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Intermittent explosions of dissipative solitons and noise-induced crisis

Jaime Cisternas* and Orazio Descalzi
Complex Systems Group, Facultad de Ingeniería y Ciencias Aplicadas, Universidad de los Andes, Avenida Monseñor Alvaro del Portillo 12455, Las Condes, Santiago, Chile

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Dissipative solitons show a variety of behaviors not exhibited by their conservative counterparts. For instance, a dissipative soliton can remain localized for a long period of time without major profile changes, then grow and become broader for a short time—explode—and return to the original spatial profile afterward. Here we consider the dynamics of dissipative solitons and the onset of explosions in detail. By using the one-dimensional complex Ginzburg-Landau model and adjusting a single parameter, we show how the appearance of explosions has the general signatures of intermittency: the periods of time between explosions are irregular even in the absence of noise, but their mean value is related to the distance to criticality by a power law. We conjecture that these explosions are a manifestation of attractor-merging crises, as the continuum of localized solitons induced by translation symmetry becomes connected by short-lived trajectories, forming a delocalized attractor. As additive noise is added, the extended system shows the same scaling found by low-dimensional systems exhibiting crises [J. Sommerer, E. Ott, and C. Grebogi, Phys. Rev. A **43**, 1754 (1991)], thus supporting the validity of the proposed picture.

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I. INTRODUCTION

In nonlinear systems, the onset of chaotic behaviors is usually accompanied by some form of intermittency [1]: a more or less regular dynamics that becomes punctuated by short bursts of different, wilder behavior. For instance, in fluid experiments, the laminar state is interrupted by short-lived turbulent currents that break the order and symmetry of the flow. The durations of these bursts are not all equal, but are highly irregular. Intermittent bursts can also interrupt chaotic behavior. Such dynamics can be described as a sudden enlargement or destruction of the chaotic attractor, and has received the name *crisis* [2].

Dissipative solitons modeled by the complex Ginzburg-Landau (CGL) equation can show chaos and intermittency [3–5]. In particular, explosions are irregular periods of rapid growth that are followed by sudden collapse to the initial profile. They have been studied by many types of theoretical methods and also found experimentally in the context of nonlinear optics [6].

As this article shows, the mechanism behind the onset of explosions is an instance of crisis: the chaotic attractors that represent the localized solitons at different points in space become connected by trajectories and now form a single delocalized attractor, within which the system wanders ergodically.

The analogy can be further expanded using additive noise, which forces the system to escape from the attractors. Noise-induced explosions were reported by the authors in [7]. As an outcome of the addition of noise, the distribution of times between explosions becomes wider and more concentrated at smaller times. As in low-dimensional cases, there is a scaling relation between characteristic time, distance to criticality, and noise intensity [8]. Therefore, noise intensity can play a similar role to distance to criticality.

*jecisternas@miuandes.cl

II. THE MODEL

As discussed in Sec. I, we are interested in the dynamics of dissipative solitons. We will use the one-dimensional complex Ginzburg-Landau equation with cubic and quintic terms for the complex amplitude $A(x,t)$. As it is normally written in nonlinear optics,

$$\begin{aligned} iA_t + \frac{\tilde{D}}{2}A_{xx} + |A|^2A + \tilde{\nu}|A|^4A \\ = i\tilde{\delta}A + i\tilde{\varepsilon}|A|^2A + i\tilde{\beta}A_{xx} + i\tilde{\mu}|A|^4A, \end{aligned} \quad (1)$$

where $|A|^2 = A^*A$, and t,x are the propagation and spatial variables, respectively. This equation is invariant under the following transformations: spatial reflection $A(x,t) \to A(-x,t)$; spatial translation $A(x,t) \to A(x+\Delta,t)$; and phase rotation $A(x,t) \to A(x,t)\exp(i\theta),(\Delta,\theta \in \mathbb{R})$. Equation (1) has been proposed as a model of passively mode-locked lasers (see, for instance, the review by Grelu and Akhmediev [9] and its references). Parameters $\tilde{D},\tilde{\nu},\tilde{\delta},\tilde{\varepsilon},\tilde{\beta},\tilde{\mu}$ are all real and constant, and reflect the properties of the nonlinear media as well as the injection and the dissipation of energy. This equation has proved to be a valuable model, in particular in the study of dissipative solitons. In recent years, it has shown a variety of very complex phenomena [3,4,10], some of them observed in experiments [6].

Now we consider a more convenient way to write Eq. (1) in a form that is more familiar to hydrodynamics,

$$\partial_t A = D\partial_x^2 A + \mu A + \beta|A|^2A + \gamma|A|^4A, \quad (2)$$

with parameters that are directly related to those of Eq. (1): $D = \tilde{\beta} + i\tilde{D}/2, \mu = \tilde{\delta}, \beta = \tilde{\varepsilon} + i, \gamma = \tilde{\mu} + i\tilde{\nu}$, which are constant and homogeneous. All of these parameters are complex with the exception of μ, which measures the distance to the onset of linear instability (also known as linear growth rate) that is going to be used as our main bifurcation parameter.

For the purpose of numerical integration, we work on a periodic domain,

$$x \in [-L/2, L/2),$$

with L large enough so the solutions remain truly localized at all times. Any other choice of boundary conditions would be equally effective as long as the soliton stays far from the boundaries (the effect of boundary conditions on localized solutions was studied in [11]).

Localized and symmetric initial profiles without phase singularities were used:

$$A(x,0) = A_0(x)$$

(the effect of initial conditions with phase singularities has been studied in [12]).

There are wide regions in the space of parameters (μ, D, β, γ) where stable localized solutions exist. Such localized solutions have tails that decay to zero at least exponentially fast, so their properties are essentially independent of the domain size L or the boundary conditions.

We will focus on two quantities that will be essential in the characterization of the solitons: the "energy" of the soliton,

$$Q(t) \stackrel{\text{def}}{=} \int_{-L/2}^{L/2} |A(x,t)|^2 dx, \tag{3}$$

and the coordinate of the "center of mass" of the soliton,

$$x_{\text{cm}}(t) \stackrel{\text{def}}{=} \frac{L}{2\pi} \arg\left(\frac{\int_{-L/2}^{L/2} |A(x,t)|^2 \exp\left(\frac{i2\pi x}{L}\right) dx}{\int_{-L/2}^{L/2} |A(x,t)|^2 dx} \right). \tag{4}$$

This definition respects the periodicity of the domain and works also when the background [the value of $A(x,t)$ away from the core of the soliton] is nonzero.

As reported in Refs. [13,14], the quantities Q, x_{cm} will exhibit constant, oscillatory, quasiperiodic, or intermittent behaviors depending on the value of parameter μ. In particular for generic explosions, the coordinate of the center of mass will show rapid jumps followed by long periods of time with very little or no changes. In two spatial dimensions, localized structures exhibiting similar phenomena have recently been reported [15].

Other works, most notably [5], have pushed the idea that explosions are essentially a manifestation of low-dimensional phenomena, such as Shilnikov bifurcation. Although the picture presented here is different, we do use a low-dimensional description based on Q, x_{cm}.

The strategy followed in the next section is the study of the onset of explosions when parameter μ is inside the neighborhood of the critical value μ_c.

III. NUMERICAL SCHEME

The one-dimensional CGL equation was integrated from a localized initial condition $A_0(x)$ (basically a Gaussian profile) using a split-step Fourier method. With the exception of μ, all of the other parameters were kept fixed in all of the simulations reported in this article. The size of the domain L was chosen large enough so that the amplitude was identically zero outside the core of the dissipative soliton, and thus its tails do not interact (no finite box effects).

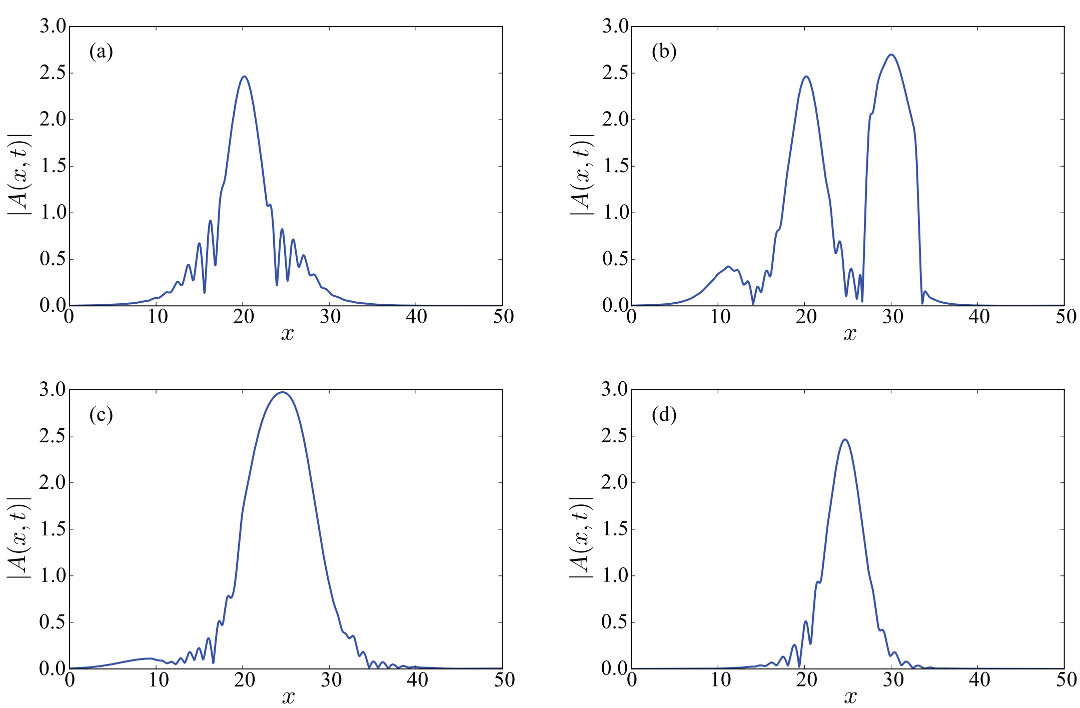

FIG. 1. (Color online) Evolution of the profile of a dissipative soliton exhibiting a typical strong asymmetric explosion. The parameter $\mu = -0.2137$ was chosen slightly larger than the critical value $\mu_c = -0.213\,75$. Other parameters are specified in the text. Four snapshots are shown: (a) the soliton begins exhibiting oscillating tails, (b) one of the tails grows and develops a secondary peak, (c) the two peaks coalesce and form a taller and broader symmetric pulse, and (d) the large pulse decays and becomes a soliton similar to the original profile but shifted in space.

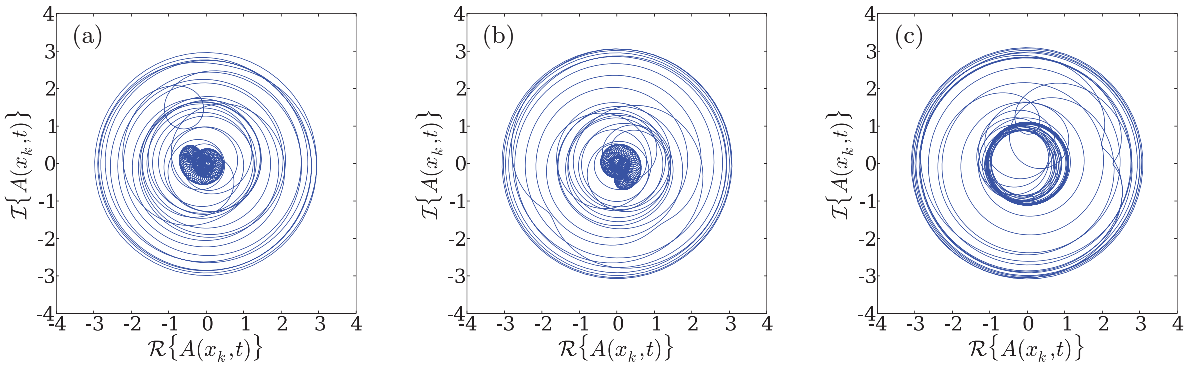

FIG. 2. (Color online) Phase-plane representation of the time evolution of three different explosions (a)–(c), at a single point in space $x_k \approx 20$ and for a fixed parameter $\mu = -0.2130$. Real and imaginary parts of the amplitude $A(x_k,t)$ at that point x_k show, for all of the explosions, a common pattern: a small meandering oscillation of frequency 2.6 Hz, then a sudden growth to a larger oscillation of radius $|A| \approx 3.2$ and frequency 5.6 Hz, followed by a decay to another small oscillation. Maxima differ because the point x_k is not always located where the explosions reach their peaks.

The simulations were carried out using a 1024-node spatial grid of size $dx \approx 0.05$. The time discretization was $dt = 0.005$ and the runs typically involved 2×10^6 iterations, so the total time was of the order of $T = 1 \times 10^4$, including several thousands of explosions in each run. By removing the first half of the time series, we checked that our results were not contaminated by transients.

We also checked that the spatial grid was fine enough and that the results were not artifacts of the spatial and temporal discretizations. First, we verified that the spatial power spectrum converged exponentially fast at all times, thus indicating that finer structures (not captured in the current discretization) were not relevant. Second, we repeated some of the simulations using a finer spatial discretization (the number of grid points was doubled, so dx was reduced to a half and dt was consequently decreased to a fourth to enforce numerical stability), resulting only in tiny changes in the location of the transitions.

Although for some specific cases we used other localized initial shapes, and obtained basically the same results, no systematic effort was developed to find other stable solutions; hence we could not say that the explosive solitons are the only stable solutions.

IV. EXPLOSIONS

Explosions of dissipative solitons were found numerically by Akhmediev's and Soto-Crespo's groups several years ago and experimentally by Cundiff *et al.* [6] more recently. In general, they appear for negative and small μ.

For $\beta = 1 + 0.8i, \gamma = -0.1 - 0.6i$, and $D = 0.125 + 0.5i$ (this choice is held throughout the article), explosions are observed, $\mu_c < \mu < 0$, with $\mu_c = -0.213\,75$.

The details of the transition have been published elsewhere [13,14] and for $\mu > \mu_c$ can be described as a sequence of the following behaviors: Fig. 1(a) shows the initial dissipative soliton, the base or silent state, with tails that oscillate wildly; Fig. 1(b) shows a secondary peak that grew from the right tail, breaking the left-right symmetry; Fig. 1(c) shows a short-lived, large, roughly symmetric pulse that appears when the two peaks nucleate; and Fig. 1(d) shows the final dissipative soliton, shifted in space with respect to the initial profile, but otherwise similar. The explosions repeat again and again at irregular times, even in the absence of noise. As far as the outcomes of our simulations have shown, the above-mentioned picture is generic. Perfectly symmetric explosions have only been observed transiently, and are eventually followed by asymmetric explosions [5]. For parameter μ closer to zero, double or *almost symmetric* explosions have been observed with both tails growing at roughly the same time [16].

Of particular relevance in the present context is Fig. 1(c), which depicts a large pulse of maximum amplitude, $|A| \approx 3$. Similar large transient pulses have been shown in [5, Fig. 1] and [16, Fig. 8(c)]. They show that the explosive growth is not unbounded, but approaches a well-defined state: for a given set of parameters, the tall pulses achieved during the explosions were consistently similar in height, width, and duration. We claim that in Fig. 1(c), the state $A(x,t)$ is in the neighborhood of an *unstable periodic orbit*. In Fig. 2, representations of the complex oscillations are depicted for three explosions, showing how the radius (for a fixed point in space) grows and apparently approaches an unstable orbit.

The onset of explosions as μ is increased is depicted in Fig. 3. For $\mu < \mu_c$ [see Fig. 3(a)], the soliton has oscillating tails (actually quasiperiodic) but does not explode, so its energy $Q(t)$ remains bounded. For $\mu \gtrapprox \mu_c$ [see Fig. 3(b)], the tails become weakly chaotic and so does $Q(t)$ [14]. For $\mu \gtrapprox \mu_c$ [see Fig. 3(c)], the soliton develops large symmetric oscillations that punctuate the regime of oscillating tails. After some transients (some thousands of time units), symmetric explosions are followed by strongly asymmetric explosions. For $\mu > \mu_c$ [see Fig. 3(d)], explosions become more frequent, but always irregular.

One of the typical effects of explosions is a shift in the location of the soliton. We tracked the center of mass of the soliton as defined by Eq. (4). Figure 4 shows the location of the center of mass of the soliton, as it switches during the fast explosions. Jumps make the soliton wander randomly in space, therefore in the long run the soliton location follows a "deterministic" diffusion [17,18]. Figure 5 considers a longer time horizon and several slightly different initial conditions with time evolutions that diverge and wander in the periodic spatial domain.

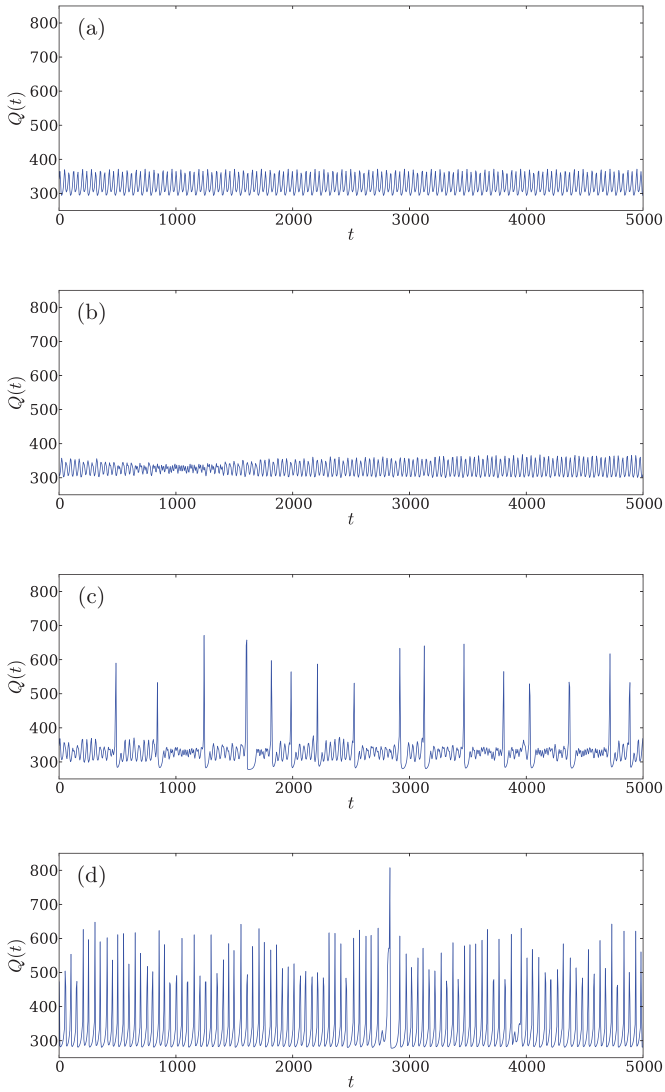

FIG. 3. (Color online) Time evolution of energy $Q(t)$, defined in Eq. (3), for four different values of parameter μ (other parameters take constant values specified in the text): (a) $\mu = -0.2140$, quasiperiodic, no explosions; (b) $\mu = -0.213\,85$, weakly chaotic, no explosions; (c) $\mu = -0.2137$, explosions at irregular intervals; and (d) $\mu = -0.2130$, frequent irregular explosions.

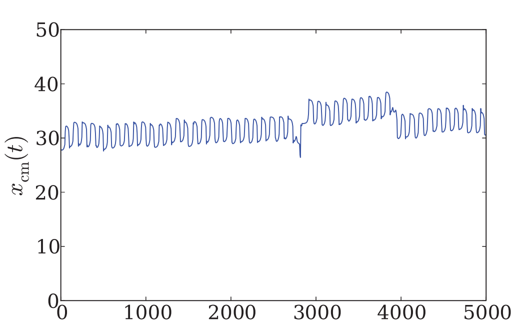

FIG. 4. (Color online) Evolution of the location of the center of mass of the dissipative soliton, defined in Eq. (4), in the regime of frequent explosions, for $\mu = -0.213$. Most explosions follow an alternating left-right-left-right pattern.

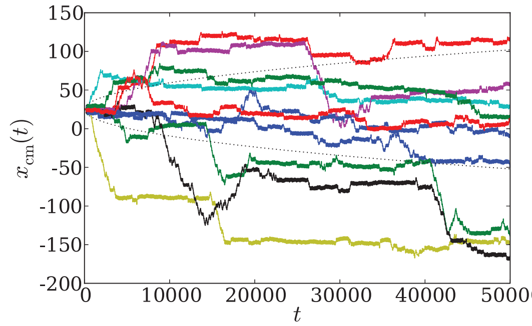

FIG. 5. (Color online) Trajectories of the center of mass for several nearly identical initial conditions, for linear parameter $\mu = -0.213$. Assuming normal diffusion, one can estimate a diffusion coefficient $D_{\rm cm} \approx 0.11$ from measurements of $\Delta x_{\rm cm}^2/\Delta t$. Trajectories in the periodic domain of size $L = 50$ were "unwrapped" to make their separation more apparent. Dotted curves represent $x_{\rm cm} = x_{\rm cm}(0) \pm \sqrt{D_{\rm cm} t}$.

The jumps experienced by the center of mass can be analyzed using the framework of continuous-time random walks (CTRW) introduced by Montroll and Weiss [19]. In its basic form, the theory of CTRW assumes that the size of the jumps and the waiting times between jumps are independent. In our case, both distributions have bounded moments: there are typical length (comparable to the width of the soliton) and time scales (see Fig. 6), so the motion of the center of mass is essentially diffusive, $|\Delta x_{\rm cm}|^2 \sim \Delta t$, and characterized solely by an effective diffusion coefficient $D_{\rm cm}$. In Fig. 5, we also plotted $x_{\rm cm} = x_{\rm cm}(0) \pm \sqrt{D_{\rm cm} t}$ with coefficient $D_{\rm cm}$ estimated from the samples. A more complete analysis of the diffusive motion of the soliton will be the subject of a forthcoming article.

The distribution of observed times depicted in Fig. 6 shows a large variability, induced by the weak chaos of the "silent" behavior and not by the explosion itself that, as we will see, has a fairly deterministic structure.

The characteristic time between explosions must clearly depend on the distance to the transition point. For μ slightly larger than $\mu_c = -0.213\,75$, explosions are very rare, and as values of μ get closer to zero, explosions become more and more frequent.

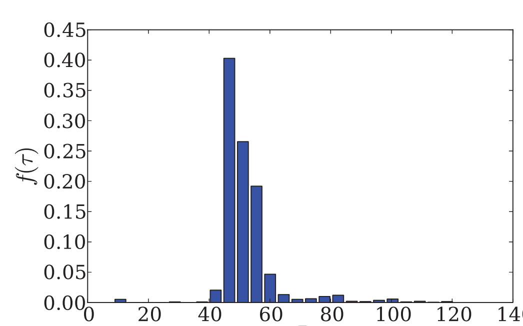

FIG. 6. (Color online) Distribution of times between explosions (residence times of *precrisis* attractor) for $\mu = -0.213$. This distribution is far from exponential and suggests that there is some memory effect induced by the structure of the attractor.

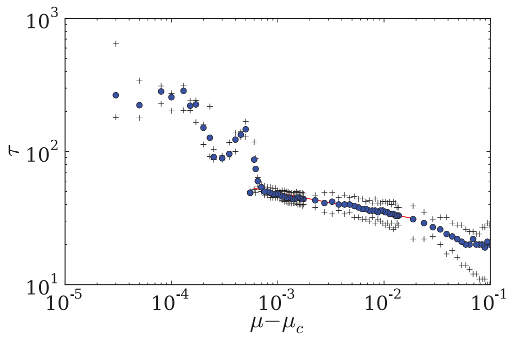

FIG. 7. (Color online) Characteristic time τ between explosions as a function of $\mu - \mu_c$, distance to the critical parameter $\mu_c = -0.213\,75$. The + symbols and the blue dot in each vertical line are the quartiles of the observed histogram for each value of μ. Very close to the onset of explosions $\mu \gtrapprox \mu_c$ (left-hand side of this figure), the histograms are rather broad and their medians do not follow a trend, but farther away from μ_c, a linear trend with slope $\alpha = 0.204$ can be identified (indicated by thin red line).

Collecting the statistics for a variety of μ values, one could verify in Fig. 7 that there are two main regions: immediately after the transition the characteristic times depend sensitively on μ in a nonmonotonic way, and for small but finite $\mu - \mu_c$, the expected time follows a power relation,

$$\tau \sim |\mu - \mu_c|^{-\alpha}, \tag{5}$$

at least in a region not too close to μ_c. As the distribution of times between explosions is far from Gaussian, we use the median as a measure of characteristic time τ.

In [5], the authors have proposed the idea that explosions are essentially a manifestation of low-dimensional phenomena such as Shilnikov bifurcation: a homoclinic bifurcation of a saddle-focus equilibrium state. The Shilnikov mechanism provides an explanation for the intermittency and chaos. However, in [14], we have shown that the "base" soliton is not a hyperbolic point, but is already a chaotic state. Also, typical explosions are strongly asymmetric, so after one explosion the soliton does not return to the original profile but to a shifted copy.

A more plausible mechanism behind the explosions could be a "bubbling" transition [20–22] that results from changes in the basin of a chaotic attractor that make the attractor transversally unstable and susceptible to small noise. Trajectories no longer remain in a small neighborhood of the attractor, but develop sporadic excursions away from it. On the contrary, our chaotic solitons do not require noise to explode [3–5,16]. Changes in the numerical precision of the time integration do not significantly shift the location of μ_c, nor the dynamics of the solitons before or after the transition. This fact gives support to the idea that the onset of explosions is a structural change of the chaotic attractor.

The bubbling transition can take place when an invariant manifold exists, typically induced by certain symmetry of the system. Assuming that in our case the invariant manifold where the chaotic attractor lies is induced by reflection symmetry, the departures from the attractor would be asymmetric and lead to shifts in the position of the soliton. However, the soliton is chaotic and instantaneously asymmetric, even before it becomes explosive. As shown in [14], the symmetric soliton loses stability in a Neimark-Sacker bifurcation, and a nonsymmetric soliton emerges with quasiperiodic and then chaotic tails (through a cascade of torus-doubling bifurcations), before the onset of explosions (as parameter μ is increased). Therefore the onset of explosions does not break the reflection symmetry, but only "magnifies" the instantaneous asymmetry of the chaotic soliton.

Equation (5) has been deduced heuristically for low-dimensional systems and suggests a connection with the *theory of crises* [2]. (For a comparison between crises and bubbling transitions, we refer the reader to [23].) The fit, although limited, suggests that localized structures may have many features of low-dimensional systems. Several limitations may hinder the quality of the fit. In the first place, infrequent explosions make it difficult to estimate τ reliably and therefore the identification of best-fit parameters becomes difficult. In the second place, close to μ_c, the system becomes very sensitive to the discretization and the choice of numerical scheme.

It is interesting to notice that even if characteristic times depend on the structure of the chaotic attractor, i.e., the dissipative soliton, the exponent α in Eq. (5) depends basically on the eigenvalues of the unstable periodic orbit [2]. Estimations of the critical exponent α from time series could be obtained using the procedure explained in [2] based on expanding and contracting eigenvalues of the mediating unstable periodic orbit [cf. Fig. 1(c)]. The expanding eigenvalue has already been estimated in [16] from the characteristic time it takes for the exploded soliton to collapse to its original profile.

The aforementioned theory of crises [2] may provide a mechanism of attractor destruction at μ_c responsible for the onset of explosions. The sudden changes (as parameter μ is varied) of the structure of the attractor (that in Ref. [14] was shown to be representable as a torus) can be explained by the theory to be collisions of the chaotic attractor with unstable periodic orbits or other attractors. The intermittency of the soliton explosions is precisely one of the signatures of crisis.

Three types of crises have been documented: attractor destruction, attractor enlargement, and attractor merging. In the first type, trajectories remain in the chaotic precritical attractor only during a transient period and then move somewhere else. For the second type, trajectories stay close to the precritical attractor for some time and then burst into larger chaotic motion before being reinjected into the precritical attractor. The evidence presented in this work indicates that the onset of explosions of dissipative solitons corresponds to the third type, *attractor merging*, that takes place in the presence of some symmetry, when two or more chaotic attractors simultaneously collide with one or more unstable periodic orbits.

In the case of explosions of dissipative solitons, *translation* symmetry allows the existence of an infinite number of radiating solitons (weakly chaotic attractors) connected by explosions (long trajectories) that pass close to large symmetric pulse (unstable periodic orbit). As it can be appreciated from Figs. 1 and 2, the explosions are fairly regular trajectories that go first along the stable manifold and then along the unstable manifold of an unstable periodic orbit, until they reach their final attractor. A more thorough study of the unstable periodic orbits using continuation techniques could potentially reveal subtle connections with other localized states and will be the subject of future research.

As both the numbers of precrisis attractors and unstable periodic orbits are infinite (every translated copy of a soliton solution is also a solution), a graphical representation of the merging is not as easy as in systems with only two attractors.

Crises have been found behind the onset of spatiotemporal chaos. Reference [24] presents a crisis that explains chaotic bursts in a reaction-diffusion system. Reference [25] shows a chaotic pulse that appears at the center of a finite domain with reflection symmetry. References [26–28] show a chaotic attractor captured by a truncated Fourier expansion of the Kuramoto-Sivashinsky equation. We believe the present work has the additional feature of capturing a crisis that results from the collision of an infinite number of chaotic attractors in the presence of a continuous symmetry.

In the context of symmetry-breaking transitions, explosions can be understood as an increase in the symmetry of a chaotic attractor [29]: a chaotic soliton that was reflection invariant *on average* becomes also translation invariant *on average* after the transition.

In the next section, we test the applicability of the crisis framework by adding weak noise to the CGL.

V. NOISE AND SCALING

Some level of noise is always present in physical phenomena. Numerical noise can also present a challenge in the study of physical models. For instance, in this work, we have shown how even when using a symmetric initial condition, in the long term the accumulative noise (induced by numerical discretization of space and time) can break the reflection symmetry and lead to asymmetric soliton explosions.

Now the addition of noise in a controlled manner, modeled by fluctuations in the parameters or as an additional term in the governing equation, can serve to emphasize the applicability of crises theory to the phenomena of soliton explosions. As proved by Sommerer *et al.* [8,30] (see also Arecchi *et al.* [31]), close to criticality, the characteristic times between explosions, the distance to the criticality, and the intensity of the added noise are connected through a generic scaling relation.

We will be studying the effects of additive noise,

$$\partial_t A = D\partial_x^2 A + \mu A + \beta|A|^2 A + \gamma|A|^4 A + \eta\xi, \quad (6)$$

with $\xi(x,t)$ being a complex white noise in space and time,

$$\langle \xi^*(x,t)\xi(x',t')\rangle = 2\delta(x-x')\delta(t-t')$$

and

$$\langle \xi(x,t)\xi(x',t')\rangle = 0.$$

Parameter η controls the intensity of the noise. In [7], it was shown how additive noise induces the soliton escape from its chaotic attractor via explosive trajectories (see [14] for a characterization of the soliton as a chaotic attractor) even before criticality. That work suggested the use of η as a new bifurcation parameter.

A first effect of noise is the increased "porosity" of the precrisis attractor [14]. For the noise-free scenario, depicted by Fig. 6, the state of the soliton followed trajectories of certain duration before making large excursions, i.e., explosions. Now in the presence of noise, points inside the attractor are all equally likely at any time to lead to excursions, so the residence times τ become exponentially distributed, as indicated in Fig. 8.

Sommerer *et al.*, in [8,30], proposed a scaling relation for the typical residence time,

$$\tau \sim \eta^{-\alpha} g\left(\frac{\mu-\mu_c}{\eta}\right), \quad (7)$$

where η is the noise intensity and $g(\cdot)$ is a nonuniversal function that depends on the system and the distribution of the noise. The exponent α is the same one that appears in Eq. (5).

Several sets of simulations are summarized in Fig. 9(a). As we plot $(\mu-\mu_c)/\eta$ versus $\tau\eta^\alpha$ in Fig. 9(b), points for different values of η should fall on a single curve that gives the graph of the function $g(\cdot)$.

Although the fit of the relations deduced by Sommerer *et al.* to the intermittent onset of explosions is only approximate, it indicates that the general picture of crises theory is correct.

A number of explanations behind the lack of a more precise fit between the restricted scaling of Eq. (7) and our numerical results come to mind. First of all, one should mention that the present model is an extended system, so any analogy with low-dimensional systems should be qualified. Sommerer *et al.* [8,30] mentioned other several possible complications that

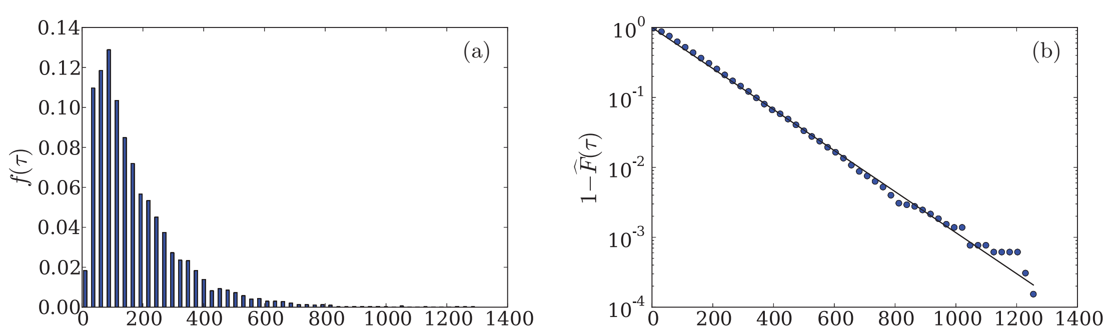

FIG. 8. (Color online) (a) Distribution of times between explosions for $\mu = -0.2200, \eta = 0.001$. Compare with the noiseless results in Fig. 6. (b) The cumulative distribution function $\hat{F}(\tau) = \int_0^\tau f(t)dt$ clearly shows a decay characteristic of an exponential distribution: noise smears the details of the chaotic precrisis attractor and induces explosions at a constant rate.

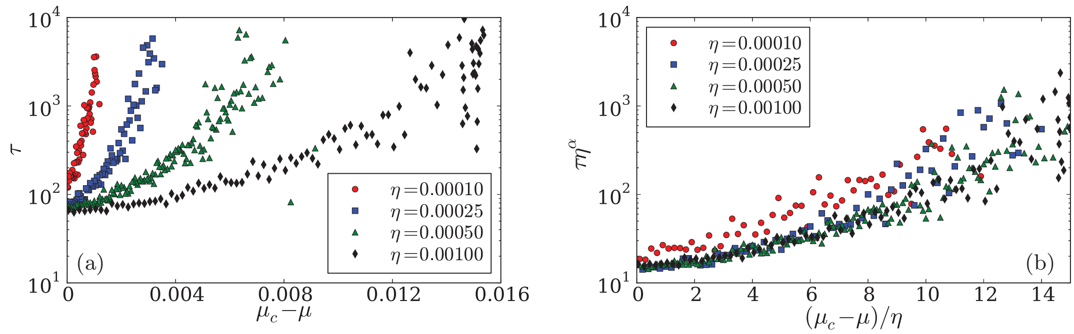

FIG. 9. (Color online) (a) Characteristic time between explosions τ as a function of parameter μ and noise intensity η. (b) Scaled representation suggested by Eq. (7) for $\mu_c = -0.213\,75, \alpha = 0.204$. Most of the points fall within the graph of a nonuniversal function $g(\cdot)$ as predicted by Eq. (7).

may render the scaling an oversimplification: true data points may have a large dispersion, and distribution of times may not be exponential; there could also be other competing routes of escape besides attractor merging. And even if the scaling is correct, it is not easy to estimate correctly α and μ_c, so the scaling would always seem approximate (one could make changes to μ_c and α in order to improve the fit, but this choice would obviously make the whole approach meaningless).

Nonetheless, we believe that the overall picture provided by crises theory is correct: dissipative solitons show some weak chaos before the onset of explosions, and explosions appear when two or more of these attractors merge; explosions transiently approach a large symmetric pulse that corresponds to an unstable periodic orbit; intermittency is a signature of the crisis; the anticipation of explosions by noise is an instance of noise-induced crisis; and there exists a nontrivial scaling between interexplosion times, noise intensity, and distance to criticality.

The addition of noise makes clear that the mechanism behind intermittent explosions is the same transition to chaos observed in many other chaotic low-dimensional systems.

VI. CONCLUSIONS

In previous works, we have studied the dynamics of dissipative solitons and their transitions between stationary, periodic, quasiperiodic, and then weakly and strongly chaotic regimes, in which explosions play a major role. In this work, the main focus was the onset of explosions and its intermittent character, with and without external noise.

This intermittency was presented as a strong suggestion of the presence of an attractor-merging crisis. This phenomenon appears when the multiple chaotic attractors induced by translational symmetry collide and merge. The resulting attractor restores the translational symmetry and includes the precritical attractors (localized solitons) and the explosion trajectories.

This mechanism was emphasized by the controlled addition of noise. A scaling relation between the characteristic time between explosions, noise intensity, and distance to criticality was found. This relation was deduced in the context of crises theory, and its verification in the case of soliton explosions gives another compelling evidence to the hypothesis that explosions result from interactions of chaotic attractors.

Finally, although rich and complex, the explosive behavior of dissipative solitons can be effectively described using the theoretical framework of low-dimensional systems.

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[1] Y. Pomeau and P. Manneville, Comm. Math. Phys. **74**, 189 (1980).
[2] C. Grebogi, E. Ott, F. Romeiras, and J. A. Yorke, Phys. Rev. A **36**, 5365 (1987).
[3] J. M. Soto-Crespo, N. Akhmediev, and A. Ankiewicz, Phys. Rev. Lett. **85**, 2937 (2000).
[4] N. Akhmediev, J. M. Soto-Crespo, and G. Town, Phys. Rev. E **63**, 056602 (2001).
[5] N. Akhmediev and J. M. Soto-Crespo, Phys. Rev. E **70**, 036613 (2004).
[6] S. T. Cundiff, J. M. Soto-Crespo, and N. Akhmediev, Phys. Rev. Lett. **88**, 073903 (2002).
[7] C. Cartes, O. Descalzi, and H. R. Brand, Phys. Rev. E **85**, 015205(R) (2012).
[8] J. C. Sommerer, E. Ott, and C. Grebogi, Phys. Rev. A **43**, 1754 (1991).
[9] P. Grelu and N. Akhmediev, Nat. Photon. **6**, 84 (2012).
[10] W. Chang, A. Ankiewicz, N. Akhmediev, and J. M. Soto-Crespo, Phys. Rev. E **76**, 016607 (2007).
[11] O. Descalzi and H. R. Brand, Phys. Rev. E **81**, 026210 (2010).
[12] O. Descalzi and H. R. Brand, Phys. Rev. E **72**, 055202(R) (2005).
[13] O. Descalzi, C. Cartes, J. Cisternas, and H. R. Brand, Phys. Rev. E **83**, 056214 (2011).
[14] J. Cisternas, O. Descalzi, and C. Cartes, Cent. Eur. J. Phys. **10**, 660 (2012).
[15] C. Cartes, J. Cisternas, O. Descalzi, and H. R. Brand, Phys. Rev. Lett. **109**, 178303 (2012).
[16] O. Descalzi and H. R. Brand, Phys. Rev. E **82**, 026203 (2010).

[17] T. Geisel and J. Nierwetberg, Phys. Rev. Lett. **48**, 7 (1982).
[18] T. Geisel and J. Nierwetberg, Phys. Rev. A **29**, 2305 (1984).
[19] E. Montroll and G. Weiss, J. Math. Phys. **6**, 167 (1965).
[20] P. Ashwin, J. Buescu, and I. Stewart, Phys. Lett. A **193**, 126 (1994).
[21] S. C. Venkataramani, B. R. Hunt, E. Ott, D. J. Gauthier, and J. C. Bienfang, Phys. Rev. Lett. **77**, 5361 (1996).
[22] S. C. Venkataramani, B. R. Hunt, and E. Ott, Phys. Rev. E **54**, 1346 (1996).
[23] G. Kubo, R. Viana, S. Lopes, and C. Grebogi, Phys. Lett. A **372**, 5569 (2008).
[24] J. Elezgaray and A. Arneodo, Phys. Rev. Lett. **68**, 714 (1992).
[25] M. Sauer, J. Leonardy, and F. Kaiser, Chaos Solitons Fract. **4**, 1461 (1994).
[26] M. Münkel and F. Kaiser, Physica D **98**, 156 (1996).
[27] J. Moehlis and E. Knobloch, Physica D **151**, 99 (2001).
[28] E. L. Rempel and Abraham C.-L. Chian, Phys. Rev. E **71**, 016203 (2005).
[29] P. Chossat and M. Golubitsky, Physica D **32**, 423 (1988).
[30] J. C. Sommerer, W. L. Ditto, C. Grebogi, E. Ott, and M. L. Spano, Phys. Rev. Lett. **66**, 1947 (1991).
[31] F. Arecchi, R. Badii, and A. Politi, Phys. Lett. A **103**, 3 (1984).