Stability of three-dimensional dust acoustic waves in a dusty plasma with two opposite polarity dust species including dust size distribution

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Propagation of dust acoustic waves (DAWs) with the effect of power law dust size distribution (DSD) in a magnetized dusty plasma with opposite polarity dust is studied. Using a reductive perturbation method, a Zakharov-Kuznetsov equation appropriate for describing three-dimensional DAWs is derived. The compressive and rarefactive solitons are possible in the present model. Due to the DSD effect, a soliton with a smaller amplitude and width and a larger velocity is observed. The stability criterion for obliquely propagating DAWs in such plasma using small-k expansion method is investigated. The growth rate of instability is derived and analyzed under the effect of power law DSD. It is found that the growth rate of instability is strongly affected by the power law DSD. The relevance of these findings to space plasma phenomena is briefly discussed.

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I. INTRODUCTION

In the last decades, dusty plasma began to have a great interest for researchers because of its important role in explaining many space and astrophysical phenomena, as well as many industrial and physical applications [1]. In dusty plasmas, the presence of charged dust particles influences significantly the plasma characteristic features. The dust grains play a significant role in plasma wave dynamics [1]. It is normal to consider the dusty plasma model with negatively charged dust only; however, there are many cases in which dust particles have both negative and positive charges [2–5]. Dusty plasmas with two opposite polarity dusts have been found in different regions of space, e.g., Jupiter's magnetosphere [3,4], cometary tails [4], Earth's mesosphere [5], etc. The consideration of negatively charged dust is due to the fact that in low-temperature plasmas, the collection of plasma particles (electrons and ions) is the only important charging process. However, there are some other more important charging processes by which dust grains become positively charged [2,6–8]. The principal mechanisms of such processes are photoemission in the presence of a flux of ultraviolet photons [6], thermionic emission induced by radiative heating [7], secondary emission of electrons from the surface of the dust grains [2,8], etc. Using Sagdeev potential analysis (SPA) [9], Ivlev and Morfill [10] investigated the role of the ion distribution (Boltzmann or highly energetic cold ions) in the characteristics of dust acoustic waves (DAWs) with negative dust species. They stated that the allowed solitons are supersonic, and in dense clouds the width of the Mach number range remains finite for the Boltzmann ions but tends to zero for highly energetic ones. Also, they concluded that the charge variation is not important in rarefied particle clouds but becomes crucial if the particle number density is sufficiently high. Later on, Popel *et al.* [11] presented a study of arbitrary-amplitude DAWs in a three-component dusty plasma including the possibility of changing the dust species polarity (either positive or negative dust grains). Their model is applied to interpret the features of two different layered structures known as noctilucent clouds (NLC) and polar mesosphere summer echoes (PMSE) in Earth's mesosphere. However, they [11] started their analysis with SPA to examine the possible Mach number regime where DAWs would propagate; in the end, they expand the Sagdeev potential for the small-amplitude limit to get the analytical expression for the produced solitons. The final equation has the form of a Korteweg–de Vries (KdV) equation. Verheest and Hellberg [12] studied a fully general description of nonlinear electrostatic modes in plasmas with an arbitrary number of constituents. They showed that arbitrary-amplitude modes can be described by a SPA, although explicit expressions will be available for some simpler power law dependency constituents. On the other hand, weakly nonlinear modes can be treated by the reductive perturbation technique (RPT) [13], which leads to nonlinear evolution equations. The nonlinear evolution equation enables us to study the basic characteristics of the nonlinear waves [11,12,14]. Also, from the first step of the RPT, the linear dispersion relation is obtained, from which the pressure and inertial effects needed to sustain the wave modes can be defined [12]. When a comparison between the SPA and the RPT is carried out [12,14], full agreement is found between the two descriptions when the difference between the linear phase velocity and the velocity of the nonlinear structure is small [12].

On the other hand, Chow *et al.* [2] have shown that due to the size effect on secondary emission, insulating dust grains with different sizes can have opposite polarity, with smaller ones being positive and larger ones being negative. The opposite situation, i.e., massive positive and lighter negative dust grains, is also possible by triboelectric charging [7,15]. This is predicted from the observations of dipolar electric fields perpendicular to the ground, with the negative pole at higher altitudes, generated by dust devils [16] and sand storms [17]. The formation of these dipolar electric fields means that negatively charged dust particles are blown upward with convection, while positively charged dust particles remain at the surface due to gravity. The coexistence of opposite polarity charged dust particles is also observed in laboratories [18–20]. It may be noted here that the case of same sized

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dust particles with opposite polarity may also occur by photoemission if the photoemission yields of the dust material are very different [21]. Sakanaka and Shukla [22] considered a simple four-component dusty plasma with opposite polarity dust fluids. They discussed the creation and the possible regimes for observing large-amplitude solitons and double layers. Mamun and Shukla [23] studied solitary potentials in cometary dusty plasmas with positive and negative dust particles. Later, Shukla [24] considered a dusty plasma with opposite polarity cold dust fluids, and he studied the linear dispersive dust Alfvén waves and the associated dipolar vortex. El-Taibany *et al.* [25] investigated the modulational instability of DAWs propagating in a four-component dusty plasma with opposite polarity dust particles. Mamun [26] studied the basic properties of arbitrary-amplitude solitary potential structures in a dusty plasma with opposite polarity dust particles. Recently, Mamun [27] has investigated the nonlinear propagation of fast and slow dust-magnetoacoustic perturbation modes in an opposite polarity dusty plasma medium consisting of both positively and negatively charged dust fluids. He showed that the fast (slow) dust-magnetoacoustic mode is propagating as compressive (rarefactive) solitary waves.

Very recently, Shukla [28,29] examined linear DAWs in a dusty plasma and discussed the possibility of a twisted dust acoustic vortex beam carrying orbital angular momentum. So it may be worth mentioning that one of the motives of the present study is to investigate the instability of three-dimensional DAWs in a four-component magnetized dusty plasma including the opposite polarity dust grains. For this purpose, we will employ the small-k expansion perturbation method [30] in dusty plasmas. However, there are few instability studies concerned with dusty plasma systems. For example, Mamun [31] discussed the instability of three-dimensional DAWs propagating obliquely to a magnetized dusty plasma. He found that the oblique external magnetic field leads to unstable DAW structures. Later, Mamun *et al.* [32] studied the properties of finite-amplitude DAWs and their instabilities in a three-component magnetized dusty plasma system. They illustrated that the inclusion of nonthermal ions and dust temperature effects modifies the nature of the produced DAWs in a dusty plasma, although they demonstrated that the variation of the nonthermal parameter has no effect on the DAW stability criterion. Moreover, El-Taibany *et al.* [33] used the small-k expansion perturbation method to study the three-dimensional stability of dust ion acoustic waves in a magnetized multicomponent dusty plasma containing negative heavy ions and stationary variable-charge dust particles. They proved that a higher growth rate corresponds to a higher wave amplitude and the unstable solitary waves were produced where negative ions are present. Recently, Akhter *et al.* [34] studied three-dimensional DAWs and their instabilities by employing the same perturbation technique in a magnetized dusty plasma. They found that the basic features of the DAW and its instability criterion or its growth rate are significantly modified by the presence of opposite polarity dust particles and external magnetic field.

However, actual observations show that the dust grain size ranges from nanometers to millimeters unless they are man-made [1]. Thus the radii r of the dust grains are not the same, but they vary within the range $[r_1,r_2]$, where r_1 (r_2) is the lower (upper) limit. The most widely applicable dust size distribution (DSD) is the power law distribution because of its various applications in space plasmas [35–38]. It is remarked that the DSD may be discrete or continuous; however, the continuous model is the most reasonable [36,38]. The differential form of the power law DSD can be written as $n_{dj}(r)dr = Kr^{-\beta}dr$, where $n_{dj}(r)dr$ is the number density of the dust grains per unit volume with radii in the range from r to $r+dr$, β is the power law index, and K is the normalization constant. The total number density of all grains is given by $N_{\text{tot}} = \int_{r_1}^{r_2} n_{dj}(r)dr$. The mass and the charge of the jth dust grain can be approximated as [35] $m_{dj} = K_m r_j^3, Z_{dj} = K_z r_j$, where K_m ($\simeq \frac{4}{3}\pi\rho_d$) is the mass constant with the assumption that the mass density of the dust grains ρ_d is the same for all grains, K_z ($\simeq 4\pi\epsilon_o \frac{V_o}{e}$) is the charge constant when we take the electric surface potential of the dust grains V_o to be constant at equilibrium, and ϵ_o is the permittivity of free space [36–38]. To the best of our knowledge, no study has focused on the instability of rotating nonlinear DAWs including DSD in a four-component dusty plasma with opposite polarity dusts. Accordingly, the second purpose of this paper is to study the effect of power law DSDs on the instability criterion of three-dimensional DAWs in the proposed model.

This paper is organized as follows: In Sec. II, we apply a RPT [13] in order to derive a Zakharov-Kuznetsov (ZK) equation appropriate for describing nonlinear DAW propagation. In Sec. III, which includes the effect of the power law DSD, the basic characteristics of DAW propagation are investigated. In Sec. IV, we apply the small-k expansion method to derive the growth rate DAW instability. Section V is devoted to the conclusion.

II. GOVERNING EQUATIONS

We consider the DAW propagation in a fully ionized magnetized dusty plasma system composed of positively and negatively charged dust particles and Boltzmann ions and electrons in the presence of an external static magnetic field $B_o\hat{\mathbf{z}}$, where B_o is the strength of the applied magnetic field. Thus, at equilibrium, we have

$$n_{io} + \sum_{j=1}^{N_p} Z_{pj} n_{pjo} = n_{eo} + \sum_{j=1}^{N_n} Z_{nj} n_{njo}\,, \qquad (1)$$

where n_{io},n_{pjo} ,n_{eo}, and n_{njo} are, respectively, ion, positive dust, electron, and negative dust number densities at equilibrium. Z_{pj} (Z_{nj}) represents the charge state of positive (negative) dust particles. Assume that the positive (negative) dust grain has N_p (N_n) different radii within the range $[r_{s1},r_{s2}]$, where r_{s1} is the lower limit and r_{s2} is the upper limit, with $s = p(n)$ for positive (negative) dust. The dynamics of DAWs [1,34,39] in such a magnetized dusty plasma system are governed by

$$\frac{\partial n_{sj}}{\partial t} + \nabla \cdot (n_{sj}\mathbf{u}_{sj}) = 0, \qquad (2)$$

$$\frac{\partial \mathbf{u}_{nj}}{\partial t} + (\mathbf{u}_{nj}\cdot\nabla)\mathbf{u}_{nj} = \mu_{nj}\nabla\phi - \mu_{nj}\omega_{cn}(\mathbf{u}_{nj}\times\hat{\mathbf{z}}), \qquad (3)$$

$$\frac{\partial \mathbf{u}_{pj}}{\partial t} + (\mathbf{u}_{pj}\cdot\nabla)\mathbf{u}_{pj} = -\mu_{pj}\nu\nabla\phi + \mu_{pj}\omega_{cp}(\mathbf{u}_{pj}\times\hat{\mathbf{z}}), \tag{4}$$

$$\nabla^2\phi = \mu_e e^{\sigma\phi} - \mu_i e^{-\phi} + \sum_{j=1}^{N_n} Z_{nj}n_{nj} - \alpha\sum_{j=1}^{N_p} Z_{pj}n_{pj}. \tag{5}$$

It is noted that since ions and electrons are lighter than dust grains and have larger velocities than dust species, it is reasonable to consider them in thermal equilibrium obeying a Boltzmann distribution.

The following normalizations are used:

$$\begin{aligned} &n_e \to n_e/n_{eo},\quad n_i \to n_i/n_{io},\quad n_{nj}\to n_{nj}/n_{no},\\ &n_{pj}\to n_{pj}/n_{po},\quad Z_{nj}\to Z_{nj}/Z_{no},\quad Z_{pj}\to Z_{pj}/Z_{po},\\ &\phi\to e\phi/T_i,\quad u_{nj}\to u_{nj}/c_d,\quad u_{pj}\to u_{pj}/c_d,\\ &t\to t\omega_p,\quad \nabla\to\lambda_D\nabla, \end{aligned}$$

where

$$\begin{aligned} c_d &= \sqrt{\frac{Z_{no}T_i}{m_{no}}},\quad \omega_p = \sqrt{\frac{4\pi e^2 Z_{no}^2 n_{no}}{m_{no}}},\\ \lambda_D &= \sqrt{\frac{T_i}{4\pi e^2 Z_{no}n_{no}}},\quad Z_{no} = \frac{1}{n_{no}}\sum_{j=1}^{N_n} Z_{nj}n_{nj},\\ Z_{po} &= \frac{1}{n_{po}}\sum_{j=1}^{N_p} Z_{pj}n_{pj},\quad n_{no} = \sum_{j=1}^{N_n} n_{nj},\\ n_{po} &= \sum_{j=1}^{N_p} n_{pj},\quad \omega_{cn} = \frac{Z_{no}eB_o}{m_{no}\omega_p},\quad \omega_{cp} = \frac{Z_{po}eB_o}{m_{po}\omega_p}, \end{aligned}$$

with

$$\begin{aligned} \sigma &= \frac{T_i}{T_e},\quad \alpha = \frac{Z_{po}n_{po}}{Z_{no}n_{no}},\quad \mu_{nj} = \frac{Z_{nj}}{m_{nj}},\quad \mu_{pj} = \frac{Z_{pj}}{m_{pj}}\\ \mu_e &= \frac{n_{eo}}{Z_{no}n_{no}},\quad \mu_i = \frac{n_{io}}{Z_{no}n_{no}},\quad \nu = \frac{Z_{po}m_{no}}{Z_{no}m_{po}}. \end{aligned}$$

The dust dynamic time τ_d is experimentally estimated to be about 10^{-2} s [1,10], and the dust charging time τ_{ch} for positive dust grains including four charging currents, photoemission, secondary electrons, and the electron and ion currents, is 2×10^{-5} s [14]. In addition, τ_{ch} for negative dust grains is about 10^{-8}–10^{-6} s, depending on the included charging currents [40]. Therefore, it is clear that $\tau_{ch} \ll \tau_d$ in both cases, which means the dust grain reaches an equilibrium dust charge number very quickly, and this enable us to consider constant dust charge in the present model. In other words, the dust charge is always close to the equilibrium value. So we consider a constant dust charge [10].

It is remarked that the applied external magnetic field is static, so $\nabla\times\mathbf{B} = \nabla\times(B_o\hat{\mathbf{z}}) = \mathbf{0}$. In addition, Allen [41] concluded that in the presence of the charge carries of both signs, the Hall effect is reduced. Therefore, the conduction and the induced currents could be ignored here.

Now, using a RPT [13], we introduce the following stretched coordinates:

$$X = \epsilon^{1/2}x,\quad Y = \epsilon^{1/2}y,\quad Z = \epsilon^{1/2}(z-\lambda t),\quad T = \epsilon^{3/2}t, \tag{6}$$

where ϵ is a small parameter measuring the amplitude of the wave perturbation. λ is the normalized velocity of the moving frame to be determined later self-consistently. The plasma parameters n_{sj}, $\mathbf{u}_{sj}$, and ϕ can be expanded as a power series in ϵ as [36]

$$\begin{aligned} n_{sj} &= n_{sjo} + \epsilon n_{sj1} + \epsilon^2 n_{sj2} + \cdots,\\ u_{sxj} &= \epsilon^{3/2}u_{sxj1} + \epsilon^2 u_{sxj2} + \cdots,\\ u_{syj} &= \epsilon^{3/2}u_{syj1} + \epsilon^2 u_{syj2} + \cdots,\\ u_{szj} &= \epsilon u_{szj1} + \epsilon^2 u_{szj2} + \cdots,\\ \phi &= \epsilon\phi_1 + \epsilon^2\phi_2 + \cdots. \end{aligned} \tag{7}$$

Substituting Eqs. (6) and (7) into the set of Eqs. (2)–(5) and collecting terms of the same powers of ϵ, for the lowest orders, we get

$$\begin{aligned} n_{nj1} &= \frac{-\mu_{nj}n_{njo}}{\lambda^2}\phi_1,\quad n_{pj1} = \frac{\nu\mu_{pj}n_{pjo}}{\lambda^2}\phi_1,\\ u_{nxj1} &= \frac{-1}{\omega_{cn}}\frac{\partial\phi_1}{\partial Y},\quad u_{nyj1} = \frac{1}{\omega_{cn}}\frac{\partial\phi_1}{\partial X},\quad u_{nzj1} = \frac{-\mu_{nj}}{\lambda}\phi_1,\\ u_{pxj1} &= \frac{-\nu}{\omega_{cp}}\frac{\partial\phi_1}{\partial Y},\quad u_{pyj1} = \frac{\nu}{\omega_{cn}}\frac{\partial\phi_1}{\partial X},\quad u_{pzj1} = \frac{\nu\mu_{pj}}{\lambda}\phi_1. \end{aligned} \tag{8}$$

The linear dispersion relation is calculated as

$$\lambda^2 = \frac{\sum_{j=1}^{N_n}\frac{n_{njo}Z_{nj}^2}{m_{nj}} + \alpha\nu\sum_{j=1}^{N_p}\frac{n_{pjo}Z_{pj}^2}{m_{pj}}}{\mu_i+\sigma\mu_e}. \tag{9a}$$

Now, applying the power law DSD, we obtain

$$\lambda = \left[\frac{\left(K_nK_{zn}^2r_{n1}^{-\beta} - \alpha\nu K_pK_{zp}^2r_{p1}^{-\beta}\right)(1-R^{-\beta})}{\beta K_m(\mu_i+\sigma\mu_e)}\right]^{1/2}, \tag{9b}$$

where K_{zn} (K_{zp}), K_n (K_p), and r_{n1} (r_{p1}) are the charge constant, the normalization constant, and the minimum radius of negative (positive) dust, respectively. R represents the ratio of the maximum to the minimum dust radius. The monosized dust particles case is obtained by considering that all dust particles have the same radius $\overline{r_{n,p}} = [(1-\beta)r_{n1,p1}(R^{2-\beta}-1)]/[(2-\beta)(R^{1-\beta}-1)]$ for negative (n) and positive (p) dust particles. Figure 1 shows the dependence of the velocity ratio Λ (the ratio of the velocity corresponding to the case of including DSD to that corresponding to the monosized case with the average radius $\overline{r_{n,p}}$) on the dust radius ratio R and the power law index β. It is clear that the velocity is larger than that of the monosized dust case. This behavior coincides with what has been observed for a three-component dusty plasma with only negative dust species and including DSD [36]. Also, the velocity ratio Λ increases as R increases but decreases as β increases.

Collecting terms of powers $\epsilon^{3/2}$ from Eqs. (2)–(5), the x and y components of the second-order perturbed perpendicular

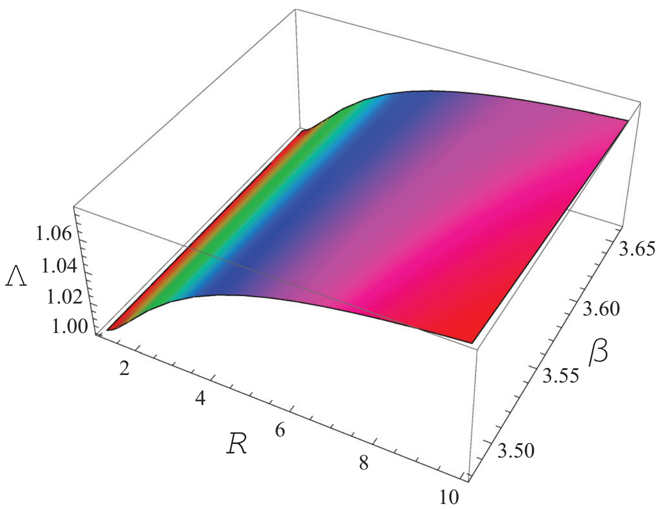

FIG. 1. (Color online) The dependence of the velocity ratio Λ on the dust radius ratio R and the power law index β for $l_\zeta = 0.3$, $l_\eta = 0.7$, $u_o = 1.1$, $\delta = 11$, $\omega_{cn} = 0.05$, $r_{n1} = 10^{-5}$, $r_{p1} = 5 \times 10^{-4}$, and $\sigma = 0.1$.

velocities are given by

$$u_{nxj2} = \frac{-\lambda}{\mu_{nj}\omega_{cn}^2}\frac{\partial^2\phi_1}{\partial Z\partial X}, \quad u_{nyj2} = \frac{-\lambda}{\mu_{nj}\omega_{cn}^2}\frac{\partial^2\phi_1}{\partial Z\partial Y},$$
$$u_{pxj2} = \frac{\lambda\nu}{\mu_{pj}\omega_{cp}^2}\frac{\partial^2\phi_1}{\partial Z\partial X}, \quad u_{pyj2} = \frac{\lambda\nu}{\mu_{pj}\omega_{cp}^2}\frac{\partial^2\phi_1}{\partial Z\partial Y}. \tag{10}$$

For the $O(\epsilon^2)$ perturbed quantities, Eqs. (2)–(5) lead to the following system of equations:

$$-\lambda\frac{\partial n_{nj2}}{\partial Z} + n_{njo}\left(\frac{\partial u_{nxj2}}{\partial X} + \frac{\partial u_{nyj2}}{\partial Y} + \frac{\partial u_{nzj2}}{\partial Z}\right) + \frac{\partial n_{nj1}}{\partial T} + n_{nj1}\frac{\partial n_{nj1}}{\partial Z} = 0,$$
$$-\lambda\frac{\partial n_{pj2}}{\partial Z} + n_{pjo}\left(\frac{\partial u_{pxj2}}{\partial X} + \frac{\partial u_{pyj2}}{\partial Y} + \frac{\partial u_{pzj2}}{\partial Z}\right) + \frac{\partial n_{pj1}}{\partial T} + n_{pj1}\frac{\partial n_{pj1}}{\partial Z} = 0,$$
$$-\lambda\frac{\partial u_{nzj2}}{\partial Z} - \mu_{nj}\frac{\partial\phi_2}{\partial Z} + \frac{\partial u_{nzj1}}{\partial T} + u_{nzj1}\frac{\partial u_{nzj1}}{\partial Z} = 0,$$
$$-\lambda\frac{\partial u_{pzj2}}{\partial Z} + \nu\mu_{pj}\frac{\partial\phi_2}{\partial Z} + \frac{\partial u_{pzj1}}{\partial T} + u_{pzj1}\frac{\partial u_{pzj1}}{\partial Z} = 0,$$
$$(\mu_i + \sigma\mu_e)\phi_2 + \sum_{j=1}^{N_n} Z_{nj}n_{nj2} - \alpha\sum_{j=1}^{N_p} Z_{pj}n_{pj2} - \nabla^2\phi_1 - \frac{1}{2}(\mu_i - \sigma^2\mu_e)\phi_1^2 = 0. \tag{11}$$

Solving Eqs. (11) with the aid of Eqs. (8) and (10) and eliminating the second-order perturbed quantities, the evolution equation appropriate for describing the nonlinear DAWs in a magnetized dusty plasma with two opposite polarity dusts is the following ZK equation:

$$\frac{\partial\phi_1}{\partial T} + A\phi_1\frac{\partial\phi_1}{\partial Z} + B\frac{\partial^3\phi_1}{\partial Z^3} + C\left(\frac{\partial^3\phi_1}{\partial Z\partial X^2} + \frac{\partial^3\phi_1}{\partial Z\partial Y^2}\right) = 0, \tag{12}$$

where

$$A = \frac{-\mu_{nj}^2 - \mu_{pj}^2(\nu - 1)}{2\lambda(\mu_{nj} - \mu_{pj})} + \frac{\lambda(\mu_i - \sigma^2\mu_e)}{2(\mu_i + \sigma\mu_e)}, \tag{13}$$

$$B = \frac{\lambda}{2(\mu_i + \sigma\mu_e)}, \tag{14}$$

$$C = B + \frac{\lambda^3}{2(\mu_{nj} - \mu_{pj})}\left(\frac{1}{\mu_{nj}\omega_{cn}^2} - \frac{1}{\mu_{pj}\omega_{cp}^2}\right). \tag{15}$$

III. SOLITARY WAVE ANALYSIS

To study the properties of DAWs propagating in a direction making an angle δ with the Z axis, i.e., with the external static magnetic field lying in the $(Z - X)$ plane, we first rotate the coordinate axes (X, Z) by an angle δ and make use of the following transformation of the independent variables [30]:

$$\zeta = X\cos\delta - Z\sin\delta, \quad \xi = X\sin\delta + Z\cos\delta, \quad \eta = Y, \quad \tau = T. \tag{16}$$

Applying these transformations to the ZK, Eq. (12), yields

$$\frac{\partial\phi_1}{\partial\tau} + \delta_1\phi_1\frac{\partial\phi_1}{\partial\xi} + \delta_2\frac{\partial^3\phi_1}{\partial\xi^3} + \delta_3\phi_1\frac{\partial\phi_1}{\partial\zeta} + \delta_4\frac{\partial^3\phi_1}{\partial\zeta^3} + \delta_5\frac{\partial^3\phi_1}{\partial\xi^2\partial\zeta} + \delta_6\frac{\partial^3\phi_1}{\partial\xi\partial\zeta^2} + \delta_7\frac{\partial^3\phi_1}{\partial\xi\partial\eta^2} + \delta_8\frac{\partial^3\phi_1}{\partial\zeta\partial\eta^2} = 0, \tag{17}$$

where

$$\delta_1 = A\cos\delta, \quad \delta_2 = B\cos^3\delta + C\sin^2\delta\cos\delta,$$
$$\delta_3 = -A\sin\delta, \quad \delta_4 = -B\sin^3\delta - C\cos^2\delta\sin\delta,$$
$$\delta_5 = 2C\left(\sin\delta\cos^2\delta - \tfrac{1}{2}\sin^3\delta\right) - 3B\cos^2\delta\sin\delta,$$
$$\delta_6 = -2C\left(\sin^2\delta\cos\delta - \tfrac{1}{2}\cos^3\delta\right) + 3B\sin^2\delta\cos\delta,$$
$$\delta_7 = C\cos\delta, \quad \delta_8 = -C\sin\delta. \tag{18}$$

Now, we look for a steady state solution of the ZK equation in the form

$$\phi_1 = \phi_0(\rho), \tag{19}$$

where $\rho = \xi - M\tau$ and M is the Mach number normalized by the dust acoustic speed c_d. So the ZK equation in the steady state form leads to

$$-M\frac{\partial\phi_0}{\partial\rho} + \delta_1\phi_0\frac{\partial\phi_0}{\partial\rho} + \delta_2\frac{\partial^3\phi_0}{\partial\rho^3} = 0. \tag{20}$$

Now, using the appropriate boundary conditions, namely, ϕ_0 and its derivatives vanish as ρ goes to infinity, Eq. (20) has the following solution:

$$\phi_0(\rho) = \phi_m\,\mathrm{sech}^2\left(\frac{\rho}{W}\right), \tag{21}$$

where ϕ_m and W are the amplitude and the width of the solitary wave, respectively, which are given by

$$\phi_m = 3M/\delta_1, \tag{22}$$

$$W = 2\sqrt{\delta_2/M}. \tag{23}$$

The solitary wave solution exists only if it has a real width W, i.e., when $\delta_2 > 0$. Also, the solitary wave is compressive (rarefactive) if $\delta_1 > 0$ $(\delta_1 < 0)$. It is clear from Eqs. (9), (13)–(15), (22), and (23) that the amplitude and the width of

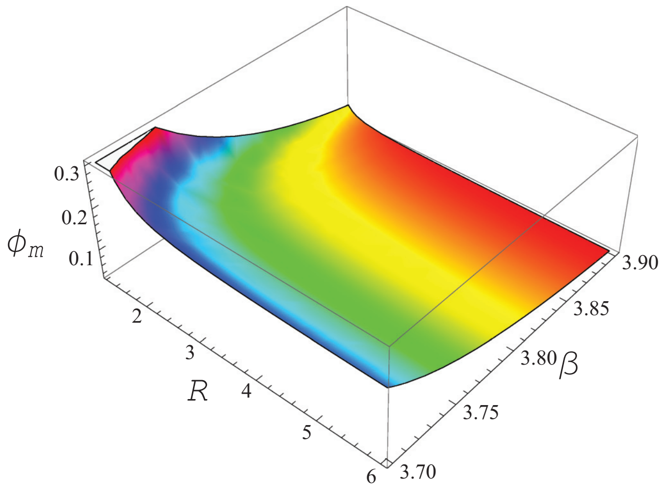

FIG. 2. (Color online) The dependence of the amplitude ϕ_m of the compressive solitary wave on the dust radius ratio R and the power law index β for $l_\zeta = 0.3, l_\eta = 0.7$, $u_o = 1.1$, $\delta = 11$, and $\omega_{cn} = 0.05$.

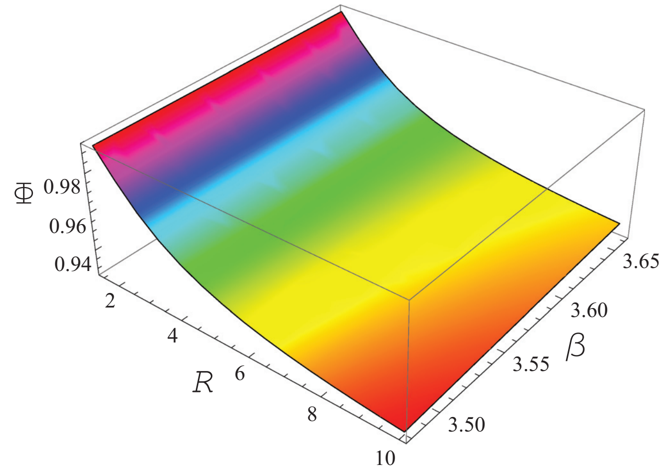

FIG. 4. (Color online) The dependence of the amplitude ratio Φ on the dust radius ratio R and the power law index β for $l_\zeta = 0.3$, $l_\eta = 0.7$, $u_o = 1.1, \delta = 11$, and $\omega_{cn} = 0.05$.

the solitary wave depend strongly on the power law DSD. Figures 2 and 3 show the effect of both R and β on the amplitude ϕ_m of the compressive and the rarefactive solitary waves, respectively. The dependence of the amplitude ratio Φ (the ratio of the amplitude including the DSD effect to that corresponding to the monosized case with the average radius $\overline{r_{n,p}}$) on the dust radius ratio R and the power law index β is shown in Fig. 4. It can be easily seen that the amplitude is smaller than that of the monosized dust case. Also, the amplitude ratio Φ decreases as R increases but slightly increases as β increases. On the other hand, Figs. 5 and 6 show the effect of both R and β on the width W of the compressive and rarefactive solitary waves, respectively. They indicate that W increases as both R and β increase. Moreover, Fig. 7 indicates that the width ratio Δ (the ratio of the width including the DSD to that corresponding to the monosized case) increases as both R and β increase. Moreover, the width is smaller than that of the monosized dust case. It is noted here that the two types of soliton solutions, compressive and rarefactive, are possible in the present model. However, only one soliton type is possible for the three-component model: the rarefactive (compressive) soliton corresponding to negative (positive) dust grains. In addition, the decrement of the soliton amplitude due to including the DSD agrees with that observed in a three-component dusty plasma [36]. Contrary to what has been observed in a three-component dusty plasma with DSD [36], the soliton width here decreases by incorporating the DSD effect.

IV. INSTABILITY ANALYSIS

Now, let us apply the small-k expansion perturbation method [30] to study the stability of obliquely propagating DAWs. We assume that

$$\phi_1 = \phi_0(\rho) + \psi(\rho,\zeta,\eta,\tau), \tag{24}$$

where ϕ_0 is defined by Eq. (21) and ψ, for a long wavelength plane wave perturbation in a direction with cosines (l_ζ,l_η,l_ξ),

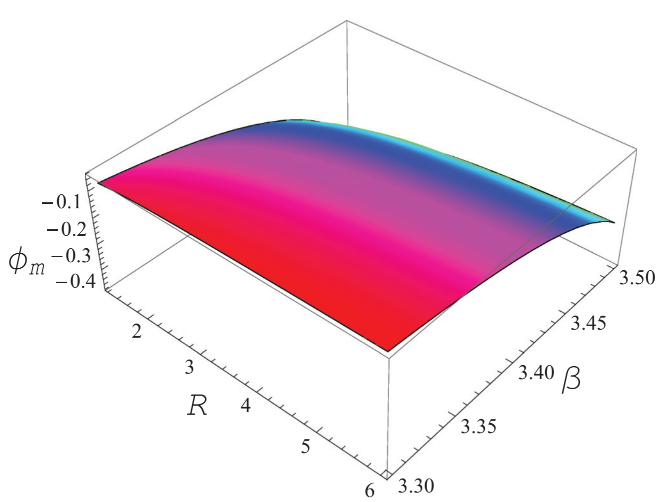

FIG. 3. (Color online) The dependence of the amplitude ϕ_m of the rarefactive solitary wave on the dust radius ratio R and the power law index β for $l_\zeta = 0.3, l_\eta = 0.7$, $u_o = 1.1$, $\delta = 11$, and $\omega_{cn} = 0.05$.

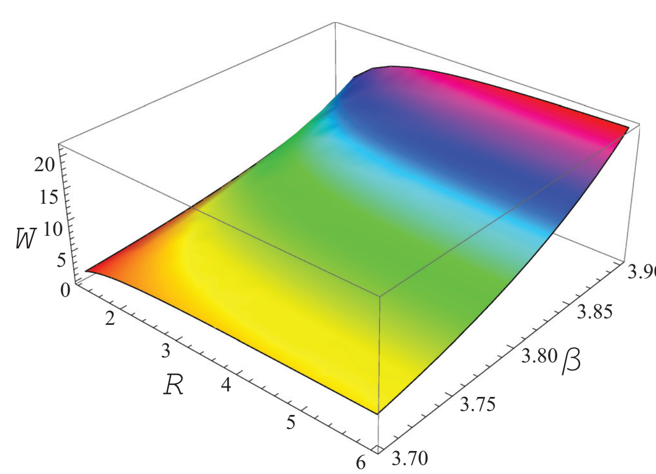

FIG. 5. (Color online) The dependence of the width W of the compressive solitary wave on the dust radius ratio R and the power law index β for $l_\zeta = 0.3$, $l_\eta = 0.7, u_o = 1.1$, $\delta = 11$, and $\omega_{cn} = 0.05$.

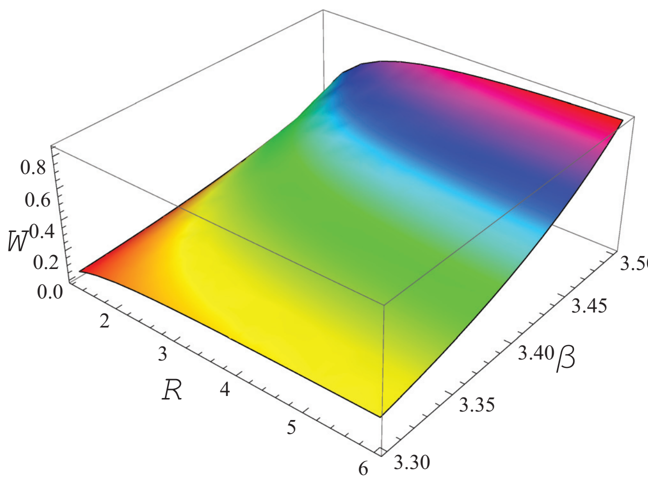

FIG. 6. (Color online) The dependence of the width W of the rarefactive solitary wave on the dust radius ratio R and the power law index β for $l_\zeta = 0.3$, $l_\eta = 0.7, u_o = 1.1$, $\delta = 11$, and $\omega_{cn} = 0.05$.

is given by

$$\psi(\rho,\zeta,\eta,\tau) = \varphi(\rho)e^{i[k(l_\zeta\zeta + l_\eta\eta + l_\xi\rho) - \omega\tau]}, \tag{25}$$

with $l_\zeta^2 + l_\eta^2 + l_\xi^2 = 1$. For small k, $\varphi(\rho)$ and ω can be expanded as

$$\varphi(\rho) = \varphi_0 + k\varphi_1 + k^2\varphi_2 + \cdots, \tag{26}$$

$$\omega = k\omega_1 + k^2\omega_2 + \cdots. \tag{27}$$

Substituting Eq. (24) into Eq. (17) and linearizing with respect to ψ, the linearized ZK equation becomes

$$\frac{\partial\psi}{\partial\tau} - M\frac{\partial\psi}{\partial\rho} + \delta_1\phi_0\frac{\partial\psi}{\partial\rho} + \delta_1\psi\frac{\partial\phi_0}{\partial\rho} + \delta_2\frac{\partial^3\psi}{\partial\rho^3} + \delta_3\phi_0\frac{\partial\psi}{\partial\zeta} + \delta_4\frac{\partial^3\psi}{\partial\zeta^3} + \delta_5\frac{\partial^3\psi}{\partial\rho^2\partial\zeta} + \delta_6\frac{\partial^3\psi}{\partial\rho\partial\zeta^2} + \delta_7\frac{\partial^3\psi}{\partial\rho\partial\eta^2} + \delta_8\frac{\partial^3\psi}{\partial\zeta\partial\eta^2} = 0. \tag{28}$$

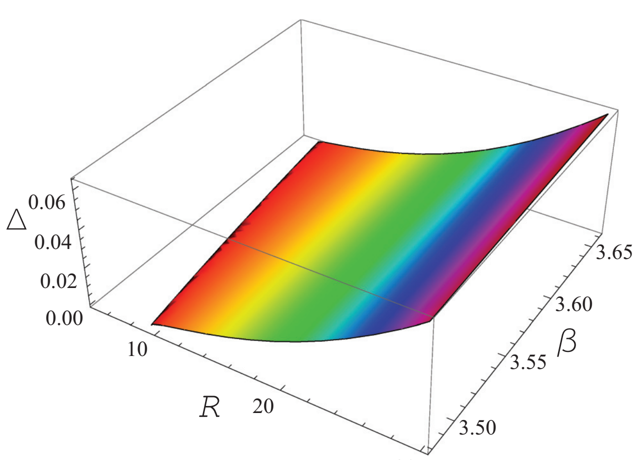

FIG. 7. (Color online) The dependence of the width ratio Δ on the dust radius ratio R and the power law index β for $l_\zeta = 0.3$, $l_\eta = 0.7$, $\delta = 11$, and $\omega_{cn} = 0.05$.

Substituting Eqs. (25)–(27) into Eq. (28) and equating the coefficients of the same powers of k, at zeroth order, we get

$$(-M + \delta_1\phi_0)\varphi_0 + \delta_2\frac{d^2\varphi_0}{d\rho^2} = \widetilde{C}, \tag{29}$$

where $\widetilde{C}$ is the integration constant. It is clear from Eq. (20) that the homogeneous part of this equation has two linearly independent solutions, namely [31],

$$f = \frac{d\phi_0}{d\rho}, \quad g = f\int^\rho \frac{d\rho}{f^2}. \tag{30}$$

Therefore, the general solution of this zeroth order, Eq. (29), can be written as

$$\varphi_0 = C_1 f + C_2 g - \widetilde{C} f\int^\rho \frac{g}{\delta_2}d\rho + \widetilde{C} g\int^\rho \frac{f}{\delta_2}d\rho, \tag{31}$$

where C_1 and C_2 are the integration constants. Now, evaluating all integrals, the general solution of the zeroth-order equation, for φ_0 not tending to $\pm\infty$ as $\rho \to \pm\infty$, can be finally simplified to

$$\varphi_0 = C_1 f. \tag{32}$$

The first-order equation, obtained from Eqs. (26), (27), and (32), can be expressed, after integration, by

$$(-M + \delta_1\phi_0)\varphi_1 + \delta_2\frac{d^2\varphi_1}{d\rho^2} = iC_1\left[\alpha_1 + \beta_1\tanh^2\left(\frac{\rho}{W}\right)\right]\phi_0 + C_3, \tag{33}$$

where C_3 is another integration constant and α_1 and β_1 are given by

$$\alpha_1 = \omega_1 + Ml_\xi - \frac{1}{2}\phi_m\mu_1 + \frac{2}{W^2}\mu_2, \tag{34}$$

$$\beta_1 = \frac{1}{2}\phi_m\mu_1 - \frac{6}{W^2}\mu_2, \tag{35}$$

where

$$\mu_1 = (\delta_1 l_\xi + \delta_3 l_\zeta), \quad \mu_2 = (3\delta_2 l_\xi + \delta_5 l_\zeta). \tag{36}$$

Similarly, the general solution of the first-order equation, for φ_1 not tending to $\pm\infty$ as $\rho \to \pm\infty$, is given by

$$\varphi_1 = K_1 f + \frac{iC_1W^2}{8\delta_2}\left[(\alpha_1 + \beta_1)\rho f + 2\left(\alpha_1 + \frac{1}{3}\beta_1\right)\phi_0\right], \tag{37}$$

where K_1 is an arbitrary constant. The second-order equation, obtained from Eq. (28), is given as

$$\left(-M\frac{d}{d\rho} + \delta_1\frac{d}{d\rho}\phi_0 + \delta_2\frac{d^3}{d\rho^3}\right)\varphi_2 = Q, \tag{38}$$

where

$$Q = i\omega_2\varphi_0 + i(\omega_1 + Ml_\xi - \mu_1\phi_0)\varphi_1 + \mu_3\frac{d\varphi_0}{d\rho} - i\mu_2\frac{d^2\varphi_1}{d\rho^2}, \tag{39}$$

$$\mu_3 = 3\delta_2 l_\xi^2 + 2\delta_5 l_\zeta l_\xi + \delta_6 l_\zeta^2 + \delta_7 l_\eta^2. \tag{40}$$

The existence of the solution of Eq. (38) requires that Q must be orthogonal to the kernel of the adjoint operator to operator L, which is given by

$$L = -M\frac{d}{d\rho} + \delta_1 \frac{d}{d\rho}\phi_0 + \delta_2 \frac{d^3}{d\rho^3}. \tag{41}$$

Thus, we obtain the following consistency condition:

$$\int_{-\infty}^{\infty} \phi_0 Q d\rho = 0. \tag{42}$$

Substituting φ_0 and φ_1 from Eqs. (32) and (37), respectively, into Eq. (42), we obtain the following dispersion relation:

$$\omega_1 = \Omega - M l_\xi + \sqrt{\Omega^2 - \Lambda}, \tag{43}$$

where

$$\Omega = \tfrac{2}{3}(\mu_1 \phi_m - 2\mu_2/W^2), \tag{44}$$

$$\Lambda = \frac{16}{45}\left(\mu_1^2\phi_m^2 - \frac{3\mu_1\mu_2\phi_m}{W^2} - \frac{3\mu_2^2}{W^4} + \frac{12\delta_2\mu_3}{W^4}\right). \tag{45}$$

Hence, from Eq. (43), we obtain an instability condition in the form

$$\Lambda - \Omega^2 > 0. \tag{46}$$

Thus, using Eqs. (18), (22), (23), (36), (40), (44), and (45), we obtain the following instability criterion:

$$S_i > 0, \tag{47}$$

where

$$S_i = l_\zeta^2\left(F_j - \frac{5}{3}\sec^2\delta\right) + l_\eta^2(F_j \cot^2\delta + 1), \tag{48}$$

$$F_j = \frac{\omega_{cn}^2}{\sum_{j=1}^{N_n} n_{njo} m_{nj} + \omega_{cn}^2} + \frac{\omega_{cp}^2}{\alpha\nu \sum_{j=1}^{N_p} n_{pjo} m_{pj} + \omega_{cp}^2}. \tag{49}$$

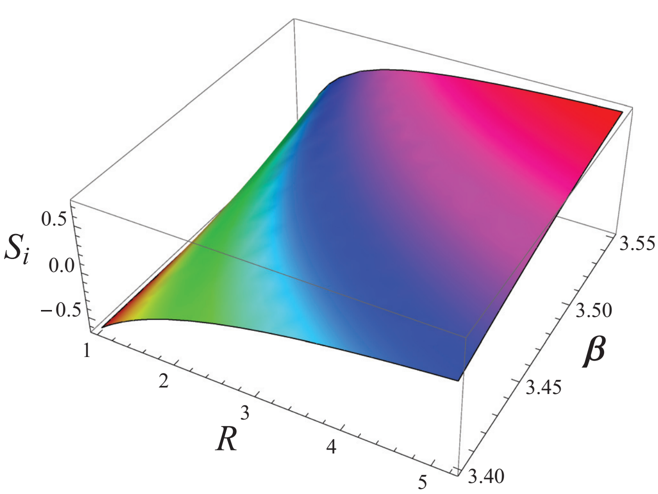

FIG. 8. (Color online) The variation of S_i with the dust radius ratio R and the power law index β for $l_\zeta = 0.3$, $l_\eta = 0.7$, $\delta = 11$, and $\omega_{cn} = 0.05$.

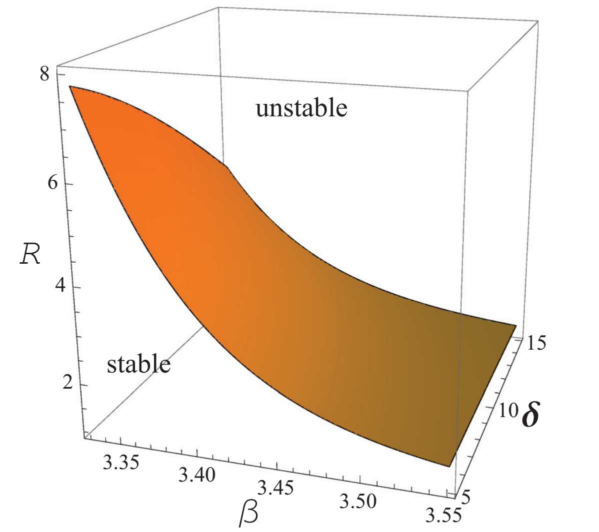

FIG. 9. (Color online) The $S_i = 0$ surface plot via the variation of R, β, and δ for $l_\zeta = 0.3$, $l_\eta = 0.7$, and $\omega_{cn} = 0.05$.

It is clear from the expression for S_i that the instability criterion, (47), depends strongly on DSD. For continuous power law DSD, we can express F_j as

$$F_j = \frac{(4-\beta)\omega_{cn}^2}{K_n K_m r_{n1}^{4-\beta}(R^{4-\beta}-1) + (4-\beta)\omega_{cn}^2} + \frac{(4-\beta)\omega_{cp}^2}{\alpha\nu K_p K_m r_{p1}^{4-\beta}(R^{4-\beta}-1) + (4-\beta)\omega_{cp}^2}. \tag{50}$$

A graphical representation of S_i as a function of both the dust radius ratio R and the power law index β is shown in Fig. 8. It can easily be seen that S_i increases as both R and β increase. The fact that S_i has positive (negative) values refers to unstable

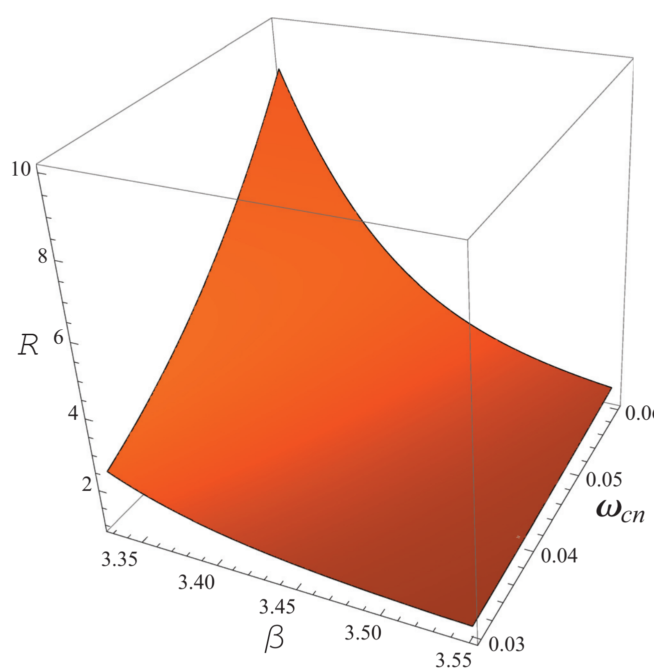

FIG. 10. (Color online) The $S_i = 0$ surface plot via the variation of R, β, and ω_{cn} for $l_\zeta = 0.3$, $l_\eta = 0.7$, and $\delta = 11$.

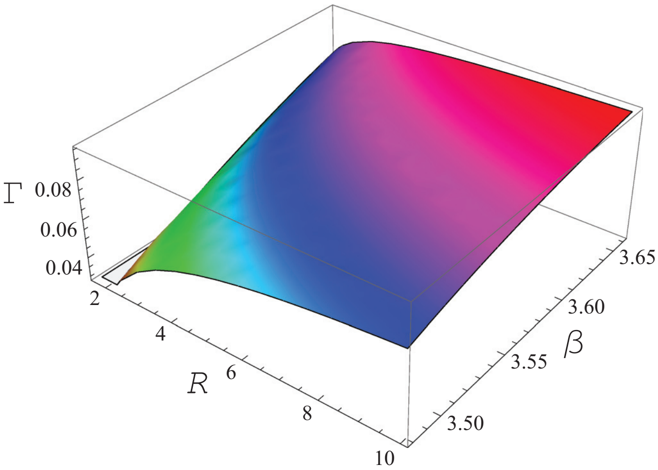

FIG. 11. (Color online) The dependence of the growth rate of the instability of DAWs Γ on the dust radius ratio R and the power law index β for $l_\zeta = 0.3, l_\eta = 0.7$, $\delta = 11$, and $\omega_{cn} = 0.05$.

(stable) perturbation of DAWs. However, it is important to study the effect of the system parameters on the surface of $S_i = 0$. Figures 9 and 10 indicate the parametric regimes of R,β,δ, and ω_{cn} above (below) the surface $S_i = 0$ correspond to unstable (stable) perturbation of DAWs.

As the instability criterion $S_i > 0$ is satisfied, the corresponding growth rate $\Gamma = \sqrt{\Lambda - \Omega^2}$ of the instability is given by

$$\Gamma = \frac{2u_o}{\sqrt{15}} \frac{\csc\delta\, S_i^{1/2}}{(F_j \cot^2\delta + 1)}. \tag{51}$$

The dependence of the growth rate Γ on R,β,l_ζ,l_η, and ω_{cn} is illustrated in Figs. 11 and 12. Figure 11 indicates that Γ increases as both R and β increase. Moreover, Γ changes drastically against R variation; that is, Γ increases rapidly for small values of R, then it becomes nearly constant for larger values of R. Also, it is clear that Γ is very sensitive to small changes in β. Figure 12 shows that Γ increases as both l_ζ and l_η increase. Moreover, Γ has a linear dependence on changes of both l_ζ and l_η. Figure 13 indicates that the larger the cyclotron frequency ω_{cn} is, the higher the growth rate Γ is. In other words, Γ increases as the strength of the external magnetic field B_o increases.

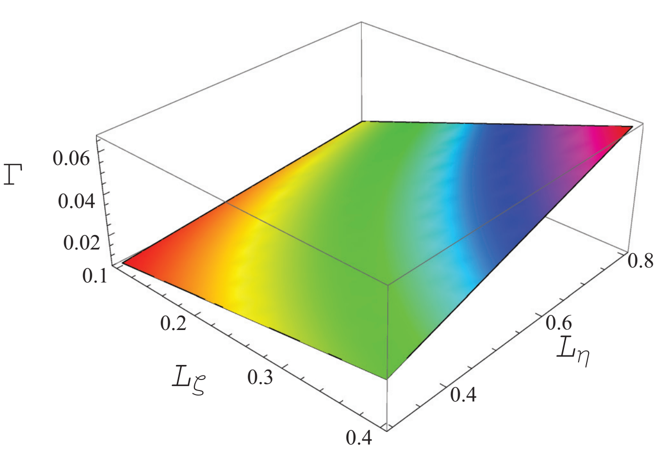

FIG. 12. (Color online) The dependence of the growth rate Γ on l_ζ and l_η for $R = 3$, $\beta = 3.48$, $\delta = 11$, and $\omega_{cn} = 0.05$.

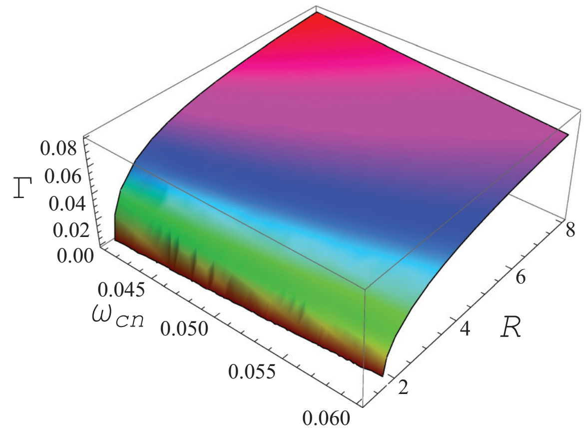

FIG. 13. (Color online) The dependence of the growth rate Γ on R and ω_c for $l_\zeta = 0.3$, $l_\eta = 0.7$, $\delta = 11$, and $\beta = 3.48$.

V. CONCLUSION

The stability of obliquely propagating three-dimensional DAWs is studied with the effect of power law DSD [35–38] in a magnetized dusty plasma including two opposite polarity dusts. The RPT [13] is applied, and a ZK equation describing DAWs is obtained. The solution of the ZK equation proves the existence of both compressive and rarefactive solitons in the present model. Then, the stability of DAW is studied using a small-k expansion perturbation method [30]. The expression for the growth rate of the wave instability is deduced.

It is found that by including the DSD effect in the present model, we get a smaller amplitude and width but with larger velocity soliton solution in comparison with the monosized case. Similar behaviors for the soliton amplitude and velocity are observed in a three-component dusty plasma with the DSD effect. However, the response of the soliton width is reversed here versus the DSD effect. Moreover, using numerical investigations, the soliton (velocity ratio Λ), the amplitude ratio Φ, and the width ratio Δ (decreases) increase as the power law index β increases.

Moreover, we conclude that the growth rate of the DAW instability Γ is strongly affected by the power law DSD. Γ increases as β or the dust radii ratio R increases. However, it is shown that Γ increases linearly as both l_ζ and l_η (cosines of obliqueness angle) increase. In addition, Γ increases as the strength of the external magnetic field B_o increases.

Finally, this work is very useful for the explanation of collective phenomena and the stability criterion of DAWs in magnetized dusty plasmas which are observed in some astrophysics environments, e.g., Jupiter's magnetosphere [3,4], cometary tails [4], and Earth's mesosphere (NLC, PMSE) [5,11,25], where the DSD effect is important. On the other hand, it is reported [10] that if the traveling potential barrier is too high, the particles cannot get across it, and they are reflected by the wave front. The reflected precursor flux upstream of the soliton leads to the formation of a shock wave which we expect may be a possible extension for the present work.

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