Reverse resonance in stock prices of financial system with periodic information

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(Received 28 February 2013; revised manuscript received 6 June 2013; published 15 July 2013)

We investigate the stochastic resonance of the stock prices in a finance system with the Heston model. The extrinsic and intrinsic periodic information are introduced into the stochastic differential equations of the Heston model for stock price by focusing on the signal power amplification (SPA). We find that for both cases of extrinsic and intrinsic periodic information a phenomenon of reverse resonance emerges in the behaviors of SPA as a function of the system and external driving parameters. Moreover, in both cases, a phenomenon of double reverse resonance is observed in the behavior of SPA versus the amplitude of volatility fluctuations, by increasing the cross correlation between the noise sources in the Heston model.

DOI: 10.1103/PhysRevE.88.012811 PACS number(s): 89.65.−s, 05.40.−a, 02.50.−r

I. INTRODUCTION

A new field called econophysics [1,2] has arisen to investigate the financial markets by the methods of stochastic dynamics. The most basic model of econophysics is a geometric Brownian motion model, which is early proposed to describe the stochastic dynamics of stock prices [3,4]. However, this model cannot agree with some statistical characteristics of actual financial data, such as the fat tails (i.e., showing non-Gaussian distribution of returns) [2,5], long-range memory, and volatility clustering [6]. Afterward, in order to make up these deficiencies, many valuable models have been developed to picture the dynamics of the stock price, for instance, the Black-Scholes option pricing model [7], the ARCH model [8], GARCH model [9], and Heston model [10]. Peculiarly, in recent years, researchers have paid more attention to the Heston model. The Heston model well describes the dynamics of stock price in actual financial markets, which consists of two coupled stochastic differential equations. The equations represent the dynamics of stock price and volatility by the log-normal geometric Brownian motion stock process and the Cox-Ingersoll-Ross mean-reverting process, respectively. Since good agreements between the Heston model and financial data were found, the Heston model has been widely verified and used to analyze dynamics of stock price in financial markets as follows. Since Bouchaud and Cout obtained the approximative effective potential for stock market crash or bubble [11], the enhancement of the lifetime of a metastable state induced by the noise has been discussed by analyzing the mean escape time in a modified Heston model with a cubic nonlinearity [12,13]. Moreover, the same effect was found by studying the statistical properties of the hitting times in different models for stock market evolution [14,15]. After solving the escape problem for the Heston model, exact expressions for the survival probability and the mean exit time have been obtained [16,17]. Recently, the Heston model has also been used to discuss the effects of the delay time on the stability of financial markets [18] and on the risks and returns of stock investment in financial markets [19]. Furthermore, besides the theoretical analysis of the Heston model, the agreement between theoretical data obtained with this model and actual financial data also attracts a wide range of studies. After the seminal paper by Drăgulescu *et al.* [20], where an analytic formula for the time-dependent probability distribution of returns was obtained, further investigations using the Heston model were carried out. Specifically, studies on the probability distribution of returns for the three major stock market indexes (Nasdaq, S&P500, and Dow-Jones) [21], the exponential distribution of financial returns obtained from actual financial data [22], the probability density distribution of the logarithmic returns of the empirical high-frequency data of DAX and its stocks [23], and the typical price fluctuations of the Brazilian São Paulo Stock Exchange Index [24] were done.

The environment of actual financial markets contains many kinds of extrinsic periodic information, for example, periodic daily information, weekly information, periodic financial index information, and conference information. Furthermore, the roles of extrinsic periodic information have been discussed on capital structure decisions of the firm [25], implied about an investor with a higher risk aversion or a longer investment horizon [26], and analyzed on the Warsaw Stock Exchange in Poland [27]. At the same time, an actual financial market is also driven by its corresponding intrinsic periodic information, e.g., information of economic cycles. In addition, the effects of intrinsic periodic information of economic cycle have also been found in property markets [28], investigated in state highway capital expenditure [29], discussed for financial stability [30], analyzed on the World Trade Web [31], studied in continuous time evolving economic models [32], discovered for income distribution [33], and demonstrated by the research of fluctuation-dissipation theory [34]. Therefore, the roles of extrinsic and intrinsic periodic information in financial markets need to be further investigated.

Since Benzi *et al.* [35–37] found that warm climate could significantly enhance the response of the Earth's climate to the weak perturbations caused by the Earth's orbital eccentricity, stochastic resonance (SR) has been found in many dynamic systems driven by a combination of a periodic signal and noise. Then the SR has obtained comprehensive applications and discoveries in various fields, such as bistable systems [38–41], linear systems [42–44], biological systems [45], chemical systems [46], ecosystems [47,48], a tunnel diode [49–51], etc. For a comprehensive review, see Ref. [52]. Even in some financial systems, SR has also been discussed in a bistable model with

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information-carrying signal for financial market crashes and bubbles [53] and demonstrated via the signal-to-noise ratio on the specific example of an interacting-agent model of speculative activity [54]. Particularly, a phenomenon of reverse resonance has been found in a constant-potential system driven by multiplicative dichotomous noise and an input oscillatory signal [55], in a linear system driven by a multiplicative multistate noise and an input temporal oscillatory signal [56], and in a delay bistable dynamical system [57,58].

In this paper, we introduce a periodic function into the Heston model to describe the periodic information. Then the signal power amplification (SPA) [52,59,60] is employed to investigate the stochastic resonance induced by the extrinsic and intrinsic periodic information. In Sec. II, a Heston model with extrinsic and intrinsic periodic information is presented. In Sec. III, the statistical properties of stock price returns are discussed in the case of extrinsic and intrinsic periodic information. The analysis of SPA on the extrinsic and intrinsic periodic information are given in Sec. IV. In Sec. V, a brief conclusion ends the paper.

II. HESTON MODEL WITH PERIODIC INFORMATION

The Heston model is defined by the following coupled Ito stochastic differential equations [10]:

$$\begin{aligned} dr(t) &= \left(\mu - \frac{\nu(t)}{2}\right)dt + \sqrt{\nu(t)}d\xi(t), \\ d\nu(t) &= a(b-\nu(t))dt + c\sqrt{\nu(t)}d\eta(t), \end{aligned} \tag{1}$$

where $r(t)$ describes the log of the stock price, $\nu(t)$ denotes the volatility of the stock price, a denotes the mean reversion of the $\nu(t)$, b denotes the long-run variance of the $\nu(t)$, c is often called the volatility of volatility, i.e., it is the amplitude of volatility fluctuations. The deterministic solution of the $\nu(t)$ process has an exponential transient with characteristic time equal to a^{-1}, after which the process tends to its asymptotic value b [61], $\xi(t)$ and $\eta(t)$ are correlated Wiener processes.

Intuitively, each financial market is a part of economic system and surrounds in a variety of economic entities and political organizations. The economic entities and political organizations will generate multiple extrinsic periodic information to influence investors. Finally, the stock price in a financial market will be impacted by many kinds of extrinsic periodic information. At the same time, each financial market is influenced by its corresponding intrinsic periodic information. Each stock of a financial market is also impacted by the information of economic cycle of the corresponding company. The information of economic cycles over several months or years has four stages: (i) expansion, i.e., an increase in production and prices under the condition of low interests rates; (ii) crisis, i.e., stock exchanges crash and multiple bankruptcies occur in firms; (iii) recession, i.e., drops occur in prices and in output, with high interests rates; (iv) recovery, i.e., stock prices recover as the fall in prices and incomes. As large as a country and as small as a corporation, their development is motivated by their own and the integrated economic cycles. In addition, the synchronization between periodic information in financial markets and the willingness of the investor's investment can induce financial market crashes or bubbles (i.e., SR), because investor's sentiment can greatly magnify the effects of the extrinsic and intrinsic periodic information. Conversely, antisynchronization [62] between them can lead to price consolidation in financial markets (i.e., a phenomenon of reverse resonance). In this condition, a wait-and-see approach of most investors is induced, due to inconsistent between periodic information and investment expectations, i.e., changes in the stock price are relatively stable.

The $d\xi(t)$ in Eq. (1) can be understood as the interference of external environment on the stock price. When the periodic information can not be ignored, the $d\xi(t)$ is influenced by the extrinsic periodic information. Just allowing for the roles of extrinsic periodic information on stock price, the $d\xi(t)$ is approximatively simplified to contains two parts: the sum of extrinsic (i.e., multiplicative) periodic information $A\sin(\Omega t)dt$ and Wiener process $d\xi'(t)$, i.e., $d\xi(t) = A\sin(\Omega t)dt + d\xi'(t)$, where A is the amplitude of extrinsic (i.e., multiplicative) periodic information and Ω is frequency of extrinsic periodic information. As for intrinsic periodic information, the growth rate μ is oscillated. The μ is consisted of the information of economic cycles $A_e\cos(\Omega_e t + \phi_e)$ and the growth rate μ_e, i.e., $\mu = \mu_e + A_e\cos(\Omega_e t + \phi_e)$, where A_e is the amplitude of intrinsic (i.e., additive) periodic information, Ω_e is the frequency of intrinsic periodic information, and ϕ_e is the initial phase difference between extrinsic and intrinsic periodic information. Then Eq. (1) becomes

$$\begin{aligned} dr(t) &= \left[\mu_e + A_e\cos(\Omega_e t + \phi_e) - \frac{\nu(t)}{2}\right]dt \\ &\quad + \sqrt{\nu(t)}A\sin(\Omega t)dt + \sqrt{\nu(t)}d\xi'(t), \\ d\nu(t) &= a(b-\nu(t))dt + c\sqrt{\nu(t)}d\eta(t), \end{aligned} \tag{2}$$

where $\xi'(t)$ and $\eta(t)$ are correlated Wiener processes and have the following statistical properties:

$$\begin{aligned} \langle d\xi'(t)\rangle &= \langle d\eta(t)\rangle = 0, \\ \langle d\xi'(t)d\xi'(t')\rangle &= \langle d\eta(t)d\eta(t')\rangle = \delta(t-t')dt, \\ \langle d\xi'(t)d\eta(t')\rangle &= \langle d\eta(t)d\xi'(t')\rangle = \lambda\delta(t-t')dt, \end{aligned} \tag{3}$$

λ denotes the cross correlation coefficient between $\xi'(t)$ and $\eta(t)$, other parameters are the same as Eq. (1). Let $x(t) = r(t) + \mu_e t$, then Eq. (2) is changed as

$$\begin{aligned} dx(t) &= \left[A_e\cos(\Omega_e t + \phi_e) - \frac{\nu(t)}{2}\right]dt \\ &\quad + \sqrt{\nu(t)}A\sin(\Omega t)dt + \sqrt{\nu(t)}d\xi'(t), \\ d\nu(t) &= a(b-\nu(t))dt + c\sqrt{\nu(t)}d\eta(t). \end{aligned} \tag{4}$$

III. STATISTICAL PROPERTIES OF STOCK RETURNS

In this section, the statistical properties of stock price returns (Δx) are numerically simulated with Eq. (4). The stock price returns are defined as [63,64]

$$\Delta x = x_i - x_{i-1}, \tag{5}$$

where x_i is the logarithmic price of ith time point ($i = 1,2,3,\ldots$).

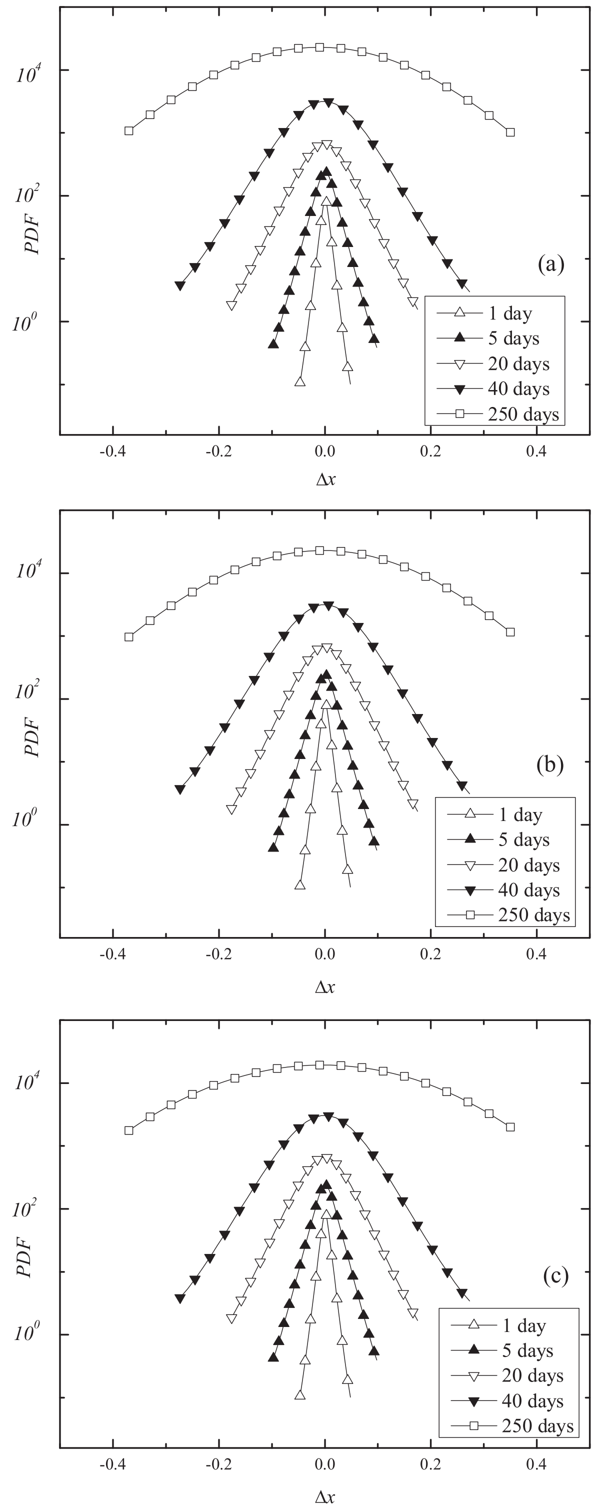

FIG. 1. The PDF versus stock price returns (Δx) with different trading days for $A = 0$ and $A_e = 0$ in (a), $A = 0.05$, $\Omega = 0.05$, and $A_e = 0$ in (b) and $A_e = 0.05$, $\Omega_e = 0.05$, and $A = 0$ in (c).

In order to compare the probability density function (PDF) of Δx with that from other literatures, we choose the values of parameters in Ref. [21] and use $\Delta t = 0.01$ as a unit to describe a trading day. Let $\lambda = 0.0$, $a = 24/2.525$, $b = 0.02/2.525$, $c = 0.94/2.525$, and initial position $x_0 = c/a \approx 0.039$, because each year has about 252.5 trading days averagely obtained from 1982–2001 Dow Jones data in Ref. [20]. The PDF is numerically simulated over 10^5 paths with a multiple of periods based on Eqs. (4) and (5), and the results are plotted in Figs. 1–3.

TABLE I. Comparison of the kurtosis of the Dow Jones data in Refs. [65,66] with Fig. 1 at various trading days.

trading days	Dow Jones	Fig. 1(a)	Fig. 1(b)	Fig. 1(c)
1	69.26	69.280	69.278	69.072
5	19.68	19.730	19.728	19.459
20	7.80	7.850	7.844	7.429
40	6.02	6.052	6.045	5.431
250	−0.33	−0.318	−0.320	−0.578

The plots of PDF versus stock price returns (Δx) under different trading days are shown for different periodic information in Fig. 1. For the case of no periodic information ($A = 0$ and $A_e = 0$), the same results as Fig. 2 in Refs. [20,21] and Fig. 1 in Refs. [65,66] are shown in Fig. 1(a). For small extrinsic or intrinsic periodic information in Fig. 1(b) or 1(c), we find good agreements between PDF and that in Fig. 1(a). In addition, it is difficult to find the difference between Figs. 1(a) and 1(b) [or Fig. 1(c)] visually. To quantitatively compare Fig. 1(a) with Fig. 1(b) [or Fig. 1(c)], we obtain the kurtosis of 1982–2001 Dow Jones data from Refs. [65,66] and calculate the kurtosis of Fig. 1 as shown in Table I. We can find that the deviation of kurtosis is very small between Dow Jones data and Figs. 1(a) [1(b) or 1(c)] at various trading days and also conforms to the Table I in Ref. [66]. It shows that the model [Eq. (4)] is consistent with the actual financial market. Then in order to discuss the effects of extrinsic and intrinsic periodic information on the PDF, the calculate results are plotted on Figs. 2 and 3, respectively.

Just considering the roles of extrinsic periodic information (i.e., $A_e = 0.0$) on PDF, we present the discussion in detail for Fig. 2. To discuss the influences of A for shorter and longer trading days, Figs. 2(a) and 2(b) are given, and the results show that increment of A very weakly reduces the peak value of PDF for shorter trading days [i.e., five trading days in Fig. 2(a)], conversely visibly reduces the peak value of PDF for longer trading days [e.g., 250 trading days in Fig. 2(b)]. Moreover, to discuss the effects of Ω for shorter and longer trading days, Figs. 2(c) and 2(d) are also given. Figures 2(c) and 2(d) show that whether shorter [in Fig. 2(c)] or longer trading days [in Fig. 2(d)], increments of frequency of extrinsic periodic information weaken the peak value of PDF, and the behavior for longer trading days is more apparent than for shorter trading days. In other words, the increments of strength and frequency of extrinsic periodic information weakly impacts the stability of returns for shorter trading but visibly reduces the stability of returns for longer trading. Here the stability of returns is described by the peak value of the PDF. In addition, from a financial point of view, the longer trading days are, the longer investors are influenced by extrinsic periodic information in actual financial market. Then, an increase of strength or frequency of extrinsic periodic information increases investment uncertainty due to the diversity of investors' assessment in extrinsic periodic information. An

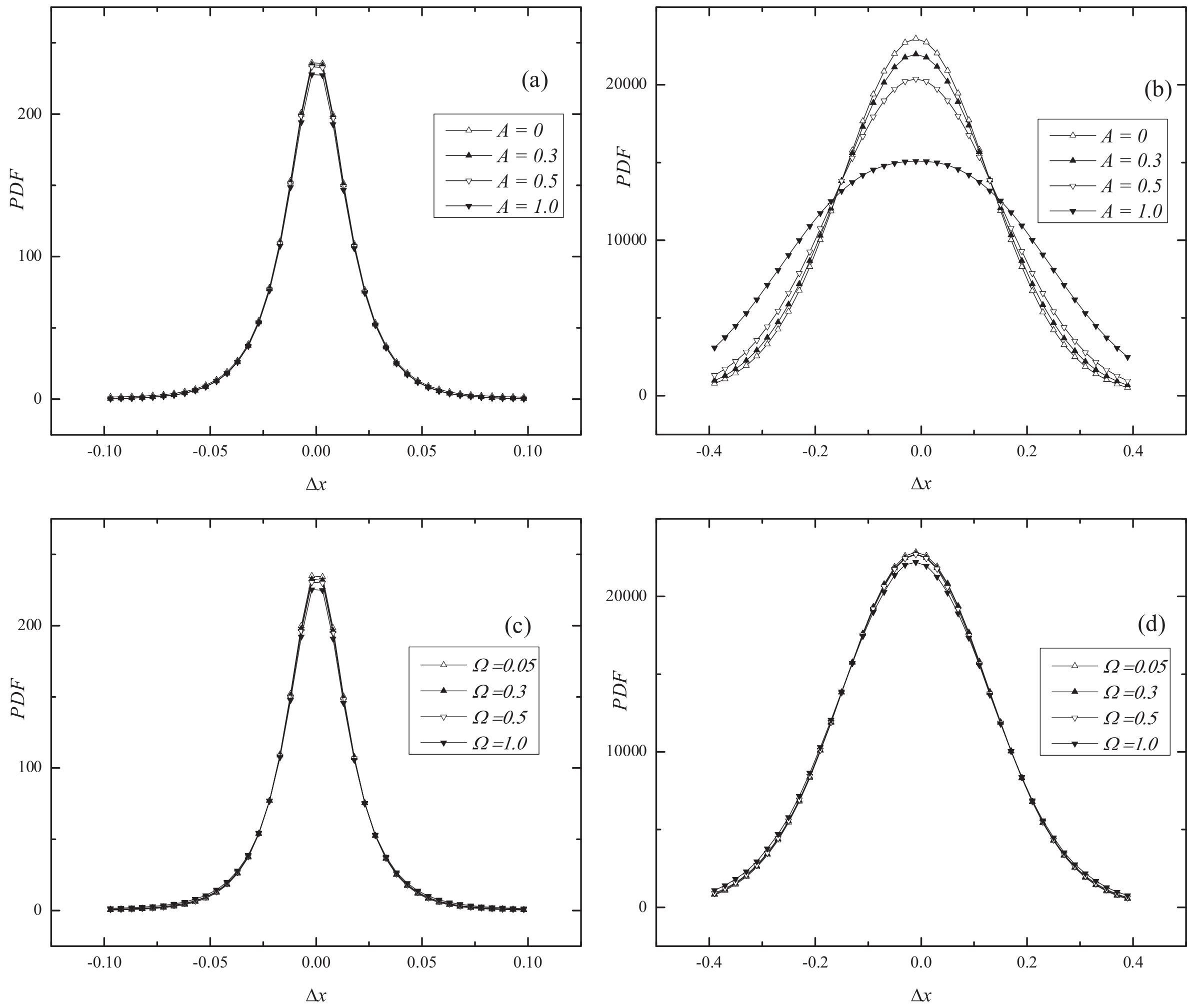

FIG. 2. The PDF versus Δx with $\Omega = 0.05$ and different values of A for five trading days in (a) and 250 trading days in (b); with $A = 0.1$ and different values of Ω for five trading days in (c) and 250 trading days in (d).

increase of investment uncertainty also reduces the stability of investment returns.

Only considering the roles of intrinsic periodic information (i.e., $A = 0.0$) on PDF, we present the discussion in detail in Fig. 3. To discuss the influences of A_e for shorter and longer trading days, the curves of PDF versus Δx are plotted in Figs. 3(a) and 3(b). One can see that increments of A_e very weakly influence the peak value of PDF for shorter trading days (e.g., five trading days) in Fig. 3(a), but distinctly reduces the peak value of PDF for longer trading days (e.g., 250 trading days) in Fig. 3(b). Moreover, to discuss the roles of Ω_e for shorter and longer trading days, the curves of PDF versus Δx are also plotted in Figs. 3(c) and 3(c). One can see that for shorter trading days, increments of Ω_e weakly reduce the peak value of PDF in Fig. 3(c), but for longer trading days, increments of Ω_e weakly enhance the peak value of PDF in Fig. 3(c). Obviously from a financial point of view, the longer trading days are, the longer investors are influenced by intrinsic periodic information in actual financial market. Meanwhile, the stronger amplitude of information of economic cycles is, the larger absolute value of long-term return is, i.e., the lower probability of zero return is. In addition, for the higher the frequency of information of economic cycles, the time for long-term investment is easily across multiple cycles to cause higher probability of zero return.

In a word, an increase of both amplitude and frequency for extrinsic and intrinsic periodic information reduces the stability of returns, except for the roles of frequency of intrinsic information for longer trading days. In addition, as increasing the trading days, the difference for the presence of information or not is enhanced. The same behavior in Refs. [20,21,65,66] shows that as increasing the trading days, the discrepancy between the empirical and the theoretical cumulative distributions increases

IV. SIGNAL POWER AMPLIFICATION

In order to investigate the stochastic resonance in the market system [Eq. (4)], the SPA is employed to characterize the stochastic resonance of the system [52,59,60]

$$\eta = 4A^{-2}|\langle e^{i\Omega t} X(t)\rangle|, \tag{6}$$

where $X(t)$ is obtained from the ensembles average over the stochastic path $x(t)$ realizations. Through integrating Eq. (4) with a forward Euler algorithm, $x(t)$ can be obtained. After fast Fourier transformation of $X(t)$, we obtain the amplitude

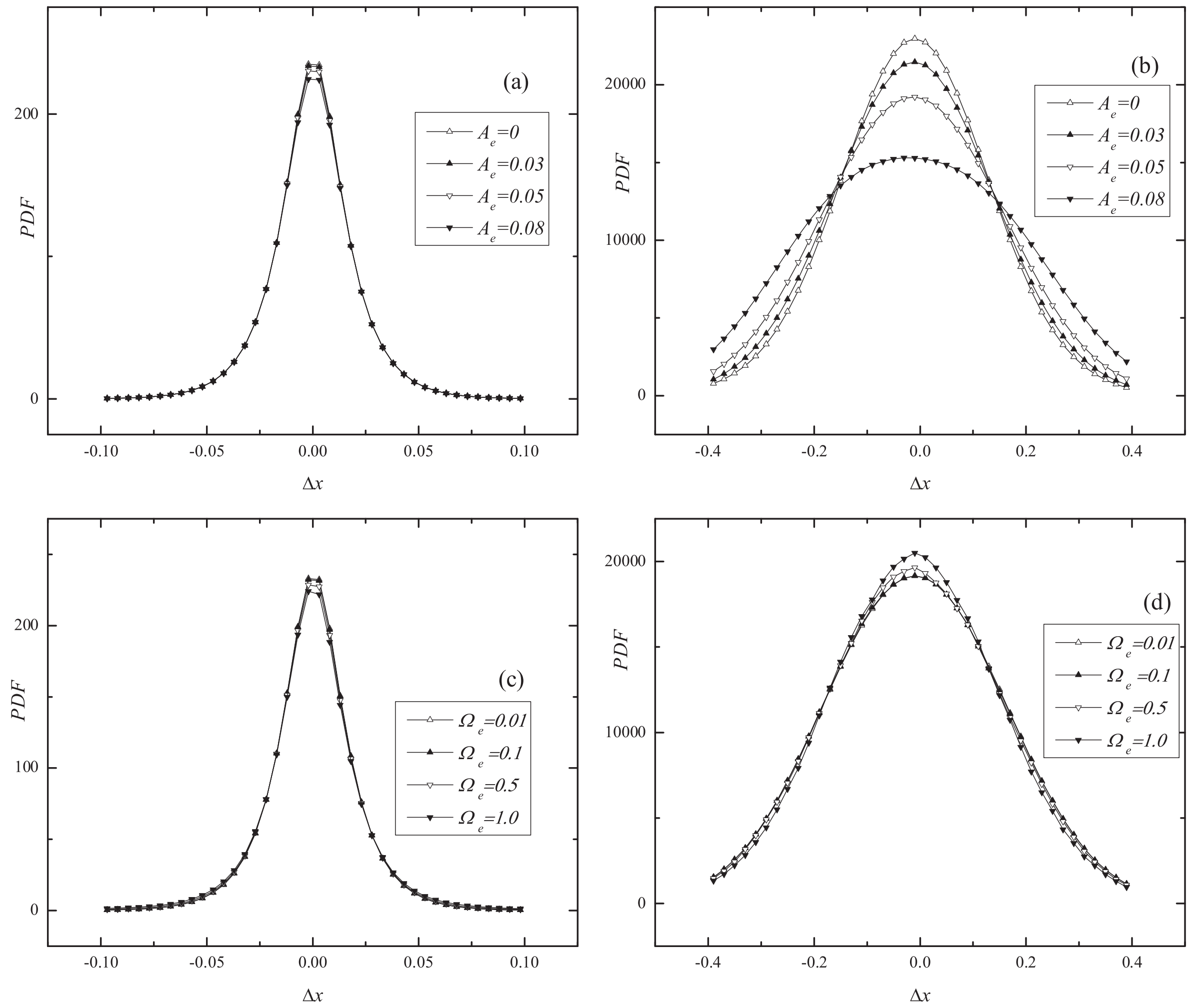

FIG. 3. The PDF vs Δx with $\Omega_e = 0.05$ and different values of A_e for five trading days in (a) and 250 trading days in (b); with $A_e = 0.1$ and different values of Ω_e for five trading days in (c) and 250 trading days in (d).

of the first harmonic of the output information and η from Eq. (6).

In the following, in addition to analytical parameters, we fix parameters $a = 2$, $b = 0.05$, $c = 1$ due to relative stability of financial system from Ref. [12,13] and parameters $A = 0.05$, $\Omega = 0.05$, $A_e = 0.05$, $\Omega_e = 0.05$ due to good agreements between model and other literatures [20,21,65,66] (see Fig. 1). After numerically simulating the signal power amplification, we discuss the stochastic resonance for the case of extrinsic or intrinsic periodic information, respectively.

A. Extrinsic periodic information

Only considering the roles of extrinsic periodic information (i.e., let $A_e = 0.0$), Eq. (4) becomes

$$\begin{aligned} dx(t) &= -\frac{v(t)}{2}dt + A\sqrt{v(t)}\sin(\Omega t)dt + \sqrt{v(t)}d\xi'(t), \\ dv(t) &= a(b - v(t))dt + c\sqrt{v(t)}d\eta(t). \end{aligned} \tag{7}$$

Then SPA η can be numerically calculated based on Eqs. (6) and (7), and results are shown in the Figs. 4–7.

The SPA η versus the mean reversion a, the long-run variance b, and the amplitude of volatility fluctuations c for different λ are plotted in Figs. 4(a)–4(c), respectively. In Fig. 4(a), the SPA monotonically decreases as increasing a for nonpositive λ, but for positive λ has a minimal value, which indicates a phenomenon of reverse resonance [55–57]. In Fig. 4(b), the SPA monotonically increases as increasing b for nonpositive λ, but has a minimum for positive λ too. Meanwhile, in Fig. 4(a) [and 4(b)] the minimum value increases and moves to shorter a (and larger b) when the values of λ increases. In Fig. 4(c), the SPA monotonically increases as increasing b for nonpositive λ, but for positive λ, the SPA first displays a minimum value and then two minimum values (i.e., a phenomenon of double reverse resonance) as λ increases. In addition, in Fig. 4(c) the minimum values are away from each other as λ increases. From previous description, we can find that the SPA η displays reverse-resonance behavior as a function of a, b, and c, respectively, i.e., there is an optimal value of a, b, and c for minimum extrinsic periodic information output. Especially, η vs c presents double reverse-resonance phenomenon as λ increases. From a financial point of view, an increase of the mean reversion a reduces the action time of extrinsic periodic information. An increase of b and c enhances the volatility of the stock price, and stock in higher volatility implies more investors who easily

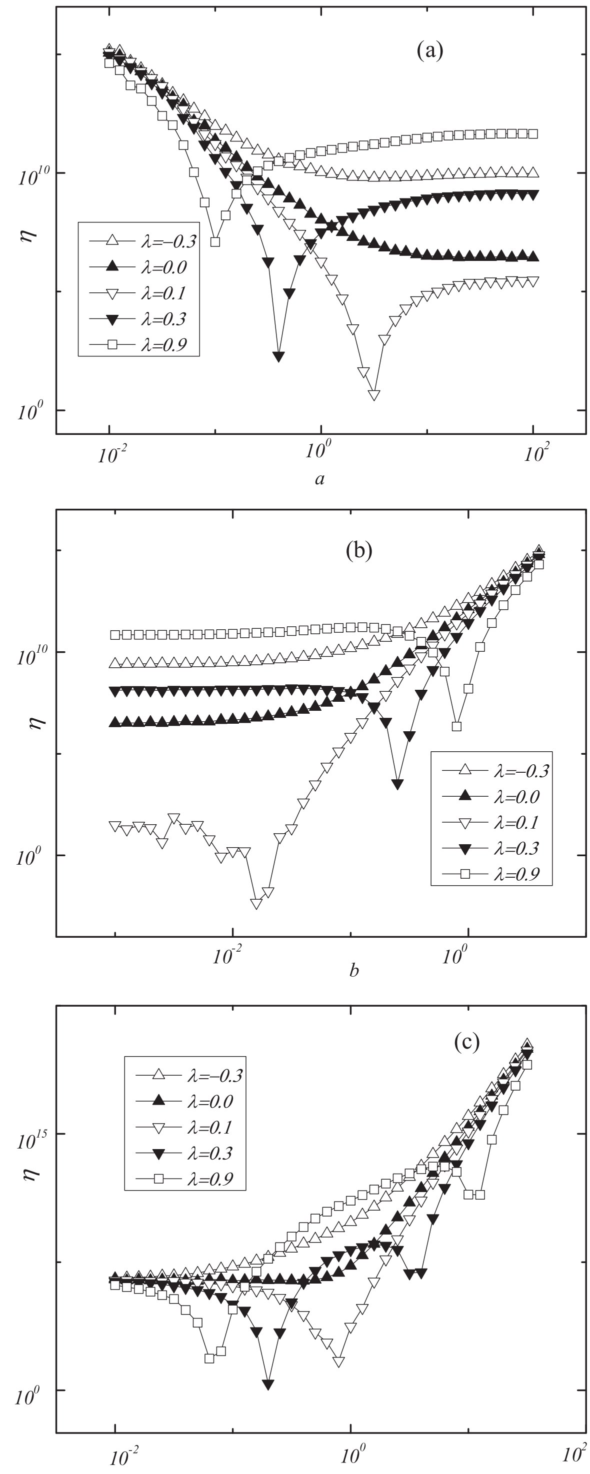

FIG. 4. The SPA η vs a in (a), vs b in (b) or vs c in (c) with different values of λ.

amplify the effects of extrinsic periodic information. Hence the monotonic behaviors of Fig. 4 well agree with actual financial market. From a physical point of view, the presence of the noise should have two roles [55–57]: it drives the motion of the particles, and at the same time reduces its motion. When the frequency of the input information is in the vicinity of the intrinsic frequency, either stochastic resonance or reverse resonance is induced because of the synchronization

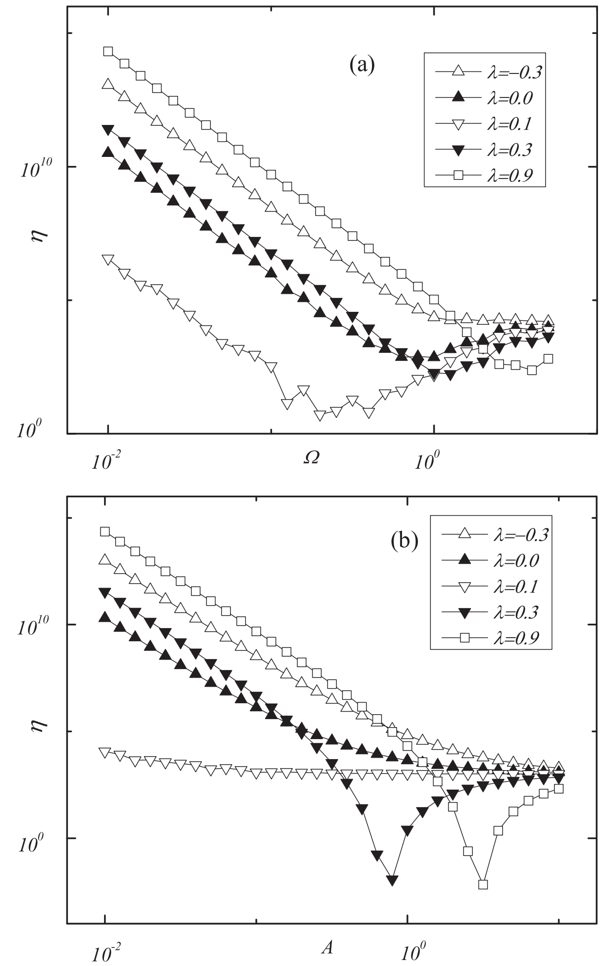

FIG. 5. The SPA η vs Ω in (a) or vs A in (b) with different values of λ.

and antisynchronization corresponding to two roles of noise, respectively. Analogously, the presence of noise originated from investors should also have two roles in financial system. As a, b, c increase, the intrinsic frequency of stocks in a finance system achieves to the vicinity of the frequency of the extrinsic periodic information ($\Omega = 0.05$). Here the intrinsic frequency of stocks can be approximatively obtained from the Eq. (20) in Ref. [20]. Then "good" and "bad" information are synch with stock falling and rising, respectively. At this time the motion of stock price is weakened. Consequently, the reverse-resonance behaviors in Fig. 4 result from antisynchronization between a stochastic time scale (determined by financial system) and a deterministic time scale (determined by the extrinsic periodic information). Particularly, double reverse-resonance phenomenon in Fig. 4(c) emerges in two regions corresponding to the lower and higher volatility. In addition to the reasons in antisynchronization, this is also because risk and return weakly attract investors for some stocks related to double reverse resonance, e.g., low risk with relatively low profit in lower volatility and high risk with low profit in higher volatility. Meanwhile, the Fig. 4(b) in Ref. [12] indicates that increase in λ is associated with decrease in the volatility. For fixing the intrinsic frequency of volatility of stocks corresponding to the frequency of information ($\Omega = 0.05$) in reverse resonance, an increase of λ is associated with decrease in a and increase in b

or c. Therefore the nonmonotonic behavior is consistent with Fig. 4.

Then to analyze roles of the extrinsic periodic information on SPA, the results are presented in Fig. 5. For negative λ, the η monotonically decreases as increasing Ω in Fig. 5(a) and A in Fig. 5(b). As increasing the λ, a minimum value of η vs Ω or A emerged in Fig. 5(a) or 5(b), respectively. From a financial point of view, an increase in Ω reduces the time of "good" or "bad" information impacting investors, so that amplified action of investors to information is weakened by the increase in Ω, as the monotonic decreasing behaviors in Fig. 5(a). The presence of nonmonotonic behaviors originate from antisynchronization, which is induced by varying Ω to near intrinsic frequency of system (determined by financial system parameters). We analogously find increasing Ω reduces SPA and the SR emerges at some driving frequency in physical systems [57,67]. Meanwhile, an increase in A implies an increase in the scope of information dissemination, i.e., the asymmetry of information to investors is reduced. It is suggested that the amplified effects of extrinsic periodic information by investors are weakened as increasing A, as the monotonic decreasing behaviors in Fig. 5(b). At some values of Ω, investors are impacted by a certain amount of periodic force of information, and the volatility of stock price is enhanced, i.e., intrinsic frequency of original stock is influenced. Then the antisynchronization can be induced at some value of A, as the nonmonotonic behaviors in Fig. 5(b). The similar behaviors can be found in some systems [57,60].

In the end of the Sec. IV A, to understand the effects of λ on stochastic resonance, the results are shown in Figs. 6 and 7. A minimum value of SPA η versus λ is displayed in Figs. 6 and 7.

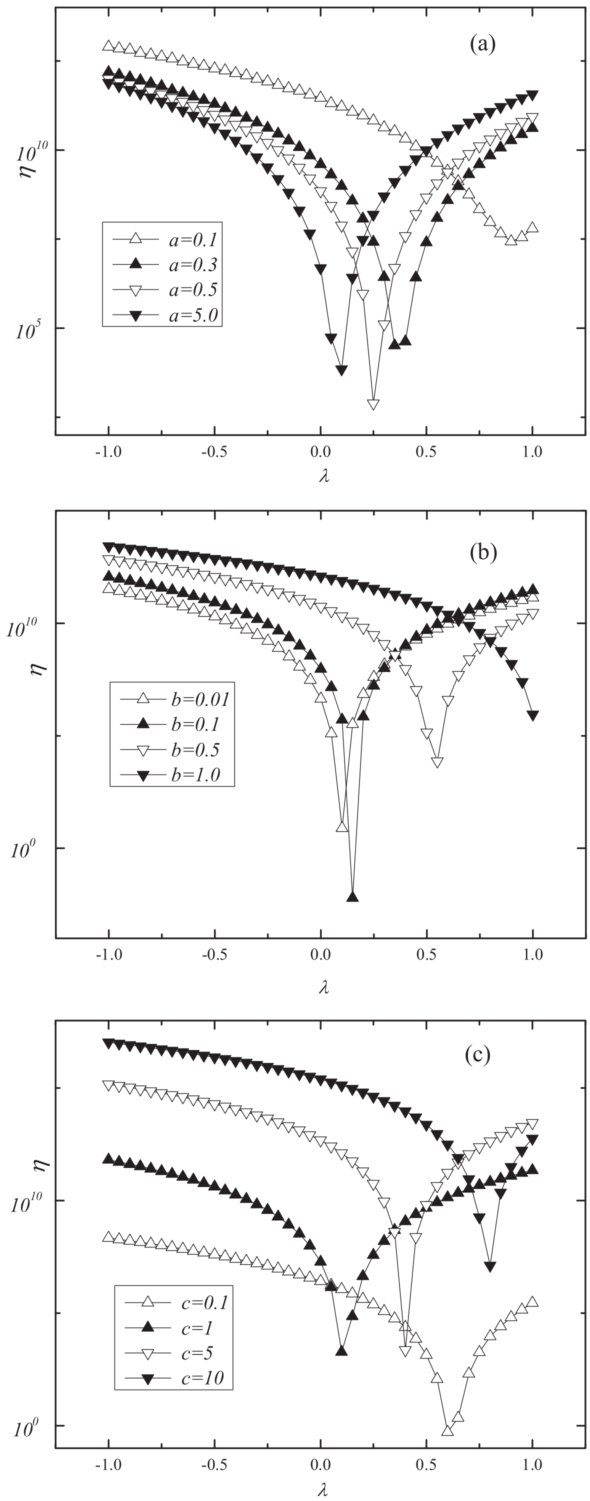

FIG. 6. The SPA η vs λ with different values of a in (a), b in (b), or c in (c).

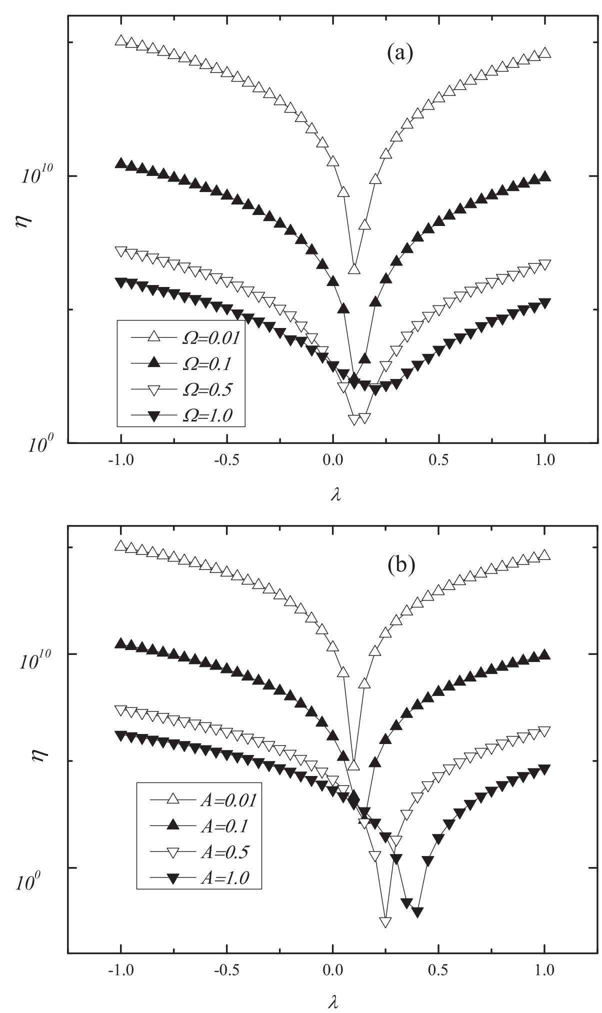

FIG. 7. The SPA η vs λ with different values of Ω in (a) or A in (b).

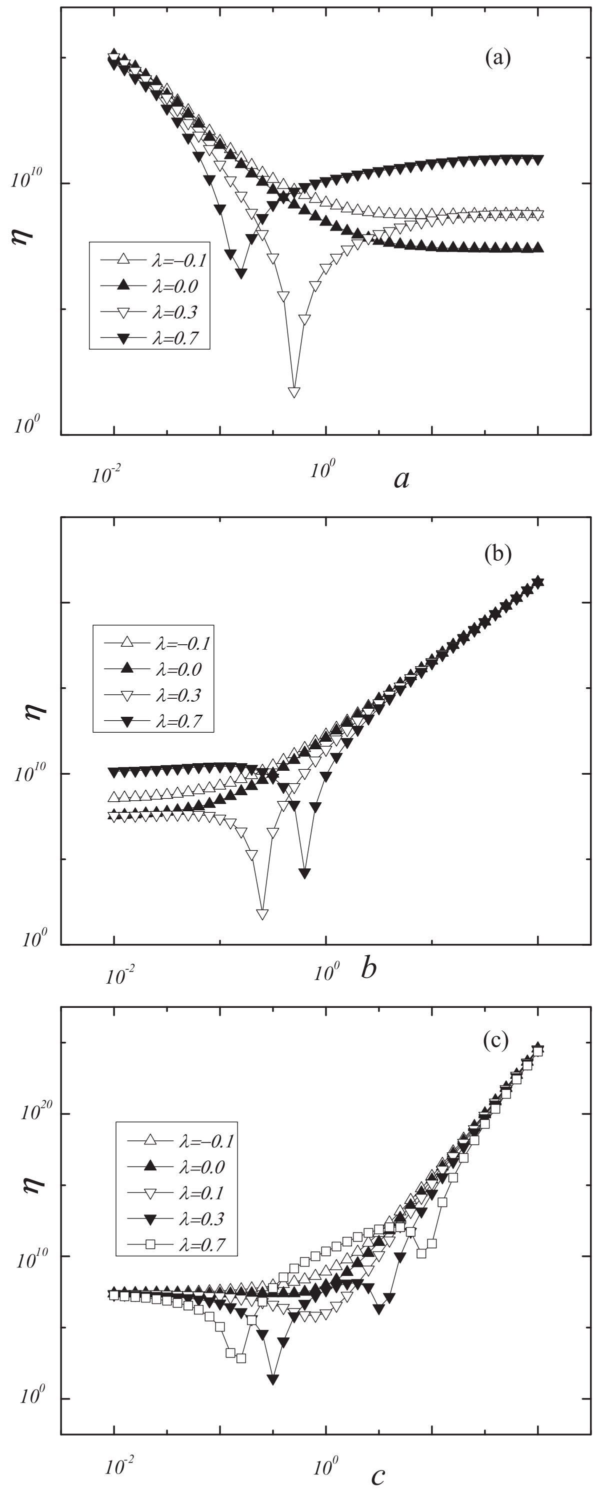

FIG. 8. The SPA η vs a in (a), vs b in (b), or vs c in (c) for different values of λ.

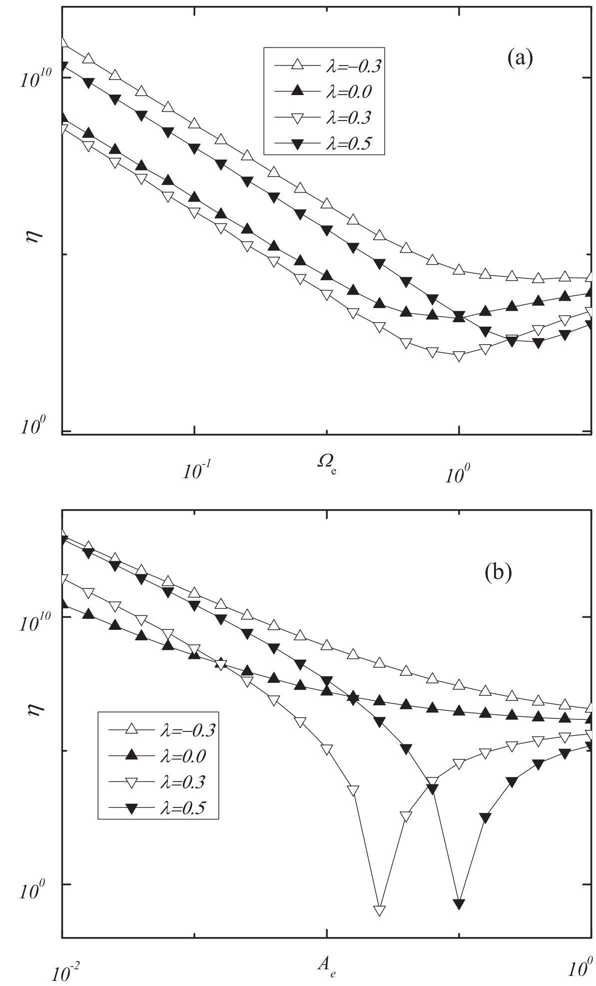

FIG. 9. The SPA η vs Ω_e in (a) or vs A_e in (b) for different values of λ.

As increasing a in Fig. 6(a), the minimum value moving to shorter λ first decreases and then increases. As increasing b in Fig. 6(b), the minimum value moving to longer λ first decreases and then increases. As increasing c in Fig. 6(c), the increasing minimum value first moves to smaller λ and then to larger λ. As increasing Ω in Fig. 7(a) or A in Fig. 7(b), the minimum value first decreases and then increases. In other words, A phenomenon of reverse resonance is observed in the behavior of SPA versus λ with varying a, b, c, Ω, and A in Figs. 6(a), 6(b), 6(c), 7(a), and 7(b), respectively. From a financial point of view, the monotonic decreasing behaviors are induced by an increase in λ, due to a decrease in volatility. For nonmonotonic behaviors, the reason can be found from description of Fig. 4. In addition, a positive correlation is more easy to cause reverse resonance as shown in Figs. 4 and 5, and a phenomenon of reverse resonance is easily produced at positive correlation as shown in Figs. 6 and 7. This is because a positive correlation is associated with low volatility [12] and positive skewness of stock returns [10,68] besides antisynchronization. Then the influences of information to rational investors are weakened for some stocks in low risk and high profit concerning low volatility and positive stock returns. Finally, to lock the frequency of information ($\Omega = 0.05$) corresponding to the intrinsic frequency of stock in volatility at reverse-resonance, the movement of the minimum value is reasonable in Figs. 4–7.

B. Intrinsic periodic information

Only considering the roles of intrinsic periodic information (Let $A = 0.0$) and fixing $\phi_e = 0$, the Eq. (4)

reads

$$dx(t) = \left[A_e \cos(\Omega_e t) - \frac{\nu(t)}{2}\right]dt + \sqrt{\nu(t)}d\xi(t),$$
$$d\nu(t) = a(b - \nu(t))dt + c\sqrt{\nu(t)}d\eta(t). \quad (8)$$

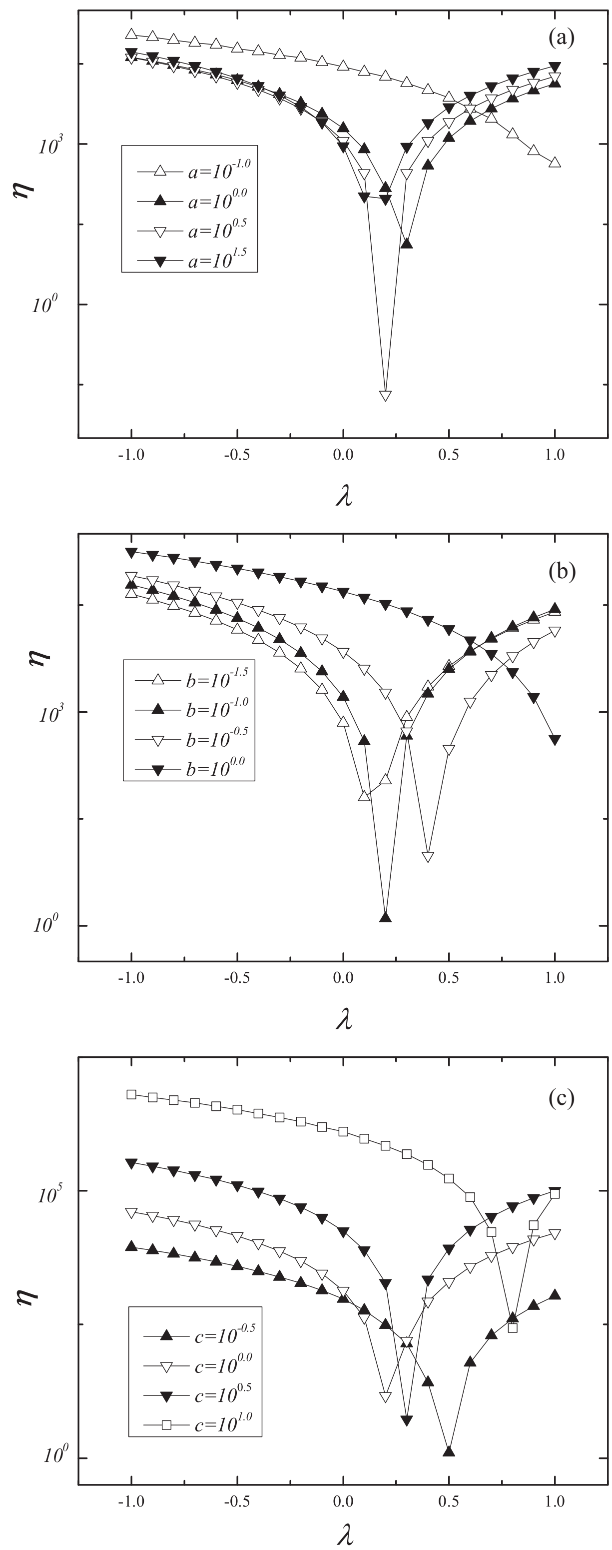

FIG. 10. The SPA η vs λ for different values of a in (a), b in (b), or c in (c).

Then SPA η can be numerically calculated based on Eqs. (6) and (8) and results are shown in the Figs. 8–11.

To compare with Fig. 4, the effects of volatility on SR are presented in Fig. 8. We can also find that, for $\lambda \leqslant 0$ monotonic increase in SPA is related to a decrease in a and an increase in b and c in Figs. 8(a)–8(c), respectively. Furthermore, for $\lambda > 0$ a reverse resonance is induced in Figs. 8(a) and 8(b), and double reverse resonance is observed in Fig. 8(c). These results are the same as Fig. 4, i.e., whether for extrinsic or intrinsic information, the η shows the same characteristics on the parameters in volatility. The monotonic and nonmonotonic behaviors can be understood similarly from description in Fig. 4. Especially the nonmonotonic behaviors are also induced by the antisynchronization, when the intrinsic frequency of stocks is close to the frequency of intrinsic periodic information originating from their corresponding companies. In other words, a stock rising and falling are synchronized with recession and expansion in its corresponding companies. In the following for comparing with Fig. 5, we also show the roles of the intrinsic periodic information on SR in Fig. 9, and find that reverse resonance is observed in the behavior of η versus A_e and Ω_e as shown in Figs. 9(a) and 9(b). Obviously, the monotonic and nonmonotonic behaviors are the same as

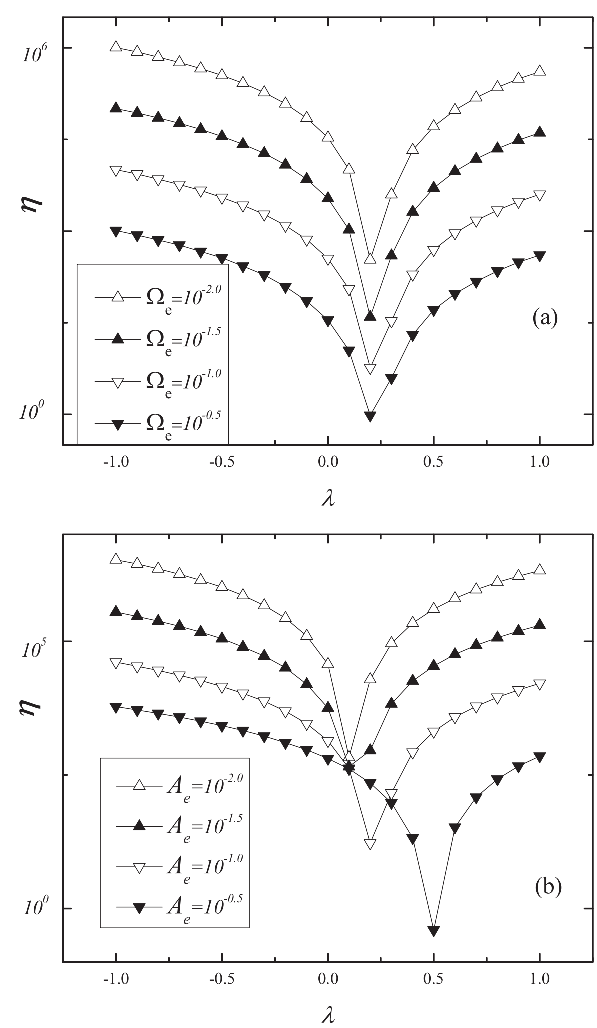

FIG. 11. The SPA η vs λ for different values of Ω_e in (a) or A_e in (b).

Fig. 5. For detailed reason of the behaviors, we can understand from previous discussion in Fig. 5.

In the end of the Sec. IV B, to understand the influences of λ on stochastic resonance for the case of intrinsic periodic information, the results are plotted in Figs. 10 and 11. For comparing with Figs. 6 and 7, as one can see that a phenomenon of reverse resonance for η versus λ also emerge in Figs. 10 and 11 and are easily induced at positive correlation. For monotonic and nonmonotonic behaviors and movement of the minimum value, the reason can be understood from previous description.

V. CONCLUSION

In this paper, we have investigated the stochastic resonance of the stock prices in finance system with the Heston model. The extrinsic and the intrinsic periodic information are introduced to the stochastic differential equations of the Heston model for stock price, respectively. First, we discussed the probability density function of stock price returns. Our numerically simulated results are compared with that in literatures, and good agreements are found between them. Then, the SPA η in the Heston model is numerically simulated. The numerically simulated results are found as follows: (i) For the case of extrinsic periodic information, a phenomenon of reverse resonance emerges, when η is a function of the mean reversion a, long-run variance b, the amplitude c of volatility fluctuations, the amplitude A, and frequency Ω of extrinsic periodic information, respectively, for $\lambda > 0$ (λ is the correlated strength between two Wiener processes of the stock price and the volatility); (ii) For the case of intrinsic periodic information, a phenomenon of reverse resonance also emerges, when the η is a function of a, b, c, the amplitude A_e, and frequency Ω_e of intrinsic periodic information, respectively, for $\lambda > 0$; (iii) For the cases both extrinsic and intrinsic periodic information, as increasing λ, a phenomenon of double reverse resonance is produced when the η is a function of c.

ACKNOWLEDGMENTS

This work was supported by the National Natural Science Foundation of China (Grant No. 11165016) and the program of IRTSTYN in Yunnan Province.

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