Microwave guiding along double femtosecond filaments in air

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Microwave guiding along double parallel lines of femtosecond-laser-generated plasma filament has been demonstrated over a distance of about 8 cm in air, corresponding to a maximum microwave signal intensity enhancement more than sixfold the free-space propagation. It is shown that the operating frequency and the line electric width influence the propagation coefficient of microwaves propagating along this transmission line. Based on channeling microwaves along this line and by measuring and comparing the propagated microwave signals, the basic parameters of laser-generated plasma filament, namely, its electron density and conductivity, are obtained.

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I. INTRODUCTION

Laser filamentation in air is a well-known phenomenon that has been extensively studied in the past two decades [\[1\]](#page-3-0). Researchers have shown that such ultrashort laser pulses can propagate tens of meters in air and up to several kilometers in the atmosphere, beyond the Rayleigh range of the laser beam [\[2\]](#page-3-0). The physical origin of the laser filamentation in air is now well understood. Even if many physical effects come into play during the propagation of the pulse in the filament, the process can be described by the action of mainly two nonlinear physical effects: the optical Kerr effect, which acts against diffraction and tends to focus the beam on itself, and multiphoton ionization, which limits the intensity. The dynamic balance between the self-focusing and defocusing by the plasma yields a narrow and long plasma column (filament) up to tens of meters or more [\[3\]](#page-3-0). Remote sensing [\[4\]](#page-3-0), lighting discharge control [\[5\]](#page-4-0), and guide energy in the form of electric current and microwave radiation in air [\[6–8\]](#page-4-0) are potential applications of the laser filamentation in air.

The transfer of microwave radiation pulses in atmospheric air using the laser plasma as a guiding structure has been proposed in Refs. [\[9,10\]](#page-4-0). More recently, a project using a femtosecond-laser-induced filament to guide energy in the form of electric current and microwave radiation has started [\[6\]](#page-4-0). It is of great interest since it offers a solution to the high natural divergence of the microwave radiation by confining the radiation and maintaining a high-energy density over long distances. In addition, it can be used as a simple and nonintrusive method to diagnose the generated plasma filament. The duration of the microwaves that can be transmitted over this virtual femtosecond plasma transmission line is limited by the lifetime of the plasma, which could be increased from tens to hundreds of nanoseconds by using a second laser pulse to sustain the plasma [\[11,12\]](#page-4-0). Several possible guiding configurations of femtosecond plasma transmission lines (waveguides) are studied both theoretically $[12–16]$ and experimentally $[6–8]$. The implementation of a cylindrical plasma waveguide whose walls are formed by multiple filamentation of a femtosecond-laser beam, with the use of a 100-TW laser facility, was explored in Ref. [\[6\]](#page-4-0).

Channeling of microwave radiation in double-transmissionline geometry, in which the plasma filament played the role of one of the conductors, was experimentally demonstrated in Ref. [\[7\]](#page-4-0), while guiding microwaves along a single femtosecond plasma filament was recently demonstrated in Ref. [\[8\]](#page-4-0).

In this work we study experimentally the guiding of microwave radiation along two parallel lines of femtosecondlaser-generated plasma filaments. The separation distance between the two filaments (the electrical width) and the operating frequency are shown to influence the propagation coefficient of the microwaves propagating along this virtual transmission line. In addition, as a direct result of microwave channeling along the two plasma filaments combined with a simple physical model, we introduce a simple, fast, and nonintrusive experimental method to obtain the basic parameters of a femtosecond-laser-generated plasma filament.

II. EXPERIMENTAL SETUP

The experimental setup for the transmission of microwaves along two parallel lines of femtosecond-laser-generated plasma filaments is presented in Fig. [1.](#page-1-0) In our experiments the plasma filaments are produced by splitting a femtosecond-laser pulse with a total energy of 3.6 mJ into two paths and then are focused in air by using two $f = 100$ cm lenses. The femtosecond-laser pulse has a central wavelength of 800 nm, a repetition rate of 1 kHz, and a pulse duration of 50 fs at full width at half maximum. The radius of the beam is 4 mm at $1/e^2$ of intensity. The two splitting laser pulses have the same energy of 1.78 mJ and the paths length of the two laser pulses is adjusted to be the same to ensure the simultaneous existence of the two parallel filaments. In the experiment, the plasma length and diameter were also measured by imaging its fluorescence emission with a CCD camera and the results show that filament length and diameter are about 10 cm and 150 μ m, respectively.

Microwave radiation of a circularly polarized continuouswave microwave source is coupled by a coax-to-waveguide adaptor to a *Ka*-band rectangular waveguide working as an open-ended waveguide, i.e., the microwave source. To match the standard waveguide with the double line, a pair of copper tips is used to match the microwave signal to the double lines. The filaments serve as two conductors of the double line in which microwave radiation propagates and the microwave

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FIG. 1. (Color online) Schematic of the experimental setup for the double plasma transmission line geometry.

radiation is attached to the plasma channel (filament) just about 10 mm after its origin. The signal that transmits through the double filaments is detected with a linear antenna in the middle of the two filaments and perpendicular to the filaments as shown in the inset of Fig. 1. The antenna is the inner core of a 50- Ω coaxial cable 10 mm in length that can move along the propagation direction in the middle of the filaments by using a one-dimensional translation stage; in this way we can change the distance between the transmitting and receiving systems. The microwave intensity propagated along the double filaments is measured by a 2.5-GHz bandwidth oscilloscope (Tektronix TDS7254B, with a 20-GHz sampling rate) having a 50-Ω impedance resistance. The experiments utilize verylow-power microwaves, thereby avoiding any modification of the plasma filament, and the detecting antenna is placed in the far field radiation zone of the source. The waveguide used in the experiment is a WR-90 rectangular waveguide and the dominant operating mode is TE_{10} . The direction of the electric field for this TE_{10} mode is pointed in the same direction the rf cable enters the waveguide-to-coax adaptor.

III. PHYSICAL MODEL

The two-wire transmission line is actually a system of two coupled single-wire lines, which have been called surface wave transmission lines [\[17\]](#page-4-0). If the excitation of the line is symmetrical, the radiating fields of the currents in the two wires compensate for each other to a large extent, provided the spacing *D* between the wires is very small compared to the wavelength *λ*. The main idea behind the proposal of using laser-induced filaments to form a microwave guiding structure is based on the property that wave propagation through plasma is only possible for frequencies higher than the plasma frequency. The typical measured electron density in a laser-induced filament in air is between 10^{15} and 10^{17} cm⁻³, resulting in a plasma frequency $f_{pl} = 300 - 900$ GHz. This equation gives the upper limit of the electromagnetic frequency that can be propagated along this virtual transmission line.

For the sake of simplicity, the plasma filament will be modeled as a line with constant radius and uniform electron density. Let us consider the case where there are two parallel conducting cylinders of the same radius *r*, having the same conductivity and separated by distance *D*. The propagation constant γ along the line is defined as [\[18\]](#page-4-0)

where α is the attenuation constant, β is the phase constant, $C_{eq} = \pi \varepsilon_0 / \ln(D/r)$ is the capacitance per unit length, $L_{eq} = (\mu_0/\pi) \ln(D/r)$ is the inductance per unit length, R_{eq} = $2/\pi \sigma_{pl}r^2$ is the resistance per unit length, $\varepsilon_0 = 10^{-9}/36\pi$ is the free space permittivity, $\mu_0 = 4\pi \times 10^{-7}$ is the free space permeability, ω is the angular frequency, and σ_{pl} is the plasma conductivity defined as [\[19\]](#page-4-0)

$$
\sigma_{pl} = \omega_{pl}^2 v_c \varepsilon_0 / (\omega^2 + v_c^2),\tag{2}
$$

where $v_c = 3.91 \times 10^{-9} N_a T_e^{0.5}$ is the effective rate of elastic collisions of the electrons and neutral particles of the medium, N_a is the concentration of neutral particles, T_e is the electron temperature in eV, and ω_{pl} is the plasma frequency, which can be defined by [\[20\]](#page-4-0)

$$
\omega_{pl} = (N_e e^2 / m_e \varepsilon_0)^{1/2},\tag{3}
$$

where N_e is the electron density of the free electron and e and *me* are the charge and mass of an electron, respectively. For plasma filaments in the atmosphere, where the air is weakly ionized, the collision frequency is about 10^{12} s⁻¹ [\[15\]](#page-4-0). Both the plasma frequency and collision frequency determine the conductivity of the plasma channel $[19]$. Equation (1) gives

$$
\alpha \approx 0.5 R_{eq} \sqrt{C_{eq}/L_{eq}},\tag{4}
$$

$$
\beta = \omega \sqrt{L_{eq} C_{eq}},\tag{5}
$$

$$
V_{ph} = \omega/\beta = 1/\sqrt{L_{eq}C_{eq}} = c,\tag{6}
$$

where V_{ph} is the phase velocity and c is the speed of light. The attenuation constant α and phase constant β in Eqs. (4) and (5) are in Np*/*m and rad*/*s, respectively. Since the phase velocity of the propagating wave is equal to the speed of light according to Eq. (6) , the microwave pulse can keep attaching to the plasma filament and the plasma recombination at the wire-trailing end does not affect propagation of microwave radiation.

Equation (4) is valid as long as the two lines are electrically close together. The conventional rule asserts that when equal magnitude but opposite sign currents exist on opposite sides of the two-wire line, radiation does not occur [\[21\]](#page-4-0); however, this rule is valid only when the two wires are electrically close together. The radiation resistance must be taken into account when the line's electrical width, which represents the ratio between the wires' separation to wavelength D/λ , cannot be neglected. The radiation resistance of the two-wire line depends on the ratio between the wires' separation and the wavelength and can be defined as [\[22\]](#page-4-0)

$$
R_{\rm rad} \approx 240\pi^2 (D/\lambda)^2. \tag{7}
$$

The radiation resistance can be neglected and the dominating loss is due to plasma loss when the electrical width is very small $(D \ll \lambda)$. The radiation resistance of the line will rise and consequently the radiated power from the line goes up, which leads to higher losses when the electrical width increases. In this case Eq. (4) can be written as

$$
\alpha_t \approx 0.5 R_t \sqrt{C_{eq}/L_{eq}},\tag{8}
$$

$$
\gamma = \alpha + j\beta = \sqrt{(R_{eq} + j\omega L_{eq})j\omega C_{eq}},\tag{1}
$$

where $R_t = R_{eq} + R_{rad}$ is the total line resistance.

FIG. 2. (Color online) Microwave signal response of the reception antenna obtained with the double filaments alone (blue dashed line), with the microwave source alone (green dash-dotted line), and with the microwave guiding in the double line (red solid line).

IV. RESULTS AND DISCUSSION

Figure 2 presents the output signals of the oscilloscope for a double line of femtosecond-laser plasma filaments measured at a length of about 40 mm from the copper tips. The transmitted radiation has a frequency of 10 GHz and the distance between the two tips is ∼1 cm. In the figure the blue dashed line is obtained when the microwave source is switched off, hence enabling detection of the radiation of the two plasma filaments, i.e., from multipole moments inside the filaments oscillating at the plasma frequency [\[23\]](#page-4-0). When the microwave source is switched on, in the absence of the double filaments, the observed signal is a continuous one (the green dash-dotted line in the figure), which represents the free space propagation of the microwave. When the microwave source is switched on and the plasma filaments are produced by a femtosecond-laser pulse, an enhanced pulse (red solid line) is observed in the oscilloscope, which indicates that the propagation coefficient of microwave radiation is increased. This enhanced microwave pulse is synchronized with the laser pulse or the formation of the plasma lines. The microwave pulse duration is approximately 0.5 ns in all experiments and is independent of the double-line length and the time response of the oscilloscope of \sim 150 ps.

The microwave pulse duration is believed to be determined by the time at which the plasma density inside the filament is high enough to radiate a sufficient amount of the propagating microwave along it. In other words, the detected microwave signal in the experiments is due to the radiation of a microwave propagating along the line, thus when the plasma initial density is high, the radiated microwave from the line will be sufficient to be detected, while for a longer time scale the density will be lower and the detection system (antenna) cannot observe the radiated microwaves. The plasma density decays by an order of magnitude during the first 0.5 ns [\[24\]](#page-4-0). The results shown in Fig. 2 confirm that the microwave pulse transmitted along the double line is due to the guiding effect. The enhancement in transmission of the microwave signal due to the guiding of the radiation along the plasma channel, defined as the ratio of the

FIG. 3. (Color online) Dependence of the microwave pulse amplitude V_{max} on the length of the double line.

signal intensity of the guided wave over free propagation, is over sixfold in the present conditions.

Figure 3 shows the dependence of the amplitude of a pulse's microwave signal on the double-line length beginning at a distance of 30 mm from the copper tips, shown in Fig. [1.](#page-1-0) One can see that the amplitude decreases linearly with increasing distance between the receiving and transmitting systems. The starting point of signal detection at 35 mm as shown in Fig. 3 is located in the far field zone, which can be represented by the relation $\sim 2d^2/\lambda$, where *d* is the largest dimension of the radiator, in this case 2.286 cm for the open-ended waveguide. According to this equation, the frequency of investigation, ∼10 GHz, which corresponds to a wavelength *λ* of 3 cm, equates to a far field of ∼3.48 cm, while the ending point of signal detection is determined by the filament length.

The dependence of microwave loss on the propagating frequency and the line's electrical width is investigated. Figure 4 shows the relationship between loss in the detected voltage, which represents the difference between the initial

FIG. 4. (Color online) Dependence of the microwave loss on the microwave frequency and the line electrical width.

value of the signal voltage and the detected one, and the frequency for the two parallel lines of the femtosecond plasma transmission line. As shown in the figure, the general trend is that the higher the frequency, the higher the plasma loss $\alpha \propto \sqrt{f}$, as Eq. [\(4\)](#page-1-0) implicitly shows. This trend can explain using Eq. (3) ; as the frequency increases, plasma conductivity *σpl* will decrease, which leads to a higher plasma surface resistance and higher signal loss.

At the same time, since the spacing between the two plasma filaments $(D = 1$ cm) in our setup is comparable to the central microwave's wavelength ($\lambda = 3$ cm), the radiation resistance is not negligible and the radiated power from the line goes up, which leads to higher losses. Figure [4](#page-2-0) shows that increasing the operating frequency will increase the electrical width (*D/λ*) and the radiation loss $\alpha_{rad} \propto f^2$ will be present and have higher values for larger electric widths, as Eq. [\(7\)](#page-1-0) predicts. The loss of detected microwave signal is due to higher microwave radiation that affects the propagating microwave along the line and as a consequence the part of the propagating microwave that will be radiated and detected by the antenna will be lower.

V. MEASUREMENT OF PLASMA PARAMETERS

In addition to confining the microwave radiation and maintaining a high-energy density over long distances, the double plasma transmission line can be used as a simple and non-intrusive method to diagnose the generated plasma filament. Characterizing the laser-generated plasma filament in a quantitative manner is of crucial importance for both the fundamental understanding of the nonlinear optical pulse propagation and applications. Based on channeling microwaves along the two plasma filaments as demonstrated above, one can obtain the basic parameters of the plasma. For this purpose, Eq. [\(4\)](#page-1-0) can be rewritten as

$$
\alpha \approx A/N_e
$$
, $A = 1/15\pi d_{pl}^2 \mu e \ln(D^2/d_{pl}^2)$, (9)

where $\mu = em_e^{-1}v_c^{-1}$ is the electron mobility and $\sigma_{pl} = N_e \mu e$. The plasma dynamics immediately after the end of the laser pulse producing the plasma is determined by recombination. The concentration N_e after the ionizing laser pulse changes as [\[25\]](#page-4-0)

$$
N_e(t) = N_{e0}/(1 + N_{e0}\beta_{ep}t),
$$
\n(10)

where N_{e0} is the initial free electron concentration (at the laser pulse end), *t* is time, and β_{ep} is the electron-ion recombination coefficient. The microwave signal intensity $I(z,t)$ at the output from the double line of length *z* is described by the expression

$$
I(z,t) = I_0 \exp(-2zA/N_{e0}) \exp(-2zA\beta_{ep}t),
$$
 (11)

where I_0 is the microwave field intensity coupled to the double line at the input port $(z = 0)$. In this case, we can assume that the maximum amplitude V_{max} of the output signal of the microwave is proportional to the microwave pulse energy, i.e., to the time integral from the microwave radiation intensity [\[26\]](#page-4-0),

$$
V_{\text{max}}(z) = \int_0^\infty I(z, t)dt = I_0 \exp(-2zA/N_{e0})/zA\beta_r.
$$
 (12)

Let us estimate the value of the initial electron density under our experimental conditions using the values of the maximum detected voltage at two different points along the line as shown in Fig. [3,](#page-2-0) for example, $V_{\text{max}} = 12 \text{ mV}$ at $z = 5$ cm and $V_{\text{max}} = 2.5$ mV at $z = 8$ cm. Equation (12) allows us to calculate the initial average plasma electron density and conductivity of the filament to be \sim 10¹⁶ cm⁻³ and 300 Ω^{-1} m⁻¹, respectively. Using any other two points along the lines gives us approximately the same value of the initial electron density since the microwave signal decays linearly along the whole length of the line as shown in Fig. [3.](#page-2-0) This measured plasma electron density corresponding to the initial plasma electron density is in good qualitative agreement with previous experimental measurements of the plasma electron density using other measurement methods (see, for example, Refs. [\[24,25,27,28\]](#page-4-0)).

VI. CONCLUSION

It has been shown that two parallel lines of laser plasma filaments can behave like two parallel transmission lines and support microwave radiation. Microwave guiding along double parallel lines of femtosecond-laser-generated plasma filaments has been demonstrated over a distance of about 8 cm in air, corresponding to a maximum microwave signal intensity enhancement more than sixfold over free space propagation. It is shown that the line's electrical width and the operating frequency influence the propagation of microwaves along this virtual transmission line. Based on the microwave channeling along this virtual transmission line, we introduced a simple and nonintrusive method to obtain the basic parameters of a laser-generated plasma filament, such as an average electron density and a conductivity of $\sim 10^{16}$ cm⁻³ and 300 Ω^{-1} m⁻¹, respectively.

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