Persistence of uphill anomalous transport in inhomogeneous media

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For systems out of equilibrium and subjected to a static bias force it can often be expected that particle transport will usually follow the direction of this bias. However, counterexamples exist where particles exhibit uphill motion (known as *absolute negative mobility*, ANM), particularly in the case of coupled particles. Examples in single particle deterministic systems are less common. Recently, in one such example, uphill motion was shown to occur for an inertial driven and damped particle in a spatially symmetric periodic potential. The source of this anomalous transport was a combination of two periodic driving signals which together are asymmetric under time reversal. In this paper we investigate the phenomena of ANM for a deterministic particle evolving in a periodic and symmetric potential subjected to an external unbiased periodic driving and nonuniform *space-dependent* damping. It will be shown that this system exhibits a complicated response behavior as certain control parameters are varied, most notably being enhanced parameter regimes exhibiting ANM as the static bias force is increased. Moreover, the solutions exhibiting ANM are shown to be, at least over intermediate time periods, superdiffusive, in contrast to the solutions that follow the bias where the diffusion is normal.

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I. INTRODUCTION

The dynamics of systems modeling the evolution of single driven and damped particles continues to be of interest. One reason is the rich behavior present in such models. Another is that these relatively simple systems allow for the analysis and observation of real physical phenomena with only minimal resources. In particular, the transport of particles in symmetric and periodic potential landscapes has attracted considerable interest [1,2]. Such potentials lend themselves to a vast number of applications including Josephson junctions [3], charge density waves, nanoengines [4], and transport in biological systems [5].

The prototypical equation for such models takes the form

$$\ddot{q} = -\gamma\dot{q} - V'(q) + F(t), \qquad (1)$$

where γ is the damping parameter, $V(q)$ is the system potential, and $F(t)$ is a time-dependent driving; V and F are both usually bounded and periodic. The dot and prime denote differentiation with respect to time t and coordinate q, respectively. The symmetry properties of these models are now well understood [6,7]. In short, if the system potential and the external driving satisfy certain spatial and temporal symmetries, then each trajectory will have a counterpart whose velocity is of the same magnitude but of different sign. This has important consequences for the net flow (often called the *current* and defined more precisely later) as, if each trajectory has a counterpart whose velocity differs only by a change of sign, then the net flow will be zero. Thus, a number of studies have investigated the effects, with respect to the net flow, when these symmetries are broken. For example, numerous studies have considered the driven and damped dynamics of a particle evolving in a periodic but asymmetric potential [8–11]. They observed a nonlinear response behavior to changes in the driving amplitude, including multiple current reversals. In the Hamiltonian limit $\gamma = 0$, the focus has been on how these asymmetries influence the sticking episodes to regular transport supporting islands in the chaotic part of phase space [12–14]. Obtaining directed particle transport in systems with zero-average forces has become known as the *ratchet effect* [15].

Recently, [16] considered an alternative to the more common spatially uniform damping. They studied a system of the form given by Eq. (1), with symmetric potential and driving, where the constant coefficient of friction γ was replaced by a space-dependent term. They found that the frictional inhomogeneity mimics the role played by asymmetric potentials and/or external driving forces, resulting in nonzero net flow, i.e., the ratchet effect. Motivations for such studies come from a variety of sources. For example, in Josephson junctions a phase-dependent damping can represent an interference term between the pair and quasiparticle currents [17,18] (in the latter the authors also give a thorough phase-space analysis of such a junction).

An interesting extension to problems with an externally modulated potential comes when a dc bias is introduced, serving as a constant tilt to the potential landscape [19–25]. These studies have examined the fascinating phenomena of *absolute negative mobility* (ANM) where a particle can travel in the direction opposite to a constant applied force. Most of the studies so far have looked at noise-induced ANM. Further, it was proven that for the overdamped dynamics of Brownian particles, where inertial effects are negligible, the solutions may not exhibit ANM [22].

Studies of these inherently biased systems are important as they find application in a number of areas. For example, in the transport of biomolecules where the separation of particles may be desirable [26], this separation becomes inherently difficult when the particles are working against an additional load. Therefore, finding regimes where particles move against an applied load becomes extremely important. Further, ANM has recently been observed experimentally in the domain of Josephson junctions where the related phenomenon is known as *negative absolute resistance* [27]. The authors were able

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verify theoretical predictions obtained from a model of a damped Brownian particle in one dimension [22,23].

Less common are works on the ANM phenomena in single particle deterministic systems, i.e., the noiseless case. This has been detailed in only a few studies, for example [19,22–24]. A recent study [28], in an attempt to mimic the roll played by noise in previous works, considered a "vibrational motor"—a system where additional driving terms yield stochastic-like (yet deterministic) dynamics. ANM was observed in this system in regimes where it was solely induced by noise (when the additional driving terms are absent).

To the author's knowledge, the effect of absolute negative mobility has not been observed in systems with a frictional inhomogeneity. Illustrating such an effect will be the focus of the present study. In particular, we investigate the transport processes of single particles evolving in a symmetric and periodic potential, subjected to an unbiased external ac driving and a static dc bias. It will be shown that a frictional nonuniformity can induce the phenomena of absolute negative mobility. Moreover, the mechanism that allows for such an astonishing effect is different from those presented to date, and this will also be discussed.

The paper is organized as follows. In the next section we outline the system under investigation, and discuss some of its important properties. Here, the main observable of interest, i.e., *particle current*, will also be presented. Numerical results, pertaining to the particle current, will then be presented in Sec. III. A discussion then follows in Sec. IV on the mechanism and phase-space structures that allow for the occurrence of ANM in the system under consideration. Further, the dynamics will be characterized in terms of rates of diffusion. We finish with a summary of the results.

II. SYSTEM

We study the dynamics of a driven and damped particle evolving in a symmetric and periodic "washboard" potential. The potential, in addition to the time-periodic modulations of its inclination, will also be subjected to a static dc-bias force. Further the strength of the damping will be space dependent. The equation of motion for this system is given by

$$\ddot{q} = -\gamma(q)\dot{q} + A\cos(\omega t) + \cos(q) + F, \qquad (2)$$

where $q = q(t)$ represents the spatial coordinate of the particle at time t, and with potential $V(q) = -\sin(q)$, and $\gamma(q) = \gamma[1 - \lambda\sin(q + \phi)]$, both of spatial period $L = 2\pi$. The particle is driven by a zero average time-periodic driving force of amplitude A and frequency ω, and the magnitude of the static bias force is represented by the parameter F. In addition, the space-dependent damping is regulated by three parameters, namely γ, λ, and ϕ, which control the maximal amplitude of the damping coefficient, determine the systems inhomogeneity, and determine the phase difference between the potential and the damping coefficient (which are of the same period).

As a physical realization of such a system consider a resistively and capacitively shunted Josephson junction subjected to ac and dc currents. The corresponding equations of motion, shown in dimensionless form in Eq. (2), model the phase difference across the junction. The first and third terms on the right-hand side model the quasiparticle and Josephson currents, respectively, while the second and fourth terms model the ac and bias currents, respectively. The ϕ-dependent term (often called the "$\cos\varphi$" term) accounts for interference between the Cooper-pair and quasiparticle currents [17,18]. The term ϕ can be regarded as a phase shift in the tunneling of quasiparticles across the junction. As mentioned in the introduction, ANM has been observed in a Josephson junction experimentally [27].

It is worth examining the symmetry properties of this system for the special cases related to $F = 0$. These properties determine whether or not a current (defined in this section) can emerge in the ensemble dynamics. As stated in [6], the breaking of each system symmetry is required before a current can emerge. Consider these special cases: first $F = 0$ and $\lambda = 0$, and second $F = 0$, $\lambda \neq 0$, and $\phi = n\pi$ $(n \in \mathbb{Z})$. In both cases the transformation $q \to -q + \pi$, $t \to t + T/2$ $(T = 2\pi/\omega)$ yields a new trajectory with an average velocity which differs from that of the original only by sign. Thus a zero current results. This transformation for $\lambda \neq 0$ and $\phi \neq n\pi$ $(n \in \mathbb{Z})$ does not necessarily produce counterpropagating trajectories and thus the necessary conditions for the ratchet effect to occur have been created. The dynamical effects of $\phi \neq n\pi$ $(n \in \mathbb{Z})$ are now discussed.

Following similar lines to the discussion of [29], we outline how the frictional nonuniformity can be used to induce the ratchet effect in the absence of a dc-bias force. Figure 1 shows the potential $V(q)$, and the strength of the coefficient of damping (for two values of the phase ϕ) over one spatial period L. With $\phi = 0$ the damping coefficient is symmetric about the potential minimum with the result that neither motion to the left nor to the right is favored with respect to the nonlinear damping term. With a nonzero phase ($\phi = 0.35$ in the this case) symmetry with respect the potential minimum is broken. Looking at the curve corresponding to $\phi = 0.35$ in Fig. 1 it can be seen that the damping coefficient to the left of the potential minimum is, on average, smaller than that to the right of the potential minimum, thus favoring motion to the left. It is this mechanism that allows for the emergence of a nonzero current [16]. The ratchet effect, induced by this mechanism, will be exploited in the present work.

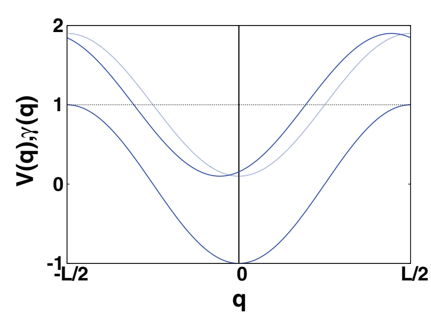

FIG. 1. (Color online) Shown, over one spatial period L, are the potential $V(q)$ (bottom curve) and the corresponding nonuniform coefficient of friction $\gamma(q)$ for two values of the phase ϕ; the curve with minimum centered at $q = 0$ corresponds to phase $\phi = 0$, while the curve with a minimum centered to the left of $q = 0$ results when $\phi = 0.35$; here $\lambda = 0.9$.

To gain a quantitative perspective of how the frictional inhomogeneity parameter λ influences the dynamics we compute the current v. That is, we calculate the time-averaged mean velocity for an ensemble of initial conditions, i.e.,

$$v = \frac{1}{T_s} \int_0^{T_s} dt \langle p(t) \rangle, \tag{3}$$

where T_s is the simulation time and the ensemble average is given by

$$\langle p(t) \rangle = \frac{1}{N} \sum_{n=1}^{N} p_n(t), \tag{4}$$

with N being the number of initial conditions. Numerical results related to the current will be presented in the next section.

III. CURRENT

In this section we discuss the numerically computed current. The initial conditions have been chosen such that the $q_n(0)$ are uniformly distributed in the potential well centered at the origin, with $p_n(0) = 0$ for all $n \in N$. For computation of the long-time average, numerical integration is performed using a fourth-order Runge-Kutta method, over a simulation time interval $T_s = 10^5$ ($\approx 1.6 \times 10^4 \times T_0$ with $T_0 = 2\pi$ being the period duration of harmonic oscillations about a potential minimum) with step size $dt = 0.01$. The ensemble average is calculated using an ensemble of $N = 1000$ initial conditions.

Figure 2 shows the current, as a function of λ, for different values of the static bias force F. For $F = 0$ the current is in the main close to zero. However, there exists a window of λ values such that motion to the left is promoted ($0.37 \lesssim \lambda \lesssim 52$). The direction of the current in this window is most certainly induced, not only by the choice of λ, but also by the specific choice of the phase $\phi = 0.35$ (see Fig. 1). Moreover, another choice of ϕ can induce a current that moves to the right. The sharp transitions to/from the window of nonzero current relate to the existence of different attractors in phase space. This can be observed in the corresponding bifurcation diagram (not shown). See below for details in the case $F = 0.1$. Note that motion to the left in this case does not qualify as negative mobility as the particle is not working against an external load. This can only happen for $F \neq 0$. Increasing the bias force to $F = 0.1$, we see that a window, of significant extent, opens which supports ANM. Outside of this window the current follows the bias; i.e., there is a positive current. Importantly, most of the λ values inside this window of negative mobility produce a zero current when the static bias force is switched off. Thus, one can conclude that this effect of negative mobility is induced by the static bias force, rather than through a carefully chosen phase ϕ; i.e., it is the tilting of the potential that results in uphill motion [30]. The reason for the fluctuations of v in this window is due to the coexistence of attractors supporting transport in opposite directions (see Fig. 3). However, for $0.95 \lesssim \lambda \lesssim 1.08$ there is single attractor in phase space supporting uphill motion. This helps explain why the current remains constant within this window, and further why the current has an increased magnitude.

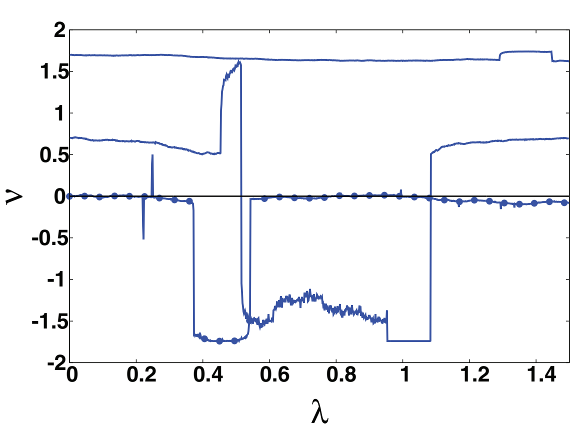

FIG. 2. (Color online) The current, computed for three values of the static bias force F, as a function of λ. The curve with dots corresponds to a dc bias of $F = 0.0$, the middle curve to $F = 0.1$, and the upper curve to $F = 0.2$. The remaining parameters are $\gamma = 0.108$, $\phi = 0.35$, $A = 1.512$, and $\omega = 0.58$.

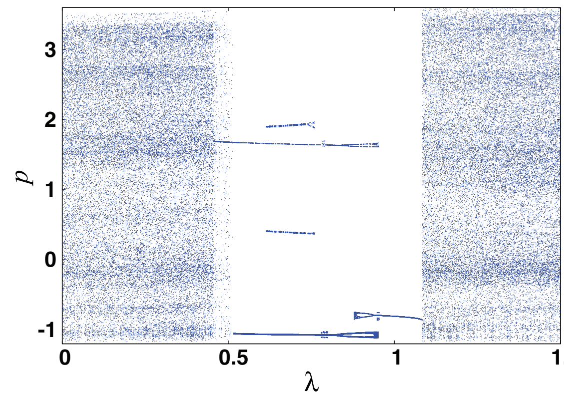

FIG. 3. (Color online) Bifurcation diagram, as a function of λ, for $F = 0.1$. The remaining parameters are given in Fig. 2.

Such windows of absolute negative mobility exist for $F < F_{\mathrm{crit}} \approx 0.17$. Remarkably, as F is increased from $F = 0$, the size of the window supporting ANM grows (with respect to its extent in the λ domain) approaching almost three times the $F = 0$ size. However, this behavior eventually ceases and as $F \to F_{\mathrm{crit}}$ from below, the windows become of smaller and smaller extent. Beyond F_{crit}, solutions exhibiting negative mobility no longer exist, and instead follow the direction of the bias force, resulting in a positive current. An example of this is shown for $F = 0.2$ where the current is $v \approx 1.75$ for the entire range of λ.

To see that the occurrence of ANM in this system is indeed dependent on the frictional inhomogeneity, consider again the curve related to $F = 0.1$ in Fig. 2. Starting from zero frictional inhomogeneity ($\lambda = 0$), the current is positive; i.e., the current is in the direction of the bias. Upon increasing λ this remains true until $\lambda \approx 0.51$ where there is an abrupt change in the direction of the current. The current then remains negative with increasing λ until a second critical value $\lambda \approx 1.08$ where the current again becomes positive. Thus, for the constant dc bias $F = 0.1$, ANM is possible only for certain values of the frictional inhomogeneity parameter λ.

In the next section, we will discuss the phase-space structures that allow for such counterintuitive motion. Moreover, the mechanism that produces uphill motion will also be discussed.

IV. ABSOLUTE NEGATIVE MOBILITY

In this section it is our aim to gain further understanding of the phase-space structures that facilitate this uphill motion, and to look more closely at the underlying mechanism that allows for negative mobility.

Let us first consider the bifurcation diagram for the curve related to $F = 0.1$ in Fig. 2. The results, obtained by stroboscopically sampling trajectories after each period of the driving (omitting a transient), are contained in Fig. 3. It can be seen that for the range of λ considered, this system supports aperiodic chaotic solutions and periodic solutions. With regard to the window of ANM observed in Fig. 2, the corresponding window in Fig. 3 supports only periodic solutions. In contrast, it would appear that the solutions following the direction of the dc-bias evolve chaotically. This behavior would help explain why, for the majority of λ values (with $F = 0.1$), the current in the window of ANM is of greater magnitude than for the λ values corresponding to a positive current.

Just like for the current, the transition from chaotic motion to periodic motion is abrupt. Sharp transitions from chaotic to periodic motion (and vice versa) related to the transition from downhill to uphill motion have been observed before in the case of coupled particle [31]. Moreover, the exact reasons behind current reversals in general single-particle systems of the form Eq. (1) remains open to debate [8,10,32].

Now let us turn our attention to the actual mechanism that allows a particle to run uphill. For simplicity, we will look at a parameter set corresponding to the window of period 1 orbits seen in Fig. 3. An example trajectory, with starting time $t \approx 9958.3$ coinciding with a change from positive to negative external driving, for this parameter set is given in Fig. 4 (top panel). Initially the driving (middle panel) becomes negative $[F(t) < 0]$, while at the same time the damping strength (bottom panel) is approaching its minimum. Importantly, when the damping strength reaches its minimum, the driving strength is also close to its minimum of $-A$; that is, the driving strength has attained almost its maximal amplitude, but in the direction opposite to the bias. This coordination between driving and damping allows the particle to run almost freely uphill. Subsequently, at $t \approx 9961$ the damping coefficient attains its maximal value. However, the particle is being driven against the bias (by a driving force that is still close to it minimum value), and continues to be so even after the damping coefficient has oscillated once more between its minimum and maximum values.

As the driving becomes positive at $t \approx 9963.5$ (indicated by the vertical lines in the figure), the damping coefficient is approaching its minimum. This results in a slowing down of the particle's ascent. Eventually the particle's uphill motion ceases and it then follows the direction of the bias (see inset in Fig. 4). Importantly, this turning point occurs in the final stages (in the course of a single period of the external driving) of positive driving, resulting in only short intervals of downhill motion. This behavior continues in a periodic fashion allowing the particle to travel large distances in the direction opposite to the applied dc-bias force.

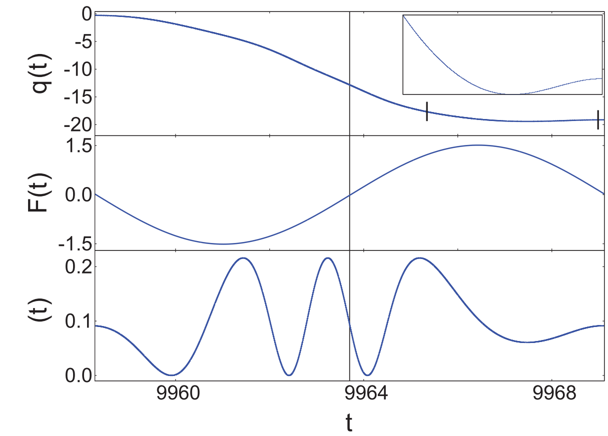

FIG. 4. (Color online) The constituent parts of a solution exhibiting negative mobility over the course of a single period of the external driving for $F = 0.1$, $\lambda = 1.0$, and the remaining parameters as in Fig. 2. The top panel shows the evolution of the coordinate q, the middle panel shows the time-periodic driving, and the bottom panel shows the space-dependent damping. The vertical line in each panel divides the figures into two segments; left segments correspond to $F(t) < 0$, and the right segments $F(t) > 0$. The coordinate q in the top panel is shown mod(8360). The inset in the top panel is a magnification of the portion of the curve between the two small vertical lines.

This mechanism, while sharing some of the characteristics of absolute negative mobility seen in previous studies in that it depends on fine tuning of the external driving for said effect to occur, is unique as it relies on the nonuniform damping to aid the uphill motion.

To further characterize the motion we now look at the mean-squared displacement for ensembles of particles, i.e., the rate of diffusion, given by

$$\sigma_q^2(t) = \langle (q - \langle q \rangle)^2 \rangle, \tag{5}$$

where $\langle \ldots \rangle$ indicates averages over ensemble. Typical *normal* diffusion processes exhibit a linear relationship with time, that is,

$$\sigma_q^2(t) \sim t^{\alpha} \tag{6}$$

with $\alpha = 1$. However, with $\alpha \neq 1$ the diffusion becomes *anomalous*—either superdiffusive ($\alpha > 1$) or subdiffusive ($\alpha < 1$) [33]. Figure 5 shows the temporal evolution of $\sigma_q^2(t)$ for five representative λ values taken from Figs. 2 and 3. These values are $\lambda = 0$ (zero frictional inhomogeneity, chaotic motion), $\lambda = 0.2$ (nonzero frictional inhomogeneity, chaotic motion), $\lambda = 0.67$ (regular coexisting attractors, uphill motion), $\lambda = 1$ (single periodic attractor, uphill motion), and $\lambda = 1.3$ (chaotic motion).

The early temporal evolution of σ_q^2, starting with every initial condition in the same potential well, is similar in all cases. Subsequently, at around $t \approx 10^2$ different regimes become apparent. The three λ values associated with chaotic (downhill) dynamics quickly settle into normal diffusive motion with an exponent $\alpha \approx 1$ (lines A, B, and E in Fig. 5). Interestingly, for the λ values where ANM was observed, the fitted exponent

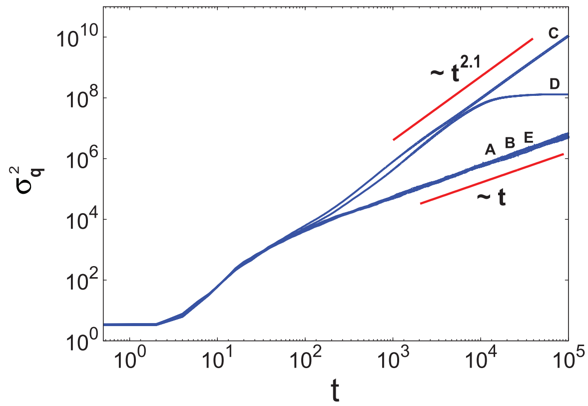

FIG. 5. (Color online) Log-log plot showing the temporal evolution of the particle mean-squared displacement for five values of the inhomogeneity parameter λ: (A) $\lambda = 0$, (B) $\lambda = 0.2$, (C) $\lambda = 0.67$, (D) $\lambda = 1.0$, and (E) $\lambda = 1.3$. The curves A, B, and E are only distinguishable upon magnification. The (red) fitted lines indicate normal diffusion (lower line) and superdiffusion (upper line).

($\alpha \approx 2.1$) shows that the motion is superdiffusive over a number of decades. In fact, superdiffusion persists for the entire simulation in the case of $\lambda = 0.67$. This is not the case for $\lambda = 1$, where, after a number of decades, there is no diffusion. The reason for this is that in phase space, when $\lambda = 1$, only a single period 1 attractor exists, meaning that eventually all initial conditions evolve to this attractor. Thus, each trajectory undergoes the same motion resulting in the rate of diffusion becoming zero. In contrast, for $\lambda = 0.67$ there exists three attractors in phase space—a period 1 and a period 2 attractor exhibiting downhill motion, and a period 1 orbit exhibiting uphill motion. These counterpropagating attractors yield the superdiffusive motion (remember diffusion here is ensemble averaged).

V. SUMMARY

We have studied the driven and damped dynamics of single particles evolving in a tilted periodic and symmetric potential (the tilt being induced by a static dc-bias force). Unlike previous studies of such systems where the damping coefficient remains constant, the system explored here contains a damping coefficient that is space dependent. It has been shown that introducing a frictional inhomogeneity can result in some interesting dynamics, most notably being the appearance of absolute negative mobility, i.e., solutions that run against an external load.

In more detail, with a zero dc bias, a phase difference between the equally periodic potential and nonuniform damping breaks a spatial symmetry of the system and allows for the emergence of a nonzero current, as can be seen in Fig. 2. Increasing the (positive) dc bias from zero has two unexpected results. First, the presence of the frictional inhomogeneity allows the particle to work against a significant load up to a critical value of the dc bias $F = F_c$. Second, the current-response behavior as a function of the inhomogeneity parameter λ and dc-bias value F is quite remarkable. As F is increased from $F = 0$ the window of λ values exhibiting ANM increases to almost three times its $F = 0$ size, before shrinking to zero at $F = F_c$.

In addition, a heuristic explanation of the underlying mechanism producing such solutions has revealed that the uphill motion relies on the space-dependent damping, and not just the frequency of the driving, in contrast to previous studies. Further analysis has revealed that the uphill motion is superdiffusive, at least on intermediate time periods, whereas the downhill motion exhibits normal diffusion.

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