Nonstationary magnetosonic wave dynamics in plasmas exhibiting collapse

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In a Lagrangian fluid approach, an explicit method has been presented previously to obtain an exact nonstationary magnetosonic-type wave solution in compressible magnetized plasmas of arbitrary resistivity showing competition among hydrodynamic convection, magnetic field diffusion, and dispersion [Chakrabarti *et al.*, Phys. Rev. Lett. **106**, 145003 (2011)]. The purpose of the present work is twofold: it serves (i) to describe the physical and mathematical background of the involved magnetosonic wave dynamics in more detail, as proposed by our original Letter, and (ii) to present an alternative approach, which utilizes the Lagrangian mass variable as a new spatial coordinate [Schamel, Phys. Rep. **392**, 279 (2004)]. The obtained exact nonlinear wave solutions confirm the correctness of our previous results, indicating a collapse of the magnetic field irrespective of the presence of dispersion and resistivity. The mean plasma density, on the other hand, is less singular, showing collapse only when dispersive effects are negligible. These results may contribute to our understanding of the generation of strongly localized magnetic fields (and currents) in plasmas, and they are expected to be of special importance in the astrophysical context of magnetic star formation.

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I. INTRODUCTION

The dynamics of waves and/or oscillations in various physical systems are very important to the understanding of underlying physical phenomena where the parameters of the medium are random functions of coordinates [1]. Such problems are of particular importance for the physics of strongly nonlinear compressional waves in magnetized plasmas where the magnetic field is strongly inhomogeneous [2,3]. The interaction of the magnetic field with large-scale magnetohydrodynamic (MHD) waves is an important factor contributing to the energy balance and the *dynamics* of the strongly nonlinear waves. This is why we stress here the importance of the study of strongly inhomogeneous media as a necessary step toward the understanding of nonlinear dynamics in magnetized plasmas. However, the extracted physics could also be of interest from the point of view of general physics areas, such as liquid crystals, fluid with vortices, accretion disks, molecular clouds, etc.

Analytical solutions obtained through the introduction of Lagrangian coordinates [4–7], expressing the space-time evolution of waves and/or oscillations in hydrodynamics and other collective systems, such as plasmas in the fluid description, have gotten a great deal of attention recently due to their capability of predicting interesting novel physical phenomena. The development of singular structures at a finite time [4,8] has been one of the interesting nonlinear phenomena in hydrodynamics and plasmas. This is because, at the breaking or collapse event, initial regular patterns corresponding to field variables can be completely destroyed and thus lead to the formation of singular patterns. Typical examples include the formation of drops and the breakup of jets [9–12], the expansion of plasma into vacuum [13–16], etc. Utilizing a Lagrangian fluid description, exact nonlinear space-time-dependent solutions have been obtained in different physical contexts, including complex plasma (e.g., dusty plasma) [17], the formation of nonuniform structures in charged particle beams [18], the development of instability and the wave breaking limit [19], etc.

Studies on the generation of magnetic fields have been an active area of research in different areas of physics over the past years, especially astrophysical environments and laser-plasma interaction. Thus magnetic fields are found to play a central role in every scale of different plasma systems. A number of different physical mechanisms have now been identified as a source of magnetic field generation in plasmas. Among those are the Biermann battery [20], the Weibel instability [21], the inverse Faraday effect [22,23], the current filamentation instability [24], and the ponderomotive force of an intense laser beam [25–28].

More recently, new astrophysical observations have indicated that magnetic fields play a larger role in the birth of stars than previously thought [29,30]. There is an indication that the picture of star formation in which giant clouds of gas and dust collapse inward due to self-gravity is too simple and must be supplemented by processes preceding this gravitational collapse process, such as the magnetosonic-type collapse process described in this paper. The paper is consequently organized as follows: In Sec. II we present the basic equations and solve them in Sec. III in the style of our previous letter, but with a larger emphasis on their physical and mathematical background. Section IV is devoted to a numerical presentation of the results, and Sec. V to an alternative proof by making use of the Lagrangian mass variable. As an application, we offer in Sec. VI a picture of magnetic star formation being supported analytically by our present analysis, and we finish with a summary in Sec. VII.

II. BASIC EQUATIONS

The basic equations which describe the magnetosonic-type compressional dispersive waves in an electron-ion collisional magnetoplasma are the momentum equation for electron and ion fluid,

$$m_e n_e\left(\frac{\partial}{\partial t}+\mathbf{v}_e\cdot\nabla\right)\mathbf{v}_e = -n_e e\left(\mathbf{E}+\frac{1}{c}\mathbf{v_e}\times\mathbf{B}\right)+m_e n_e \nu_{ei}(\mathbf{v}_i-\mathbf{v}_e), \quad (1)$$

$$m_i n_i\left(\frac{\partial}{\partial t}+\mathbf{v}_i\cdot\nabla\right)\mathbf{v}_i = n_i e\left(\mathbf{E}+\frac{1}{c}\mathbf{v}_i\times\mathbf{B}\right)+m_i n_i \nu_{ie}(\mathbf{v}_e-\mathbf{v}_i). \quad (2)$$

The continuity equations for both species is

$$\frac{\partial n_j}{\partial t}+\nabla\cdot(n_j\mathbf{v}_j)=\mathbf{0}, \quad (3)$$

and the following Maxwell equations are

$$\nabla\times\mathbf{E}=-\frac{1}{c}\frac{\partial\mathbf{B}}{\partial t}, \quad (4)$$

$$\nabla\times\mathbf{B}=\frac{4\pi\mathbf{J}}{c}=-\frac{4\pi e}{c}(n_e\mathbf{v}_e-n_i\mathbf{v}_i), \quad (5)$$

where ν_{ei} (ν_{ie}) is the constant (velocity-independent) electron-ion (ion-electron) collision frequency, j denotes e (i) for electrons (ions), and the remaining symbols have their usual meaning. In writing Eqs. (1) and (2), we have assumed magnetic pressure to be large compared to kinetic pressures of plasma species, and thus the plasma is treated as cold. In addition, in Eq. (5) we have neglected the displacement current compared to particle current since we are interested in low-frequency mode, where ω is much smaller than the electron plasma frequency, ω_{pe}. In such a situation, we can assume the quasineutrality condition $n_e\sim n_i\sim n$. Here all the analysis will be done in one spatial variable x. Subsequently, we assume that all the variables are functions of x and t. Furthermore, we will assume the magnetic field to be along the z direction, and the propagation and inhomogeneity are in the x direction.

Using all those conditions stated above from the continuity equations for both species, we find that

$$\frac{\partial}{\partial x}\left[n(v_{ix}-v_{ex})\right]=0\;\Rightarrow\; v_{ix}=v_{ex}=u, \quad (6)$$

where we have assumed that $v_{ix}(0,t)=v_{ex}(0,t)=0$. This implies that the conduction current flows along y, the direction perpendicular to the plasma motion. In view of these continuity equations, we have

$$\left(\frac{\partial}{\partial t}+u\frac{\partial}{\partial x}\right)n=-n\frac{\partial u}{\partial x}, \quad (7)$$

whereas momentum equations (1) and (2) can be rewritten as

$$m_e\left(\frac{\partial}{\partial t}+u\frac{\partial}{\partial x}\right)\mathbf{v}_e = -e\left(\mathbf{E}+\frac{1}{c}\mathbf{v_e}\times\mathbf{B}\right)+m_e\nu_{ei}(\mathbf{v}_i-\mathbf{v}_e), \quad (8)$$

$$m_i\left(\frac{\partial}{\partial t}+u\frac{\partial}{\partial x}\right)\mathbf{v}_i = e\left(\mathbf{E}+\frac{1}{c}\mathbf{v}_i\times\mathbf{B}\right)+m_i\nu_{ie}(\mathbf{v}_e-\mathbf{v}_i). \quad (9)$$

Here it should be noticed that the left-hand sides of Eqs. (7)–(9) contain a common convective operator. This nonlinear operator can be transformed into a linear operator if we introduce Lagrangian variables (ξ,τ) through the following transformation relation:

$$\xi=x-\int_0^\tau u(\xi,\tau')d\tau',\quad \tau=t. \quad (10)$$

With this transformation, the derivative operators are transformed into

$$\left(\frac{\partial}{\partial t}+u\frac{\partial}{\partial x}\right)\equiv\frac{\partial}{\partial\tau},\quad \frac{\partial}{\partial x}\equiv\left[1+\int_0^\tau\frac{\partial u}{\partial\xi}d\tau'\right]^{-1}\frac{\partial}{\partial\xi}. \quad (11)$$

Using these transformations, the continuity equation (7) can now be expressed as

$$\frac{\partial}{\partial\tau}\left[n\left(1+\int_0^\tau\frac{\partial u}{\partial\xi}d\tau'\right)\right]=0,$$
$$\Rightarrow\;\frac{n(\xi,\tau)}{n(\xi,0)}=\left[\left(1+\int_0^\tau\frac{\partial u}{\partial\xi}d\tau'\right)\right]^{-1}=\frac{\partial\xi}{\partial x}. \quad (12)$$

Expressing momentum equations (8) and (9) in terms of Lagrangian variables and combining the equations, we have

$$\frac{\partial}{\partial\tau}(m_e\mathbf{v}_e+m_i\mathbf{v}_i)=\frac{e}{c}(\mathbf{v}_i-\mathbf{v}_e)\times\mathbf{B}, \quad (13)$$

where we have used $m_e\nu_{ei}=m_i\nu_{ie}$. The x and y components are

$$\frac{\partial u}{\partial\tau}=\frac{eB(x)}{(m_e+m_i)c}(v_{iy}-v_{ey}) \quad (14)$$

and

$$\frac{\partial}{\partial\tau}(m_e v_{ey}+m_i v_{iy})=0, \quad (15)$$

respectively. From Eq. (15), it is evident that the total momentum is conserved along the y direction. Taking $v_{ey}(\xi,0)=v_{iy}(\xi,0)=0$, we find

$$v_{iy}=-\frac{m_e}{m_i}v_{ey}. \quad (16)$$

If we take the magnetic field associated with the wave under study to be along the z direction, i.e., $\mathbf{B}=B(x,t)\hat{e}_z$, where $\hat{e}_z$ is the unit vector along the z direction, we can also confirm that the current flows along the y direction. Now from Eq. (5) we have

$$ne(v_{iy}-v_{ey})=-\frac{c}{4\pi}\frac{\partial B}{\partial x}. \quad (17)$$

Substituting $(v_{iy}-v_{ey})$ in Eq. (14), we obtain

$$\frac{\partial u}{\partial\tau}=-\left[\frac{cB}{4\pi(m_e+m_i)n}\right]\frac{\partial\xi}{\partial x}\frac{\partial B}{\partial\xi}. \quad (18)$$

The evolution equation for the magnetic field can further be expressed by taking a curl in the electron momentum equation (8),

$$\nabla\times\mathbf{E}+\frac{1}{c}\nabla\times(\mathbf{v_e}\times\mathbf{B}) = -\frac{m_e}{e}\left[\nabla\times\left(\frac{\partial}{\partial t}+u\frac{\partial}{\partial x}\right)\mathbf{v}_e\right]+\frac{m_e\nu_{ei}}{e^2}\left[\nabla\times\frac{\mathbf{J}}{n}\right]. \quad (19)$$

We can simplify the above equation further by using Eq. (4) and writing $\nabla \equiv \hat{e}_x(\partial/\partial x)$ and rearranging as

$$\left(\frac{\partial}{\partial t}+u\frac{\partial}{\partial x}\right)B+B\frac{\partial u}{\partial x} = -\frac{cm_e}{e}\frac{\partial}{\partial x}\frac{\partial v_{ey}}{\partial \tau}+\frac{m_e c^2 \nu_{ei}}{4\pi e^2}\frac{\partial}{\partial x}\left(\frac{1}{n}\frac{\partial B}{\partial x}\right). \quad (20)$$

In Eq. (17), substituting v_{iy} from Eq. (16) we have

$$\left(1+\frac{m_e}{m_i}\right)v_{ey}=\frac{c}{4\pi en}\frac{\partial B}{\partial x}. \quad (21)$$

Substituting v_{ey} in Eq. (20) (in terms of the Lagrangian variable), we have

$$\frac{\partial B}{\partial \tau}+\frac{Bn}{n(\xi,0)}\frac{\partial u}{\partial \xi} = \frac{c^2}{4\pi e^2}\left(\frac{m_e m_i}{m_e+m_i}\right)\frac{n}{n(\xi,0)}\frac{\partial}{\partial \xi}\frac{\partial}{\partial \tau}\left(\frac{1}{n(\xi,0)}\frac{\partial B}{\partial \xi}\right) + \frac{m_e c^2 \nu_{ei}}{4\pi e^2}\frac{n}{n(\xi,0)}\frac{\partial}{\partial \xi}\left(\frac{1}{n(\xi,0)}\frac{\partial B}{\partial \xi}\right). \quad (22)$$

Now we normalize Eqs. (7), (18), and (22) by $\hat{n}=n/n_0$, $\hat{u}=u/v_A$, $\hat{B}=B/\sqrt{4\pi n_0(m_e+m_i)v_A^2}$, $\hat{\xi}=\xi/L$, and $\hat{\tau}=\tau v_A/L$, with n_0, v_A, and L denoting a constant equilibrium density, the Alfvén velocity, and an arbitrary length scale, respectively. Hereafter, hats will be removed for simplicity of notation. Equations (7), (18), and (22), respectively, become

$$\frac{\partial}{\partial \tau}\left(\frac{1}{n}\right)=\frac{1}{n(\xi,0)}\frac{\partial u}{\partial \xi}, \quad (23)$$

$$\frac{\partial u}{\partial \tau}=-\frac{1}{2n(\xi,0)}\frac{\partial B^2}{\partial \xi}, \quad (24)$$

and

$$\frac{\partial B}{\partial \tau}=-\frac{Bn}{n(\xi,0)}\frac{\partial u}{\partial \xi}+\epsilon\frac{n}{n(\xi,0)}\frac{\partial}{\partial \xi}\frac{\partial}{\partial \tau}\left(\frac{1}{n(\xi,0)}\frac{\partial B}{\partial \xi}\right) +\eta\frac{n}{n(\xi,0)}\frac{\partial}{\partial \xi}\left(\frac{1}{n(\xi,0)}\frac{\partial B}{\partial \xi}\right), \quad (25)$$

where $\epsilon=(\delta/L)^2$ is the dispersion parameter arising from the electron's finite mass, $\eta=(m_e c^2/4\pi n_0 e^2)(\nu_{ei}/L v_A)$ is the dissipation parameter which arises due to collision, and δ is the skin depth defined by $\delta=\sqrt{c^2 m_e m_i/[4\pi(m_e+m_i)n_0 e^2]}$. Equations (23) and (25) can now be combined to give the following equation in a more compact form as

$$\frac{\partial}{\partial \tau}\left(\frac{B}{n}\right)=\frac{1}{n(\xi,0)}\frac{\partial}{\partial \xi}\left[\frac{1}{n(\xi,0)}\frac{\partial}{\partial \xi}\left(\epsilon\frac{\partial B}{\partial \tau}+\eta B\right)\right]. \quad (26)$$

Furthermore, Eqs. (23) and (24) can be combined to give

$$\frac{\partial^2}{\partial \tau^2}\left(\frac{1}{n}\right)=-\frac{1}{2n(\xi,0)}\frac{\partial}{\partial \xi}\left(\frac{1}{n(\xi,0)}\frac{\partial B^2}{\partial \xi}\right). \quad (27)$$

Thereafter, the couple of partial differential equations (26) and (27) have to be solved for n and B. We will solve them by the method of separation of variables in the next section. Without proof, we mention that our coupled equations (26) and (27) can also be derived by a similar analysis, which makes use of the resistive magnetohydrodynamic (MHD) equations, of a $1/n$-dependent resistivity and of a generalized Ohm's law in which the $\partial(j/n)/\partial\tau$ term is kept. This implies, as one can see by inspection, that our approach is valid if the dispersion parameter ϵ is sufficiently small, satisfying $(v_A/c)\leqslant\epsilon\ll(m_e/m_i)$. The dispersion parameter may hence be a small quantity only (later, for illustration, we allow a larger value: $\epsilon=0.1$) and the Alfvén velocity may not be too high asking for a sufficiently dense plasma. From this it follows that at a later stage when the magnetic field explodes (see later), the MHD approximation, and with it our model, breaks down and physical corrections are inevitable. Candidates for the latter are pressure and dissipation with the electronic and ionic subsystem and more generally kinetic and gyrokinetic effects, the consideration of which is, however, beyond the scope of the present paper.

III. SOLUTION

In this section, we present the solution obtained by Chakrabarti *et al.* [8] for the sake of analytical completeness. Proposing the separation of variable ansatz, we have

$$n(\xi,\tau)=N(\xi)\phi(\tau), \quad B(\xi,\tau)=b(\xi)\psi(\tau), \quad (28)$$

where $n(\xi,0)=N(\xi)\phi(0)$ and $B(\xi,0)=b(\xi)\psi(0)$ with $\psi(0)\neq 0$ and $\phi(0)\neq 0$. Substituting these in Eqs. (26) and (27), we have

$$-\frac{\phi^2(0)}{\psi^2}\frac{d^2}{d\tau^2}\left(\frac{1}{\phi}\right)=\frac{1}{2}\frac{d}{d\xi}\left[\frac{1}{N(\xi)}\frac{db^2}{d\xi}\right], \quad (29)$$

$$\frac{\phi^2(0)}{\epsilon\dot{\psi}+\eta\psi}\frac{d}{d\tau}\left(\frac{\psi}{\phi}\right)=\frac{1}{b}\frac{d}{d\xi}\left[\frac{1}{N(\xi)}\frac{db}{d\xi}\right], \quad (30)$$

where the dot over ψ implies the derivative with respect to time. In the above equations, the left-hand side is a function of τ only, whereas the right-hand side is a function of ξ only. Therefore, an equality can be maintained if each side is a constant. Thus we have

$$-\frac{\phi^2(0)}{\psi^2}\frac{d^2}{d\tau^2}\left(\frac{1}{\phi}\right)=\alpha, \quad (31)$$

$$\frac{1}{2}\frac{d}{d\xi}\left[\frac{1}{N(\xi)}\frac{db^2}{d\xi}\right]=\alpha, \quad (32)$$

$$\frac{\phi^2(0)}{\epsilon\dot{\psi}+\eta\psi}\frac{d}{d\tau}\left(\frac{\psi}{\phi}\right)=\beta, \quad (33)$$

$$\frac{1}{b}\frac{d}{d\xi}\left[\frac{1}{N(\xi)}\frac{db}{d\xi}\right]=\beta, \quad (34)$$

where α,β are two separation constants to be determined later. First let us solve the spatial part. From Eq. (32), we find

$$\frac{db^2}{d\xi}=2\alpha N\xi+c_1, \quad (35)$$

where c_1 is constant, and since $B=0$ at $\xi=0$, then $c_1=0$. Therefore, we have

$$\frac{1}{N}\frac{db}{d\xi}=\frac{\alpha\xi}{b}. \quad (36)$$

Substituting this in Eq. (34) and solving for $b(\xi)$, we have

$$b(\xi) = \sqrt{\frac{\alpha}{\beta}}\frac{\xi}{\sqrt{\xi^2+\xi_0^2}}, \tag{37}$$

where ξ_0 is an integration constant. One can easily find out the spatial density distribution, which is

$$N(\xi) = \frac{1}{\beta}\frac{\xi_0^2}{\left(\xi^2+\xi_0^2\right)^2}. \tag{38}$$

To solve the temporal part of the solution, we considered Eqs. (31) and (33), from which we obtain

$$\frac{1}{\psi^2}\frac{d^2}{d\tau^2}\left(\frac{1}{\phi}\right) = -\bar{\alpha}, \quad \frac{1}{\psi}\frac{d}{d\tau}\left[\psi\left(\frac{1}{\phi}-\epsilon\bar{\beta}\right)\right] = \bar{\beta}\eta, \tag{39}$$

where $\bar{\alpha} = \alpha/\phi^2(0)$, $\bar{\beta} = \beta/\phi^2(0)$. It should be noticed that finite ϵ destroys the direct integrability of Eq. (39). A way around this dilemma was proposed in Ref. [8] through the integrating factor, introduced by

$$\frac{1}{\psi}\frac{d}{d\tau} \equiv f(\theta)\frac{d}{d\theta}, \tag{40}$$

where f is an unknown function that will be determined later. With this, Eq. (39) can be written as

$$f\frac{d}{d\theta}\left[\psi\left(\frac{1}{\bar{\beta}\phi}-\epsilon\right)\right] = \eta. \tag{41}$$

After a little algebra we can show that the above equation reduces to

$$\frac{d^2\ln\psi}{d\theta^2} = \frac{\alpha}{\beta}\frac{\psi}{\lambda f}, \tag{42}$$

where we choose the function $f(\theta)$ such that

$$f\left(\frac{1}{\bar{\beta}\phi}-\epsilon\right) = \frac{\lambda}{\psi}, \tag{43}$$

and λ is a constant to be determined later. Now substituting Eq. (43) into Eq. (41), we have

$$f\frac{d}{d\theta}\left(\frac{\lambda}{f}\right) = \eta, \tag{44}$$

whose solution is $f = f_0\exp(-\eta\theta/\lambda)$. If $\eta = 0$, we can show that $f = 1$ so that $f_0 = 1$. Also for $\eta = 0$ we have found $\lambda = (1-\beta\epsilon)/\beta$. If we look at Eq. (43), then at $t = 0$ we have

$$f_0\left(\frac{1}{\bar{\beta}\phi(0)}-\epsilon\right) = \frac{\lambda}{\psi(0)}. \tag{45}$$

The value of λ is consistent with Eq. (45). We have $\psi(0) = \phi(0) = 1$. Therefore, $\bar{\alpha} = \alpha$ and $\bar{\beta} = \beta$, and Eq. (42) becomes

$$\frac{d^2\ln\psi}{d\theta^2} = \frac{\alpha}{\beta\lambda}\psi\exp\left(\frac{\eta\theta}{\lambda}\right). \tag{46}$$

Now letting $\ln\psi = -2\Psi - \eta\theta/\lambda$ and $\bar{\theta} = \theta/\sqrt{\beta\epsilon-1}$, a solution for ψ can be written as

$$\psi = \frac{2}{\alpha}\text{sech}^2\left(\frac{\theta}{\sqrt{\beta\epsilon-1}}\right)\exp\left[-\eta\beta\left(\frac{\theta}{1-\beta\epsilon}\right)\right]. \tag{47}$$

The above solution must satisfy the condition $\psi(0) = 1$ (since $\theta = 0$ implies $\tau = 0$), resulting in $\psi(0) = 1 = 2/\alpha$. This implies that one of the separation constants is $\alpha = 2$ and the solution for ψ becomes

$$\psi(\tau) = \text{sech}^2\left(\frac{\theta}{\sqrt{\beta\epsilon-1}}\right)\exp\left[-\eta\beta\left(\frac{\theta}{1-\beta\epsilon}\right)\right], \tag{48}$$

and the corresponding $\phi(\tau)$ solution is given by

$$\phi(\tau) = \frac{1}{\beta\epsilon+(1-\beta\epsilon)\cosh^2\bar{\theta}\exp[-(2\eta\beta\bar{\theta})/\sqrt{\beta\epsilon-1}]}. \tag{49}$$

It can be noted here also that $\phi(0) = 1$ is satisfied, which is our requirement. Here the variable θ is a monotonic function of time τ, which can be determined. Let us first write the complete solution for the magnetic field and the density, which are given by

$$B(\xi,\tau) = \sqrt{\frac{2}{\beta}}\left\{\frac{\xi}{\sqrt{\xi^2+\xi_0^2}}\right\}\text{sech}^2\left(\frac{\theta}{\sqrt{\beta\epsilon-1}}\right) \times \exp\left[\frac{\eta\beta\theta}{\beta\epsilon-1}\right], \tag{50}$$

$$n(\xi,\tau) = \frac{1}{\beta}\left\{\frac{\xi_0^2}{\left(\xi^2+\xi_0^2\right)^2}\right\}\left[\beta\epsilon+(1-\beta\epsilon) \times\cosh^2\left(\frac{\theta}{\sqrt{\beta\epsilon-1}}\right)\exp\left(-\frac{2\eta\beta\theta}{\beta\epsilon-1}\right)\right]^{-1}. \tag{51}$$

The relation between ξ and x can also be determined as given below,

$$\xi = \frac{x}{\beta\epsilon+(1-\beta\epsilon)\cosh^2\bar{\theta}\exp[-2\eta\beta\bar{\theta}/\sqrt{\beta\epsilon-1}]}. \tag{52}$$

The relation between time τ and auxiliary variable θ are related as

$$\tau = \frac{\sqrt{\beta\epsilon-1}}{4}\left[e^{-2\bar{\eta}\bar{\theta}}\left(\frac{\sinh 2\bar{\theta}+\bar{\eta}\cosh 2\bar{\theta}}{1-\bar{\eta}^2}\right) - \frac{\bar{\eta}}{1-\bar{\eta}^2} - \frac{1}{\bar{\eta}}(e^{-2\bar{\eta}\bar{\theta}}-1)\right], \tag{53}$$

where

$$\bar{\theta} = \frac{\theta}{\sqrt{\beta\epsilon-1}}, \quad \bar{\eta} = \frac{\eta\beta}{\sqrt{\beta\epsilon-1}}. \tag{54}$$

Still one more separation constant, namely β, needs to be fixed. For that we use the condition for the conservation of density,

$$\int_{-\infty}^{+\infty} n(\xi,0)d\xi = 1. \tag{55}$$

We find $\pi/(2\beta\xi_0) = 1$. ξ_0 can be taken as unity without any loss of generality. So the solution given above is complete with $\beta = \pi/2$.

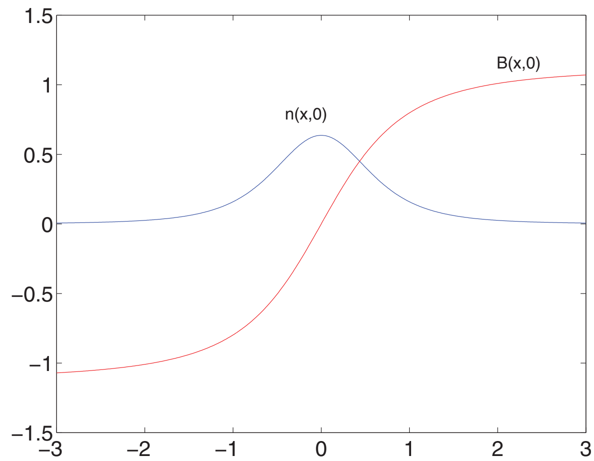

FIG. 1. (Color online) Initial density and magnetic field configuration similar to the Harris current sheet solution.

IV. ANALYSIS

First let us see the behavior of the initial solution, i.e., at $\tau = 0 = \theta$, which implies $\xi = x$ and the solutions are

$$n(x,0) = \frac{2}{\pi}\frac{1}{(1+x^2)^2}, \tag{56}$$

$$B(x,0) = \frac{2}{\sqrt{\pi}}\frac{x}{\sqrt{(1+x^2)}}. \tag{57}$$

At $t = 0$, the magnetic field and density, respectively, look like $\tanh\varsigma$ and $\mathrm{sech}^4\varsigma$ if we set $\xi \sim \sinh\varsigma$. Therefore, our system of equations includes a Harris current sheetlike state [31] as an initial plasma configuration, as indicated in Fig. 1.

Let us first analyze the solutions given in Eqs. (50)–(53) in an ideal dispersionless and dissipationless situation, i.e., $\epsilon = \eta = 0$. Then we find for $\bar{\theta} = -i\theta$,

$$B(\xi,\tau) = \frac{2}{\sqrt{\pi}}\left\{\frac{\xi}{\sqrt{\xi^2+1}}\right\}\frac{1}{\cos^2\theta}, \tag{58}$$

$$n(\xi,\tau) = \frac{2}{\pi}\left\{\frac{1}{(\xi^2+1)^2}\right\}\frac{1}{\cos^2\theta}, \tag{59}$$

with

$$\xi = \frac{x}{\cos^2\theta}, \qquad \tau = \frac{1}{4}(\sin 2\theta + 2\theta). \tag{60}$$

It may be noted that at $\theta = \pi/2$ (or, equivalently, $\tau = t = \pi/4$), the magnetic field and density solutions possess singularities depicted in Fig. 2. This is because, physically, in the absence of dispersion and dissipation, convective nonlinearity being operative feeds energy to short scales in the system, and eventually the density and magnetic field get compressed and ultimately blow up. The critical time is one-fourth of the wave period. It is to be noted also that when $\epsilon = 0$, the hyperbolic function becomes periodic.

Next, we take ϵ finite but $\eta = 0$, i.e., dispersion is present but dissipation is absent. With this, if we examine the solutions, then we find

$$B(\xi,\tau) = \frac{2}{\sqrt{\pi}}\left\{\frac{\xi}{\sqrt{\xi^2+1}}\right\}\mathrm{sech}^2\bar{\theta}, \tag{61}$$

$$n(\xi,\tau) = \frac{2}{\pi}\left\{\frac{1}{(\xi^2+1)^2}\right\}\frac{1}{\beta\epsilon + (1-\beta\epsilon)\cosh^2\bar{\theta}}, \tag{62}$$

$$\xi = \frac{x}{\beta\epsilon + (1-\beta\epsilon)\cosh^2\bar{\theta}}, \quad \tau = \frac{\sqrt{\beta\epsilon - 1}}{4}(\sinh 2\bar{\theta} + 2\bar{\theta}). \tag{63}$$

The corresponding figure is displayed in Fig. 3.

Next, we take the complementary condition as discussed above, i.e., $\epsilon = 0$ but $\eta \neq 0$, which means dispersion is absent but dissipation is operative. With this, if we examine the solutions, then we find

$$B(\xi,\tau) = \frac{4}{\sqrt{\pi}}\left\{\frac{\xi}{\sqrt{\xi^2+1}}\right\}\frac{e^{-2\eta\beta\theta}}{1+\cos 2\theta}, \tag{64}$$

$$n(\xi,\tau) = \frac{4}{\pi}\left[\frac{1}{(\xi^2+1)^2}\right]\frac{e^{-2\eta\beta\theta}}{1+\cos 2\theta}, \tag{65}$$

$$\xi = \frac{xe^{-2\eta\beta\theta}}{1+\cos 2\theta}, \quad \tau = \frac{1}{4}\left[e^{2\eta\beta\theta}\left(\frac{\sin 2\theta + \eta\beta\cos 2\theta}{1+\beta^2\eta^2}\right) - \frac{\eta\beta}{1+\beta^2\eta^2} + \frac{1}{\eta\beta}(e^{2\eta\beta\theta}-1)\right], \tag{66}$$

which is correct in time behavior due to dissipation. We see that finite η alone cannot prevent the existence of singularity either in density or in the magnetic field, as indicated in Fig. 4.

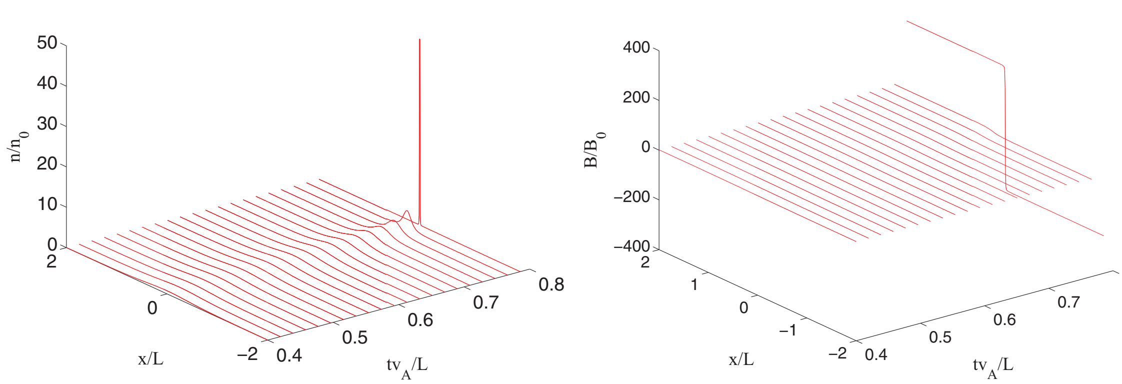

FIG. 2. (Color online) Time evolution of the density and magnetic field in the absence of dispersion and dissipation ($\epsilon = \eta = 0$). The figure shows singularity in density and magnetic field arising at time $\pi/4$ in units of Alfvén transit time.

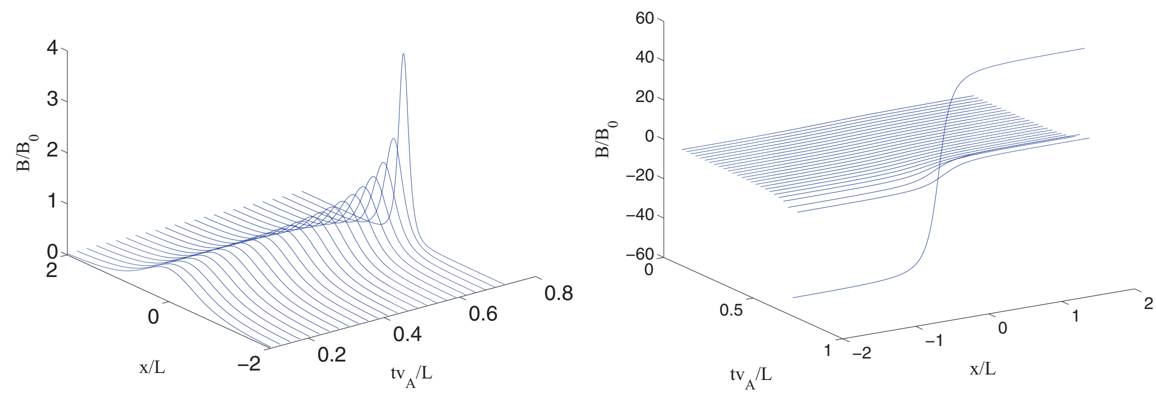

FIG. 3. (Color online) Time evolution of the density and magnetic field in the absence of resistivity ($\eta = 0$) with finite dispersion ($\epsilon = 0.1$). The figure shows that finite dissipation cannot stop singularity in density and magnetic field, it merely delayed the singularity.

If both ϵ,η are present, the solutions once again can be rewritten as

$$B(\xi,\tau) = \frac{2}{\sqrt{\pi}}\left\{\frac{\xi}{\sqrt{\xi^2+1}}\right\}\text{sech}^2\bar{\theta}\exp(\bar{\eta}\bar{\theta}), \tag{67}$$

$$n(\xi,\tau) = \frac{2}{\pi}\left\{\frac{1}{(\xi^2+1)^2}\right\}[\beta\epsilon + (1-\beta\epsilon) \times \cosh^2\bar{\theta}\exp(-2\bar{\eta}\bar{\theta})]^{-1}. \tag{68}$$

The relation between ξ,x and τ,θ can also be expressed as

$$\xi = \frac{x}{\beta\epsilon + (1-\beta\epsilon)\cosh^2\bar{\theta}\exp(-2\bar{\eta}\bar{\theta})}, \tag{69}$$

$$\tau = \frac{\sqrt{\beta\epsilon - 1}}{4}\left[e^{-2\bar{\eta}\bar{\theta}}\left(\frac{\sinh 2\bar{\theta} + \bar{\eta}\cosh 2\bar{\theta}}{1-\bar{\eta}^2}\right) - \frac{\bar{\eta}}{1-\bar{\eta}^2} - \frac{1}{\bar{\eta}}(e^{-2\bar{\eta}\bar{\theta}} - 1)\right], \tag{70}$$

where

$$\bar{\eta} = \frac{\eta\beta}{\sqrt{\beta\epsilon - 1}}, \quad \text{and} \quad \bar{\theta} = \frac{\theta}{\sqrt{\beta\epsilon - 1}}. \tag{71}$$

In Fig. 5, it is shown that dispersion may arrest the density singularity but not the magnetic field. With regard to the question of why the magnetic field and the density behave differently when dispersion is active—the former still collapses whereas the latter stays finite—we admit that we do not have a final, conclusive solution, as the interplay between hydrodynamic convection, dispersion, and dissipation is a rather complex one governed by nonlinearity. One explanation we have in mind is that dispersion acts as usual (for magnetosonic waves), preventing the density from collapsing. For the magnetic field, on the other hand, since resistivity is assumed to scale like $1/n$, being diminished at maximum density, the field diffusion is more or less offset in this region and hence the magnetic field behaves as in the ideal case and collapses.

V. SOLUTION BY THE LAGRANGIAN MASS VARIABLE

In this section, we are going to provide an additional proof of the solution described in the previous section by means of a more simplified description that utilizes the Lagrangian mass variable. The system of coupled differential Eqs. (26) and (27) can be substantially simplified without loss of generality by switching to the Lagrangian mass variable [7]. Introduction of Lagrangian mass variable ζ instead of ξ yields the mathematical operator

$$\frac{\partial}{\partial\zeta} = \frac{1}{n(\xi,0)}\frac{\partial}{\partial\xi}, \tag{72}$$

which can be obtained from the transformation relation: $\zeta = \int^{\xi} n(\xi',0)d\xi'$. Moreover, if one introduces for convenience the

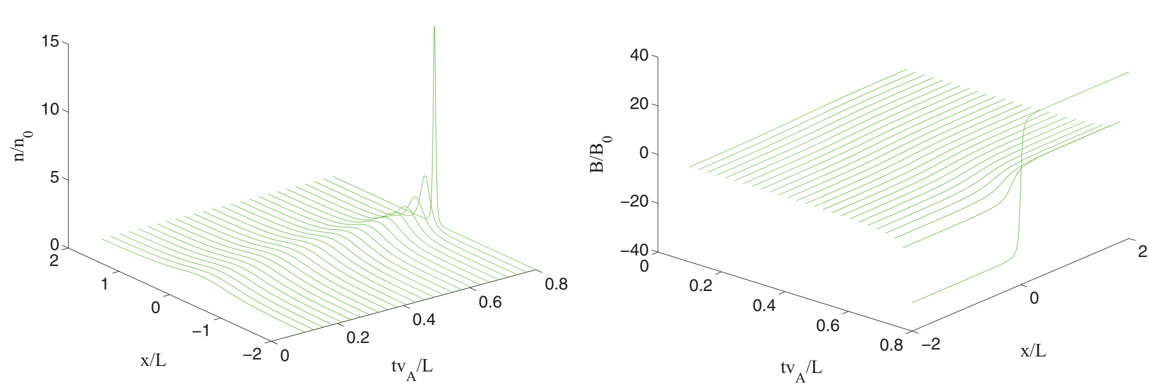

FIG. 4. (Color online) Time evolution of the density and magnetic field in the absence of dispersion ($\epsilon = 0$) with finite dissipation. The figure shows that finite dissipation cannot remove singularity in density and magnetic field.

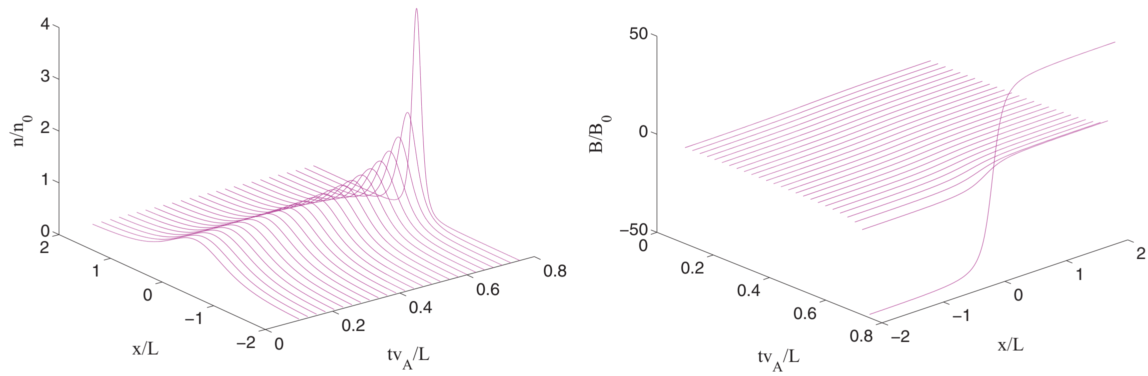

FIG. 5. (Color online) Time evolution of the density and magnetic field in the presence of both dispersion and dissipation ($\epsilon = 0.1$, $\eta = 0.03$). The figure shows that neither finite dispersion nor resistivity can remove magnetic field singularity, whereas for finite dispersion density, singularity is removed.

specific volume $V(\zeta,\tau) = 1/n(\zeta,\tau)$ [which is the Jacobian of the transformation from (x,t) to (ζ,τ)], Eqs. (26) and (27) can be written as

$$\frac{\partial}{\partial\tau}(BV) = \frac{\partial^2}{\partial\zeta^2}\left[\epsilon\frac{\partial B}{\partial\tau} + \eta B\right], \tag{73}$$

$$\frac{\partial^2 V}{\partial\tau^2} = -\frac{1}{2}\frac{\partial^2 B^2}{\partial\zeta^2}. \tag{74}$$

It is worth mentioning that the space-dependent coefficient $n(\xi,0)$ has now disappeared, leaving the system in a substantially simpler form. Through this simpler system of equations, we expect a more straightforward solution that may be obtained, and the earlier solution presented in the previous section can be recovered. A solution of Eqs. (73) and (74) can be attempted by a separation of the variable ansatz. We set $V(\zeta,\tau) = P(\tau)U(\zeta)$ and $B(\zeta,\tau) = Q(\tau)\mathcal{B}(\zeta)$, and then the above two partial differential equations reduce to four ordinary differential equations as

$$\frac{1}{[\epsilon(dQ/d\tau) + \eta Q]}\frac{d}{d\tau}(PQ) = \frac{1}{U\mathcal{B}}\frac{d^2\mathcal{B}}{d\zeta^2} = \beta, \tag{75}$$

$$\frac{1}{Q^2}\frac{d^2P}{d\tau^2} = -\frac{1}{2U}\frac{d^2\mathcal{B}^2}{d\zeta^2} = -\alpha, \tag{76}$$

where α and β are the separation constants, and if we choose $\alpha = 2$ and $\beta = \pi/2$, then we are in accordance with the systems described in the previous sections. Now we can obtain the spatial and temporal solutions, keeping the constant β and setting $\alpha = 2$. This may recover the previous solution, and one can easily go beyond that with the various parameter values of α and β. Concentrating on the temporal part of the equations, we have

$$\frac{d^2P}{d\tau^2} + 2Q^2 = 0, \tag{77}$$

$$\frac{d}{d\tau}(PQ) = \beta\left(\epsilon\frac{dQ}{d\tau} + \eta Q\right). \tag{78}$$

First, the nonresistive part of the solution is obtained by setting $\eta = 0$. Then $Q(\tau)$ can be easily obtained as

$$Q = \frac{1-\beta\epsilon}{P-\beta\epsilon}, \tag{79}$$

where the initial condition $P(0) = 1 = Q(0)$ is used. Substitution of Q in Eq. (77) and using a straightforward algebra leads to

$$\frac{dP}{d\tau} = \pm\frac{2(1-\beta\epsilon)}{\sqrt{P-\beta\epsilon}}, \tag{80}$$

where the negative branch of the square root is taken and $P > \beta\epsilon$ is assumed. A further integration gives

$$P(\tau) = \beta\epsilon + [(1-\beta\epsilon)^{3/2} - 3(1-\beta\epsilon)\tau]^{2/3}. \tag{81}$$

The above solution indicates that $P(\tau)$ is decreasing with increasing time, reaching $P(\tau_c) = \beta\epsilon$ with infinite slope i.e., $dP/d\tau \to \infty$, as τ approaches τ_c $(= \sqrt{1-\beta\epsilon}/3)$. Since P stands for specific volume, i.e., $1/P$ for density, one immediately sees that the density is compressed (i.e., increases) in time but stays finite as long as the dispersion parameter ϵ is finite. On the other hand, if the dispersion disappears, i.e., $\epsilon \to 0$, then P becomes zero and the density diverges. The time τ_c is known as density collapse time. Turning back to the magnetic field solution from Eq. (79), we observe that $Q \to \infty$ in either case in this limit. Therefore, magnetic field collapse is independent of dispersion. Thus dispersion is able to stop the density collapse but not that of the magnetic field.

Next we can concentrate on the solution with finite resistivity, i.e., $\eta \neq 0$. Finite resistivity in Eq. (78) destroys the direct integrability. A way out of this difficulty was proposed in a previous paper [8] by introducing an integrating factor defined as

$$\frac{1}{Q(\tau)}\frac{d}{d\tau} = f(\theta)\frac{d}{d\theta}. \tag{82}$$

Proceeding in the same way as in Sec. III, we can obtain an explicit solution for P,Q. But here we will show that these equations, indeed, yield the same solution that we obtained before. Considering $f(\theta)$ to be exponential, $f = \exp(A\theta)$ (as obtained in a previous paper [8]), and integrating Eq. (78), we have

$$Q(\theta) = \frac{1}{(P-\beta\epsilon)}\left[(1-\beta\epsilon) + \frac{\beta\eta}{A}\{1-\exp(-A\theta)\}\right]. \tag{83}$$

Notice here that in the limit $\eta \to 0$, we get back our nonresistive solution (79). Here we recall our solution for ϕ

and ψ obtained before. In the current notation, these can be written as $P = 1/\phi$ and $Q = \psi$. Moreover, the solutions can be expressed as

$$P - \beta\epsilon = (1-\beta\epsilon)\cosh^2\left(\frac{\theta}{\sqrt{\beta\epsilon - 1}}\right)\exp\left(-\frac{2\eta\beta}{\beta\epsilon - 1}\theta\right), \tag{84}$$

$$Q = \text{sech}^2\left(\frac{\theta}{\sqrt{\beta\epsilon - 1}}\right)\exp\left(\frac{\eta\beta}{\beta\epsilon - 1}\theta\right). \tag{85}$$

It can be easily seen that these two previously obtained solutions will satisfy Eq. (83) when $A = \beta\eta/(\beta\epsilon - 1)$. Now if we can show that these solutions will satisfy Eq. (77), then our proof is complete. For this we can switch on the differential operator $d/d\tau \to d/d\theta$, which is

$$\frac{d}{d\tau} = \text{sech}^2\left(\frac{\theta}{\sqrt{\beta\epsilon - 1}}\right)\exp(2A\theta)\frac{d}{d\theta}. \tag{86}$$

Using this and Eq. (85), we have

$$\frac{d^2P}{d\theta^2} + \left[2A - \frac{2}{\sqrt{\beta\epsilon - 1}}\tanh\left(\frac{\theta}{\sqrt{\beta\epsilon - 1}}\right)\right]\frac{dP}{d\theta} + 2\exp(-2A\theta) = 0. \tag{87}$$

This equation is indeed satisfied by Eq. (84). This confirms the correctness of the results obtained in Sec. III as well as in Ref. [8] with respect to the temporal behavior, which is the essential part because it decides the collapse behavior.

Now we concentrate on the solution of the spatial part. For this, from Eqs. (76) and (75) we obtain

$$\frac{d^2\mathcal{B}}{d\zeta^2} - \frac{\beta}{4}\mathcal{B}\frac{d^2\mathcal{B}^2}{d\zeta^2} = 0. \tag{88}$$

The above nonlinear equation can be simplified by using the dependent variable transformation $H = \mathcal{B}^2$. From Eq. (88) we obtain

$$\frac{d^2H}{d\zeta^2} = \frac{1}{H(2-\beta H)}\left(\frac{dH}{d\zeta}\right)^2. \tag{89}$$

One integration can easily be done to obtain

$$\frac{dH}{d\zeta} = c_2\sqrt{\frac{H}{1-\beta H/2}}, \tag{90}$$

from which the spatial solution is given implicitly for $[\zeta = \zeta(\mathcal{B}^2)]$ by

$$\sqrt{\frac{2}{\beta}}\left[\sin^{-1}\sqrt{\frac{\beta\mathcal{B}^2}{2}} + \sqrt{\frac{\beta\mathcal{B}^2}{2}\left(1 - \frac{\beta\mathcal{B}^2}{2}\right)}\right] = c_2\zeta + c_3, \tag{91}$$

where c_2 and c_3 are integration constants. Since $\mathcal{B} = 0$ at $\zeta = 0$, therefore $c_3 = 0$, but $c_2 \neq 0$. We now have

$$\frac{\partial x}{\partial\zeta} = \frac{1}{n(\xi,0)}\frac{\partial x}{\partial\xi} = \frac{1}{n(\xi,\tau)} \equiv V(\zeta,\tau), \tag{92}$$

from which, by using Eqs. (76), we obtain

$$x = \frac{1}{4}P(\tau)\frac{d\mathcal{B}^2}{d\zeta}. \tag{93}$$

At this point we will again use Eq. (90) to obtain the derivative of the magnetic field occurring on the right-hand side of the above equation. Therefore substituting $d\mathcal{B}^2/d\zeta$ in Eq. (93), we have

$$\frac{4x}{P(\tau)} = \frac{c_2\mathcal{B}}{\sqrt{1-\beta\mathcal{B}^2/2}}, \tag{94}$$

from which we find

$$\mathcal{B} = \frac{4x/(c_2P)}{\sqrt{1+\frac{\beta}{2}[4x/(c_2P)]^2}}. \tag{95}$$

It may be noted here that $\beta = \pi/2$ and the constant c_2 physically signifies the maximum value of the magnetic field gradient at $\zeta = 0$. We can choose this constant $c_2 = 2\sqrt{\pi}$ without any loss of generality. This choice exactly matches the spatial magnetic field profile obtained before at $\tau = 0$. Initially, i.e., at $\tau = 0$, $P(0) = 1$, and putting the value of c_2 mentioned above, we have

$$\mathcal{B}(x,0) = \frac{2}{\sqrt{\pi}}\frac{x}{\sqrt{1+x^2}}, \tag{96}$$

which is obtained before as indicated in Eq. (57). Therefore, we reproduce the earlier results exactly by Lagrangian mass variables. In Eq. (95), $P(\tau)$ can be evaluated from (84), and a relation that connects τ and θ is given in Eq. (53).

VI. A FIRST APPLICATION: COLLAPSE OF DENSITY AND MAGNETIC FIELD AS A SEED FOR MAGNETIC STAR FORMATION

The applicability of the present model is restricted to physical systems that admit in the allowed parameter range an initial Harris-type current sheet solution, which acts as a seed and undergoes amplification and collapse, such as the one described below in this section. This is a consequence of the separation ansatz (28). Other solutions of (26) and (27) and (73) and (74), respectively, may be obtained by a different ansatz, such as one that allows for nonlinear traveling-wave solutions.

As a first application, we refer now to the early stage of star formation and to the role that the magnetic field is playing. The simple picture of star formation calls for giant clouds of gas and dust to collapse inward due to gravity, growing denser and hotter until igniting nuclear fusion. The problem is that only a small fraction of cloud material forms stars, such that there must be additional forces that hinder the contraction process. Magnetic fields and turbulence are the two leading candidates. Numerical simulations of magnetohydrodynamic turbulence [32–34] indicate that a weak magnetic field, corresponding to super-Alfvenic turbulence, will be tangled by turbulent eddies, and one should not expect a correlation between field orientations inside molecular clouds and those in the surrounding intercloud medium (ICM). On the other hand, a strong magnetic field (sub-Alfvenic turbulence) can channel turbulent flows and preserve field orientation over large length scales. To give some typical numbers, molecular clouds have a density of $n > 10^5$ cm^{-3} and a linear size below 1 parsec (pc), whereas ICM has $n \sim 1$ cm^{-3} and an (accumulation) length of several hundred pc (e.g., 200 pc). Using optical sub-mm polarimetry, Li *et al.* [29,30] studied 25 dense patches in the

Orion molecular cloud region, each one about a light-year in size, together with the corresponding ICM region. Their result was that "even though the core separations exceed the core size by as much as a factor of 100, they are for the most part magnetically connected, i.e., the core's mean field directions are similar" and, moreover, these directions are close to the mean-field direction seen in the ICM. Their conclusion is that, comparing this result with molecular cloud simulations, only the sub-Alfvenic cases result in field orientations consistent with the observations. This may be considered as a hint that a contraction process such as that described in the present paper, being mediated by a strong magnetic field, may be the origin of this novelty in star formation. By adjusting the parameters, such as n_0, B_0, dispersion, and dissipation, the local amplification of the density of 10^5 together with the alignment of the magnetic field in both regions are reasonably well described by and conform with our model, especially when it is supplemented by more realistic effects, such as pressure, kinetic effects, or stability effects that will keep the magnetic field finite. We may hence conclude that the collapse process described in this paper can act as a seed of matter clumping in the Universe prior to the onset of gravitation and turbulence.

VII. SUMMARY

In conclusion, we emphasize that our analysis of a compressional MHD kind of collapse has been oriented toward a simple macroscopic situation in which nonlinearity, time dependence, dispersion, and resistive dissipation are treated on an equal footing, resulting in an exact solution of the governing equations. The solution shows a competition among hydrodynamic convection, magnetic field diffusion, and dispersion and includes a Harris-like current sheet state as an initial plasma configuration. This results in an unbounded amplification of magnetic field and a bounded amplification of plasma density at a finite time. This is because the dispersive effect is found to halt the collapse of density but not of the magnetic field, whereas resistivity alone can neither halt density nor magnetic field collapse. Such a collapse process associated with magnetosonic-type compressional dispersive waves can be a possible mechanism for the generation of magnetic fields whose physical origin lies solely on the electron and ion currents.

These types of nonlinear, dispersive, and dissipative solutions represent a new class of transient solutions in magnetized plasmas and may arise in a manifold of similar physical situations [35]. We should stress that the results previously obtained by the same authors in Ref. [8] are shown to be correct, and they are reproduced here by presenting solutions through another method that utilizes a Lagrangian mass variable. Finally, we would like to mention that this magnetic field collapse opens the possibility of very many follow-up studies with more general fluid physics, e.g., in terms of generalized Ohm's law, kinetic descriptions, etc., to determine the sturdiness of the effect described, and that more investigations in this direction are necessary to improve our understanding of the generation of strong magnetic fields in plasmas, especially in astrophysical environments.

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