Spontaneous scale-free structure in adaptive networks with synchronously dynamical linking

Wu-Jie Yuan,[1,2] Jian-Fang Zhou,[1] Qun Li,[1] De-Bao Chen,[1] and Zhen Wang[2,3,*]
[1]*College of Physics and Electronic Information, Huaibei Normal University, Huaibei 235000, China*
[2]*Department of Physics, Hong Kong Baptist University, Kowloon Tong, Hong Kong*
[3]*Center for Nonlinear Studies and Beijing–Hong Kong–Singapore Joint Center for Nonlinear and Complex Systems (Hong Kong), Institute of Computational and Theoretical Studies, Hong Kong Baptist University, Kowloon Tong, Hong Kong*
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Inspired by the anti-Hebbian learning rule in neural systems, we study how the feedback from dynamical synchronization shapes network structure by adding new links. Through extensive numerical simulations, we find that an adaptive network spontaneously forms scale-free structure, as confirmed in many real systems. Moreover, the adaptive process produces two nontrivial power-law behaviors of deviation strength from mean activity of the network and negative degree correlation, which exists widely in technological and biological networks. Importantly, these scalings are robust to variation of the adaptive network parameters, which may have meaningful implications in the scale-free formation and manipulation of dynamical networks. Our study thus suggests an alternative adaptive mechanism for the formation of scale-free structure with negative degree correlation, which means that nodes of high degree tend to connect, on average, with others of low degree and vice versa. The relevance of the results to structure formation and dynamical property in neural networks is briefly discussed as well.

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I. INTRODUCTION

Many real-world systems, such as biological metabolic uptakes and information transmission in society, can be featured by a complex network [1,2]. To explore its basic mechanism, a great number of studies have been proposed from theoretical viewpoints and application of technology [3–11]. In particular, the adaptive network (AN), which is a dynamical entity with interplay between topology structure and dynamics, attracted great interest in recent years [12,13]. Along this line, some scholars began to focus on the synchronization by considering the coevolution of dynamical states and topological structures in the AN [14–21]. A typical example is to identify schemes of enhancing or stabilizing synchronization via changing the coupling strengths [16–20,22–25].

Moreover, since real systems grow by adding new connections [1,26–29], the dynamical features of a complex network have also been extensively studied under the growth model by the preferential attachment mechanism [30–34]. However, the addition of links in connection with synchronization in AN has been investigated little (namely, the feedback of network dynamics has not been considered) [35–38]. Thus, it becomes of particular interest to expect that the growth of links may have a certain adaptive relationship with dynamical synchronization behavior [39–41]. For example, Ref. [39] showed that the locking process was associated with the emergence of a scale-free degree distribution in the network connectivity. Reference [40] further unveiled that the emergence of modular and scale-free structures was associated with a striking enhancement of local synchronization. Motivated by the anti-Hebbian learning rule in neural systems, i.e., the synaptic links (or coupling strengths) are established (or strengthened) between two neurons when asynchronous states occur between them [42–45], in the present work, we propose a simple yet generic adaptive model to achieve global synchronization of the network. We investigate the spontaneous structure in the adaptive model and how structure and dynamics coevolve during the course of achieving synchronization. By means of systematic and scientific investigation, we show that (1) when global synchronization (i.e., the same dynamical process for all the nodes) is achieved, the AN spontaneously forms a scale-free structure and (2) the adaptive process produces two nontrivial power-law behaviors of average dynamical deviation from mean activity of the network and negative degree correlation. Moreover, we show that the vertices whose dynamics approaches the mean activity of the network ultimately become hub nodes. In the remainder of this paper we will first describe the adaptive model; subsequently, we will present the main results of numerical simulations, and finally we will summarize our conclusions.

II. ADAPTIVE MODEL

We consider N coupled identical chaotic oscillators, and the state of oscillator i can be expressed as follows:

$$\dot{X}_i = F(X_i) + c\sum_{j=1}^{N} G_{ij}[H(X_j) - H(X_i)], \quad (1)$$

where $F(X)$ denotes the dynamics of an individual oscillator, $H(X)$ is the coupling function, c represents the coupling strength, and $G = (G_{ij})$ is the coupling matrix. If G_{ij} is adopted directly by the binary adjacency matrix $A = (A_{ij})$, the possibly resulting heterogeneity in the degree distribution will suppress synchronization [46]. In order to enhance the synchronizability of the possibly resulting heterogeneous network, G_{ij} is normalized by the degree k_i of node i, as adopted in Ref. [7]. Namely, $G_{ij} = A_{ij}/k_i$ for $k_i \neq 0$; otherwise, $G_{ij} = 0$. Actually, the normalization will not affect the following qualitative results. In the AN, the adjacency matrix A is determined by the collective synchronization

*Corresponding author: zhenwang0@gmail.com

properties of the network. Thus, A is a matrix with time-dependent connection [i.e., $A_{ij}(t)$].

To achieve global synchronization of the network, it is reasonable to assume that each node tries to synchronize with the mean field of the network by increasing its connections. We assume that the nodes whose dynamics is far from the mean field are more likely to be linked. With respect to the adaptive rule of link growth, the following mechanisms exist.

(i) *Growth*. Start from a nonconnected network with an initial dynamical condition which is randomly chosen from the chaotic attractor. At every time step h, two random nodes i and j are selected if no link exists between them.

(ii) *Preferential attachment*. The probability of producing a new connection between nodes i and j is proportional to the rate of their dynamical differences with the mean activity of the network, namely,

$$p = \frac{\Delta_i}{\Delta_j} / \sum_{i,j} \frac{\Delta_i}{\Delta_j} \quad (\Delta_i \geqslant \Delta_j), \tag{2}$$

where $\Delta_i = \langle |H(X_i) - \frac{1}{N}\sum_{k=1}^{N} H(X_k)| \rangle$ and $\Delta_j = \langle |H(X_j) - \frac{1}{N}\sum_{k=1}^{N} H(X_k)| \rangle$. Here, $\langle \cdot \rangle$ denotes the average over the time scale h of making connections. In Eq. (2), we take the node with the large dynamical difference as node i and take the other node as node j, and hence, $\Delta_i \geqslant \Delta_j$.

Based on the above rule, it is clear that the node whose dynamics is far from the mean activity of the network is more likely to connect with a node whose dynamics approaches the mean activity, which is very consistent with the behavior characteristics in real-world systems, such as technological and biological networks [47,48]. This means that the two nodes are likely to form a link if one node is synchronous and the other node is asynchronous (comparing with the mean field). In the sense of dynamical difference from the mean field, the adaptive rule can be regarded as an extension of the anti-Hebbian learning rule in neural systems, which could make a contribution to the complete hierarchical synchronization in a heterogeneous network [16]. Indeed, many extensions of the original anti-Hebbian learning rule have been experimentally found in real neural systems [42–45], and today, the anti-Hebbian learning rule is often rephrased [49]. In addition, motivated by the Barabási-Albert scale-free network model [30,31], we expect that our adaptive scheme can spontaneously produce a heterogeneous scale-free structure due to hierarchical synchronization within the anti-Hebbian linking rule.

In general, given a complex network with a fixed number of vertices, its synchronizability can be improved by increasing the number of edges [6]. In addition, the synchronizability depends strongly on coupling strength and coupling function [50–53]. Here, we consider that the adjacency matrix A is always symmetrical, namely, $A_{ij}(t) = A_{ji}(t)$. The matrix A_{ij} is adaptively acquired with one large enough coupling strength c, which can lead to global synchronization in the dynamical network. To broaden the region of coupling strength c leading to the global synchronization [16], the coupling function $H(X) = X = (x,y,z)$ is adopted in the present work. In our model, when the system is synchronizable, it will approach the global synchronization state for many large networks. This general character could be illustrated by a chaotic

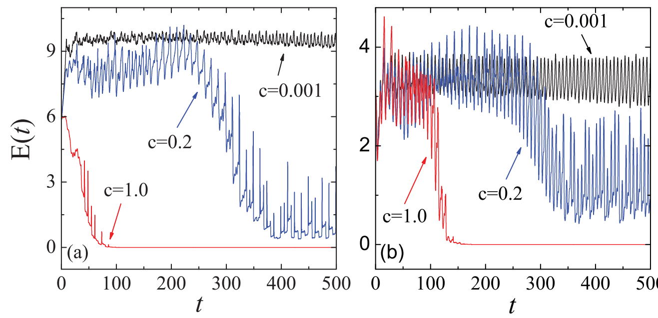

FIG. 1. (Color online) The synchronization error $E(t)$ for different coupling strengths c, indicating transition to synchronization in the ANs of (a) Rössler oscillators and (b) the food web model. Depicted results are obtained for $N = 1000$ and $h = 0.02$. Here, for simplicity, we suppose that the new connection is always added between the nodes with the maximum difference $\Delta_{\max}$ and the minimum difference $\Delta_{\min}$ if the connection does not already exist; otherwise, there is no new connection at this time step.

Rössler oscillator, $X = (x,y,z)$ and $F(X) = [-0.97y - z, 0.97x + 0.15y, z(x - 8.5) + 0.4]$, and a chaotic food web model, $F(X) = [x - 0.2xy/(1 + 0.05x), -y + 0.2xy/(1 + 0.05x) - yz, -10(z - 0.006) + yz]$, whose chaotic diagrams are similar [54]. Irrespective of either form for $F(X)$, we believe that qualitatively similar behavior would be displayed.

III. RESULTS AND ANALYSIS

In this work, we use the fourth Runge-Kutta method with a time step equal to 0.02. When the synchronization error $E(t)$ satisfies the condition $E(t) = \langle |X_i - \langle X_i \rangle| \rangle < 10^{-8}$ (where $\langle \cdot \rangle$ denotes the average over the whole network), the global synchronization is realized [55–57]. We first inspect how the synchronization behavior depends on the coupling strength in the network. Figure 1 features the characteristic results obtained for different values of coupling strength c. As evidenced in both panels, for small value of c the appearance of synchronization in ANs is very difficult. However, with increasing coupling strength, both oscillator models can reach global synchronization, especially for large value of c. These results thus suggest that the strong coupling is beneficial for the synchronization of networks. In what follows we will further examine the characteristics of the synchronization state.

Results presented in Fig. 2 depict the generated network structure under the state of global synchronization. Clearly, Fig. 2(a) shows, in the region with a large degree, that the network coevolves spontaneously into a heterogeneous structure with the same power-law degree distribution $P(k) \sim k^{-\gamma}$ for both oscillator models. This is because the network gradually achieves global synchronization under our proposed connections rule (see Fig. 1). Importantly, this scaling is robust to different network sizes N, as demonstrated in Fig. 2(b), indicating that the system organizes itself into a scale-free stationary state similar to the model of Barabási *et al.* [30,31]. Note that we show the degree distribution corresponding to a nonsynchronous state ($c = 0.001$; see Fig. 1) in the inset of Fig. 2(a). Clearly, the distributions are exponential for both Rossler and food web models. This exponential decrease is faster than power-law depression with the increase of the

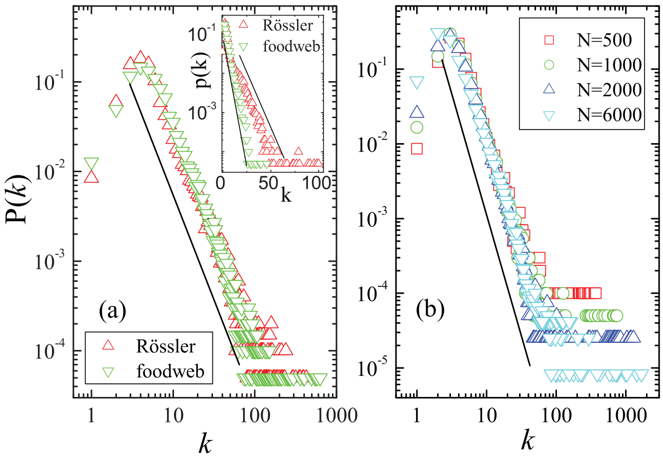

FIG. 2. (Color online) (a) Degree distributions $P(k)$ as a function of k in a log-log plot for our ANs of Rössler (upward triangles) and food web (downward triangles) models at $N = 1000$ and $h = 0.02$ and (b) their dependence on different network size N at $h = 0.2$. Results in (a) and (b) are averaged over 20 realizations of the ANs with random initial conditions. For clarity, only the results of Rössler oscillators are shown in (b). The solid lines in (a) and (b) are linear fittings, which respectively confirm their scale-free properties. Depicted results are obtained for $c = 1.0$, which can lead to global synchronization of the network (see Fig. 1). For comparison, the inset in (a) gives the degree distributions $p(k)$ in a linear-log plot for our ANs of Rössler (upward triangles) and food web (downward triangles) models at $N = 1000$, $h = 0.02$, and $c = 0.001$, which cannot lead to global synchronization of the network (see Fig. 1). The solid lines in the inset in (a) are linear fittings, which confirm their exponential properties.

degree. This is because the selected nodes for each connection are more random under the nonsynchronous state. Therefore, the resulting network is less heterogeneous. Namely, the degree distribution demonstrates a faster decrease with the degree.

Next, we focus on the property of power-law degree distribution corresponding to the global synchronization state of the network. It is found that the degree distribution exponent γ depends on two parameters: coupling strength c and time step h of making connections. One can see from Fig. 3 that the exponent γ monotonously increases with the coupling strength c. This can be attributed to the fact that the oscillators achieve faster synchronization under stronger coupling (see Fig. 1 for better understanding). In this sense, the nodes with the minimum difference $\Delta_{\min}$ are often selected to build connections with those hub nodes, which in turn accelerates the formation of a more heterogeneous structure. However, with continuous increase of coupling strength c, the exponent γ will reach a saturation value due to the saturation of the synchronization velocity under sufficiently large values of c. Moreover, we show in Fig. 3 that the value of γ at large h is greater than that of small h. This is because, for a large linking time step h, a more heterogeneous network is formed within the longer coevolution time for achieving global synchronization.

In the coevolutionary model, an interplay exists between the dynamical synchronous process and network topology. It thus becomes interesting to investigate how the final structure of the network depends on the synchronous precess in our work.

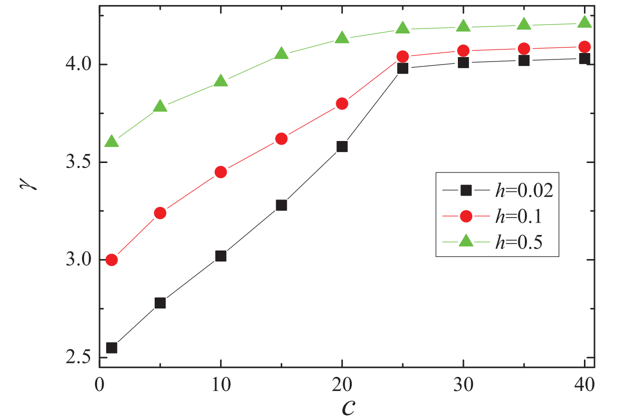

FIG. 3. (Color online) The degree exponent γ as a function of coupling strength c under different values of the linking time step h. Results are averaged over 20 independent realizations. Since the results for the Rössler oscillators and the food web model are nearly the same as shown in Fig. 2(a), we only show the results for the Rössler oscillators. The depicted results are obtained for $N = 1000$.

Here, we pay much attention to the dependence of degree k on the dynamical synchronous difference with the mean activity of the network (for the vertices possessing degree k). To simplify the discussion, we define an average dynamical difference as follows:

$$\langle \Delta(k) \rangle = \frac{1}{Np(k)} \sum_{i \in (k_i = k)} \frac{1}{\tau} \int_0^\tau \Delta_i(t) dt, \tag{3}$$

where τ is the time length for achieving global synchronization. The average difference $\langle \Delta(k) \rangle$ represents the average deviation strength (ADS) of the vertices possessing degree k from the mean activity of the network during the course

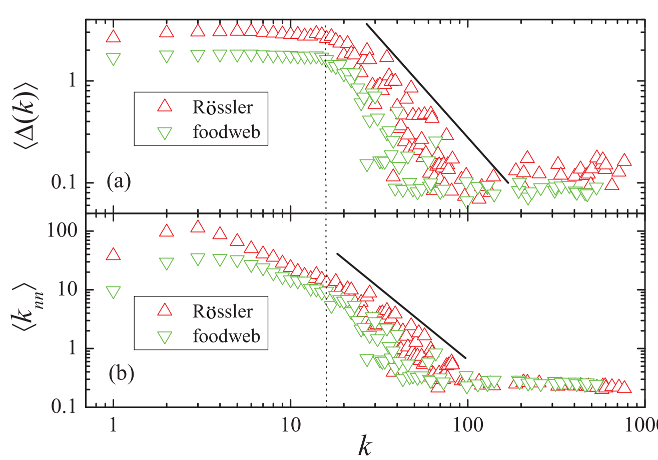

FIG. 4. (Color online) (a) The ADS $\langle \Delta(k) \rangle$ of vertices with degree k from the mean activity of the whole network and (b) the mean degree $\langle k_{nn} \rangle$ of the nearest neighbors of the vertices as a function of the degree k in a log-log plot for Rössler oscillators (upward triangles) and food web model (downward triangles). Results are averaged over 20 independent realizations. The dotted line represents the thresholds $k_t = 14$ in (a) and (b) for both oscillator models. The solid lines give slopes 1.91 in (a) and 2.43 in (b), which confirms their power-law decreases. The depicted results are obtained for $N = 1000$, $c = 1.0$, and $h = 0.14$.

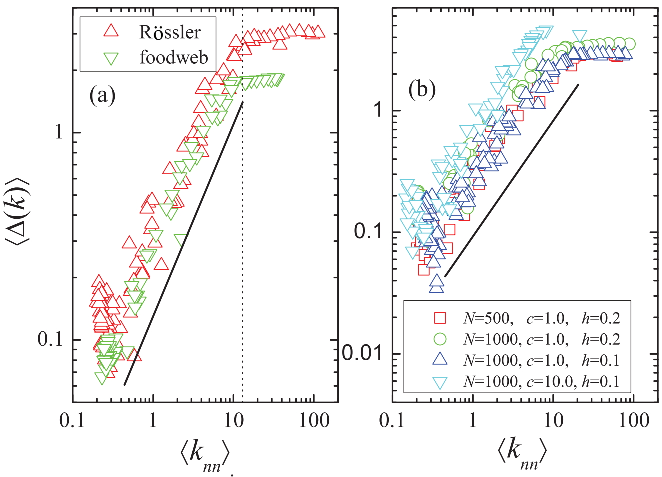

FIG. 5. (Color online) (a) The ADS $\langle\Delta(k)\rangle$ as a function of $\langle k_{nn}\rangle$ in a log-log plot for Rössler oscillators (upward triangles) and the food web model (downward triangles) with $N = 1000$, $c = 1.0$, and $h = 0.14$ and (b) ADS $\langle\Delta(k)\rangle$ vs $\langle k_{nn}\rangle$ for different coevolutionary network parameters N, c, and h. Results in both panels are averaged over 20 realizations on the ANs with random initial conditions. The dotted line denotes the thresholds of $\langle k_{nn}\rangle$ at $k_t = 14$ in (a) for both oscillator models. For clarity, only the results of the Rössler oscillators are shown for different coevolutionary network parameters in (b). The solid lines in (a) and (b) have a slope of 0.78, confirming the power-law behaviors.

of achieving synchronization. In Fig. 4(a), it is clear that a degree threshold point k_t exists for both oscillator models. When $k < k_t$ (i.e., in the small k region), the ADS $\langle\Delta(k)\rangle$ only has a small variation with degree k. However, for $k > k_t$, an almost-power-law $\langle\Delta(k)\rangle \sim k^{-s}$ decrease appears with $s = 1.91 \pm 0.02$, which indicates that the vertex with larger degree processes the less ADS during the course of achieving synchronization. In such a case, the vertices possessing the least deviation from the mean activity of the network ultimately become hub vertices of the network. This result is very consistent with real-life systems, such as an acquaintance network [58], where people are more likely to make a connection with people whose view is popular (i.e., being close to the mean case of most views, which is the so-called law of "the minority is subordinate to the majority").

To characterize the degree correlation in the ANs, we also measure the mean degree $\langle k_{nn}\rangle$ of the nearest neighbors of a vertex as a function of degree k [48], and we find that the degrees of adjacent vertices appear to be negatively correlated, as demonstrated in Fig. 4(b). Thus, vertices with a large degree tend to connect, on average, with ones with a small degree. Particularly, $\langle k_{nn}\rangle$ displays an almost-power-law decrease as $\langle k_{nn}\rangle \sim k^{-q}$, with $q = 2.43 \pm 0.02$, in the large $k > k_t$ region. This negative power-law degree correlation exists widely in a great number of networks, including technological and biological networks [47,48,59]. However, the negative degree correlation in our work is caused by the anti-Hebbian adaptive connecting rule.

It is worth remarking that for two different kinds of oscillators, Rössler and food web models, the threshold point k_t is the same [see the dotted line in Figs. 4(a) and 4(b)]. Moreover, with the degree k above the threshold value k_t, the average dynamical difference $\langle\Delta(k)\rangle$ [see Fig. 4(a)] and the mean degree $\langle k_{nn}\rangle$ [see Fig. 4(b)] also display the same power-law decrease for the two oscillator models, respectively. This interesting result is consistent with the same power-law degree distribution [see Fig. 2(a)] for the two oscillators, which could be due to similar chaotic diagrams of their oscillators [54].

Combining the above $\langle\Delta(k)\rangle \sim k^{-s}$ and $\langle k_{nn}\rangle \sim k^{-q}$, we can easily gain a nontrivial power-law behavior:

$$\langle\Delta(k)\rangle \sim \langle k_{nn}\rangle^{\theta}, \tag{4}$$

with the exponent $\theta = s/q$ in the region of $k > k_t$ (i.e., $\langle k_{nn}\rangle < \langle k_{nn}\rangle|_{k=k_t}$). Figure 5(a) features an obvious scale-free property [for $\langle\Delta(k)\rangle$ vs $\langle k_{nn}\rangle$] with the exponent 0.78 ± 0.02 in both oscillator models, which is fully in agreement with the value of $\theta = s/q$. Finally, it is worth remarking that the scale-free property is robust to variation of coevolutionary parameters [including system size N, the coupling strength c, and the linking time step h; see Fig. 5(b)]. In fact, this robust property is caused by the robustness of power-law relations $\langle\Delta(k)\rangle \sim k^{-s}$ and $\langle k_{nn}\rangle \sim k^{-q}$ with exponents $s = 1.91 \pm 0.02$ and $q = 2.43 \pm 0.02$.

IV. CONCLUSION AND DISCUSSION

In sum, motivated by the anti-Hebbian learning rule in neural systems, we have studied the dynamical organization of connections via adding new links. The feedback from the dynamical synchronization shapes the network into a scale-free structure with negative degree correlation, as confirmed in many real networks, such as technological and biological networks [47,48,59]. Moreover, we show that the vertices whose dynamics approach the mean activity of the network ultimately become hub nodes, consistent with real systems as well [58]. Therefore, our model provides an alternative adaptive mechanism for the formation of scale-free structure with negative degree correlation by introducing the anti-Hebbian-like learning rule.

Our study also indicates that such an anti-Hebbian synaptic growth during neural development [42] could play an important role in generating a neuronal scale-free structure [60,61] and firing synchronization [62–64]. To focus on the effect of the addition of links induced by the anti-Hebbian learning rule, we do not consider here the removal and rewiring of links. In the future, when addition and removal or rewiring of links are both considered in ANs governed by the anti-Hebbian-like learning rule, further investigations are expected to clarify more structural and dynamical properties. In particular, the anti-Hebbian scheme from the plasticity of synaptic strengths [43–45] may provide a useful tool for understanding the mechanism of self-organized criticality described by power-law structural and dynamical properties in neural systems [65–67], which is still an unaddressed fundamental issue regarding the cellular mechanism.

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