context of weakly nonlinear theories. The model of the complex Ginzburg-Landau equation (CGLE) has been extensively used for the study of spatiotemporal chaos. The study of the CGLE, i.e., a universal description of oscillatory systems close to the onset, reveals complex patterns and underlying principles relevant for all oscillatory systems [1,5,11–13]. This complexity is often considerable. In response to this challenge, the control of STC has emerged in recent years as a problem of increasing fundamental and applied value. A current challenge in pattern-formation research is to develop control schemes that stabilize the patterned states so that a desired, otherwise unstable, solution may be realized. Since the pioneering work of Ott, Grebogi, and Yorke [14], controlling chaos has been extensively investigated. A variety of approaches, such as the Ott-Grebogi-Yorke (OGY) scheme, a feedback technique [15], a nonlinear diffusion effect [16,17], and an adaptive method [18,19], have been developed for the purpose of chaos control. Ott, Grebogi, and Yorke suggested a method to stabilize an unstable periodic orbit. The main idea of Ott, Grebogi, and Yorke consisted in waiting for a natural passage of the chaotic orbit close to the desired periodic behavior and then applying a small disturbance in order to stabilize such periodic dynamics (which would in fact be unstable for the unperturbed system).

It is generally difficult to apply the OGY idea to high-dimensional systems such as turbulence. This has suggested the development of some alternative approaches. The first was introduced by Pyragas, who has proposed an empirical control method based on implementing a delayed feedback loop called time-delay autosynchronization (TDAS) [15]. It consists in designing a proper feedback line through which a state variable is directly perturbed such as to control a periodic orbit. This method has a number of attractive properties and has been implemented successfully in a variety of experimental systems including electronic [20,21], laser [22], plasma [23], and chemical [24] systems.

The feedback can be either local [25] (at each spatial point the field at the same point at previous times is fed back) or global (at each spatial point a term proportional to the integral of the field over the spatial variable is fed back) [26]. In spite of the widespread use of the CGLE, investigations of TDAS in the context of the CGLE are relatively few. In the following, we mention some publications relevant in this context: Bleich and Socolar studied the control of STC by traveling waves for the CGLE without global terms (but in the extended TDAS scheme with multiple delay times) [27]. Harrington and Socolar investigated the corresponding two-dimensional case [28]. Moreover, control of spatiotemporal chaos has been shown to be effective in stabilizing traveling waves in CGLE in one and two spatial dimensions [29–31]. Based on previous works on the CGLE under the influence of global feedback [26,32] and on TDAS in the CO oxidation reaction [33], Beta and Mikhailov investigated global TDAS in the CGLE, focusing on the question of why for global TDAS noninvasive stabilization of uniform oscillations is not possible [34]. Aranson *et al.* [35] suggested a method of turbulence control in the CGLE without gradient force by developing a spiral wave with local feedback injection. They stabilized a structurally unstable topological defect, whose analytical expression is known, by adding an extra term in the CGLE. Boccaletti *et al.* [17,18] used another method called an adaptive method in CGLE to control turbulence.

In a recent work [36], by using the cubic-quintic CGLE to study the wave patterns in a spatially extended system,

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we found many types of spatiotemporal regimes. We found that when the Benjamin-Feir-Newell (BFN) criterion is met, the dynamics becomes chaotic and three types of chaotic behavior can be distinguished. Below the BFN line, stable and spatiotemporal intermittency states are found. We then controlled the defect turbulence regime by using the nonlinear diffusion technique [16]. We have shown that an unstable traveling hole can be stabilized in the chaotic regime.

The present work extends current research on the control of STC in extended systems. We show that the developed turbulence, in particular the defect turbulence in the system described by the one-dimensional cubic-quintic CGLE, can be suppressed by introduction of delayed global feedback. We show that in this case of turbulence there are two parameters whose variation is sufficient to suppress the chaos in the system: the intensity of the control signal and the delay time. By varying these parameters, turbulence can be suppressed.

This paper is organized as follows. In Sec. II we review some properties of the cubic-quintic CGLE and present a modified form of the model equation. In Sec. III we analyze the stability of the traveling wave. The numerical simulations of the control problem are carried out in Sec. IV. We discuss the results in Sec. V and summarize in Sec. VI.

II. MODEL EQUATION

In this section we review some properties of wave solutions of the cubic-quintic CGLE. The cubic-quintic CGLE represents an important prototypical equation since it arises generically as an envelope equation for a weakly inverted bifurcation associated with traveling waves. The one-dimensional form of this equation including dissipation and dispersion can be written as

$$\frac{\partial A}{\partial t} = (1+ic_1)\frac{\partial^2 A}{\partial x^2} + \mu A + (1-ic_3)|A|^2 A - (1-ic_5)|A|^4 A, \tag{1}$$

where $A(x,t)$ is a complex field. In writing Eq. (1) we have already transformed it into the moving frame. The value c_1 represents the linear dispersion term, while c_3 and c_5 represent the nonlinear dispersion of wave patterns; μ is the criticality parameter, t and x are evolutional and spatial variables, respectively. We consider a system in which we impose periodic boundary conditions, which can be constructed experimentally in one-spatial-dimension geometries [37].

We consider that Eq. (1) admits the periodic solution $A(x,t) = A_0 e^{i\Omega_0 t}$ with

$$|A_0|^2 = \tfrac{1}{2}(1 \pm \sqrt{4\mu + 1}) \tag{2}$$

and

$$\Omega_0 = -|A_0|^2(c_3 - c_5|A_0|^2). \tag{3}$$

In the BFN unstable regime, all the solutions are unstable. As we have seen in [36], when the condition $1 - c_1c_3 - 2|A_0|^2(1 - c_1c_5) > 0$ of the BFN instability is satisfied, uniform oscillations are unstable and the spatiotemporal chaos spontaneously develops in the system. To control the different turbulence regimes observed in the domain, a global feedback term can be introduced. The modified cubic-quintic CGLE is given by

$$\frac{\partial A}{\partial t} = (1+ic_1)\frac{\partial^2 A}{\partial x^2} + \mu A + (1-ic_3)|A|^2 A - (1-ic_5)|A|^4 A + F, \tag{4}$$

where F is a feedback term given by

$$F(t) = \alpha e^{i(\chi_0+\omega\tau)}\overline{A}(t-\tau) \tag{5}$$

with

$$\overline{A}(t) = \frac{1}{L}\int_0^L A(x,t)dx. \tag{6}$$

Here $\overline{A}(t)$ is the spatial average of the complex oscillation amplitude, L is the length of the system, α is the feedback intensity factor, χ_0 determines the phase shift between $\overline{A}$ and F, ω is the frequency of the oscillation, and τ is the time delay. When the time delay is short ($\tau \ll 1$), the slowly varying average amplitude $\overline{A}(t)$ does not significantly change within the time delay and the delays in this term could be neglected. Then we obtain the following equation:

$$\frac{\partial A}{\partial t} = (1+ic_1)\frac{\partial^2 A}{\partial x^2} + \mu A + (1-ic_3)|A|^2 A - (1-ic_5)|A|^4 A + \alpha e^{i\chi}\overline{A}(t), \tag{7}$$

where $\chi = \chi_0 + \omega\tau$. The principal effect of the delays here is providing an additional phase shift $\Delta\chi = \omega\tau$ between the average slow oscillation amplitude and the signal, which can be easily manipulated by changing the delay time. Below we analyze how the phase shift variations influence the properties of the system.

III. LINEAR STABILITY ANALYSIS

The presence of the global feedback term in Eq. (7) modifies the frequency and the amplitude of traveling waves. By injecting again the periodic solution $A(x,t) = A_1 e^{i\Omega t}$ in the modified cubic-quintic CGLE, the amplitude and the frequency of oscillations are given by

$$\begin{aligned} &\mu + |A_1|^2 - |A_1|^4 + \alpha\cos(\chi) = 0, \\ &\Omega = -c_3|A_1|^2 + c_5|A_1|^4 + \alpha\sin(\chi), \end{aligned} \tag{8}$$

where A_1 and Ω are the new values of frequency and amplitude, respectively. The resolution of Eq. (8) leads to

$$|A_1|^2 = \tfrac{1}{2}\{1 \pm \sqrt{1 + 4[\mu + \alpha\cos(\chi)]}\} \tag{9}$$

and

$$\Omega = -|A_1|^2(c_3 - c_5|A_1|^2) + \alpha\sin(\chi). \tag{10}$$

Thus global feedback shifts the solution of Eq. (10) and can be constructed by writing

$$\alpha = \frac{\Omega + |A_1|^2(c_3 - c_5|A_1|^2)}{f(\chi)}, \tag{11}$$

where $f(\chi) = \sin(\chi)$. Generally, the function $f(\chi)$ is 2π periodic and satisfies the condition $f(0) = f(2\pi)$ and for each frequency Ω of the oscillation it determines the respective value of the feedback intensity α. Equation (9) shows that when the condition

$$\alpha > -\frac{\frac{1}{4}+\mu}{\cos(\chi)} \tag{12}$$

is satisfied, uniform oscillations become impossible in the system and the uniform steady state $A = 0$ becomes stable in the class of uniform solutions. We notice that when the feedback term is suppressed ($\alpha = 0$), the frequency of uniform oscillation becomes $\Omega = \Omega_0$.

The linear stability of uniform oscillations is investigated by considering the evolution of small perturbations in the periodic solution, that is, we look for solutions of Eq. (7) of the form

$$A(x,t) = A_1 e^{i\Omega t}[1 + \varepsilon B(x,t)]. \tag{13}$$

Inserting Eq. (13) into Eq. (7), we obtain upon linearization in B

$$\begin{aligned}\frac{\partial B}{\partial t} = \mu B + (1+ic_1)\frac{\partial^2 B}{\partial x^2} - B|A_1|^2[2(-1+ic_3) \\ + 3(1-ic_5)|A_1|^2] - |A_1|^2 B^*[(-1+ic_3) \\ + 2(1-ic_5)|A_1|^2] + \alpha e^{i\chi}\overline{B}(t).\end{aligned} \tag{14}$$

Using the ansatz

$$B = B_1(t)e^{ikx} + B_2(t)e^{-ikx} \tag{15}$$

one gets a closed system of equations for $B_1(t)$ and $B_2^*(t)$, setting $B_1(t) \propto B_{10}e^{\lambda t}$ and $B_2(t) \propto B_{20}e^{\lambda t}$, where $\lambda = \lambda_1 + i\lambda_2$ is a complex value. After substituting Eq. (15) into Eq. (14) we obtain the eigenvalue equation

$$E = (C + iD - i\lambda_2)(C - iD - i\lambda_2), \tag{16}$$

where

$$E = |A_1|^4[(1-2|A_1|^2)^2 + (c_3 - 2c_5|A_1|^2)^2], \tag{17}$$

$$\begin{aligned}C &= -k^2 + \mu - \lambda_1 + 2|A_1|^2 - 3|A_1|^4,\\ D &= \Omega + |A_1|^2(2c_3 - 3c_5|A_1|^2) - c_1k^2.\end{aligned} \tag{18}$$

Equation (16) must be solved numerically for a given set of parameters. The system behavior is found depending on the values of λ_1 and λ_2. The sign of λ_1 determines the stability. When $\lambda_1 > 0$, unstable groups of modes are observed and the uniform oscillations are unstable; in contrast, for $\lambda_1 < 0$, uniform oscillations remain stable. The last unstable group of modes in the considered system has $\lambda_2 = 0$ and its wave numbers are close to a certain wave number $k \neq 0$ [see Figs. 1(a) and 1(b)]. In this case, standing waves appear because the spatial profiles of the real amplitude A and phase are periodic in space but do not undergo temporal oscillations. For a value of k, the instability corresponds to a periodic pattern with a wavelength of $2\pi/k_c$. A different bifurcation scenario exists wherein standing and traveling waves are observed as primary bifurcating solutions. In addition to standing waves, the system has oscillatory modes with $\lambda_2 \neq 0$ that correspond to periodic spatiotemporal modulation of the uniform oscillations. The value of $\lambda(k)$ is given by

$$\lambda(k) = -(k^2 - \mu - 2|A_1|^2 + 3|A_1|^4) \pm \sqrt{|A_1|^4[(1-2|A_1|^2)^2 + (c_3 - 2c_5|A_1|^2)^2] - [\Omega + |A_1|^2(2c_3 - 3c_5|A_1|^2) - c_1k^2]^2}. \tag{19}$$

The increment of growth of these modes is

$$\lambda_1(k) = -(k^2 - \mu - 2|A_1|^2 + 3|A_1|^4). \tag{20}$$

Substituting Eq. (9) into Eq. (20) leads to

$$\lambda_1(k) = -2\mu - 3\alpha\cos(\chi) - A_1^2. \tag{21}$$

Here the instability will occur by periodic spatiotemporal modulations of uniform oscillations since $\lambda_2 \neq 0$ and the most unstable modes have wave numbers close to $k_0 = 0$ [see Fig. 1(c)]. They are observed beyond the critical value

$$\alpha_f = -\frac{1}{9\cos(\chi)}(1 + 6\mu - \sqrt{1+3\mu}). \tag{22}$$

Here the instability can occur only if $\cos(\chi) < 0$. Note that the critical value for the feedback intensity does not depend on the parameters c_1, c_3, and c_5. Supposing that $\Omega = \Omega_0$ leads $|A_1| = |A_0|$. Inserting these two values into the expression in Eq. (19), we obtain

$$\lambda(k) = -(k^2 - |A_0|^2 + 2|A_0|^4) \pm \sqrt{|A_0|^4[(1-2|A_0|^2)^2 + (c_3 - 2c_5|A_0|^2)^2] - [|A_0|^2(c_3 - 2c_5|A_0|^2) + c_1k^2]^2}. \tag{23}$$

Then, by resolving the equation $\lambda(k) = 0$, we deduce the wave number k given by

$$k = \sqrt{\frac{2|A_0|^2[1 - c_1c_3 - 2|A_0|^2(1 - c_1c_5)]}{1 + c_1^2}}. \tag{24}$$

This equation is possible if $1 - c_1c_3 - 2|A_0|^2(1 - c_1c_5) > 0$ is satisfied. This condition is the BFN criterion [36].

IV. NUMERICAL SIMULATION

In this section we present the results of a numerical study of Eq. (7). We analyze the stability of the unstable wave patterns by using the global feedback term. All simulations were carried out for a one-dimensional system of length $L = 250$. Our numerical simulations are made by using a finite-difference scheme in space and the standard fourth-order Runge-Kutta algorithm in time [38]. The discrete form of the periodic

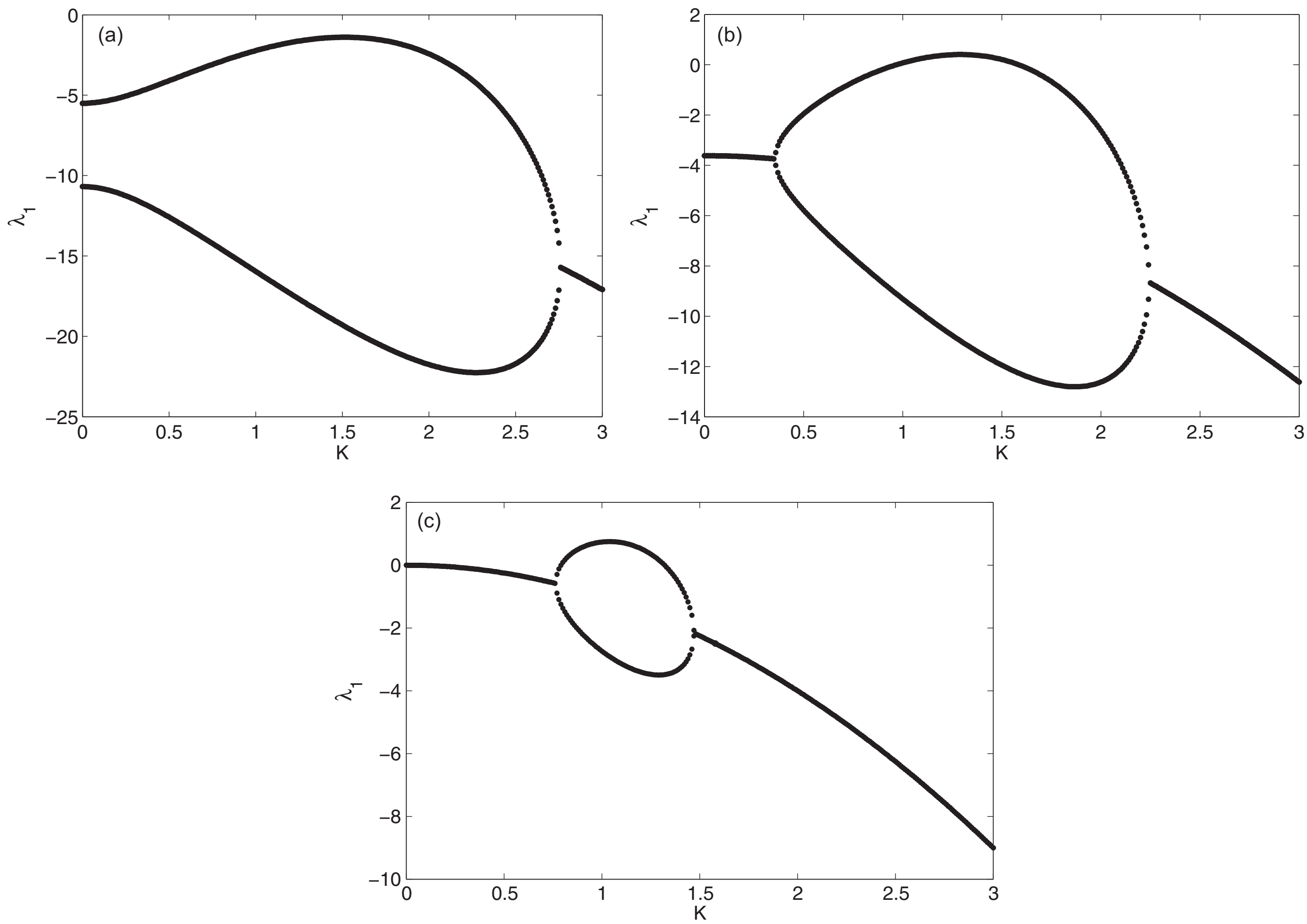

FIG. 1. Growth rate $\lambda_1(k)$ of the traveling wave as a function of wave number k for (a) $\alpha = 3.9, \chi = \pi/2$; (b) $\alpha = 5.5, \chi = 3\pi/4$; and (c) $\alpha = 9.2, \chi = 5\pi/4$ with $c_3 = 0.5$, $c_1 = 2.5$, $c_5 = 1.1$, and $\mu = 1.0$.

boundary condition that we used is $A_1 = A_n$ and $A_{n+1} = A_2$. The precision of the numerical results is examined by testing several steps of integration in space and in time. We have chosen a grid spacing $dx = 0.25$ and the typical time step was $dt = 0.01$. We apply periodic boundary conditions and use as the initial condition a traveling hole [36]. The parameter values are chosen as in Ref. [36] in order to be in the BFN unstable region specifically in the case of defect turbulence regimes. We investigate the effects of the feedback intensity value and the phase shift on the system. A sufficiently strong feedback suppresses turbulence and establishes uniform oscillations. We show that for an appropriate choice of the global feedback term and the delay time we can show that the spatiotemporal chaos becomes completely stable.

After numerical simulations we found five different types of regimes in the domain depending on the values of α and χ. These regimes are summarized in the state diagram of Fig. 2. The five regimes observed are defect turbulence, spatiotemporal intermittency, phase turbulence, standing waves, and plane waves.

Figure 3, obtained for the value of $\alpha = 0$, displays the defect turbulence regime. It is similar to the one found without feedback [36]. The wave field fluctuates irregularly and the modulus of the amplitude $|A|$ drops down to zero frequently (black areas on the right-hand side of Fig. 3). As the feedback intensity is increased starting from zero, global oscillations set in and defect turbulence regimes are replaced

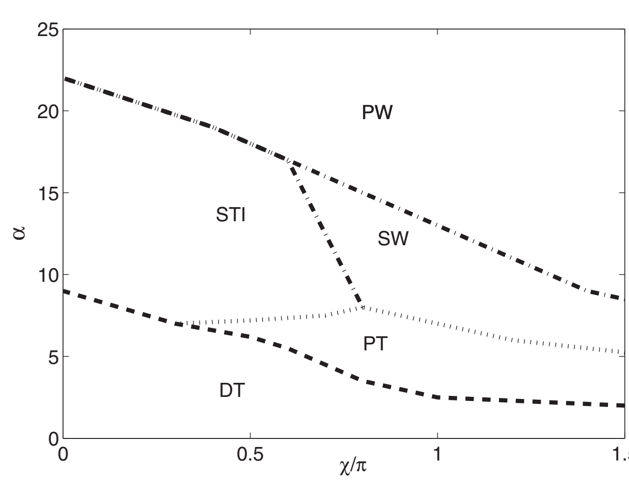

FIG. 2. Phase diagram of (χ,α) showing different types of dynamical regimes: defect turbulence (DT), spatiotemporal intermittency (STI), phase turbulence (PT), standing wave (SW), and plane wave (PW) for $c_1 = 2.5$, $c_3 = 0.5$, $c_5 = 1.1$, and $\mu = 1.0$.

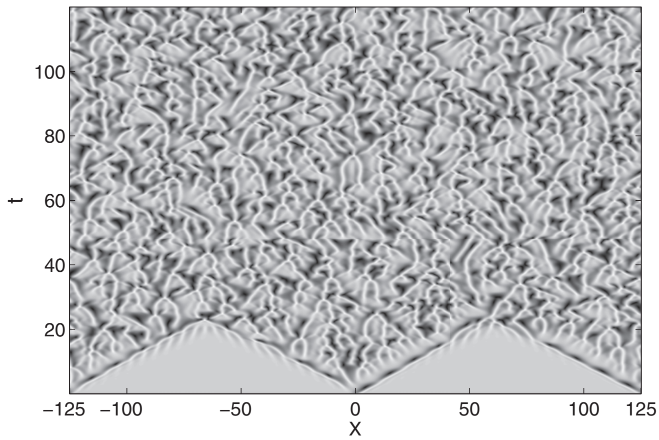

FIG. 3. Space-time variation of the wave pattern amplitude $|A|$ for $\alpha = 0.0$, $\chi = \pi/6$, $c_1 = 2.5$, $c_3 = 0.5$, and $c_5 = 1.1$, denoting defects in the system and $\mu = 1$.

by other interesting regimes until the appearance of the laminar state.

For certain values of α, amplitude defects disappear from some parts of the system and thus an intermittent state is developed. Figure 4 illustrates three examples of intermittent regimes depending on the values of α and χ of the wave amplitude $|A|$ in a one-dimensional system [36,39].

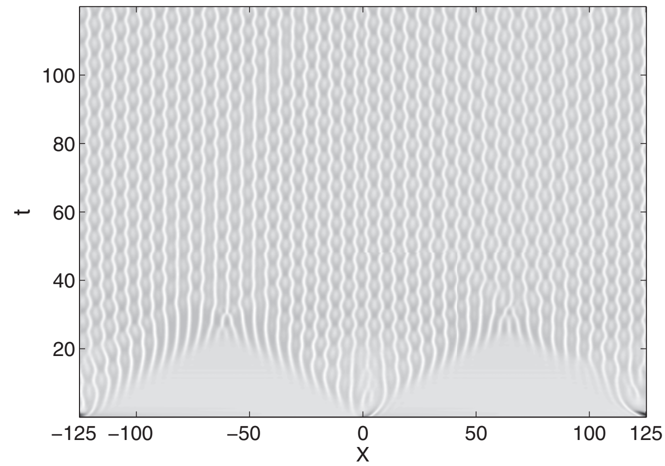

FIG. 5. Phase turbulence regime of the wave pattern amplitude $|A|$ obtained for $\alpha = 6.0$, $\chi = 7\pi/4$, $c_1 = 2.5$, $c_3 = 0.5$, $c_5 = 1.1$, and $\mu = 1.0$.

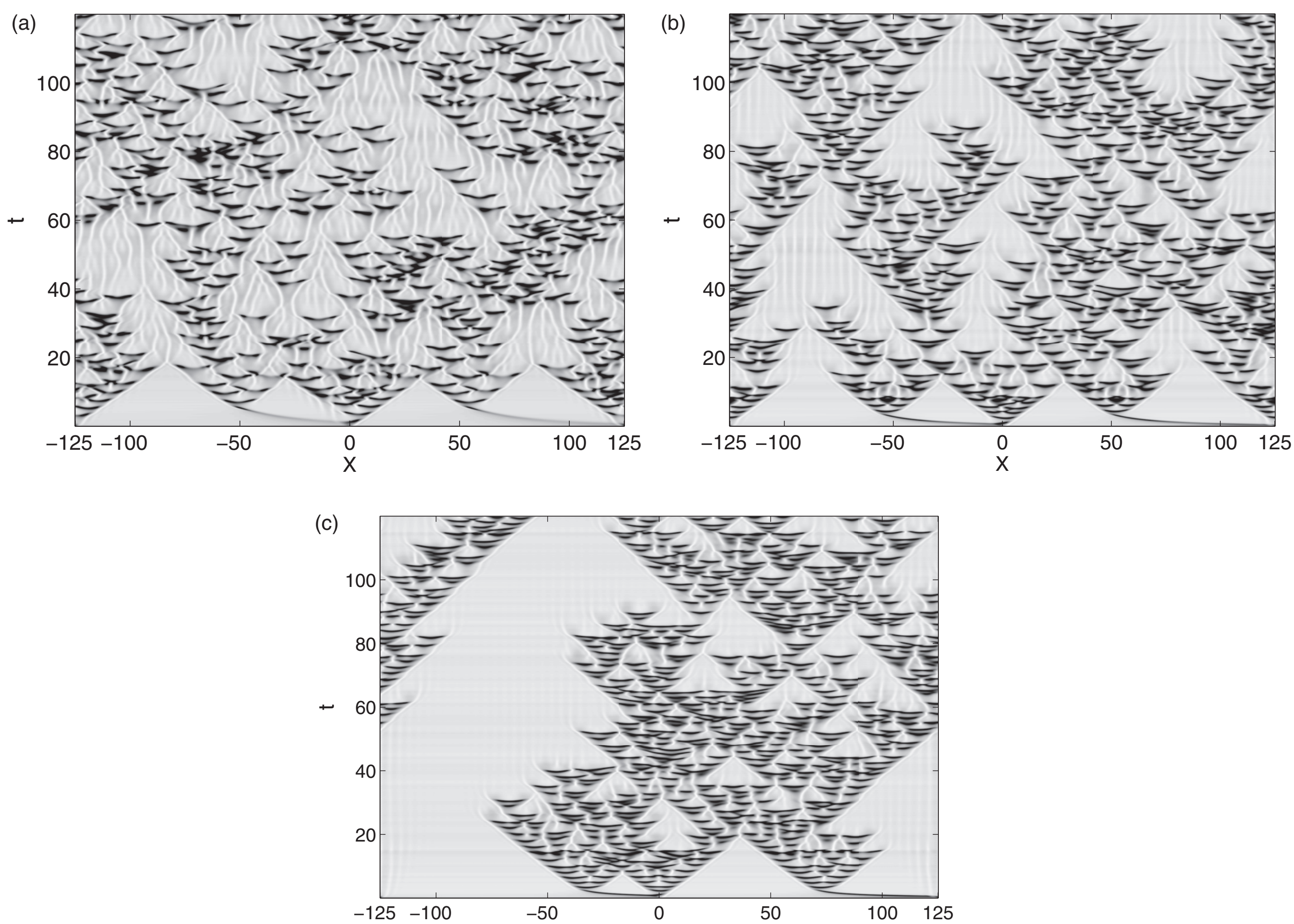

FIG. 4. Transition from defect turbulence to the spatiotemporal intermittency regime of the wave pattern amplitude $|A|$ for $c_1 = 2.5$, $c_3 = 0.5$, $c_5 = 1.1$, $\chi = \pi/6$, and $\mu = 1.0$ when (a) $\alpha = 8.0$, the coexistence between STI and PT is showing; (b) $\alpha = 12.0$, the interchange between STI and SW is showing; and (c) $\alpha = 15$, the space-time denotes the coexistence between STI and PW.

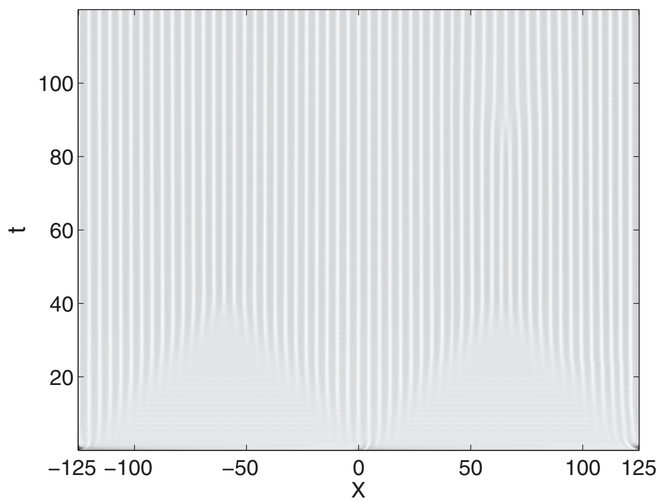

FIG. 6. Space-time variation of the wave pattern amplitude $|A|$ for $\alpha = 9.0$, $\chi = 7\pi/4$, $c_1 = 2.5$, $\mu = 1.0$, $c_3 = 0.5$, and $c_5 = 1.1$, displaying a standing wave regime where bright stripes repeatedly develop from the dark uniform state.

In Fig. 4(a), turbulent bursts occupy most of the system and laminar areas are relatively rare, while Figs. 4(b) and 4(c) show the coexistence of turbulence and laminar states filled with standing waves [Fig. 4(b)] or plane waves [Fig. 4(c)]. By further increasing the feedback intensity from the states of intermittency turbulence, uniform oscillations are observed for the phase shift in the interval $0 < \chi < 3\pi/4$. For appropriate values of α and χ, the domain is still always turbulent, but the defects disappear completely; this is the phase turbulence regime (illustrated in Fig. 5). In this regime, the amplitude is always bounded away from zero; it never reaches zero and remains saturated.

Standing waves are displayed in Fig. 6. In this type of pattern, bright stripes repeatedly develop from the dark uniform state. The stripes are only visible during relatively short intervals of each oscillation cycle. They form a spatially periodic array. Standing waves with similar properties have been observed in the model of CO oxidation under intrinsic gas phase coupling [40,41]. For appropriate values of α, standing waves and spatiotemporal intermittency disappear, the system becomes laminar, and uniform oscillations are observed (not shown).

V. DISCUSSION

Motivated by the research on the cubic-quintic CGLE, in Ref. [16] we controlled the spatiotemporal chaos in the domain [36] by using the nonlinear diffusion control term. In the present paper we study the control of spatiotemporal chaos in a spatially extended system described by a cubic-quintic CGLE with global time-delay feedback. A model of Eq. (7) was obtained by a reduction of the global feedback system in Eqs. (4) and (5) valid for relatively short delay times $\tau \ll 1$, as studied in Ref. [26]. Nonetheless, it also may be interesting to consider the global control problem without assuming that the delay time is short.

The global control implemented in the reduced model (7) and the original (4) is invasive. This means that when turbulence is suppressed and uniform oscillation is stabilized, the control signal does not vanish and effectively the system is then under the action of a uniform periodic driving force. It is known that in the chaotic dynamical systems, described by a small number of variables, stabilization of unstable periodic orbits can be achieved in a noninvasive way by using the time-delay autosynchronization proposed by Pyragas [14]. In order to show the influence of a longer time delay τ for a better comparison with previous work [34] we solve the cubic-quintic CGLE in the scaled form

$$\frac{\partial A}{\partial t} = (1 + ic_1)\frac{\partial^2 A}{\partial x^2} + (\mu - i\omega)A + (1 - ic_3)|A|^2 A - (1 - ic_5)|A|^4 A + F, \tag{25}$$

where F is a feedback term given by

$$F(t) = \alpha e^{i\chi}[\overline{A}(t-\tau) - \overline{A}(t)] \tag{26}$$

and

$$\overline{A}(t) = \frac{1}{L}\int_0^L A(x,t)dx. \tag{27}$$

The new values of the amplitude and frequency are given by

$$|A_1|^2 = \tfrac{1}{2}(1 \pm \sqrt{1 + 4\{\mu + \alpha[\cos(\chi - \Omega\tau) - \cos(\chi)]\}}) \tag{28}$$

and

$$\Omega = -\omega - |A_1|^2(c_3 - c_5|A_1|^2) + \alpha[\sin(\chi - \Omega\tau) - \sin(\chi)]. \tag{29}$$

A cubic CGLE with a similar feedback scheme was investigated in Refs. [26,34]. Motivated by experiments [33] on the control of chemical turbulence in the CO oxidation reaction on Pt(1 1 0), Beta *et al.* used the cubic CGLE as a model for spatially extended oscillatory systems to study common aspects of the control of diffusion-induced chemical turbulence by applying TDAS [34]. They showed how the strongly disordered state can be stabilized in the system. The initially unstable state undergoes several transformations successively: defect turbulence, phase turbulence, standing wave state, and uniform oscillations. The results of our numerical study of Eq. (25) are given in Fig. 7, which shows the effects of a longer time delay on the system; it displays the progressive transition from defect turbulence to a plane wave state. In the absence of feedback $\alpha = 0$ [Fig. 7(a)], the defect turbulence is observed. For small α, the weak turbulence is observed [Fig. 7(b)]. When the feedback term grows, the system displays a disordered state of phase turbulence [Fig. 7(c)], stationary standing wave patterns [Fig. 7(d)], and uniform oscillations [Fig. 7(e)], respectively. The main result is the presence of a weak turbulence regime during the transition from the defect turbulence regime to the plane wave regime. This state is not obtained in Ref. [34]. Equation (25) is used to control spatiotemporal chaos in many applications such as binary fluid convection [42], the spiral waves in the Couette-Taylor flow between counterrotating cylinders [43], chemical turbulence [7,8], electrical turbulence in the cardiac muscle [9], and the electrohydrodynamic instabilities in nematic liquid crystal [44].

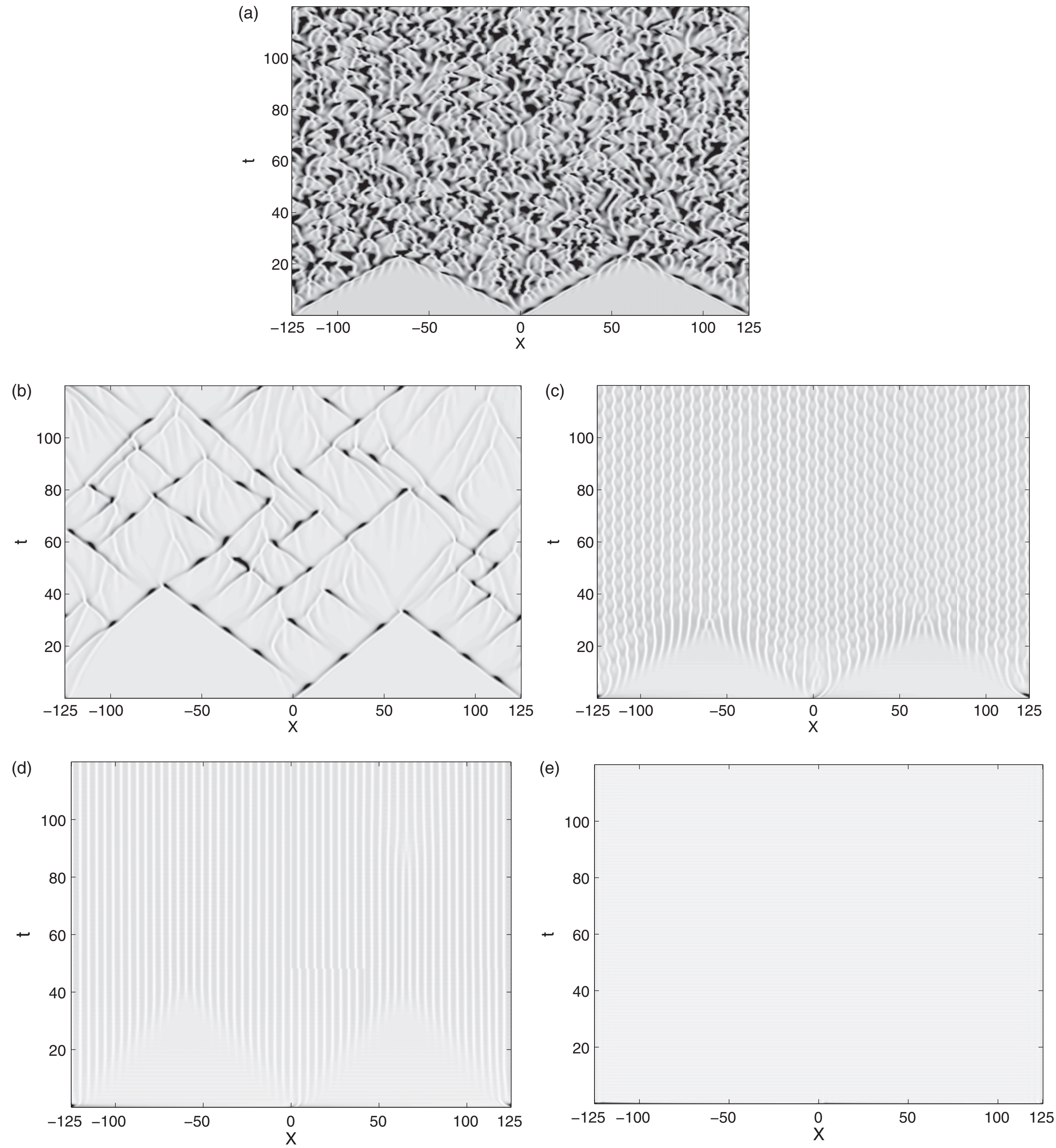

FIG. 7. Space-time variation of the wave pattern amplitude $|A|$, $\chi = \pi/2$, $c_1 = 2.5$, $c_3 = 0.5$, $\mu = 1.0$, $c_5 = 1.1$, and $\tau = 1.2$ for (a) $\alpha = 8.0$, showing DT; (b) $\alpha = 12.0$, the weak turbulence regime; (c) $\alpha = 15.0$, PT, (d) $\alpha = 18.0$, SW; and (e) $\alpha = 22.0$, PW.

VI. CONCLUSION

We have investigated the behavior of traveling waves in the defect turbulence regime described by the cubic-quintic complex Ginzburg-Landau equation influenced by the global feedback term through a TDAS scheme. This investigation is of general importance since the CGLE is generally derived from an amplitude expansion near the threshold of a bifurcation and higher-order perturbations appear naturally. We have shown that the chaotic dynamics that emerged from this prototypical equation, with periodic boundary conditions, can be controlled by a global feedback scheme.

We presented the model equation of a cubic-quintic CGLE and then the modified cubic-quintic CGLE with a feedback term. We showed analytically that the stabilization of the wave patterns in the turbulence regime depends on its growth rate. By using hole solutions as initial conditions, we simulated the modified cubic-quintic CGLE in the case of the defect turbulence regime. Then the spatiotemporal chaos regime previously obtained with the original cubic-quintic CGLE [36] was stabilized by introducing delayed global feedback in the cubic-quintic CGLE. We note that control of STC can be achieved by varying only two parameters, i.e., the intensity of the control and the delay time. These two parameters have a simple physical interpretation and could be relatively easily manipulated in an experiment. We considered the case in which the time delay is longer. As a result we also controlled the defect turbulence regime. In the scheme's transition to a plane wave regime, we observed the presence of weak turbulence in the system. Our numerical results are in good agreement with the analytical predictions of the linear stability analysis and many previous experiments. This work should prove beneficial for the development of many significant technological applications such as mixing, optical fiber manufacture, and chemical reactions leading to complex spatiotemporal dynamics. We leave for further study the examination of the possibility of stabilizing traveling waves of the cubic-quintic CGLE in two spatial dimensions using the global feedback scheme.

[1] M. C. Cross and P. C. Hohenberg, Rev. Mod. Phys. **65**, 851 (1993).
[2] P. Coullet, L. Gil, and J. Lega, Phys. Rev. Lett. **62**, 1619 (1989).
[3] H. Sakaguchi, Prog. Theor. Phys. **84**, 800 (1990).
[4] B. I. Shraiman, A. Pumir, W. van Saarloos, P. C. Hohenberg, H. Chaté, and M. Holen, Physica D **57**, 241 (1992).
[5] H. Chaté, Nonlinearity **7**, 185 (1994).
[6] S. T. Bramwell, P. C. W. Holdsworth, and J. F. Pinton, Nature (London) **396**, 552 (1998).
[7] Y. Kuramoto, *Chemical Oscillations, Waves, and Turbulence* (Springer, Berlin, 1984).
[8] M. Kim, M. Bertram, M. Pollmann, A. von Oertzen, A. S. Mikhailov, H. H. Rotermund, and G. Ertl, Science **292**, 1357 (2001).
[9] S. Sinha, A. Pande, and R. Pandit, Phys. Rev. Lett. **86**, 3678 (2001).
[10] Y. Kodama and A. Hasegawa, Opt. Lett. **17**, 31 (1992).
[11] M. van Hecke, Phys. Rev. Lett. **80**, 1896 (1998).
[12] H. Chaté and P. Manneville, Physica A **224**, 348 (1996).
[13] J. B. Gonpe Tafo, L. Nana, and T. C. Kofane, Phys. Scr. **87**, 065001 (2013).
[14] E. Ott, C. Grebogi, and J. A. Yorke, Phys. Rev. Lett. **64**, 1196 (1990).
[15] K. Pyragas, Phys. Lett. A **170**, 421 (1992).
[16] R. Montagne and P. Colet, Phys. Rev. E **56**, 4017 (1997).
[17] J. B. Gonpe Tafo, L. Nana, and T. C. Kofane, Eur. Phys. J. Plus **127**, 75 (2012).
[18] S. Boccaletti and F. T. Arecchi, Europhys. Lett. **31**, 127 (1995).
[19] S. Boccaletti and J. Bragard, Philos. Trans. R. Soc. London Ser. A **364**, 2383 (2006).
[20] K. Pyragas and Tamaševičius, Phys. Lett. A **180**, 99 (1993).
[21] D. J. Gauthier, D. W. Sukow, H. M. Concannon, and J. E. S. Socolar, Phys. Rev. E **50**, 2343 (1994).
[22] S. Bielawski, D. Derozier, and P. Glorieux, Phys. Rev. E **49**, R971 (1994).
[23] T. Fukuyama, H. Shirahama, and Y. Kawai, Phys. Plasmas **9**, 4525 (2002).
[24] P. Parmananda, R. Madrigal, M. Rivera, L. Nyikos, I. Z. Kiss, and V. Gáspár, Phys. Rev. E **59**, 5266 (1999).
[25] M. E. Bleich and J. E. S. Socolar, Phys. Lett. A **210**, 87 (1996).
[26] D. Battogtokh and A. Mikhailov, Physica D **90**, 84 (1996).
[27] M. E. Bleich and J. E. S. Socolar, Phys. Rev. E **54**, R17 (1996).
[28] I. Harrington and J. E. S. Socolar, Phys. Rev. E **64**, 056206 (2001).
[29] K. A. Montgomery and M. Silber, Nonlinearity **17**, 2225 (2004).
[30] C. M. Postlethwaite and M. Silber, Physica D **236**, 65 (2007).
[31] A. S. Mikhailov and K. Showalter, Phys. Rep. **425**, 79 (2006).
[32] D. Battogtokh, A. Preusser, and A. Mikhailov, Physica D **106**, 327 (1997).
[33] C. Beta, M. Bertram, A. S. Mikhailov, H. H. Rotermund, and G. Ertl, Phys. Rev. E **67**, 046224 (2003).
[34] C. Beta and A. S. Mikhailov, Physica D **199**, 173 (2004).
[35] I. Aranson, H. Levine, and L. Tsimring, Phys. Rev. Lett. **72**, 2561 (1994).
[36] J. B. Gonpe Tafo, L. Nana, and T. C. Kofane, Eur. Phys. J. Plus **126**, 105 (2011).
[37] I. Aranson and L. Kramer, Rev. Mod. Phys. **74**, 99 (2002).
[38] L. Nana, A. B. Ezersky, and I. Mutabazi, Proc. R. Soc. London Ser. A **465**, 2251 (2009).
[39] F. Mertens, R. Imbihl, and A. S. Mikhailov, J. Chem. Phys. **99**, 8668 (1993).
[40] M. Falcke, H. Engel, and M. Neufeld, Phys. Rev. E **52**, 763 (1995).
[41] M. Falcke and H. Engel, J. Chem. Phys. **101**, 6255 (1994).
[42] P. Kolodner, Phys. Rev. A **44**, 6448 (1991).
[43] L. Nana, A. B. Ezersky, N. Abcha, and I. Mutabazi, J. Phys.: Conf. Ser. **137**, 012006 (2008).
[44] A. Joets and R. Ribotta, in *Propagation in Systems Far from Equilibrium*, edited by J. E. Westreid, H. R. Brand, P. Manneville, G. Albinet, and N. Boccara (Springer, New York, 1988), p. 176.